(11/9)

- $X \sim N(0,1)$ ind. of $X \sim N(0,1)$

$$R = \frac{X_1}{X_2} = \int_{supp[x]} |x| \, f_{x_1}(xr) \, f_{x_2}(x) \, dx$$

$$= \int_{\mathbb{R}} |x| \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2 r^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

$$= \frac{1}{2\pi} \int_{\mathbb{R}} |x| e^{-\frac{1}{2}x^2(r^2+1)} dx$$

$$= \frac{1}{2\pi}\left(\int_{-\infty}^{0} (-x) e^{-\frac{1}{2}x^2(r^2+1)} dx + \int_0^\infty x e^{-\frac{1}{2}x^2(r^2+1)} dx \right)$$

$$= \frac{1}{\pi} \int_0^\infty x e^{-\frac{1}{2}x^2(r^2+1)} dx$$

$$= \frac{1}{\pi} \int_0^{-\infty} x e^{u} - \frac{1}{x^2(r^2+1)} du$$

$x=0 \Rightarrow u=0$

$x=\infty \Rightarrow u=-\infty$

let $u = -\frac{1}{2}x^2(r^2+1)$ $\quad \frac{du}{dx} = -x(r^2+1)$ $\quad dx = -\frac{1}{x(r^2+1)} du$

$$= \frac{1}{\pi(r^2+1)} \left[-e^u\right]_0^{-\infty} = \frac{1}{\pi(r^2+1)} = \text{Cauchy}(0,1)$$

midterm #2

- $X_1 \dots X_n \overset{iid}{\sim} f(u, \sigma^2)$

unknown

$\bar{X}$ is the average r.v

$E(\bar{X}) = u$

$S^2 = \frac{1}{n-1} \Sigma (X_i - 2)^2$

$E(\bar{X}) = u$

unbiased

$E(S^2) = \sigma^2$

- $X_1 \dots X_n \overset{iid}{\sim} N(u, \sigma^2)$

$\bar{X} \sim N(u, \frac{\sigma^2}{n})$ $\qquad S^2 = \frac{1}{n-1}\left((X_1-\bar{X})^2 + \dots + (X_n - \bar{X})^2\right)$

$S^2 \sim ?$ $\qquad \vec{Z} = \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}$ $\quad \sum_{i=1}^{n} Z_i^2 = \vec{Z}^T \vec{Z} \sim \chi^2_n$

- $\sum Z_i^2 = \sum \left(\frac{X-u}{6} \right)^2 = \frac{\sum (X_i - u)^2}{6^2} \sim X_n^2$

$\sum (X_i u)^2 = \sum X_i - \bar{X} + \bar{X} - u)^2$

$= \sum (X_i - \bar{X})^2 + 2(X - \bar{X})(\bar{X} - u) + (\bar{X} - u)^2$

$= \sum (X_i - \bar{X})^2 + 2(\sum X \bar{X} - u \sum X_i - \sum \bar{X}^2 + \sum \bar{X} u$

$+ \cancel{n\bar{X}^2} - \cancel{u}\bar{X} - \cancel{n\bar{X}^2} + \cancel{n\bar{X}u}) + n(\bar{X} + u)^2$

$= \frac{\sum (X_i - \bar{X})^2}{6^2} + \frac{n(\bar{X} - u)^2}{6^2}$

$\hookrightarrow$ Con; $\frac{(n-1)S^2}{6^2} \sim X_{n-1}^2$

- Caehn's Thm:

$Z_1 \cdots Z_n \overset{iid}{\sim} N(0,1)$

Let $Q_1 \cdots Q_K$ be scalar r.v's (Quadratic form)

$Q = \vec{z}^T B \vec{z}$ where $B_1 \cdots B_K$ are positive semi-definit

(a) $n = \sum \text{rank}(B_j)$ $\Updownarrow$

(b) Q_j's are independent $\Updownarrow$

(c) $Q_j \sim X^2 \text{ rank}(B_j)$

Def: If $\vec{v}^T A \vec{v} \geq 0$

s.t. $Z^T Z = Q_1 + \cdots + Q_K$

- $\sum Z_i^2 = \sum (Z_i - \bar{Z}) + (\bar{Z}))^2$

$\sum (Z_i - \bar{Z})^2 + \underbrace{2 \sum (Z_i - \bar{Z}) \bar{Z}} + \sum \bar{Z}^2$

- $\sum Z_i^2 = \underbrace{\sum (Z_i - \bar{Z})^2}_{Q_1} + \underbrace{\sum \bar{Z}^2}_{Q_2}$