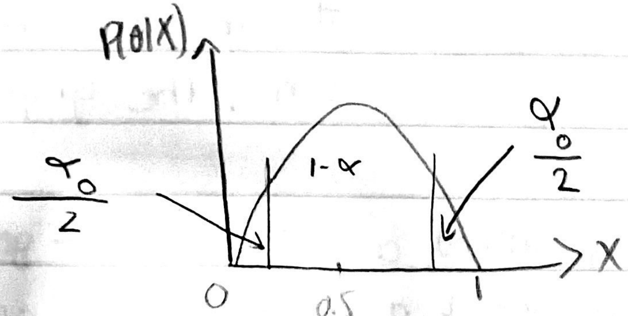
March 1.

lec 8.

$X=1$, $n=2$, $F: Bin(n, \theta)$

$P(\theta) = Beta(1,1) \Rightarrow P(\theta|X) = Beta(2,2)$




produced a 95% credible region for $\theta$.

10/15 CM

$$CR_{\theta,1-\alpha_0} := \left[ Q\left[\theta|X, \frac{\alpha_0}{2}\right], Q\left[\theta|X, 1-\frac{\alpha_0}{2}\right] \right]$$

$$= [qbeta(2.5\%, 2, 2), qbeta(97.5\%, 2, 2)]$$

$$= [0.094, 0.906]$$

$$P(\theta \in [.094, .906] | X) = 95\%$$

This is a real probability statement! The CI approach cannot give you such a statement! the CR is highly interpretable. Also, the CR is a proper subset of $\Theta$ for $\alpha_0 > 0$. This is not always true with CI's e.g. here the 95% CI for this data:

$$CI_{\theta, 95\%} = \left[0.5 \pm 1.96\sqrt{\frac{0.5 - 0.5}{2}}\right] = [-0.21, 1.21] \not\subset \Theta = (0,1)$$

The above CR is technically a two-sided CR. You can also create one-side (i.e. left-sided or right-sided) CR's.

$$CR_{L,\theta,1-\alpha} := [\text{smallest value in } \Theta \text{ or } -\infty, Q[\theta|X, 1-\alpha_0]]$$

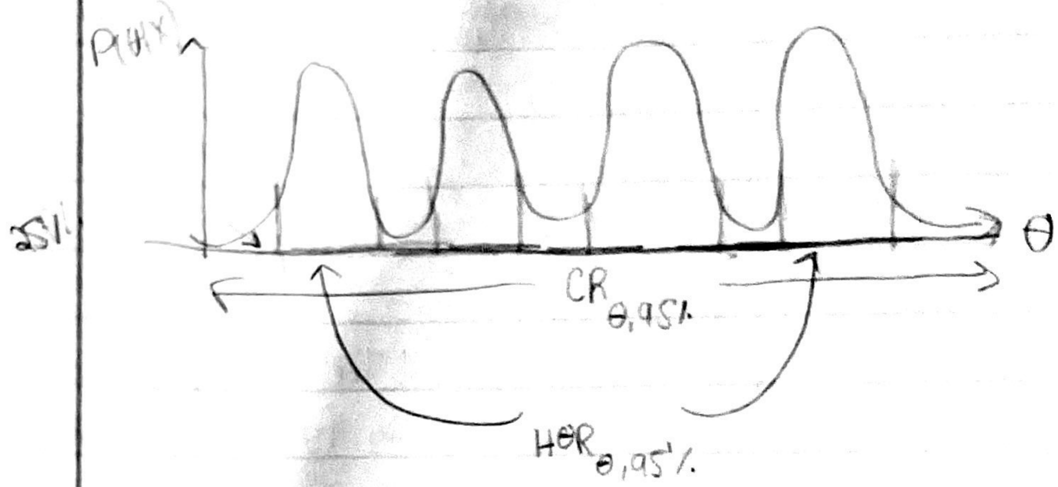
ex. in our dataset qbeta(.95%, 2, 2)

$$\hookrightarrow = [0, 0.865] = P(\theta < 0.865 | X) = 95\%$$

$$CR_{R,\theta,1-\alpha_0} := [Q[\theta|X, \alpha_0], \text{large value in } \Theta \text{ or } \infty]$$

$$= [qbeta(.05, 2, 2), 1] = [.131, 1] \Rightarrow P(\theta > .136 | X) = 95\%$$

Another approach (which we will see but not study further) is called the high density region (HDR) approach. Consider the following posterior for $\theta$:




$P(\theta \in HDR_{\theta,95\%}) = 95\%$. but it has *minimum* width of the interval pieces is minimum)

Sometimes the CR = HDR (ex in unimodal posteriors)
Disadvantages of the HDR approach. (1) it can be non-contiguous ex. in pieces (2) it's computationally intense. (3) no L or R intervals.

it's a dirty solution.

