rv ≡ random variable

PMF ≡ Prob. Mass Functn $P(x) = P(X=x)$

CDF ≡ Cumulative Distribut. Funct. $F(x) = P(X \leq x)$

r.v. has Support (x) = $Supp(X) \subseteq \mathbb{N}$

PDF = Prob. Density Funct. $\sum_{x \in Supp(X)} P(x) = 1$

$$X \sim Bern(p) = \begin{cases} 1 \rightarrow p \\ 0 \rightarrow 1-p \end{cases}$$ p ≡ parameter

$p \in (0,1)$ $p \neq 0$, $p \neq 1$

Indicator function $$\mathbb{1}_x = \begin{cases} 1 & \text{if } A \\ 0 & \text{if } A^c \end{cases}$$

$$X \sim Bern(p) = p^x (1-p)^{1-x}$$

Review

08/29/2017

A random variable x

$p(x) = P(X = x)$

$F(x) = P(X \leq x)$

$\sum p(x) = 1$

The most fundamental Bernoulli real variable

Vs $\begin{cases} 1 & \text{wp } p \\ 0 & \text{wp } 1-p \end{cases}$ p = parameter $p \in (0, 1)$

$p \neq 0$ $p \neq 1$

$X \sim Deg(c)$ Degenerate

$X \sim Deg(c) = \mathbb{1}_{x=c}$ indicator funct.

$\mathbb{1}_A = \begin{cases} 1 \text{ if } A \\ 0 \text{ if } A^c \end{cases}$

distribut

X_1 X_2 are independent

$P(X_1, X_2) = P_{X_1}(x_1) \cdot P_{X_2}(x_2)$

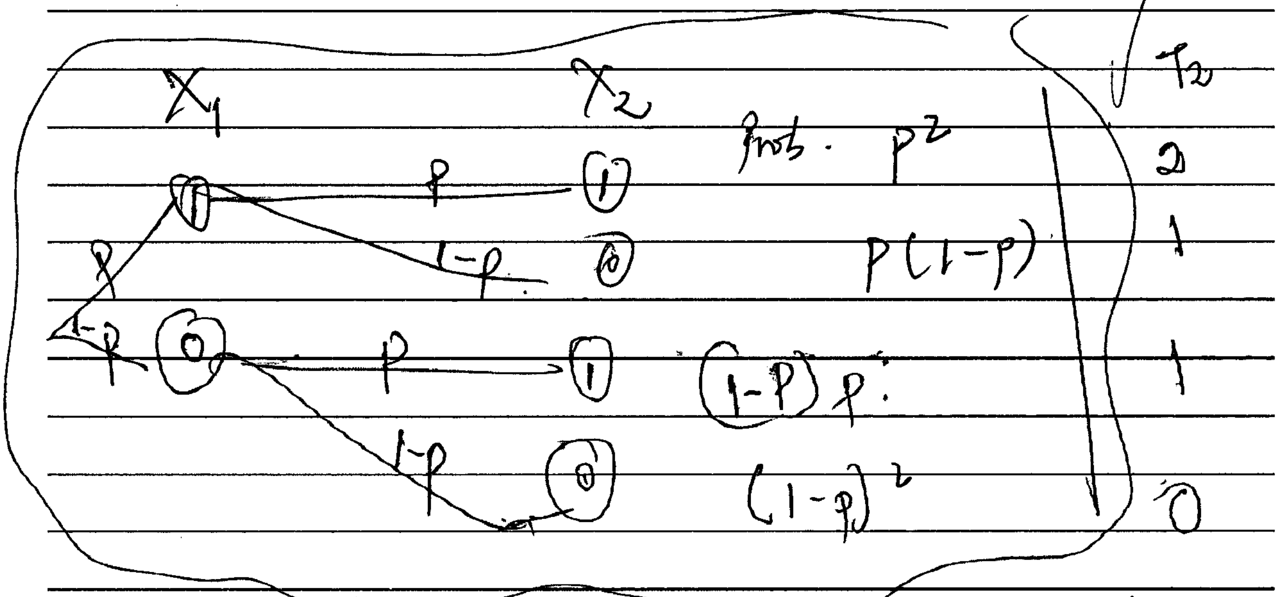
Joint mass funct

X_1 X_2 are independent

$T_2 = X_1 + X_2$

$Supp(T_2) = \{0, 1, 2\}$

$A + B = \{a+b, a \in A, b \in B\}$

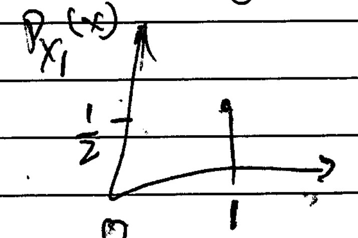


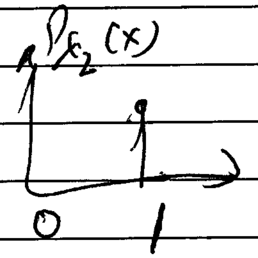
$$T_2 = \begin{cases} 2 \to p^2 \\ 1 \to 2p(1-p) \\ 0 \to (1-p)^2 \end{cases}$$

