← assume σ known

$$E[X^*|X] = E_\theta[E_{X^*}[X^*|\theta)|X]$$

$$= E_\theta[\theta|X] = \hat{\theta}_{MMSE} = \frac{\frac{\bar{x}n}{\sigma^2} + \frac{\mu_0}{\tau^2}}{\frac{n}{\sigma^2} + \frac{1}{\tau^2}}$$

$$Var[X^*|X] = \overbrace{E_\theta[\overbrace{var_{X^*}[X^*|\theta]}^{\sigma^2}|X]}^{\sigma^2}$$

$$+ \underbrace{var_\theta[E_{X^*}\underbrace{[X^*|\theta]}_{\theta}|X]}_{\sigma^2_p}$$

$$Var[Y] = E_X[var_Y[Y|X]] + var_X[E_Y[Y|X]]$$

$$E[Y] = E_X[E_Y(Y|X)]$$

$$E[Y|Z] = E_X[E_Y(Y|X)|Z]$$

$X_1, \dots, X_n$ exch $N(\theta, \sigma^2)$

$$P(\theta, \sigma^2|X) \propto P(X|\theta, \sigma^2)\underbrace{P(\theta, \sigma^2)}_{P(\theta|\sigma^2)P(\sigma^2)}$$

$$N\left(\mu_0, \frac{\sigma^2}{M}\right) \quad \text{Inv Gamma}\left(\frac{\nu_0}{2}, \frac{\nu_0\sigma_0^2}{2}\right)$$

$$P(\theta, \sigma^2) = P(\theta)P(\sigma^2)$$

$$= N(\mu_0, \tau^2)\ \text{InvGamma}\left(\frac{\nu_0}{2}, \frac{\nu_0\sigma_0^2}{2}\right)$$

$$\tau^2 \neq f(\sigma^2)$$

$$P(\theta, \sigma^2 | X) \propto P(X|\theta, \sigma^2)\, P(\theta) P(\sigma^2)$$

$$\propto (\sigma^2)^{-\frac{n}{2}}\, e^{-\frac{1}{2\sigma^2}\left((n-1)s^2 + n(\bar{X}-\theta)^2\right)}\, e^{-\frac{1}{2\tau^2}(\theta-\mu_0)^2}$$

$$\propto e^{-\frac{n}{2\sigma^2}\bar{X}^2}\, e^{\frac{n\bar{X}\theta}{\sigma^2}}\, e^{\frac{-n\theta^2}{2\sigma^2}}$$

$$= N(\theta_p, \sigma_p^2)\, K(\sigma^2|X)$$ $\not\propto$ InvGamma, $\not\propto$ Any known dist

$$P(\sigma^2|X) = C(X)\, k(\sigma^2|X)$$

$$\hat{\theta}_{map} = \arg\max P(\theta|X)$$

<u>bad idea</u>

$$P(\theta|X) = C(X)k(\theta|X)$$

$$\ln P(\theta|X) = \ln C(X) + \underbrace{\ln\left(k(\theta|X)\right)}_{g(\theta|X)}$$

$$g(\theta|X) \sim \text{Tay}(\theta, c, 2)$$

$$f(x) \approx \overset{\text{Taylor}}{\text{Tay}}(x, c, d) := \sum_{i=0}^{d} \frac{f^{(i)}(c)}{i!}(x-c)^i$$

~~center~~ free, center, degree $\in \mathbb{N}$

$g(\theta|X) \approx Tay(\theta, c, 2) \to g(c|X) + g'(c|X)(\theta - c) + \frac{g''(c|X)(\theta-c)^2}{2}$

$c = \hat{\theta}_{map}$, simplifies

$= g(\hat{\theta}_{map}|X) + \frac{g''(\hat{\theta}_{map}|X)(\theta - \hat{\theta}_{map})^2}{2}$

$P(\theta|X) \propto k(\theta|X) = e^{g(\theta|X)} \approx \underbrace{e^{g(\hat{\theta}_{map}|X)}}_{\#}$

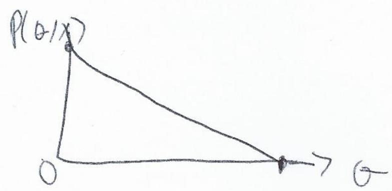
$e^{\frac{1}{2 g''(\hat{\theta}_{map})^{-1}}(\theta - \hat{\theta}_{map})^2} \propto N\left(\hat{\theta}_{map}, \left(\frac{1}{\sqrt{g''(\hat{\theta}_{map}|X)}}\right)^2\right)$ pretty!!!

