2/1/21

Lecture 1

Let X be a random variable (rv), let x be a realization from the rv.
Let Supp[X] denote the support of the rv, a subset of the reals. This is a set of all possible values that could be realized thus x belongs to this set.

- Discrete r.v.'s

$|\text{Supp}[X]| \leq |\mathbb{N}|$ finite or if infinite, it is countably infinite

$p(x) := P(X = x)$ probability mass function (PMF)

$$p : \text{Supp}[X] \to (0,1] \qquad \sum_{x \in \text{Supp}[X]} p(x) = 1$$

$F(x) := P(X \leq x)$ cumulative distribution function (CDF)

$$F : \mathbb{R} \to [0,1] \qquad P(X \leq x) = \sum_{\{y : y \in \text{Supp}[X] \,\&\, y \leq x\}} p(y) = \sum_{y=-\infty}^{x} p(y)$$

- Continuous r.v.'s

$|\text{Supp}[X]| = |\mathbb{R}|$ uncountably infinite e.g. $[0,1]$

The CDF definition remains the same. The PMF doesn't exist. And we define $f(x) := F'(x)$, the probability density function (PDF)

$$P(X \in [a,b]) = F(b) - F(a) = \int_a^b f(x)\,dx$$

$$\text{Supp}[X] = \{x : f(x) > 0\}$$

$$f : \text{Supp}[X] \to (0, \infty), \qquad \int_{\text{Supp}[X]} f(x)\,dx = 1$$

rvs are identified by their ~~[illegible]~~ CDF/PMF (discrete) or CDF/PDF (continuous) discrete (PMFs)

$X \sim \text{Bern}(p) := p^x(1-p)^{1-x} \rightarrow \text{Supp}[X] = \{0,1\}$

$X \sim \text{Binom}(n,p) := \binom{n}{x} p^x (1-p)^{n-x} \rightarrow \text{Supp}[X] = \{0,1,\ldots,n\}$

Continuous (PDFs)

$X \sim \text{Exp}(\lambda) := \lambda e^{-\lambda x} \rightarrow \text{Supp}[X] = (0,\infty)$

$X \sim N(\mu, \sigma^2) := \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \rightarrow \text{Supp}[X] = \mathbb{R}$

$\rightarrow p(1) = p^1(1-p)^{1-1} = p,\quad p(0) = p^0(1-p)^{1-0} = 1-p,\quad p(\tfrac{1}{2}) = p^{\frac{1}{2}}(1-p)^{\frac{1}{2}}$

What is p?? What is n? What is lambda? What is mu? sigsq? They are "tuning knobs", which are called parameters. p controls how often 0's or 1's occur in the Bernoulli r.v.

What are the legal values for p? What are values that "make sense" for the Bernoulli r.v.? What values respect its support?

$p \in (0,1)$ The parameter space for the parameter p.

Why not negative or greater than 1? They're not probabilities. Why not 0 or 1?

$X \sim \text{Bern}(1) = 1^x(1-1)^{1-x} = p(x)$

$p(0) = 1^1(1-1)^{1-0} = 1 \cdot 0^1 = 0$

$p(1) = 1^1(1-1)^{1-1} = 1 \cdot 0^0 = 1$

$\text{Deg}(1) := \{1 \text{ w.p. } 1$ the degenerate r.v. which is technically a r.v. but not interesting because it's not "random" it is oxymoronic

As a convention (standard), degenerate parameter values are not considered part of the parameter space.

Let θ denote unknown parameters. And let $\vec{\theta}$ (theta vector) denote multiple unknown parameters. And let Θ denote the parameter space. $\theta \in \Theta$

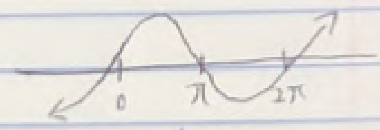
$X \sim \text{Bern}(\theta)$, $\Theta = (0,1)$

$X \sim \text{Binomial}(\theta_2, \theta_1) = \binom{\theta_2}{x} \theta_1^{x} (1-\theta_1)^{\theta_2 - x}$, $\Theta = (0,1) \times \mathbb{N}$

$\mathcal{F}_{Bern} := \{ \theta^x (1-\theta)^{1-x} : \theta \in (0,1) \}$ All possible Bernoulli r.v.'s
a "parametric model"

$\mathcal{F} := \{ p(x; \underset{\uparrow}{\vec{\theta}}) : \vec{\theta} \in \Theta \}$ e.g. $f(x;c) = \sin(cx)$

parameter(s)



discrete $\rightarrow$ $p(x_1, x_2, \ldots, x_n; \theta)$ joint mass function (JMF)

continuous $\rightarrow$ $f(x_1, x_2, \ldots, x_n; \theta)$ joint density function (JDF)

If $x_1, \ldots, x_n \overset{ind}{\sim}$ $\quad \rightarrow = p(x_1;\theta) \cdot p(x_2;\theta) \cdot \ldots \cdot p(x_n;\theta)$

$\rightarrow = f(x_1;\theta) \cdot f(x_1;\theta) \cdot \ldots \cdot f(x_n;\theta)$

independent

"multiplication rule"