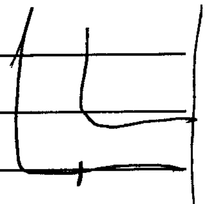
$p_T(2) = p^2 \qquad p_T(0) = (1-p)^2$

$p_T(1) = 2p(1-p)$

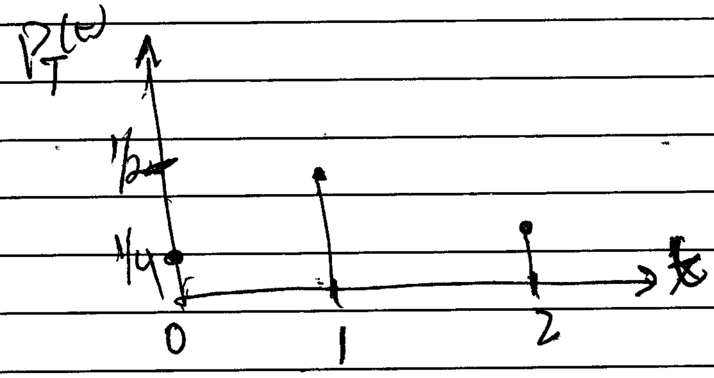
Imagine $p = \frac{1}{2}$

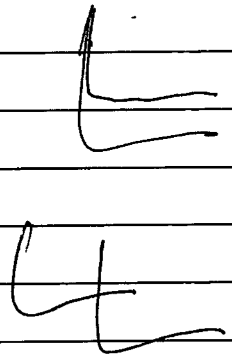






convolve





$$P(T_2 = t) = \sum_{x \in supp(X)} p_X(x)\, p_{X_2}(t-x) \qquad T_2 = X_1 + X_2$$

$$\sum_{x \in \{0,1\}} \left(p^x (1-p)^{1-x}\right)\left(p^{t-x} (1-p)^{1-t+x}\right)$$

$$= p^x \sum_{x \in \{0,1\}} (1-p)^{2-t}$$

$$p(t) = P(T_2 = t) = \sum_{x \in supp(x)} P_{X_1}(x) P_{X_2}(t-x)$$

$$= \sum_{x \in supp\{0,1\}} p^x (1-p)^{1-x} \mathbb{1}_{x \in \{0,1\}} \; p^{t-x} (1-p)^{1-t+x}$$

$$p^t (1-p)^{2-t} \sum \mathbb{1}_{x \in \{0,1\}} \mathbb{1}_{t-x \in \{0,1\}}$$

$$\underbrace{\mathbb{1}_{0 \in \{0,1\}} \mathbb{1}_{t-0 \in \{0,1\}} + \mathbb{1}_{1 \in \{0,1\}} \mathbb{1}_{t-1 \in \{0,1\}}}$$

$$\mathbb{1}_{t \in \{0,1\}} + \mathbb{1}_{t-1 \in \{0,1\}}$$

$$p(0) = (1-p)^2 \qquad p(2) = p^2 \qquad p(1) = 2p($$

$$X \sim Bern(p) = Bin(1,p) = \binom{1}{x} p^x (1-p)^{1-x}$$

$$= p^x (1-p)^x \mathbb{1}_{x \in \{0,1}$$

$$T_2 = \binom{2}{t} p^t (1-p)^{2-t}$$

Indicator, support, PMF

$$\sum_{x\in\{0,1\}} \binom{1}{x}\binom{1}{t-x} = \binom{1}{0}\binom{1}{t} + \binom{1}{1}\binom{1}{t-1} = \binom{2}{t}$$

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

$$p(t) = P(T_2 = t) = P_{X_1}(x) * P_{X_2}(x) = \sum_{x\in supp(x)} P_{X_1}(t) P_{Y_2}(t-x)$$

$$T_3 = X_1 + X_2 + X_3 = X_3 + T_2$$

$$= P_{X_3}(x) * P_{T_2}(x)$$

$$= \sum_{x\in supp(X_3)} P_{X_3}(x) P_{T_2}(t-x)$$

$$T_3 = \binom{3}{t} p^t (1-p)^{3-t}$$