Mid 2 1(j) $\mu_1 = \frac{1}{\theta - 1} \Rightarrow \frac{1}{\mu_1} = \theta - 1 \Rightarrow \theta = \frac{1}{\mu_1} + 1$

$$\Rightarrow \hat{\theta}^{mm} = \frac{1}{\hat{\mu}_1} + 1 = \frac{1}{\bar{X}} + 1$$

2m) $X_1, \ldots X_n \overset{iid}{\sim} \theta(x+1)^{-\theta-1}$

$I_n(\theta) = n I(\theta)$ $\quad \Sigma E[-\ell''] = n E[-\ell'']$

$= E[-\ell''(\theta; x_1, \ldots x_n)] = E\left[-\sum_{i=1}^{n} \ell''(\theta; x_i)\right]$

One Obs.

$I(\theta) = E[-\ell''(\theta; x)] = E_X[-\frac{1}{\theta^2}] = \frac{1}{\theta^2}$

$\ell(\theta; x) = \ln(\theta) - (\theta+1)\ln(x+1) = \ln(\theta) - \theta \ln(x+1) - \ln(x+1)$

$\ell'(\theta; x) = \frac{1}{\theta} - \ln(x+1)$

$\ell''(\theta; x) = -\frac{1}{\theta^2}$ $\quad \theta$

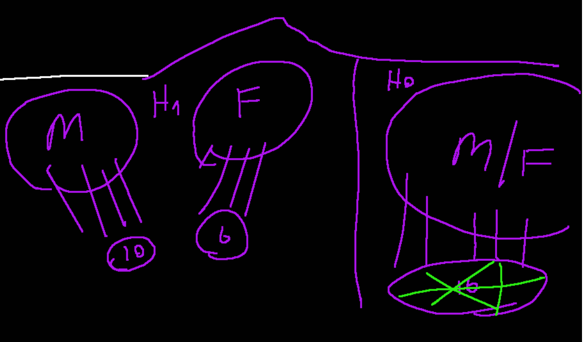
2p) let $\hat{\theta}$ be an unbiased estimator

$$Var[\hat{\theta}] \geq \frac{I(\theta)^{-1}}{n} = \frac{(\theta^{-2})^{-1}}{n} = \frac{\theta^2}{n}$$

5e) $\frac{X_1 - \mu}{\sigma_1} \sim N(0,1) \Rightarrow (\;)^2 \sim \chi^2_1$

7b) $\hat{\phi} = \sum_{j=1}^{K} \frac{(O_j - E_j)^2}{E_j} =$

HW 7 1(a) $n = n_1 + n_2 = 16$, $n_1 = 10$, $n_2 = 6$



$B_{all} = \binom{16}{10} = \binom{16}{6}$

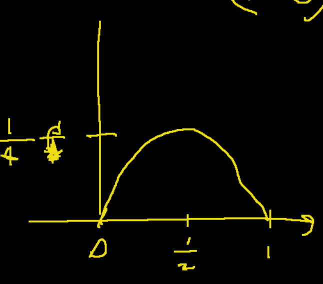
e.g $n = n_1 + n_2 = 100$, $n_1 = 50$, $n_2 = 50$ $\quad B_{all} = \binom{100}{50} = 10\underbrace{0 \ldots\ldots 0}_{29}$

$B < B_{all}$

$= 1{,}000{,}000$

Mid 1 6e) $\hat{\theta} = \bar{X}$, DGP: iid Bern

$\theta - \theta^2 = f(\theta) = \theta(1-\theta)$



$MSE[\hat{\theta}] = Var[\hat{\theta}] = Var[\bar{X}] = \frac{\theta(1-\theta)}{n}$

$\max\{MSE[\hat{\theta}]\} = \max\left\{\frac{\theta(1-\theta)}{n}\right\} = \frac{1}{4n}$

Mid 2 9(c) $\hat{\theta}_T = \frac{75}{202}$, $\hat{\theta}_C = \frac{40}{202}$ $\quad \frac{\hat{\theta}_T - \hat{\theta}_C}{\hat{SE}[\hat{\theta}_T - \hat{\theta}_C]} \xrightarrow{d} N(0,1)$

$CI_{\theta_T - \theta_C, 95\%} = \left[\hat{\theta}_T - \hat{\theta}_C \pm 1.96\, \hat{SE}[\hat{\theta}_T - \hat{\theta}_C]\right]$

