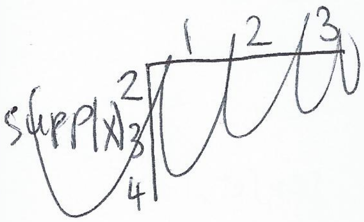
390 - 3

$$P(A_l|B) = \frac{\overbrace{P(B|A_l)\,P(A_l)}^{\to\, P(A_l, B)}}{\underbrace{\sum_{i=1}^{n} P(B|A_l)\,P(A)}_{P(B)}}$$

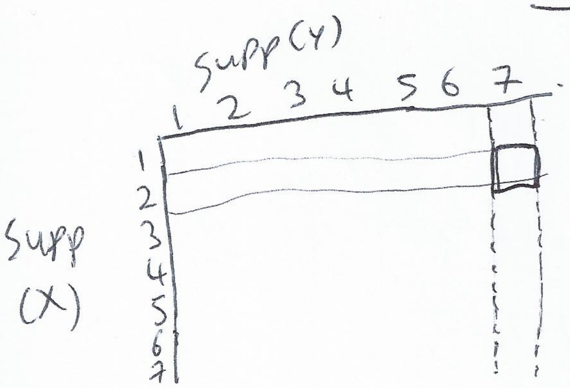
$\boxed{\text{Baye's Theorem}}$



$$P(X=2\,|\,Y=7) = \frac{P(X=2, Y=7)}{P(Y=7)}$$

$$\to \sum_{X + supp(7)} P(X, 7)$$

$$P(X|y) = \frac{P(x,y)}{P(y)}$$



disjoint

$P(Y=7) = P(X=1, Y=7) + P(X=2, Y=7)$
$+ P(X=3, Y=7) + P(X=4, Y=7)$
$+ P(X=5, Y=7) + P(X=6, Y=7)$
$+ P(X=7, Y=7)$

$$P(y) = \sum_{x \in Supp(x)} P(x,y)$$ marginalization

$$f(x,y) = \frac{f(x,y)}{f(y)} = \frac{f(y|x)\,f(x)}{f(y)}$$

$$f(y) = \int_{x \in Supp(x)} f(x,y)\,dx$$

f(y|x) =

$$P(\theta | x) = \frac{P(x | \theta)\, p(\theta)}{P(x)}$$

Prior for θ

Posterior

likelihood of x

Prior for (x)

"Marginal likelihood"

Bayseian

frequentist

$$P(\theta; x) = \frac{P(x;\theta)\, P(\theta)}{P(X)}$$

$\leftarrow$ makes no sense, either 0, or 1

θ is set.

Makes no sense because You cannot Calculate $p(x)$ without knowing what θ is

X big no, no

Coin flip $\langle 0, 1, 1 \rangle$

$P(X | \theta = 0.75) = .25 \cdot .75 \cdot .75 = .141$

$P(X | \theta = 0.5) = .5 \cdot .5 \cdot .5 = .125$

$P(\theta = 0.25)$?? $P(\theta = 0.75)$??

Principle of indifference

All prior hypothesis about θ are equally likely.

$$\underbrace{P(\theta = .25)}_{\text{idea 1}} = \underbrace{P(\theta = .75)}_{\text{idea 2}} = \frac{1}{2}$$

$$\Omega = X \times \Theta_0 \longrightarrow \Theta_0 \subseteq \Theta$$ 3-2)

$$X \longrightarrow \mathrm{Supp}[X]^n$$

⟨0,0,0⟩	⟨1,0,0⟩	⟨0,1,0⟩	⟨0,0,1⟩	110	011	101	111	$\theta = 0.25$
000	100	010	001	110	101	011	⟨0,0,0⟩	$\theta = 0.75$

Sample space $= \mathrm{supp}(x)^3$

$X = \{(0,0,0), (0,0,1), (0,1,0), (1,0,0), (0,1,1), (1,0,1), (1,1,0), (1,1,1)\}$

$P(\theta = .25) \quad P(0,0,0 \mid \theta = 0.25) = .75^3 = .421$

is $\langle 0,0,0\rangle, \theta = .25 \cap (0,0,0), \theta = .75 = \phi$?

Yes. different piece of the universe.

let $X = (0,1,1)$ what is $P(\theta = .25)$

$P(\theta = .25 \mid X = \langle 0,1,1\rangle)$

$\langle 0,1,1\rangle = \Omega_0$ Universe

Therefore

$$= \frac{P(0,1,1 \mid \theta = .25)}{\underbrace{P(0,1,1 \mid \theta = .25) + P(0,1,1 \mid \theta = .75)}_{\text{all } P\langle 0,1,1\rangle}}$$

$$= \frac{P(0,1,1 \mid \theta = .25) P(\theta = .25)}{[P(0,1,1 \mid \theta = .25)(P(\theta = .25)] + [P(0,1,1 \mid \theta = .75) P(\theta = .75)}$$

$$= \frac{.047 * \cancel{.5}}{.047 * \cancel{.5} + .141 * \cancel{.5}} = .25$$

$$P(\theta = .75 \mid \langle 0,1,1 \rangle) = .75$$

$$= P(\theta = .25)^c \mid (0,1,1) = 1 - .25 = .75$$

$$P(X) = \sum_{i=1}^{n} P(X, \theta_i) \text{ or } \int_{\Theta} P(X, \theta)\, d\theta$$

$$= \sum_{i=1}^{n} P(X \mid \theta_i) P(\theta_i) \text{ or } \int_{\Theta} P(X \mid \theta) P(\theta)\, d\theta$$

$$P(X, \theta) \neq P(X) P(\theta)$$

Ex $P(0,1,1, .25) \neq P(X)\, P(\theta = .25)$

$$.047 * .5 \neq .094 * .5$$

Imagine $\Theta = \{0.1, 0.25, 0.5, 0.75, 0.9\}$ 3-3)

$P(\theta_i) = \frac{1}{5}\ \forall i$ each θ are $\frac{1}{5}$ likely to be real θ.

data $= X = (0,1,1)$ what is $P(\theta|X)$?

$P(X|\theta) =$

$\theta = .1 \Rightarrow .009$

$\theta = .25 \Rightarrow .0469$

$\theta = .5 \Rightarrow .125$

$\theta = .75 \Rightarrow .1406$

$\theta = .9 \Rightarrow .081$

$$P(\theta|X) = \frac{P(X|\theta)\,P(\theta)}{P(X)} \propto P(X|\theta)\,P(\theta)$$

$\rightarrow$ not a function of θ

$$\frac{P(\theta = .75|X)}{P(\theta = .9|X)} = \frac{P(X|\theta = .75)\,P(\theta = .75)}{P(X|\theta = .9)\,P(\theta = .9)}$$

$$\hat{\theta}_{map} = \underset{\theta \in \Theta_0}{\operatorname{argmax}} \{P(\theta|X)\} = \underset{\theta \in \Theta_0}{\operatorname{argmax}} \left\{ \frac{P(X|\theta)P(\theta)}{P(X)} \right.$$

Bayes

$$= \underset{\theta \in \Theta_0}{\operatorname{argmax}} \{ P(X|\theta) P(\theta)$$

Maximum a posteri
(map)

In $(0,1,1)$ $\hat{\theta}_{map\,Bayes} = .75$

$\hat{\theta}_{MLE} = .66$

) not equal

because $\Theta_0 \neq \Theta = (0,1)$

we were only limited to 5 choices of θ.

Bayesian Conditionalism

$P(\theta = 0.25) = 0.5 \rightarrow P(\theta = 0.25 | \langle 0,1,1 \rangle) = 0.25$

Data comes in and updates our prior beliefs.