9/5/2017

$X_1, X_2 \overset{iid}{\sim} \text{Bern}(p)$

$T = X_1 + X_2 \sim P_{X_1}(x) * P_{X_2}^{x} = \sum_{x \in \{0,1\}} P_{X_1}(x) P_{X_2}(t-x)$

$P(2) = \underbrace{P_{X_1}(0)}_{P(X_1=0)} \underbrace{\underbrace{P_{X_2}(2)}_{P(X_2=2)}}_{\text{illegal}} + \underbrace{P_{X_1}(1)}_{P(X_1=1)} \underbrace{P_{X_2}(1)}_{P(X_2=1)}$ without indicator function

$2^n = ? \quad \sum_{i=0}^{n} \binom{n}{i}$ ~~set A s.t.~~ $|A| = n$

$\Rightarrow A = \{w_1, w_2, \cdots, w_n\}.$

$2^A = \{B : B \subseteq A\}$

$= \{B : B \subseteq A \;\&\; |B| = 0\}$

$\cup \{B : B \subseteq A \;\&\; |B| = 1\}$

$\cup \cdots \cdots$

$\cup \{B : B \subseteq A \;\&\; |B| = n\}.$

$2^A = \bigcup_{i=0}^{n} \{B : B \subseteq A \;\&\; |B| = i\}$

$2^{|A|} = |2^A| \quad \sum_{i=0}^{n} |\{B : B \subseteq A \;\&\; |B| = i\}| = \sum \binom{n}{i}$

Recall

$E(X) = \sum_{x \in Spp(X)} x\, p^{(x)}$ for discrete X

$E[g(x)] = \sum_{x \in Spp(X)} g(x)\, p(x)$ sometimes expectations doesn't exist.

let $Y = g(x) = h(x)$ s.t $X \sim Poisson(\lambda)$

$E(Y) = \sum_{x \in \{0,1,\dots\}} h(x) \frac{e^{-\lambda} \lambda^{x}}{x!} = \underbrace{h(0)(1)}_{DNE} + \sum h(x) \frac{e^{-\lambda}\lambda^{x}}{x!}$

let $Z = 1_A \sim Bern(P(A))$

$E(Z) = E(1_A) = P(A)$

$X, Y \overset{iid}{\sim} Geom(P) = (1-P)^x P$

$F_X(x) = P(X \leq x) = 1 - P(X > x)$

$= 1 - P(X \geq x+1)$

$1 - F_X(x) = P(X > x) = (1-P)^{x+1}$

$P(X > Y) = E(Z) = \sum_{x \in Spp(X)} \sum_{y \in Spp(Y)} 1_{x>y} P_{X,Y}(x,y)$

$Z = 1_{X>Y}$

$E(Z) = E(g(X,Y)) = \sum_{y \in Spp(Y)} \sum_{x \in Spp(X)} g(x,y) \underbrace{P_{X,Y}(x,y)}_{J.M.F}$

$= \sum_{x \in \{0,1,\dots\}} \sum_{y \in \{0,1,2,\dots\}} 1_{x>y} P(1-P)^x P(1-P)^y$

$P_{X,Y}(x,y) = P_X^{(x)} P_Y^{(y)}$ do to independent

$$= p^2 \sum_{y\in\{0,1,\dots\}} (1-p)^y \sum_{x\in\{0,1,\dots\}} (1-p)^x \, 1_{x>y}$$

$$= p^2 \sum_{y\in\{0,1,\dots\}} (1-p)^y \sum_{x=y+1}^{\infty} (1-p)^x$$

~~let $x' = x-(y+1) = x-y-1$~~

$$\Rightarrow x = x' + y + 1$$

$$= p^2 \sum_{y\in\{0,1,\dots\}} (1-p)^y \sum_{x'=0}^{\infty} (1-p)^{x'+y+1}$$

Recall if $r \in [0,1) \Rightarrow \sum_{x=0}^{\infty} r^x = \frac{1}{1-r}$ the geometric series

$$= \overset{p}{\cancel{p^2}} \sum_{y\in\{0,1,\dots\}} (1-p)^{2y+1} \frac{1}{\underbrace{1-(1-p)}_{\frac{1}{p}}} = p(1-p)\sum_{y\in\{0,\dots\}} (1-p)^{2y}$$

$$= p(1-p) \sum_{y=0}^{\infty} \left((1-p)^2\right)^y = \underbrace{p(1-p)\frac{1}{1-(1-p)^2}}$$

$$= \cancel{p}(1-p)\frac{1}{\underbrace{\cancel{1}-\cancel{1}+2p-p^2}_{p(2-p)}}$$

