Lecture - 07 02|18|2020

$F = Bin(n, \theta)$ with n known

$P(\theta) = Beta(\alpha, \beta) \overset{\text{if } \alpha = \beta = 1}{=} U(0,1)$

Conjugacy — hyperparameter

$P(\theta | x) = Beta(\alpha + x, \beta + n - x)$

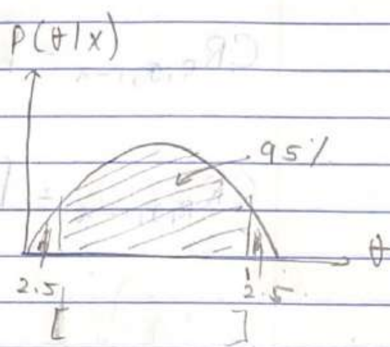
$n = \alpha + \beta$ (sample)

$P(\theta) = U(0,1) = Beta(1,1)$ (α, β)

$\alpha = 1, n = 2$

$\Rightarrow P(\theta | x) = Beta(2, 2)$

$\hat{\theta}_{MMSE} = \hat{\theta}_{MMAE} = \hat{\theta}_{MAP} = 1/2$



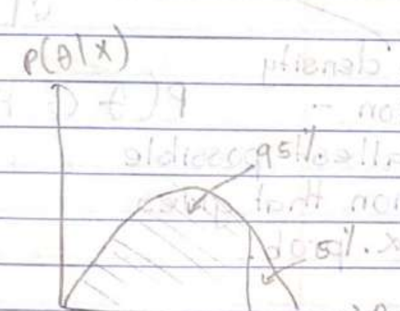
I want a region providing a confidence set for θ.

$CR_{\theta, 1-\alpha} := [\text{Quantile}[\theta|x, \alpha/2], \text{Quantile}[\theta|x, 1-\alpha/2]]$

2-side credible region.

$$\begin{cases} CR_{\theta, 95\%} := [qbeta(2.5\%, 2, 2), qbeta(97.5\%, 2, 2)] \\ = [0.09, 0.91] \end{cases}$$

$P(\theta \in CR_{\theta, 1-\alpha} | X) = 1 - \alpha.$



Left-side credible region

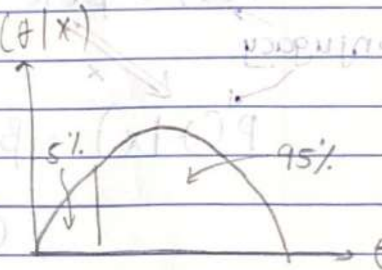
$P(\theta \in CR_{L, \theta, 1-\alpha} | X) = 1 - \alpha$

$\Rightarrow P(\theta \leq L | X) = 1 - \alpha$

$$CR_{L,\theta,1-\alpha} = \Big[\underset{\alpha \ \inf(\Theta)}{\overset{\text{smallest}}{\text{value of } \theta}} \text{ or}, \ \text{quantile}[\theta|x, 1-\alpha]\Big]$$

$$CR_{L,\theta,1-\alpha} = [0, \text{qbeta}(95\%, 2, 2)]$$

Right side credible region

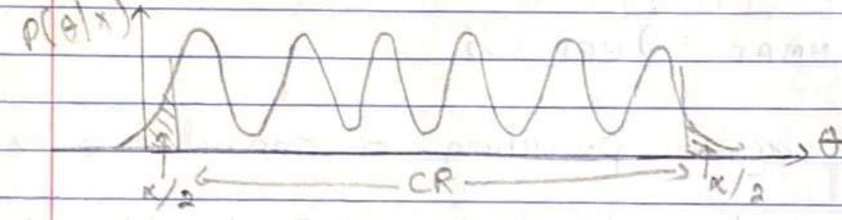


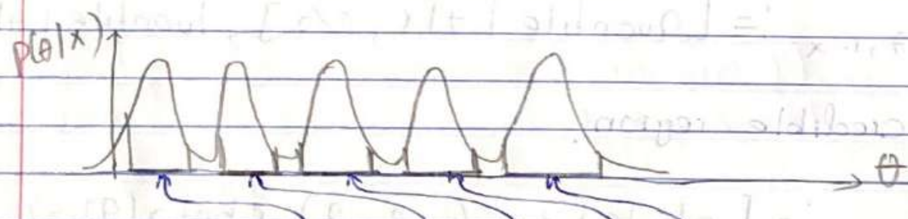
$$P(\theta \in CR_{R,\theta,1-\alpha} | x) = 1-\alpha.$$

