Lec 3

$\hat{\theta}_{MLE} := \arg\max_{\theta \in \Theta} \{ l(\theta, x) \}$

"point estimate"

$\times$

In advance class, we'll prove

$\hat{\theta}_{MLE} \overset{d}{\approx} N(\theta, SE[\hat{\theta}_{MLE}] \approx N($

In advanced class you'll prove

$$\underset{\text{r.v.}}{\hat{\theta}_{MLE}} \overset{d}{\approx} N(\theta, \underbrace{SE[\hat{\theta}_{MLE}]}_{\text{fraction of } \theta}) \underset{\text{2nd}}{\approx} N(\underset{\text{pt. est.}}{\hat{\theta}_{MLE}}, \hat{SE}[\hat{\theta}_{MLE}])$$

"estimator"

$\times$

impossible to know   approximation

estimator

$\rightarrow SE[\hat{\theta}_{MLE}] \Big|_{\theta = \hat{\theta}_{MLE}}$

estimate

MLE's allow for 3 goal of inference

① point estimator $\hat{\theta}_{MLE}$ ② confidence interval (CI): $[\hat{\theta}_{MLE} \pm Z_{\frac{\alpha}{2}} \hat{SE}(\hat{\theta}_{MLE})]$ Math 242 / Econ 382

③ Hypothesis testing   math 242 / Econ 382   retained region $(RR_\alpha) = [\theta_0 \pm Z_{\frac{\alpha}{2}} SE(\hat{\theta}_{MLE})] \Big|_{\theta = \theta_0}$

Trouble in Paradise   Example:

① $\tilde{F} \overset{iid}{=}$ Beroulli $X_1, \dots, X_n \overset{iid}{=}$ Bern (θ) $X = \langle 0, 0, 0 \rangle$ $\hat{\theta}_{MLE} = \bar{x} = 0$

$CI_{\theta, 1-\alpha} = [\bar{x} = Z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{x}(1-\bar{x})}{3}}] = \{0\}$

$RR_\alpha = \{\theta_0\}$ all they are rejected

② what if you know $\theta = [0.1, 0.2]$ Is there any way to make use of this information? No

③ Let's interpret the confident interval

$CI_{\theta, 95\%} = [0.37, 0.43]$ what is interpretation.

Andrew's theory: $P(\theta \in CI_{\theta, 0.95}) = 0.95$ wrong... but that is what you want to say.

our assumption was θ in a fixed value (parameter)

$P(0.39 \in [0.37, 0.43]) = 1$ $P(0.36 \in [0.37, 0.43]) = 0$

Valid interpretation:

(I) If I repeat the experiment multiple times, $\approx 95\%$ of the CI's will include

exp#1 [ ] #1 exp#2 [ ] #2 exp#3 [ ] #3 exp#4 [ ] #4

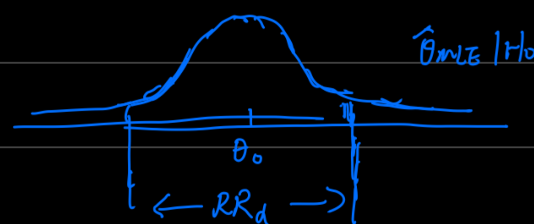
(II) Before you do the experiment $P(\theta \le CI_{\theta,1-\alpha}) \doteq 1-\alpha$.

(III) (skip)

(IV) In a hypothesis that you either reject H_0 or reject H_A

$\hat{\theta}_{MLE} \in RR_\alpha \Rightarrow$ Reject H_0

$\hat{\theta}_{MLE} \notin RR_\alpha \Rightarrow$ Reject H_A



"p-value" is defined as:

$P($seeing $\hat{\theta}_{MLE}$ "or more extreme" $| H_0$ —— $)$

$\neq P(H_0 | x)$ that what you want
probabibly

(5) $\tilde{F}$ = iid Beroulli $(\Theta = (0,1))$ $X = <0, 1, 0>$ $\hat{\theta}_{MLE} = \bar{x} = \frac{1}{3}$

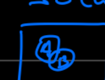
$CI_{\theta, 95\%} = \left[\frac{1}{3} \pm z_\alpha \sqrt{\frac{\frac{1}{3}\cdot\frac{2}{3}}{3}}\right] = [-0.20, 0.87]$

we know that $\theta \ge 0$! This is bad CI cuz it didn't response the parameter space (i)

why? n is too small (=3), ..., therefore $\hat{\theta}_{MLE} \not\sim N($ $) \Rightarrow$ wrong.