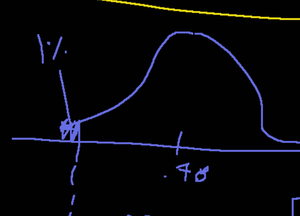
$= \left[\hat{\theta}_T - \hat{\theta}_C \pm 1.96 \right]$

$= \left[\frac{75-40}{202} \pm 1.96 \cdot \sqrt{\frac{\frac{75}{202}\left(1 - \frac{75}{202}\right)}{202} + \frac{\frac{40}{202}\left(1 - \frac{40}{202}\right)}{202}}\right]$

$\sqrt{Var[\hat{\theta}_T - \hat{\theta}_C]} = \sqrt{Var[\hat{\theta}_T] + Var[\hat{\theta}_C]} = \sqrt{\frac{\hat{\theta}_T(1-\hat{\theta}_T)}{n_T} + \frac{\hat{\theta}_C(1-\hat{\theta}_C)}{n_C}}$

Mid 1 9(e) $n = 30$

$\Phi(-2.33) = 1\%$



$z = .48 - 2.33\sqrt{\frac{.48(1-.48)}{30}}$

← RET →

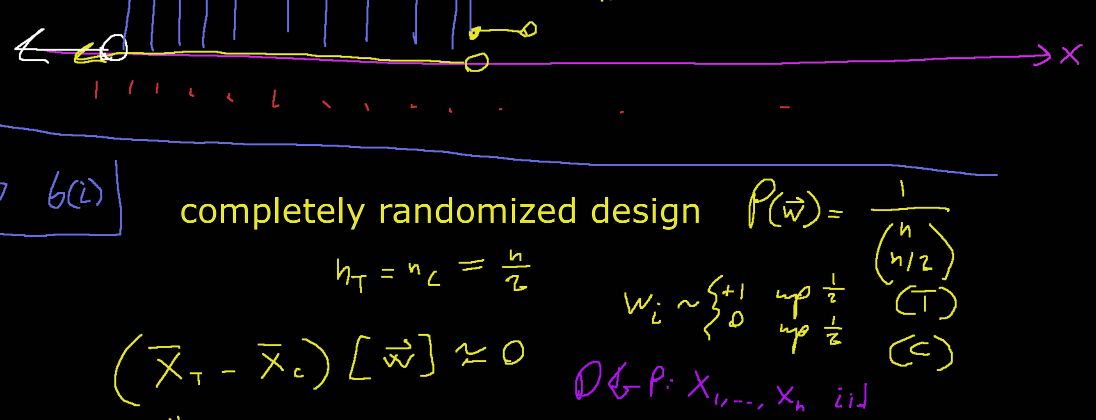
$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ $\quad \frac{1}{n_1+n_2}(\Sigma + \Sigma)$

$= \frac{1}{n_1+n_2-1}\left(\sum_{i=1}^{n_1}(x_{1i} - \bar{x}_1)^2 + \sum_{i=1}^{n_2}(x_{2i} - \bar{x}_2)^2\right)$

$= s^2_{pooled} = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-1}$

$s_{pooled}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = \ldots$

HW 7 1(h)



HW 7 6(i) completely randomized design $P(\vec{w}) = \frac{1}{\binom{n}{n/2}}$

$n_T = n_C = \frac{n}{2}$

$w_i \sim \begin{cases} +1 & wp\ \frac{1}{2} \quad (T) \\ 0 & wp\ \frac{1}{2} \quad (C) \end{cases}$

$(\bar{X}_T - \bar{X}_C)[\vec{w}] \approx 0$

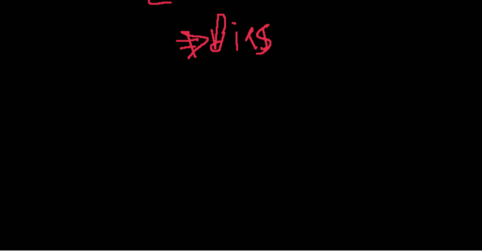
$= \frac{1}{n/2}\sum_{\{i : w_i = 1\}} x_i - \frac{1}{n/2}\sum_{\{i : w_i = 0\}} x_i$

DGP: $X_1, \ldots, X_n$ iid

$\bar{X}_T - \bar{X}_C \overset{\cdot}{\sim} N\left(0, \frac{\sigma_x^2}{n}\right)$

The X_i's in the T group have the same distr as the X_i's in the C group (due to the randomization). Now you invoke the 2-sample CLT.

$\bar{e}_T - \bar{e}_C \overset{\cdot}{\sim} N\left(0, \frac{\sigma^2}{n}\right)$



Ability (x)