I LOVEPDF

First exam - March 3
Second exam - April

We will cheat sheet for exams & final

Ex.

Let X be a random variable (rv) (are model that realize in different states), let x be a realization from the rv. Let $\text{Supp}[X]$ denote the support of the rv. This is a set of all possible values that could be realized thus x belong to this set.

represent all the states; all the probability that are accesible in that model. X will be one value from realization

- Discrete r.v's
- $|\text{supp}[X]| \leq |\mathbb{N}|$ ← (finite or if infinite, it is countably infinite)

measure the size of support

(support of random variable of X)

- $p(x) := P(X = x)$ Probability mass Function (PMF)

def.

- $p: \text{Supp}[X] \rightarrow [0,1]$ $\qquad \sum_{x \in \text{Supp}[X]} p(x) = 1$

mapping

- $F(x) = P(X \leq x)$ → cumulative distribution function (cdf)

- $F: \mathbb{R} \rightarrow [0,1]$ 0 is legal

Another formula: $\sum_{\{y:\, y \in \text{Supp}[X] \,\&\, y \leq x\}} P(y) =$

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$$= \sum_{y=-\infty}^{x} P(y)$$ why this is wrong?

The value in $supp[X]$ don't have to be spaced by 1.

- Continous random variable (rvs) (also known as contiounable ranges' $\sum P(x)=1 \;\; x \in supp[x]$ won't work)

the size of real # that are:

$|Supp[X]| = |\mathbb{R}|$ uncountably infinite ex. $[0,1]$

The CDF definition remains the same. The PMF doesn't exist. And we define $f(x) := F'(x)$ the probability density function (PDF)

$$P(X \in [a,b]) = F(b) - F(a) = \int_a^b f(x)\,dx$$

such that

$$Supp[X] = \{ x : f(x) > 0 \}$$

PDF is not a negative probability is not always negative it can never go down.

It's not a probability. It's a derivate

$$f : Supp[X] \xrightarrow{pdf} (0, \infty) \qquad \int_{Sup[X]} f(x)\,dx = 1$$

★ Random variables are identified by their CDF/PMF (discrete) or CDF/PDF (continous)

Short for Bureolli Function

$X \sim \text{Bern}(p) = p^x (1-p)^{1-x}$ → $supp[X] = \{0,1\}$

$X \sim \text{Binom}(n,p) = \binom{n}{x} p^x (1-p)^{n-x}$ → $supp[X] = \{0,1,\dots,n\}$

discrete

Binomial Function

$X \sim \text{Exp}(\lambda) = \lambda e^{-\lambda x}$ → $supp[X] = (0,\infty)$

$X \sim N(\mu, \sigma^2) = \frac{1}{2\pi\sigma^2} e^{-x\frac{1}{2\sigma^2}(x-\mu)^2}$ ↑ $supp[X] = \mathbb{R}$

continuous

$P(1) = p^1 (1-p)^{1-1} = p$

$P(0) = p^0 (1-p)^{1-0} = 1-p$

$P(\frac{1}{7}) = p^{\frac{1}{7}} (1-p)^{1/7}$

The problem is he didn't specify the $supp[X]$

what is p?

→ It's a success probability.

what is n? what is landa? what is mu?

→ They are "tuning knobs", which are called parameters. p controls how often 0's or 1's occur in the Bernoulli random variable.

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What are the legal values for p? what are values that "make sense" for the Bernoulli random variable? what value respect its support?

Ans. = $p \in (0,1)$ The parameter space for the parameter p.

Ans. why not negative or greater than 1?
→ they are not probabilities.
why not: 0 or 1?

Ex. $X \sim Bern(1) = 1^x (1-1)^{1-x} = p(x) = Deg(1) = 1$

$P(0) = 1^0 (1-1)^{1-0} = 1 \cdot 0^1 = 0$
$P(1) = 1^1 (1-1)^{1-1} = 1 \cdot 0^0 = 1$

* The degenerate random variable which is technically a random variable but not interesting because it's not "random" it is oxymoronic

* As a convention (standard), degenerates parameter values are not consider part of the parameter space.

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Let thetha denote unknown parameters. And let thetha vector ($\vec{\theta}$) denote multiple unknown parameters. And let Θ denote the parameter space. $\theta \in \Theta$

* $X \sim Bern(\theta)$, $\Theta = (0,1)$

* $X \sim Binomial(\theta_2, \theta_1) = \binom{\theta_2}{x} \theta_1^x (1-\theta_1)^{\theta_2 - x}$, $\Theta = (0,1) \times \mathbb{N}$

(your estimating the p; it's two dimensional) n p

* $F_{Ber} = \{ \theta^x (1-\theta)^{1-x} ; \theta \in (0,1) \}$ All possible Bernoulli random variable a "parametric model"

$P(\theta|x) =$ all possible PMF

* $F = \{ p(x) : \vec{\theta} \in \Theta \}$ theta

$M := \{ P(x;\vec{\theta}) : \vec{\theta} \in \Theta \}$ parameter(s)

$P(X_1, X_2, X_3 \ldots X_n : \theta)$ joint mass function

$f(x_1, x_2, \ldots x_n : \theta)$ joint density Function (JDF)

If $X_1, \ldots X_n \overset{\text{independent}}{\sim}$

$= P(X_1; \theta) \cdot P(X_2; \theta) \ldots P(X_n; \theta)$

$= F(X_1; \theta) \cdot F(X_2; \theta) \ldots F(X_n; \theta)$

"Multiplication Rule"