MATH 390.03-02 / 650 Spring 2016 Homework #10

Professor Adam Kapelner

Due 8:30AM, Wednesday, May 25, 2016
(material covered on exam, writeup not required – for extra credit only)

(this document last updated Monday 16$^{\text{th}}$ May, 2016 at 9:26pm)

Instructions and Philosophy

The path to success in this class is to do many problems. Unlike other courses, exclusively doing reading(s) will not help. Coming to lecture is akin to watching workout videos; thinking about and solving problems on your own is the actual "working out." Feel free to "work out" with others; **I want you to work on this in groups.**

Reading is still *required*. For this homework set, read about Gibbs sampling and Metropolis-Hastings sampling. Also read ch17 and the epilogue in McGrayne.

The problems below are color coded: green problems are considered *easy* and marked "[easy]"; yellow problems are considered *intermediate* and marked "[harder]", red problems are considered *difficult* and marked "[difficult]" and purple problems are extra credit. The *easy* problems are intended to be "giveaways" if you went to class. Do as much as you can of the others; I expect you to at least attempt the *difficult* problems.

Problems marked "[MA]" are for the masters students only (those enrolled in the 650 course). For those in 390, doing these questions will count as extra credit.

This homework is worth 100 points but the point distribution will not be determined until after the due date. See syllabus for the policy on late homework.

Up to 10 points are given as a bonus if the homework is typed using LaTeX. Links to instaling LaTeX and program for compiling LaTeX is found on the syllabus. You are encouraged to use `overleaf.com`. If you are handing in homework this way, read the comments in the code; there are two lines to comment out and you should replace my name with yours and write your section. The easiest way to use overleaf is to copy the raw text from hwxx.tex and preamble.tex into two new overleaf tex files with the same name. If you are asked to make drawings, you can take a picture of your handwritten drawing and insert them as figures or leave space using the "\vspace" command and draw them in after printing or attach them stapled.

The document is available with spaces for you to write your answers. If not using LaTeX, print this document and write in your answers. I do not accept homeworks which are *not* on this printout. Keep this first page printed for your records.

NAME: ____

Problem 1

These are questions about McGrayne's book, chapter 17 and the Epilogue. They are optional.

(a) [easy] What do the computer scientists who adopted Bayesian methods care most about and whose view do they subscribe to? (p233)

(b) [easy] How was "Stanley" able to cross the Nevada desert?

(c) [easy] What two factors are leading to the "crumbling of the Tower of Babel?"

(d) [harder] Does the brain work through iterative Bayesian modeling?

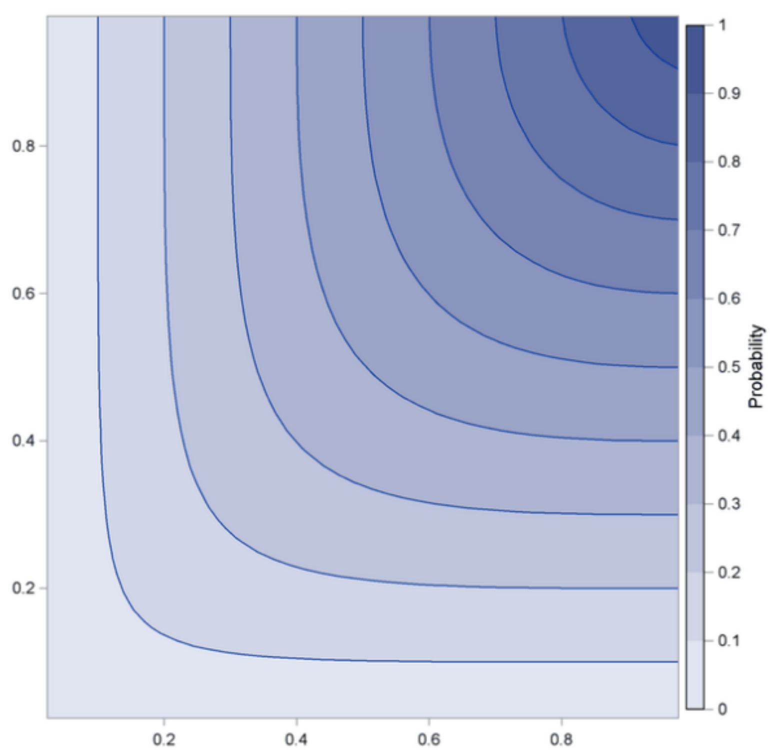
(e) [easy] According to Geman, what is the most powerful argument for Bayesian Statistics?

Problem 2

These are questions which introduce Gibbs Sampling.

(a) [easy] Outline the systematic sweep Gibbs Sampler algorithm below (in your notes).

(b) [easy] Pretend you are estimating $\mathbb{P}(\theta_1,\ \theta_2 \mid X)$ and the joint posterior looks like the picture below where the x axis is θ_1 and the y axis is θ_2 and darker colors indicate higher probability. Begin at $\langle \theta_1, \theta_2 \rangle = \langle 0.5, 0.5 \rangle$ and simulate 5 iterations of the systematic sweep Gibbs sampling algorithm by drawing new points on the plot (just as we did in class).



(c) [harder] We previously have shown that if $X \mid \theta \sim \text{Binomial}(n, \theta)$ and the prior on $\theta \sim \text{Beta}(\alpha, \beta)$, then $X \sim \text{BetaBinomial}(n, \alpha, \beta)$. Even though we proved this result, pretend like you didn't know it and create a Gibbs sampler which finds $\mathbb{P}(X)$.

(d) [easy] Consider the simple linear regression model under OLS with n data points. Assume the true $\beta_0 = 2$ and $\beta_1 = -2$ and $\sigma^2 = 1$. Let's sample $n = 3$ points. Run the code on lines 1–20 of the code at the link here which will plot the 3 data points and provide a best fit line. Do you think the best fit line does a good job?

(e) [easy] Consider the model in (d). We impose priors of $\beta_0 \sim \mathcal{N}(0, \sigma^2/m)$, $\beta_1 \sim \mathcal{N}(0, \sigma^2/m)$ and $\sigma^2 \sim \text{InvGamma}\left(\frac{n_0}{2}, \frac{n_0\sigma_0^2}{2}\right)$ which is the model we did in class. Find the kernel of the posterior.

(f) [easy] Consider the model in (d). Find the conditional distribution of σ^2. The other two conditionals are filled in below so you don't have to complete the squares:

$$\begin{aligned}
\beta_0 \mid \beta_1, \sigma^2, \boldsymbol{X}, \boldsymbol{y} &\sim \mathcal{N}\left(\frac{\bar{y}n - \beta_1 \bar{x} n}{n+m}, \frac{\sigma^2}{n+m}\right) \\
\beta_1 \mid \beta_0, \sigma^2, \boldsymbol{X}, \boldsymbol{y} &\sim \mathcal{N}\left(\frac{\sum x_i y_i - \beta_0 \bar{x} n}{m + \sum x_i^2}, \frac{\sigma^2}{m + \sum x_i^2}\right) \\
\mathbb{P}\left(\sigma^2 \mid \beta_0, \beta_1, \boldsymbol{X}, \boldsymbol{y}\right) &\propto
\end{aligned}$$

(g) [easy] Let's run 10,000 iterations of the Gibbs sampler you created in part (f) by running lines 22–78 of the code at the link here. Explain the three plots that popped up. Where would you say the chains burned in at?

(h) [easy] Now let's assess autocorrelation for the Gibbs chains in part (g) by running lines 82–87 of the code at the link here. Explain these three plots? What do we mod our chains by to thin them out so the chains represent independent samples?