$X \sim Bin(n, \theta)$

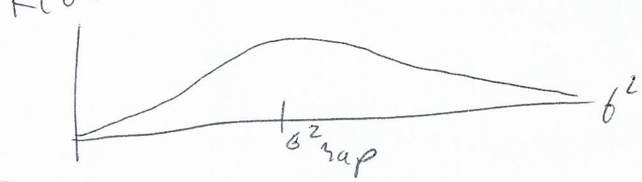
$\theta \sim Beta(1,1)$

$\theta|X \sim Beta(X+1, n-X+1) = Beta(1,4)$

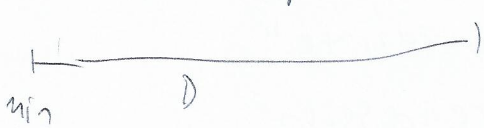
$X=0, \quad N=3$



$P(\theta, \sigma^2|X) \propto P(\theta|X, \sigma^2)\, k(\sigma^2|X)$



GRID $\begin{cases} x_1 = \min(D) \\ x_2 = \min(D) + \varepsilon \\ \vdots \\ x_n = \max(D) \end{cases}$



$\sigma^2_G \subset [0, 10000000]$ $\quad supp(\sigma^2)$

$= \{\sigma^2_1, \ldots \sigma^2_G\}$

$P(\theta_g | X) \approx \hat{c}\, k(\theta_g | X)$

$\approx \hat{f}(\theta_g | X)$

$$\hat{c} = \frac{1}{\sum_{g=1}^{G} k(\theta_g | x)}$$

$$\hat{F}(\theta_i | X) = \sum_{g=1}^{i} \hat{c}\, k(\theta_g | X)$$

$$\theta = \hat{F}^{-1}(u)$$

pick ϵ, D.

$\hat{\theta}_g$	$\hat{F}$

$$P(x^* | X) = \int\int\int\int\int P(x^* | \theta_1, \ldots \theta_5) P(X | \theta_1, \ldots \theta_5)\, d\theta_1 d\theta_2 \ldots d\theta_5$$

linear model.

Consider a bivariate distr X, Y

WLOG let Y be the "~~Leopone~~ response"
"outcome"
"dep. var"

let X be the "feature"
"covariate"
"regression"
"indep var"

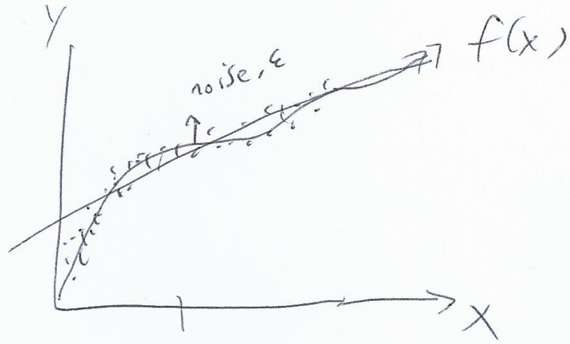
$$X \longrightarrow Y$$

function

noise (inexplicable)

$$X \xrightarrow{f,\ \varepsilon} Y$$

$$X, Y \longrightarrow \begin{matrix} (X_1, Y_1) \\ (x_2, Y_2) \\ \vdots \\ (X_n, Y_n) \end{matrix}$$



$$Y = \underbrace{f(x)}_{\text{primary}} + \underbrace{\varepsilon}_{\text{secondary}}$$

nuisance parameter

$$P(\theta | X) = \int P(\theta, \sigma^2 | X)\, d\sigma^2 \sim T$$

Primary

Secondary

gets rid of σ^2

$$\varepsilon \neq G(x)$$

$f \in \mathcal{F}$ $\quad f: \mathbb{R} \to \mathbb{R}$ $\quad |\mathcal{F}| = 2^{|\mathbb{R}|}$: Too crazy

Limit $\mathcal{F}$

$$f \in \mathcal{F} = \{\beta_0 + \beta_1 x : \beta_0 \in \mathbb{R},\ \beta_1 \in \mathbb{R}\}$$