$$P(\theta \geq R | x) = 1-\alpha.$$

$$CR_{R,\theta,1-\alpha} = [\text{Quantile}[\theta|x, \alpha], \text{ largest value of } \theta \text{ or } \alpha \inf(\Theta)]$$

$$CR_{R,\theta,1-\alpha} = [\text{qbeta}(5\%, 2, 2), 1]$$





$$HDR_{\theta,1-\alpha} = \ \cup \ \cup \ \cup \ \cup \ = [0.1, 0.3] \cup [0.5, 0.6]$$

$$\cup [0.7, 0.8] \cup [0.9, 1.0]$$

High density Region - Smallest possible region that gives $1-\alpha$ prob.

$$P(\theta \in HDR_{\theta,1-\alpha}) = 1-\alpha$$

Disadvantage of HDR.

I Computationally intense.

II Non - contiguous is strange

3rd goal of inferance :- theory testing

You wish to convince someone of something (H_a), but people currently believe a business-as-usual idea (H_0).

H_a : UFO's exist and alians have visited earth.
H_0 : UFO's deals exist and alians have not visited earth

Two ways of "proving" H_a!

I Assume H_a is true and demand evidence to the contrary. If you cannot provable evidence, H_a stands

II Even though i believe H_a, I am so confident that it's true that I am willing to suppose the opposite (H_0) and adduce evidence until everyone sees H_0 is wrong and they'll be found to conclude H_a.

In strategy II, everyone has a legend of. skeptiesm with evidence, we call that α. If the evidence doesn't meet or beat this level, we retail H_0. In science at large, we're agreed you R communal α- level.

In inferance, we wish to test theories about θ; we would like to demonstrate the following

Ⓐ $H_a: \theta \neq \theta_0 \implies H_0: \theta = \theta_0$ (two-sided test)

Ⓑ $H_a: \theta < \theta_0 \implies H_0: \theta \geq \theta_0$ (left-sided test)

Ⓒ $H_a: \theta > \theta_0 \implies H_0: \theta \leq \theta_0$ (right-sided test)

Bayesian Hypothesis testing

$P(H_0|X) < \alpha \implies$ reject H_0
accept H_a

$P(H_0|X) \geq \alpha \implies$ retain H_0

$\alpha = 5\%$ in the scientific standard.

$H_0: \theta < 0.5$ $\quad F: Bin(n, \theta)$, n known
$H_a: \theta \geq 0.5$ $\quad n = 100$, $x = 61$

$$P(\theta \leq 0.5 | X) = \int_0^{0.5} \frac{1}{\beta(62,40)} \theta^{61} (1-\theta)^{39} \, d\theta.$$

"coin is unfairly weighted towards heads"

$P(\theta) = U(0,1)$
$\implies \beta eta(62, 40)$

$= pbeta(0.5, 62, 40)$
$= 0.014 \leq 0.5\%$ Reject H_0. "The coin is accept H_a"

Notation for integrals of beta distribution
$P(X \leq x) = F(x) = pbeta(x, \alpha, \beta)$
$P(X > x) = 1 - F(x) = 1 - pbeta(x, \alpha, \beta)$

Q :-

θ : prop. of non-5-star rides. If $\theta > 25\%$ $\Rightarrow$ fire the driver; Bob does 200 rides and gets 37-non-5-star ratings.

Do we fire Bob?

$H_0 : \theta \leq 25\%$

$H_a : \theta > 25\%$

$P(\theta) = U(0,1)$

F : Bin(n, θ), n known $\Longrightarrow P(\theta \mid x) = \beta eta(38, 164)$

$n = 200, \quad x = 37$

$$Pval = P(\theta \leq 25\% \mid x) = \int_0^{0.25} \frac{1}{\beta(38,164)} \theta^{37} (1-\theta)^{163} \, d\theta .$$

$= \text{pbeta}(0.25, 38, 164)$

$= 0.98 \Longrightarrow$ Retain H_0
(Don't fire Bob)

$H_0 : \theta = \theta_0$

$H_a : \theta \neq \theta_0$

$Pval = P(\theta = \theta_0 \mid x) = 0$.
we have a problem with 2-sided tests.