$\mathcal{F}$: Bin(n, θ) with n known

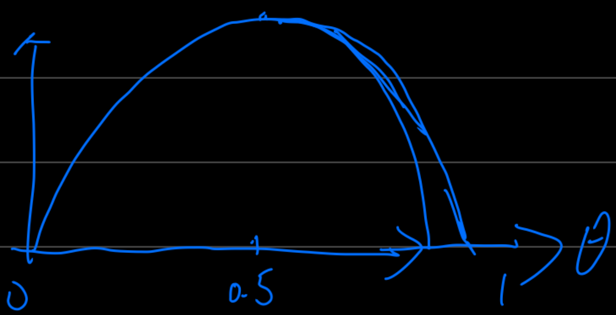
$p(\theta) = \text{Bern}(\alpha, \beta) = U[0,1] \quad (\alpha = \beta = 1)$ hyperparameter

$\Downarrow x$

$P(\theta|x) = \text{Beta}(\alpha + x, \beta + d - x) \quad n_0 = \alpha + \beta$

conjugating

$P(\theta) = U(0,1) = \text{Beta}(1,1) \qquad x = 1, n = 2 \qquad P(\theta|x) = \text{Beta}(2,2)$



$\hat{\theta}_{MMSE} = \hat{\theta}_{MMAE} = \hat{\theta} = \frac{1}{2}$

2 side situation

Credible Region $_{\theta, 1-\alpha} = [\text{Quantile}[\theta|x, \frac{\alpha}{2}], \text{Quantile}[\theta|x, 1-\frac{\alpha}{2}]]$

$P(\theta \in CR_{\theta,1-\alpha}|x) = 1-\alpha$

$CR_{\theta, 95\%} = [\text{qbeta}(2.5\%, 2, 2), \text{qbeta}(97.5\%, 2, 2)] = [0.09, 0.9]$

Left side credible region:

$P(\theta \in CR_{L\, \theta, 1-\alpha}|x) = 1-\alpha \Rightarrow P(\theta \leq L|x) = 1-\alpha.$

$CR_{L, \theta, 1-\alpha} = [$ smaller value of θ or $-\infty$ ($\approx$ inf(θ)), Quantile$[\theta|x, 1-\alpha]]$

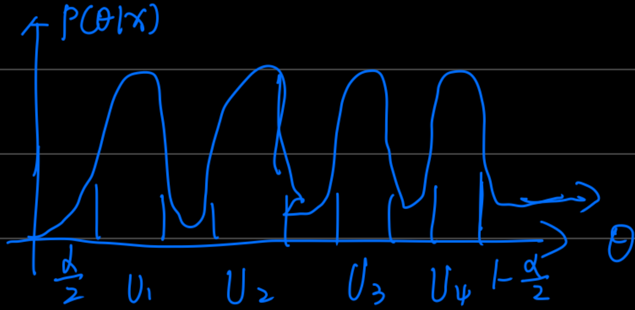
$CR_{L, \theta, 95\%} = [0, \text{qbeta}(95\%, 2, 2)]$

Right side CR:

$P(\theta \in CR_{R\, \theta, 1-\alpha}|x) = 1-\alpha \quad P(\theta \geq R|x) = 1-\alpha$

$CR_{R, \theta, 1-\alpha} = [\text{Quantile}[\theta|x, \alpha]$, larger value θ or $+\infty$ (sup(θ))$]$

$CR_{R, \theta, 95\%} = [\text{qbeta}(5\%, 2, 2), 1]$



$CR_{\theta,1-\alpha} = [$

High Density Region (HDR)

$U_1 \cup U_2 \cup U_3 \cup U_4 = [0.1, 0.3] \cup [0.5, 0.6] \cup [0.7, 0.8] \cup [0.9, 1.0]$ $\quad P(\theta \in HDR_{\theta, 1-\alpha}) = 1-\alpha$

High-Density Region Smaller possible value the given $1-\alpha$ prob

Disadvantage of HDR:

① Completely issue ② Non-contiguious is strange

3rd goal of inference: Theory testing

You wish convince someone of something (Ha) but people consider a busines usual idea (Ho).

Two ways of proving Ha:

Ⅰ assume Ha is true

if you could provide sufficient evidence, Ha stands.

Ⅱ Even though I believe Ha, I'm so confident that it's true. that I'm willing to suppose the opprosite (Ho) and show evidence until everyone sees Ho is wrong,

Ho: UFO doesn't exist Ha: UFO exists

Is strategy II, everyone has a lead of skepticlsm with evidence. We calle that α. If the evidence doesn't neeel a beat this level, we retain Ho. In science at large, we retain agreed for a common α-level

In inference, we wish a test theories θ we'd like to demonstrate the following.

Ⓐ $H_a: \theta \neq \theta_0 \Rightarrow H_0: \theta = \theta_0$ 2side Ⓑ $H_A: \theta < \theta_0 \Rightarrow H_0\ \theta \geq \theta_0$ left side

Ⓒ $H_a: \theta > \theta_0 \Rightarrow H_0: \theta \leq \theta_0$ right side

Baye's Hypothesis testing.

$P(H_0|x) < \alpha \Rightarrow$ reject H_0 / Accept H_a

$P(H_0|x) \geq \alpha \Rightarrow$ retain H_0 $\alpha = 5\%$ is the scientific standard.

$H_0: \theta < 0.5$ $H_a: \theta \geq 0.5$

$F: Bin(n, \theta)$, n known $n = 100$, $x = 61$ $P(\theta) = U(0,1) \Rightarrow P(\theta|x) = Beta(62, 40)$

$P(\theta \leq 0.5|x) = \int_0^{0.5} \frac{1}{B(62,40)} \theta^{61} (1-\theta)^{39} d\theta = pbeta(0.5, 62, 40) = 0.014 < 5\%$

Reject H_0 accept H_a

Notation for integral of beta distribution:

$P(X \leq x) = F(x) = pbeta(x, \alpha, \beta)$ $P(X \geq x) = 1 - F(x) = 1 - pbeta(x, \alpha, \beta)$

θ: proportion of non-5 star rides If $\theta \geq 25\% \Rightarrow$ fit in drive

Bob 200 rides and get 37 non-5 star

$H_0: \theta \leq 25\%$ $H_a\ \theta > 25\%$

$F: Bin(n, \theta)$ n known $n = 200$ $x = 37$ $P(\theta) = U(0,1)$ $P(\theta|x) = Beta(38, 164)$

$p(\theta \leq 25\%|x) = \int_0^{0.25} \frac{1}{B(38,164)} \theta^{37} (1-\theta)^{163} d\theta = pbeta(0.25, 38, 164) = 0.98$ retain H_0

$H_0: \theta = \theta_0$ $H_a: \theta \neq \theta_0$ prob $= p(\theta = \theta|x) = 0$ 2side tests.