Math 341/650.2
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Lec 01

Let X be a random variable (rv), let x be a realization from the rv.
Let $Supp[X]$ denote the support of the rv, a subset of real. This is a set of all possible values that could be realized thus x belongs to this set.

- Discrete rv's

$$|Supp[X]| \leq |\mathbb{N}| \text{ ; finite or if infinite, it's countably infinite.}$$

Let $P(x) := P(X=x)$; Probability Mass function (PMF)
$P: Supp[X] \longrightarrow (0,1]$

$$\sum_{x \in Supp[X]} P(x) = 1$$

$$= \sum_{\{Y : Y \in Supp[X] \;\&\; Y \leq x\}} P(Y) \overset{?}{=} \sum_{Y=-\infty}^{x} P(Y)$$

$F(x) := P(X \leq x)$, cumulative distribution function (CDF)
$F: \mathbb{R} \longrightarrow [0,1]$

- Continuous rv's

$$|Supp[X]| = \mathbb{R} \text{ ; uncountable infinities e.g. } [0,1]$$

The CDF def remains the same. The PMF doesn't exist. And we define $f(x) := F'(x)$, the Probability density function (PDF)

$$P(X \in [a,b]) = F(b) - F(a) = \int_a^b f(x)\,dx$$

$$Supp[X] = \{x : f(x) > 0\}$$

Note: PDF is not a probability, not bounded by 1. Can't be negative.

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$f: Supp[X] \longrightarrow (0, \infty)$, $\int_{Supp[X]} f(x)dx = 1$

rv's are identified by their CDF/PMF (discrete) or CDF/PDF (continuous)

discrete $\begin{cases} X \sim Bern(p) := p^x (1-p)^{1-x}, & Supp[x] = \{0, 1\} \\ X \sim Binom(n,p) := \binom{n}{x} p^x (1-p)^{n-x}, & Supp[X] = \{0, 1, \dots n\} \end{cases}$

Cont. $\begin{cases} X \sim Exp(\lambda) := \lambda e^{-\lambda x}, & supp[X] = (0, \infty) \\ X \sim N(\mu, \sigma^2) := \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, & Supp[X] = \mathbb{R} \end{cases}$

$P(1) = p^1 (1-p)^{1-1} = p$ $\quad$ $P(\frac{1}{7}) = p^{\frac{1}{7}} (1-p)^{\frac{6}{7}}$

$P(0) = p^0 (1-p)^{1-0} = 1-p$

What is p? what is n? what is λ? what is μ? σ^2? They are "tuning knobs" which are called Parameters. P controls how often 0's or 1's occur in the Bernoulli rv.

What are the legal values for p? what are values that "make sense" for the Bernoulli rv? What values respects it's support?

$p \in (0, 1)$ The parameter space for the parameter P.
Why not negative or greater than 1? They're not probabilities. why not 0 or 1?

$X \sim Bern(1) = 1^x (1-p)^{1-x} = P(x) = Deg(1)$: Σ 1 w.p 1, the degenerate rv, which is technically a rv but not interesting, coz it's not random.

$P(0) = 1^0 (1-1)^{1-0} = 1 \cdot 0^1 = 0$

$P(1) = 1^1 (1-1)^{1-1} = 1 \cdot 0^0 = 1$

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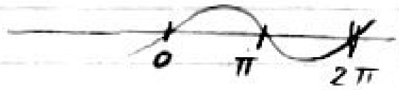
As a Convention (STD), degenerate parameter values are not considered part of the parameter space.

Let θ denote unknown parameters. Let $\vec{\theta}$ denote multiple unknown parameters. Let Θ denote the parameter space. $\theta \in \Theta$

$X \sim \text{Bern}(\theta)$
$X \sim \text{Binom}(\theta_2, \theta_1) = \binom{\theta_2}{x} \theta_1^x (1-\theta_1)^{\theta_2 - x}$, $\Theta = (0,1) \times \mathbb{N}$

$F_{\text{Bern}} := \{\theta^x (1-\theta)^{1-x} : \theta \in (0,1)\}$ 'All possible Bern rvs a "parametric model".

$F := \{P(x;\vec{\theta})\ \vec{\theta} \in \Theta\}$ e.g. $\sin(cx) = f(x;c)$
Parameter(s)
$c = 1$



$P(x_1, x_2, \dots x_n : \theta)$ joint mass function (JMF)
$f(x_1, x_2 \dots x_n : \theta)$ joint density function (JDF)

If $x_1, x_2 \dots x_n \overset{ind}{\sim}$
$P(x_1, x_2, \dots x_n) = P(x_1;\theta) \cdot P(x_2;\theta) \cdot \dots \cdot P(x_n,\theta)$
$f(x_1, x_2, \dots x_n) = f(x_1;\theta) \cdot f(x_2,\theta) \cdot \dots \cdot f(x_n,\theta)$

This is multiplication rule.