<u>Bayesian Hypothesis Testing</u>. We can immediately compute the following quantities:

Bayesian P-value $:= P(H_0|X)$, $P(H_a|X)$

threshold of "sufficient evidence"

if $P(H_0 | x) < \alpha_0$ = Reject $H_0$

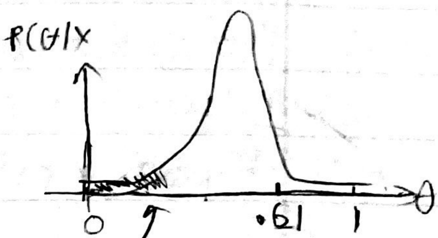
Let's recreate the hypothesis testing example from Lec. n=100. Flips of a coin where (x=61) were heads. Test if the coin is unfairly weighted towards heads

$H_a : \theta > 0.5 \Rightarrow H_0 : \theta \leq 0.5$. Assume $P(\theta) = Beta(1,1)$

To prove

$$P(\theta | x) = Beta(\underbrace{61 + 1}_{\alpha}, \underbrace{39 + 1}_{\beta})$$

prior




$P(\theta | x) = Beta(62, 40)$

$P(H_0 | x) = P(\theta \leq 0.5 | x) = \int_0^{0.5} P(\theta | x) d\theta$ posterior

$$= \int_0^{0.5} \frac{1}{\beta(62, 40)} \theta^{61} (1-\theta)^{39} d\theta$$

= pbeta(0.5, 62, 40)

= .014 = 1.4%

$\Rightarrow$ Reject $H_0$ i.e. coin is unfairly weighted towards heads

Scanned with CamScanner

$n=$ $\quad$ $x=$

Uber driver does 200 rides and get 37 non-star rating. If his true proportion of non-5 star rating is less than 25%, then Uber policy is to not fire the driver. Prove he should be fired (or not) at a 5% significant level.

$H_a: \theta > 25\%$

$\Rightarrow H_0: \theta \le 25\%$

$P(\theta) = Beta(1,1)$

$F:(n,\theta)$

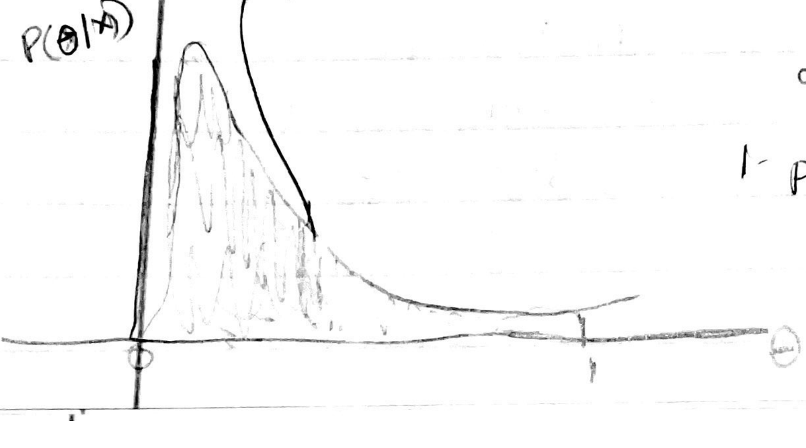
posterior $P(\theta|x) = Beta(1+37, 1+200-37)$

$Beta(\underset{\text{alpha}}{38}, \underset{\text{beta}}{164})$

$P(H_0|x) = P(\theta \le .25|x) = \int^{1} d\theta = 1 - \int_0^{0.25} d\theta$

$= 1 - pbeta(.25, \underset{\alpha \text{ alpha}}{38}, \underset{\text{Beta } \beta}{164})$

$1 - pbeta(.25, 38, 164) = .983 > 5\%$




Retain $H_0$. Don't fire him.

Let's test the coin again. Flip 100 times ($\tilde{n}$) and get 43 heads ($\tilde{x}$). Test if the coin is in unfair at 5% significant.

$H_a: \theta \neq 0.5 \Rightarrow H_0: \theta = 0.5$

$100+1-43$

$P(\theta) = Beta(1,1) \Rightarrow P(\theta|x) = Beta(44, 58)$

$P(H_0|X) = P(\theta = 0.5|x) = 0$ Reject $H_0$ always ?? Yes...

Using this approach, two sided test are always rejected (if the posterior is continous). Does this make sense? Any infinitely precise theory of theta is wrong in the real world. A coin is never exactly 50.0000000... % likely to flip heads. So, we need to slighty reframe our hypothesis using a notation of "margin of equivalence" called $\delta$ (delta).

$$H_a: \theta \notin \theta_0 \pm \delta] = H_0: \theta \in [\theta_0 \pm \delta]$$