LECTURE-'6'
02/22/16

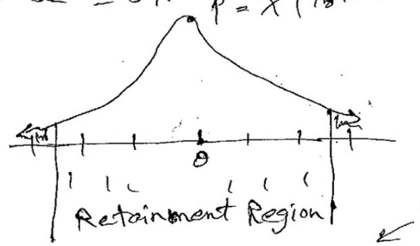
①

Two-sided

$H_0: \theta = \theta_0 \overset{e.g.}{=} 0.5$

$H_a: \theta \neq \theta_0$

Pick $\alpha \overset{e.g.}{=} 5\%$. $\hat{p} = \bar{x} \mid H_0$ is true



$\leftarrow Z$

$= [\theta \pm Z_{\alpha/2} \sqrt{\frac{\theta(1-\theta)}{n}}$

C.L.T

$P_{val} := P\left(\begin{matrix} (\hat{\theta}) \\ \text{data or} \\ \text{more extreme} \end{matrix} \middle| H_0 \text{ is true} \right) \neq P(H_0 \text{ true} \mid \text{data})$

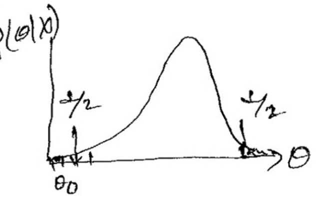
:= lowest α where you still retain

• $P(H_0 \mid X) = \underbrace{P(\theta_0 \mid X)}_{\{\text{posterior}\}} = 0$

① CR (credible region) method

If $\theta_0 \in CR \Rightarrow$ Retain H_0

If $\theta_0 \notin CR \Rightarrow$ Reject H_0



②

$P_{val} := P\left(\theta \text{ is more extreme than } \theta_0 \mid X\right)$

$\simeq 2 \min\{P(\theta > \theta_0 \mid X), P(\theta < \theta_0 \mid X)\}$

The more symmetric, the best the approximate

One-sided Hypothesis Test (Bayesian)

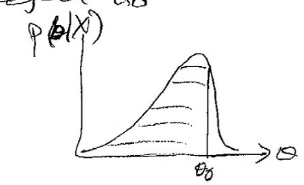
$H_0: \theta \leq \theta_0 = 0.5$

$H_a: \theta > \theta_0$

if $P(H_0 \mid X) > \alpha \Rightarrow$ Retain/Accept H_0
{very likely}

$P(H_0 \mid X) < \alpha \Rightarrow$ Reject H_0

$P_{val} = P(H_0 \mid X)$



$P(H_0 \mid X) + P(H_a \mid X) = 1$

why not let $\alpha = 50\%$

• Also; $H_0: \theta \in \Theta_0 \subset \Theta$

$H_a: \theta \in \Theta_0^c$

Two-sided, again

$H_0: \theta = \theta_0 \overset{e.g.}{=} 0.5$

$H_a: \theta \neq \theta_0 \overset{e.g.}{=} 0.5$

Pick $\alpha \overset{e.g.}{=} 5\%$

Two-sided

which we don't care (eg. 0.001)

$H_0: \theta \in [\theta_0 \pm \epsilon]$

$H_a: \theta \in [\theta_0 \pm \epsilon]^c$ $\quad \alpha = 0.05$

$\theta_0 = 0.5$, $\epsilon = 0.01$

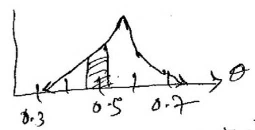
$H_0: \theta \in [.49, .51]$

$n=100$ $\quad$ # $H = 54$

$\theta \sim U(0,1) = Beta(1,1)$

$\theta | X \sim Beta(\alpha + X, \beta + n - x)$

$= Bern(55, 47)$ $\quad$ In 'R' console; $\leftarrow$ qbeta(0.025, 55, 47)



Pval =

95% CR = $[.442, .635]$

$0.5 \in$ $\Rightarrow$ Retain Ho

$P(\underbrace{\theta \in [.49, .51]}_{H_0}) = F(0.51) - F(0.49...)$

$= 0.117 \notin \alpha = 5\%$

$\Rightarrow$ Retain Ho

Frequentist:

$CI_{\theta, 95\%} = \left[\hat{\theta} \pm z_{\alpha/2}\sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}\right]$

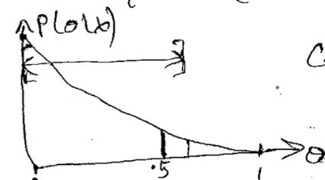
$= [.440, .640]$

Pval: $= 0.24$

$\theta \sim U(0,1) = Beta(1,1)$

$X = \langle 0, 0, 0 \rangle$ $\quad (1+3-0)$

$\theta | X \sim Beta(1,4)$



CR = $[0.006, 0.602]$

$.5 \Rightarrow$ Retain

$H_0: \theta = 0.5$

Pval: $= 2P(\theta < 0.5 | X)$

$= .13 \notin 5\%$

$\neq .125$

$H_0: \theta = [0.49, 0.51]$

$P(H_0|X) = \int_{.49}^{.51} Beta(1,4)\, d\theta = 0.01$

$< 5\%$

$\Rightarrow$ Reject

$H_0: \theta \leq 0.1$ $\quad$ severly weighted towards tails

$H_a: \theta > 0.1$ $\quad$ $P(H_0|X) = \int_0^{0.1} Beta(1,4)\, d\theta = .344 \notin 5\%$

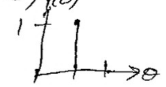
$\Rightarrow$ Retain Ho

(5)

Frequentist:

$\hat{\theta} = 0$ $\quad C.I = \left[\hat{\theta} \pm z_{\alpha/2}\sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}\right] = [0]$

$H_0 : \theta = \theta_0 = 0.5 \; (M_1) \; = \text{Deg}(0.5)$ Generate $p(\theta)$



$H_a : \theta \sim U(0,1) \; (M_2)$

Jefferey's K $= B = \frac{P(X|M_1)}{P(X|M_2)}$ Abuse of notation

Bayes Factor

$$= \frac{\int_{\Theta_{M_1}=\{0.5\}} \overbrace{p(X|\theta,M_1)}^{p(X|\theta=0.5) = \binom{n}{x}0.5^n} \overbrace{p(\theta|M_1)}^{1}\, d\theta}{\int_{\Theta_{M_2}=(0,1)} \underbrace{p(X|\theta,M_2)}_{\binom{n}{x}\theta^x(1-\theta)^{n-x}} \underbrace{p(\theta|M_2)}_{1}\, d\theta}$$

$$= \frac{(1/2)^n}{B(x+1, n-x+1)} = \frac{(1/2)^{100}}{B(55,47)} = 2.66 \in [1,3]$$

Barely worth mentioning

$$B = \frac{P(X|M_2)}{P(X|M_1)} = 2.66^{-1} < 1$$

negative evidence

E.g. $n = 100$; $X = 61$ Heads

$$\frac{(1/2)^n}{B(x+1, n-x+1)} = \frac{(1/2)^{100}}{B(62,40)} = 1.39^{-1}$$

$$B = \frac{P(X|M_2)}{P(X|M_1)} = 1.39 \text{ (Barely worth mentioning)}$$

(6)

Frequentist:

$$P_{val} := 2P(\hat{p} > .61)$$
$$= 2P(Z > 2.2)$$
$$= 0.027 < \alpha = 5\%$$
$\Rightarrow$ Reject!

$P(X|\theta) = \text{Bin}(n,\theta)$

$P(\theta) = \text{Beta}(\alpha,\beta)$ Hyperparameter

prior success

prior failure

$P(\theta|X) = \text{Bern}(X+\alpha, n-X+\beta)\, d\theta \quad \theta^{X+\alpha-1}(1-\theta)^{n-X+\beta-1}$

$E[\theta] = 0.4 = \frac{\alpha}{\alpha+\beta}$

$SE[\theta] = 0.02 = \sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}}$