$$= \frac{1-p}{2-p}.$$

Or

$$= \sum_{x\in\{0,1\cdots\}} \sum_{y\in\{0,1\cdots\}} 1_{x>y}\, p(1-p)^x p(1-p)^y$$

$$= \sum_{y\in\{0,1\cdots\}} p(1-p)^y \sum_{x\in\{0,1\cdots\}} p(1-p)^x 1_{x>y}$$

$$= \sum p(1-p)^y \underbrace{\sum_{x=y+1}^{\infty} p(1-p)^x}$$

$$= \sum_{y\in\{0,1\cdots\}} P(\underbrace{Y=y})\, P(X \geq y+1)$$

$$P(X=Y) = E(Z) = \sum_{x\in supp(X)} \sum_{y \in supp(Y)} 1_{x=y} P_{X,Y}(x,y)$$

$$Z = 1_{X=Y} \qquad = \sum_{y\in\{0,1\cdots\}} \sum p(1-p)^x p(1-p)^y 1_{x=y}$$

$$= p^2 \sum_{y\in\{0,1,\cdots\}} (1-p)^y \underbrace{\sum_{x\in\{0,1\cdots\}} (1-p)^x 1_{x=y}}_{(1-p)^y}$$

$$= p^2 \sum_{y\in\{0,1\cdots\}} \left((1-p)^2\right)^y = \frac{p^2}{1-(1-p)^2} = \frac{p}{2-p}$$

$$1 = P(X>Y) + P(X<Y) + P(X=Y)$$

$$\frac{1-p}{2-p} + \frac{1-p}{2-p} + \frac{p}{2-p}$$

$$= 1 \checkmark$$

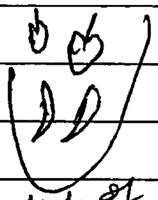
$X, Y \overset{iid}{\sim} \text{Bin}(n, p)$

$$P(X > Y) = \sum_{y \in \{0,1,\dots,n\}} P(\cancel{X} = y)\underbrace{(1 - F_X(y))}_{}$$

no closed form

$$P(X = Y) = \sum_{y \in \{0,1,..\}} \sum_{x \in \{0,1,..\}} 1_{x=y} \underbrace{\binom{n}{x} p^x (1-p)^{n-x}}_{P_X(x)} \underbrace{\binom{n}{y} p^y (1-p)^{n-y}}_{P_Y(y)}$$

$$= \sum_{y=0}^{n} \binom{n}{y}^2 p^{2y} (1-p)^{2(n-y)}$$



Basket of n Apple & Banana & Cantalopes

$P_1 + P_2 + P_3 = 1$

P_3 = prop of Cantolopes

P_1 = prop. of apples

P_2 = Prop of banana

X_1 be the # of apples drawn with replacem n times

$X_1 \sim \text{Bin}(n, P_1)$

X_2 = # bananas drawn

$X_2 = n - X_1$

X_3 = # of cantolopes drawn

let $\bar{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$

$$P(\bar{X} = \bar{x}) = P\underbrace{(X_1 = x_1, X_2 = x_2, X_3 = x_3)}_{\text{J.M.F}}$$

$$P(X_1 = x_1, X_2 = x_2) = \frac{n!}{x_1!} p^{x_1}(1-p)^{x_2} \Big|_{x_1+x_2=n}.$$

$$P(X_1 = x_1) = \frac{n!}{x_1!(n-x_1)!} p^{x_1}(1-p)^{n-x_1} \Big|_{x\in\{0,\dots\}} = \binom{n}{x_1} p^{x_1}(1-p)^{n-x_1}$$

$$P(\bar{X} = \bar{x}) = \underbrace{\frac{n!}{x_1!\,x_2!\,x_3!}}_{\binom{n}{x_1,\,x_2,\,x_3}} p_1^{x_1} p_2^{x_2} p_3^{x_3} \Big|_{x_1+x_2+x_3=n}.$$

"multichoose"

$$\bar{X} \sim \text{multinom}(n, \vec{p}) = P(\bar{x})$$

of dim k

$$= \binom{n}{x_1, x_2, \dots, x_k} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$$

$$\text{Supp}(\bar{X}) = \{\bar{x} : \bar{1}\cdot\bar{x} = n,\ \bar{x} \in \mathbb{N}_0^k\}.$$

Param Space

$$n \in \mathbb{N},\ \bar{p} \in \{\bar{e} : \bar{e}\cdot\bar{1} = 1,\ \bar{e} \in (0,1)^k\}.$$