Lecture 7;

F : Bin(n, θ) with n Known

$$P(\theta) = \underbrace{\text{Beta}(\alpha, \beta)}_{\text{hyperparameters}} \overset{\text{if } \alpha=\beta=1}{=} U(0,1)$$

$$P(\theta|X) = \text{Beta}(\underbrace{\alpha + X}_{\#\text{of Psedu Success}},\ \underbrace{\beta + n - X}_{\#\text{failure}})$$

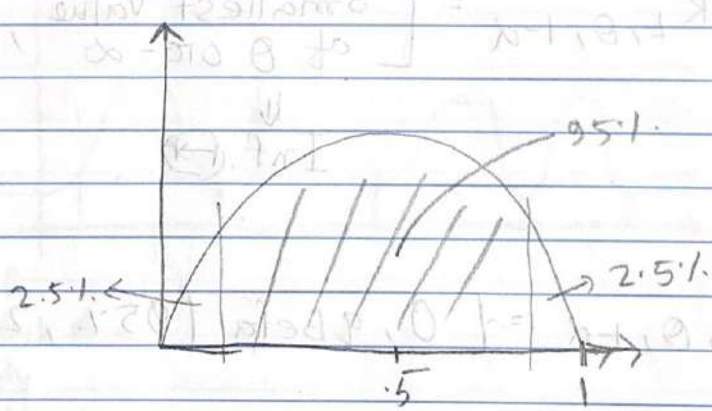
Psedue.

Conjugacy

$$\underset{\text{(Sample)}}{n} = \alpha + \beta$$

$P(\theta) = U(0,1)$ $\quad X = 1,\ n = 2$

$P(\theta|X) = \text{Beta}(2,2)$



$\alpha = \beta$

$$\hat{\theta}_{MMSE} = \hat{\theta}_{MMAE} = \hat{\theta}_{MAP} = 1/2.$$

I want, $CR_{\theta,95\%} = [\text{qbeta}[2.5\%, 2, 2], \text{qbeta}[97.5, 2, 2]]$

I want a region providing a confidence set for θ.

$$CR_{\theta,1-\alpha} := [\text{Quantile}[\theta|X, \alpha/2], \text{Quantile}[\theta|X, 1-\alpha/2]]$$

2 sided credible region.

$$P(\theta \in CR_{\theta,1-\alpha} | X) = 1-\alpha.$$

A left sided credible region:-

$$P(\theta \in CR_{L,\theta,1-\alpha} | X) = 1-\alpha$$

$$\Rightarrow P(\theta \le L | X) = 1-\alpha.$$

$$CR_{L,\theta,1-\alpha} = \left[\underset{\Downarrow \; \text{Inf}}{\text{Smallest value of } \theta \text{ or } -\infty}, \text{Quantile}[\theta|X, 1-\alpha]\right]$$

$$CR_{L,\theta,1-\alpha} := [0, \text{qbeta}(95\%, 2, 2)]$$

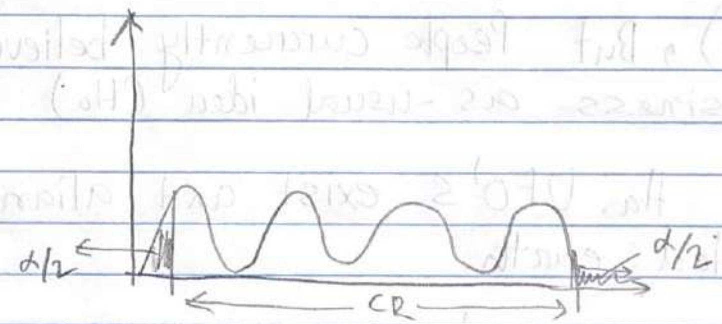
Right sided credible region:-

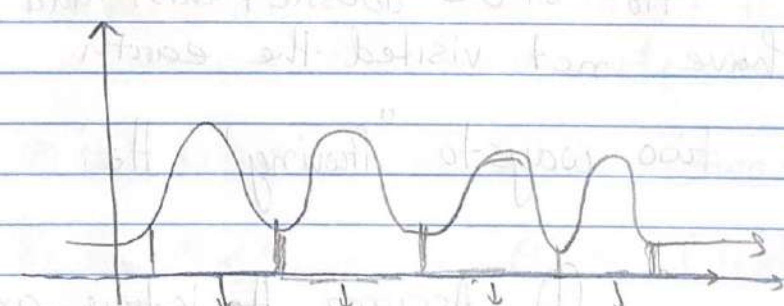
$$P(\theta \in CR_{R,\theta,1-\alpha} | X) = 1-\alpha$$

$$P(\theta \geq R | X) = 1-\alpha.$$

$$CR_{R,\theta,1-\alpha} = [\text{Quantile}[\theta|X,\alpha], \text{largest value of } \theta \text{ or } \infty]$$

$$CR_{R,\theta,1-\alpha} = [q[beta(5\%,2,2)],1].$$





$HDR_{\theta,1-\alpha} = \cup \; \cup \; \cup \; \cup = [0.1,0.3] \cup [0.5,0.6] \cup [0.7,0.8] \cup [0.9,1.0]$

