390 - 4 - 1   2/10/16 4-1

$$P(\theta = .25 \mid X_1 = 0) = \frac{\overbrace{P(X_1 = 0 \mid \theta = .25)}^{.75}\ \overbrace{P(\theta = 0.25)}^{.5}}{\underbrace{P(X_1 = 0)}\ \ .5} = .75$$

went from .5 to .75

$X_1, X_2, X_3 \overset{iid}{\sim} Bern(\theta)$

$$P(X_1 = 0 \mid \theta = 0.25)\, P(\theta = .25) + P(X_1 = 0 \mid \underset{.25}{\overset{.75}{\theta = .75}})\, P(\theta = .75)$$

$\langle 0, 1, 1 \rangle$

$$P(\theta = 0.25) \to \underbrace{P(\theta = 0.25 \mid X = 0)}_{.25} = .75 \text{ pleaded prior}$$

.5

$$P(\theta = .25 \mid X_2 = 1) = \frac{\overset{.25}{P(X_2 = 1 \mid \theta = .25)}\ \overset{.75}{P(\theta = .25)}}{P(X_2 = 1)} = .5$$

$$\underset{.25}{P(X_2 = 1 \mid \theta = .25)}\ \ \underset{.75}{P(\theta = .25)}$$

$$+ \underset{.75}{P(X_2 = 1 \mid \theta = .75)}\ \ \underset{.25}{P(\theta = .75)}$$

$$P(\theta = .25 \mid X_3 = 1) = \frac{\overset{.25}{P(X_3 = 1 \mid \theta = .25)}\ \ \overset{.5}{P(\theta = .25)}}{[P(X_3 = 1)] = \begin{bmatrix} P(X_3 = 1 \mid \theta = .25)\, P(\theta) = .25 \\ + P(X_3 = 1 \mid \theta = .75)\, P(\theta) = .75 \end{bmatrix}}$$

$$= \frac{.25 \cdot \cancel{.5}}{= [.25 * \cancel{.5}] + [.75 * .5]} = .25$$

Proof of Baysian

$$P(\theta | X_1, \dots, X_n) = \frac{P(X_1, \dots, X_n | \theta)\, P(\theta)}{P(X_1, \dots, X_n)}$$

$$P(\theta | X_2, \dots, X_n, X_1) = \frac{P(X_2, \dots, X_n | \theta)\, P(X_1|\theta) P(\theta)}{P(X_n \dots X_2 | X_1)\, P(X_1)} \quad \frac{P(X_1|\theta) P(\theta)}{P(X_1)} = P(\theta|X_1)$$

$$\frac{P(X_3, \dots, X_n | \theta) P(X_2|\theta) P(\theta|X_1)}{P(X_3, \dots, X_n | X_2, X_1) P(X_2|X_1)}$$

$$\frac{P(X_2|\theta) P(X_1/\theta) P(\theta)}{P(X_2|X_1) P(X_1)} \rightarrow \frac{P(X_1, X_2|\theta) P(\theta)}{P(X_1, X_2)}$$

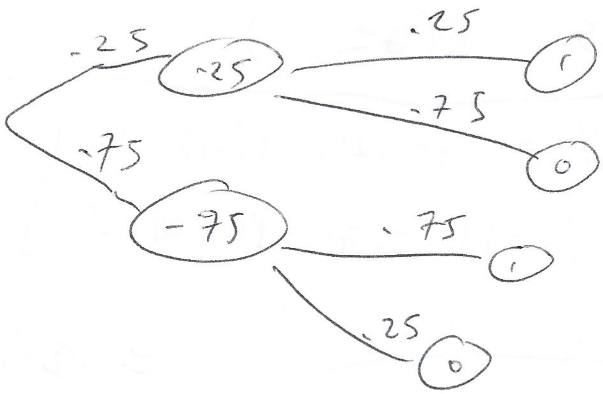
$\hat{\theta}_{MLE} = .66$ we want $X_4 | X_1, X_2, X_3 \sim Bern(\ )$

$X_4 \sim Bern(\ )$ Baysian

Posterior predictive distrib

$\theta | X_1, X_2, X_3$

X_4



.0625
.1875
$= .25$

.1875
.5625
$= .75$

$$P(x_4 \mid x_1, x_2, x_3) = \sum_{\theta \in \Theta_0} P(x_4, \theta \mid x_1, x_2, x_3)$$

$$= \sum_{\theta \in \Theta_0} P(x_4 \mid \theta, x_1, x_2, x_3) P(\theta \mid x_1, x_2, x_3)$$

Only thing we need

future datum

$$P(x^* \mid x)$$

past datum

$$= \sum_{\theta \in \Theta_0} P(x^* \mid \theta) P(\theta \mid x)$$

$$= \int_{\theta \in \Theta_0} f(x^* \mid \theta) f(\theta \mid x)\, d\theta$$

$$P(x_4 \mid \theta, x_1 x_2, x_3) = \frac{P(\theta, x_1, x_2, x_3, x_4)}{P(\theta, x_1, x_2, x_3)}$$

$$= \frac{P(x_1, x_2, x_3, x_4 \mid \theta) P(\theta)}{P(x_1, x_2, x_3 \mid \theta) \quad P(\theta)}$$

$$= \frac{P(x_4 \mid \theta) P(x_1, x_2, x_3, \mid \theta)}{P(x_1, x_2, x_3 \mid \theta)}$$

So x_1, x_2, x_3 Bern (θ)

So $x_1 \mid \theta_1, x_2 \mid \theta, x_3 \mid \theta,$ is accurate.

