Statistical Inference

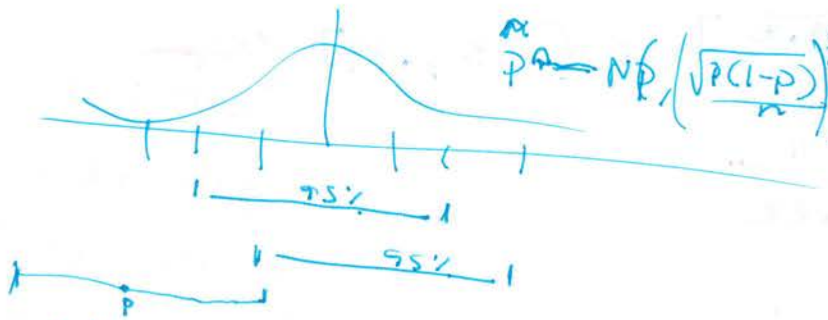
We don't know paramaterized so we want to show

① Estimate its best guess

② provide range of possible (likely) values

③ test theories about the parameter

$$\hat{p} = \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\#1's}{n} \approx p \qquad x_1, \dots, x_p \overset{iid}{\sim} Bern(p)$$



$$\hat{p} \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$$

$$P\left(p \in \hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right) = 1-\alpha$$

confidence interval of parameter p with coverage $1-\alpha$.

$$P\left(p \in \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right) \approx 1-\alpha$$

$$CI_{p, 1-\alpha} : \left[\hat{p} \pm \underbrace{z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}_{\text{margin of error}}\right]$$

$CI_{p, 95\%} = [0.47, 0.56]$

$= [0.52 \pm 0.05]$ ★ must be known ★

Interpretation of the CI

ME ① If you sample many times and compute a CI for each then the p will be in the CI $1-\alpha$ prop of the time.

① If you sample many times and compute a CI for each, the $p \in CI$ $1-\alpha$ prop of the time.

Not useful

★ No one cares if you sample many times

② Before you begin, your CI will contain p w.p $1-\alpha$.

Not useful

③ $P(p \in CI_{p, 1-\alpha}) = P(p \in [0.47, 0.57])$ Degenerate (0) or Deg(1) $\in \{0, 1\}$

~~Not useful~~

④ $P(p \in CI_{p,1-\alpha}) = 1-\alpha$ Everybody wants this

only ~~true~~ available if you are subjectivist with the right ~~pivot~~ Prior information

Ex/ Do you like mushroom?

$n = 20$ sample size 11 like mushroom

$\hat{p} = \frac{11}{20} = .55$ best guess of p "Not a representative sample"

$\alpha = 5\%$ 95% coverage $\Rightarrow z_{2.5\%} = 2$

$CI_{p,95\%} = [.55 \pm \underbrace{2 \underbrace{\sqrt{\frac{.55 \cdot .45}{20}}}_{.11}}_{.22}] = [.33, .77]$ "Does not give inference for the population of all humans."

Goal #3

Human sex ratio / proportion

~~the probability~~ Do you think P(new human baby being male) $\neq 50\%$?

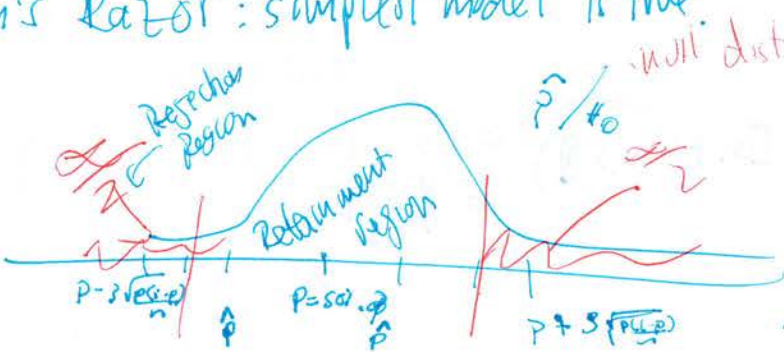
I think it Even.

$H_0: p = 50\% = p_0$, $H_a: p \neq 50\%$

null hypothesis alternative hypothesis

We need "sufficient" evidence to reject the null hypothesis?

Occam's Razor: simplest model is true



we take a sample of size n.

(n is determined beforehand)

Compute $\hat{p}$.

let $\alpha := P$(Reject H_0 | H_0 true / then is false)

Retainment Region

$$\left[p_0 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p_0(1-p_0)}{n}}\right]$$

Rejection Region is the complement of the ~~retainty~~ retainment region.

To run the test, compute $\hat{p}$

If $\hat{p} \in$ Retainment Region $\Rightarrow$ Retain H_0

If $\hat{p} \notin$ Retainment Region $\Rightarrow$ Reject H_0.

Retainment Region $n = 345$ $\alpha = 5\%$ we pick α.

$$= \left[0.5 \pm 2\sqrt{\frac{0.5(1-.5)}{345}}\right] = [.446, .554]$$

compute $\hat{p} = 169/345 = .48 \in$ Retainment Region $\Rightarrow$ Retain H_0.

Why do we need this?

testing if the coin is fair.

$H_0: p = \overbrace{0.5}$ $\rightarrow$ proportion of heads

Situation 1: $n = 100$, # head $= 51$. Fair? Yes

Situation 2: $n = 100$, # head $= 98$. Fair? No.

Situation 3. $n = 100$, # head $= 61$. Fair? at $\alpha = 5\%$

Retainment Region situation #3.

$$\left[p_0 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p_0(1-p_0)}{n}}\right] = \left[0.5 \pm 2\sqrt{\frac{0.5(0.5)}{100}}\right] = [.40 + .60]$$

61% ∉ Retainment Region so Reject H_0. (the coin is not fair).

M & M factory says 20%, are blue.

Let's test this. $\alpha = 5\%$.

$H_0 = \cancel{p} \; p = .2$

$H_a = p \neq 0.2$

$n = 270$ blues =

Retainment Region

$p_0 \pm 2\sqrt{\frac{p_0(1-p_0)}{n}} = [0.2 \pm 2\sqrt{\frac{0.2 \cdot 0.8}{27}}]$

$= [0.15, 0.25]$

blues

$\hat{p} = \frac{50}{271} = .214$

$\hat{p} \in$ Retainment Region $\Rightarrow$ Retain H_0.

Truth	Decision: Retain H_0	Reject H_0
True	✓	X type I error
False	X type II error	✓

$P(\text{type I error})$

$= P(\text{Rejecting } H_0 \mid H_0 \text{ true})$

$= \alpha$

Power (Advanced class)

$1 - p(\text{type II error}) = p(\text{Rejecting } H_0 \mid H_0 \text{ false})$

$P(\text{type II error}) =$

$P(\text{Retain } H_0 \mid H_0 \text{ false}) =$ advance class

Decision theory