Conditional probability

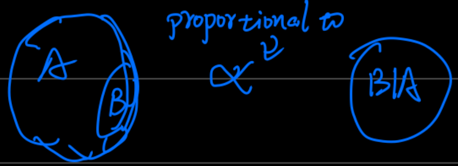
Ω (whole



A: smoking B: lung cancer (LC) Assume: $P(A) = 0.2$ $P(B) = 0.06$ $P(A.B) = 0.036$

P(lung cancer | smoking), a constant probability $= P(B|A)$

$A = \Omega' \subset \Omega$

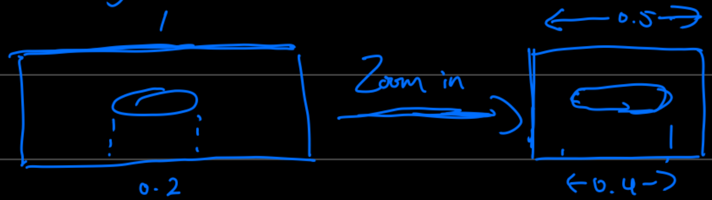


$Pr(B|A) = \frac{Pr(A,B)}{Pr(A)}$ $\quad Pr(A|B) = \frac{Pr(A,B)}{Pr(B)} = \frac{Pr(B|A)\,Pr(A)}{Pr(B)}$

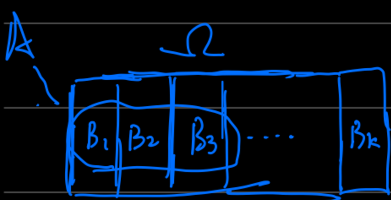
or $Pr(B|A)\cdot\frac{Pr(A)}{Pr(B)} = \frac{Pr(A,B)\,Pr(A)}{Pr(A)\cdot Pr(B)} = \frac{Pr(A,B)}{Pr(B)} = Pr(A|B)$

$\therefore Pr(B|A)\,\frac{Pr(A)}{Pr(B)} = Pr(A|B)$ Baye's Rule.

$Pr(B|A) \propto P(A,B) = \overset{\vee}{c}\, P(A,B) = \frac{P(\Omega)}{P(A)}\cdot P(A,B) = \frac{P(A,B)}{P(A)}$ $\quad P(A,B) = P(B|A)\cdot P(A)$



Zoom fraction $= \frac{\text{"1"}}{\text{"0.5"}} = 2$



s.t. $B_1 \cup B_2 \cup B_3 \cup \dots \cup B_k = \Omega$ collective exh.

But $B_i \cap B_j = \emptyset$ mutually exclusive

I can prove that $A = (A\cap B_1) \cup (A \cap B_2) \cup \dots \cup (A \cap B_k) \Rightarrow P(A) = \sum_{k=1}^{k} P(A, B_k)$

so Baye's rule $\Rightarrow P(B_i|A) = \frac{P(A|B_i)\cdot P(B_i)}{\sum_{i=1}^{k} P(A, B_i)}$ Baye's theorem.

Imagine r'v X, Y $\quad Supp[X] = \{1, 2, 3, 4\}$ $\quad supp[Y] = \{1, 2, 3, 4, 5\}$

Jmf $f(X,Y)$ $\quad P(Y=5) = \sum_{x \in Supp[x]} f(x, Y=5) = P(X=1, Y=5) + P(X=2, Y=5) + P(X=3, Y=5) + P(X=4, Y=5)$

$P(X=2|Y=5) = \frac{P(X=2, Y=5)}{P(Y=5)} = \frac{1}{4}$ $\quad P(X=x|Y=y) = \frac{P(X=x, Y=y)}{P(Y=y)}$

$P(X|Y) = \frac{P(X,Y)}{P(X)}$ ← JMF, ← PMF

Conditional mass function, CMF

Can I write the following?

$P(\theta|x) = \frac{P(x|\theta)\,P(\theta)}{P(x)}$ $\quad\theta$ is a constant i.e a degenerate r'v

thus the formula isn't useful

$\Theta \sim \{\theta \text{ wp } 1\}$ $\quad \theta|x \sim \{\theta \text{ wp } 1\}$

$$p(x) = \sum_{\theta_0 \in \Theta} P(x|\theta_0) P(\theta_0)$$

$$\subseteq \int_{\Theta} p(x|\theta_0) P(\theta_0) \, d\theta_0$$

$p(x)$ without knowing θ_0

This is unanswerable without knowing θ_0