$\theta \sim \begin{cases} .25 & \text{wp } 0.5 \\ .75 & \text{wp } 0.5 \end{cases} \rightarrow \theta \mid X \sim \begin{cases} .25 & \text{wp } .25 \\ .75 & \text{wp } .75 \end{cases}$

$$\hat{\theta}_{map} = \arg\max \{ p(\theta|x) \}$$

$= \arg\max \{ p(\theta|x) p(\theta)$

$= \arg\max \{ p(\theta|x) = \hat{\theta}_{MLE}$

if prior is the " of indifference"

★ not on test ★

want θ

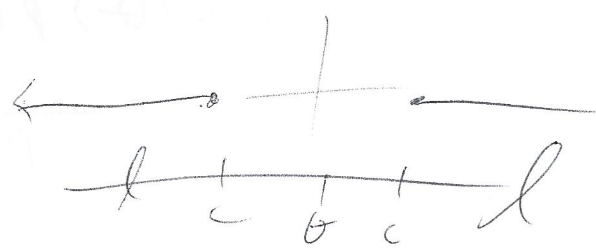
Estimate it with $\hat{\theta}$

loss $l(\theta, \hat{\theta}) = (\theta - \hat{\theta})^2$ $= |\theta - \hat{\theta}|$ absolute loss

$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ $l(x,\mu)$

$E(x-\mu)^2$

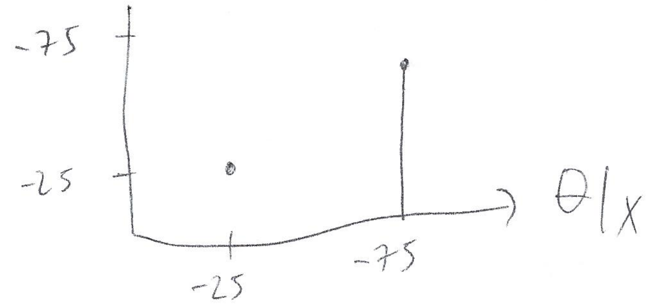
$\begin{cases} |\theta - \hat{\theta}| \le c, & 0 \\ |\theta - \hat{\theta}| > c, & A \end{cases}$



$\hat{\theta}_{map}$

Posterior mean $E(\theta|X)$ Best loss , Posterior median $\text{med}(\theta|X)$

390-4-3 $P(\theta|x)$



$$F(\theta|x) = \sum_{\theta \in \Theta_0} \theta\, p(\theta|x) = .625$$

$\notin \Theta_0$

$\theta \sim U(0,1)$

Prior

$p(x_1|\theta)\, p(x_2|\theta)\, p(x_3|\theta)$

$$P(\theta|\overset{0}{x_1}, \overset{1}{x_2}, \overset{1}{x_3}) = \frac{P(x_1 x_2, x_3|\theta)\ \overset{1}{P(\theta)}}{P(x_1, x_2, x_3)}$$

$$P(x) = \int P(x|\theta) P(\theta)\, d\theta$$

$$= \int_0^1 \underbrace{\overset{(1-\theta)}{P(x_1|\theta)}\ \overset{\theta}{P(x_2|\theta)}\ \overset{\theta}{P(x_3|\theta)}}_{\theta^2 - \theta^3}\ \overset{1}{P(\theta)}\, d\theta$$

$$= \frac{\theta^3}{3} - \frac{\theta^4}{4} \Big]_0^1 = \frac{1}{12}$$

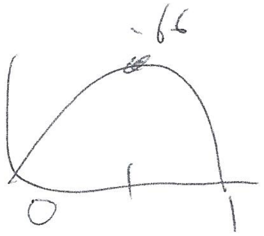
$$= 12(1-\theta)\theta^2$$

$$\hat{\theta}_{MAP} = \arg\max \{\theta^2 - \theta^3\}$$

$$\theta = [0,1]$$

$$\frac{d}{d\theta} p(\theta|x) \propto 2\theta - 3\theta^2 = 0$$

$$(2-3\theta)(\theta) = 0$$



$$\Rightarrow \hat{\theta}_{MAP} = .66 = \hat{\theta}_{MCE}$$

$$\hat{\theta}_{MMSE} = E(\theta|x) = \sum_{\theta \in \Theta_0} \theta\, p(\theta|x) = .625$$

¢

Θ_0

minimum mean square error

$$E(\theta|x) = \int_{\Theta_0} \theta\, p(\theta|x)\, d\theta = \int_0^1 \theta\, 12(1-\theta)\theta^2\, d\theta$$

$$= 12\int_0^1 (\theta^3 - \theta^4)\, d\theta = 12\left[\frac{\theta^4}{4} - \frac{\theta^5}{5}\right]_0^1 = \frac{12}{20} = .6$$

$$\hat{\theta}_{MAE} = \text{med}[\theta|x] = \left\{y : \int_0^y p(\theta|x)\, dx = .5\right\}$$

mean absolute error

$$\Downarrow$$

$$y: 12\left[\frac{\theta^3}{3} - \frac{\theta^4}{4}\right]_0^y = 0.5$$

$$4y^3 - 3y^4 = .5$$

$$y \in \{.614,\ 1.25,\ -.26 - 0.38i,\ -.26 + .38i)$$