High density Region, Smallest possible region that goes $1-\alpha$ proba

$$P(\theta \in HDR_{\theta, 1-\alpha}) = 1-\alpha.$$

Disadvantages of HDR:-

① Computationally Intense

② Non - Contiguous is strange.

3rd goal of inference :- theory testing:-

You wish to convince someone of something (Ha), But People currently believe a bussiness as -usual idea (Ho)

Ha: UFO's exist and alians have visited earth

Ho: UFO's doesnot exist and alians have not visited the earth.

two ways to "Proving" Ha:

Ⓘ Assume Ha is true and defend evidence to the contarary. If you cannot Provide evidence, Ha stands.

(ii) Even though I believe Ha, I am so confidente that it's true that I am willing to suppose the opposite and adduce evidence untill everyone see Ho is wrong and they'll be found to conclude Ho.

In stregy II, everyone has a lend of Skeptism with evidence, we call that α.
If the evidence doesn't meet or beat this level we retain Ho. In science we've agreed uppon a communal α-level.

In Inference, we wish to test theories about θ, we would like to demonstrate the following:-

(A) $H_a: \theta \neq \theta_0 \Rightarrow H_0: \theta = \theta_0$ (two sided test)

(B) $H_a: \theta < \theta_0 \Rightarrow H_0: \theta \geq \theta_0$ (left-sided test

(C) $H_a: \theta > \theta_0 \Rightarrow H_0: \theta \leq \theta_0$ (Right-sided test

Baysian Hypothesis testing:-

$P(H_0|X) < \alpha \Rightarrow$ Reject H_0 / Accept H_a.

$P(H_0|X) \geq \alpha \Rightarrow$ retain H_0.

$\alpha = 5\%$ is the scientific standard.

$H_0 : \theta < 0.5$

$H_a : \theta \geq 0.5$

$F : Bin(n, \theta)$, n known

$n = 100, \; x = 61$

$$P(\theta \leq 0.5 | X) = \int_0^{0.5} \frac{1}{B(62,40)} \theta^{61} (1-\theta)^{39} \, d\theta.$$

"Coin is unfairly weighted towards heads."

Prior.

$P(\theta) = U(0,1)$ | $P(\theta|X) = Beta(62, 40)$.

$P(\theta \leq 0.5 \mid X) = \text{Pbeta}(0.5; 62, 40)$

$= 0.014 < 5\%$.

Reject H_0. This coin is unfairly ~~by~~ weighted towards head. Accept H_a. Reject H_0.

Notation for integrals of beta distribution

$P(X \leq x) = F(x) = \text{Pbeta}(x, \alpha, \beta)$

$P(X > x) = 1 - F(x) = 1 - \text{Pbeta}(x, \alpha, \beta)$.

Question: θ ; Prop of non-5-Star rides.

If $\theta > 25\%$ ⇒ fire the driver.

BoB does 200 rides and gets 37 non-5-Star ratings, Does fire Bob?

H_0: $\theta \leq 25\%$ $\qquad P(\theta) = U(0,1)$

H_a: $\theta > 25\%$ $\qquad P(\theta|X) = \text{Beta}(\underset{x+1}{38}, \underset{n-x+1}{164})$

F: Bin (n, θ), n known

$n = 200$, $X = 37$.

$$P_{val} = P(\theta \leq 25\% \mid X)$$

$$= \int_0^{.25} \frac{1}{B(38, 164)} \theta^{37} (1-\theta)^{163} \, d\theta$$

$$\cancel{= P(b} = \text{Pbeta}(0.25, 38, 164)$$

$= .98$ ⇒ Retain H_0. Don't fire bob.

$H_0 = \theta = \theta_0$

$H_a : \theta \neq \theta_0$

~~Prob~~ =

$P_{val} = P(\theta = \theta_0 | X) = 0$

we have a preoblem with 2 sided tests.