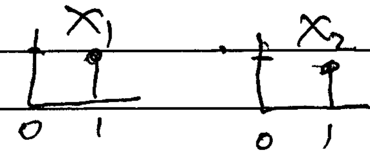
$$= p^t(1-p)^{3-t}\left(\binom{2}{t}+\binom{2}{t-1}\right)$$

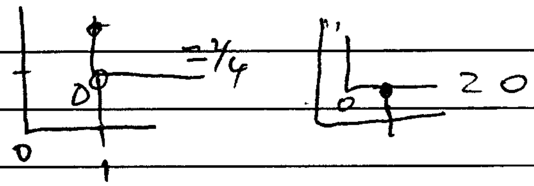
$$= \binom{3}{t} p^t (1-p)^{3-t}$$

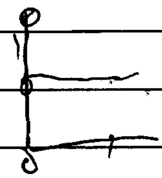
8/31/17

$X_1, X_2 \overset{iid}{\sim} Bern(\frac{1}{2})$



$T_2 = X_1 + X_2$





$$T = X_1 + X_2 \sim p_{X_1}(x) * p_{X_2}(x) = \sum_{x \in supp(X_1)} p_{X_1}(x)\, p_{X_2}(t-x)$$

$$= \sum_{x \in \{0,1\}} p^x (1-p)^{1-x} p^{t-x} (1-p)^{1-t+x}$$

$$= \sum_{x \in \{0,1\}} p^t (1-p)^{2-t} = p^t (1-p)^{2-t} \sum_2 1 = 2p^t(1-p)^{2-t}$$

$X_1, X_2 \overset{iid}{\sim} Bin(n, p)$

$$Y = X_1 + X_2 \sim p_{X_1}(x) * p_{X_2}(x) = \sum_{x \in supp(X_1)} p_{X_1}(x)\, p_{X_2}(y-x)$$

$$= \sum_{\substack{x=0 \\ x \in \{0,\dots,n\}}}^{n} \binom{n}{x} p^x (1-p)^{n-x} \underbrace{1_{x \in \{0,\dots,n\}}}_{\text{not needed}} \binom{n}{y-x} p^{y-x} (1-p)^{n-y+x} \underbrace{1_{y-x \in \{0,\dots,n\}}}_{\text{not needed}}$$

becse its 1

$$= \sum_{x\in\{0,1,\dots,y\}} \binom{n}{x} p^x (1-p)^{y-x} \binom{n}{y-x} p^{y-x}(1-p)^{n-y+x}$$

$$= p^y(1-p)^{2n-y} \underbrace{\sum_{x\in\{0\dots n\}} \binom{n}{x}\binom{n}{y-x}}_{\text{Vandermonde's Identity}}$$

$$= \binom{2n}{y} p^y (1-p)^{2n-y} = \text{Binom}(2n, p).$$

Consider $B_1, B_2, \dots \overset{iid}{\sim} \text{Bern}(p)$.

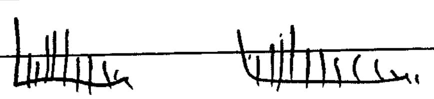
Let $X = \min_t \{B_t = 1\} - 1$

$X \sim \text{Geometric}(p) = (1-p)^x p$

$P(X=0) = p$ $\qquad$ $P(X=2) = (1-p)^2 p$

$P(X=1) = (1-p)p$ $\qquad$ $P(X=x) = (1-p)^x p$

$\text{Supp}[X] = \{0,1,2,\dots\} = \mathbb{N}_0$.



$$T_2 = X_1 + X_2 \sim p^{(t)} = p_{X_1}^{(x)} \cdot p_{X_2}^{(x)} = \sum_{x\in \text{Supp}(X_1)} p_{X_1}^{(x)}\, p_{X_2}^{(t-x)}$$

$$= \sum_{x\in\{0,1,\dots\}} (1-p)^x p\, (1-p)^{t-x} p\, \mathbb{1}_{t-x\in\{0,1,\dots\}}$$

$$= (1-p)^t p^2 \sum_{x\in\{0,1,\dots\}} \mathbb{1}_{x\le t} = (1-p)^t p^2 \underbrace{\sum_{x=0}^{t} 1}_{t+1}$$

$$= (t+1)(1-p)^t p^2 .$$

$$T_3 = X_1 + X_2 + X_3 = X_3 + T_2 = p_{X_3}^{x} \cdot p_{T_2}(x)$$

$$= \sum_{x \in supp(X_3)} p_{X_3}(x)\, p_{T_2}(t-x)$$

$$= \sum_{x \in \{0,\dots\}} (1-p)^x p\,(t-x+1)(1-p)^{t-x} p^2 \, 1_{t-x \in \{0,1,2,\dots\}}$$

$$= (1-p)^t p^3 \sum_{x \in \{0,1,\dots\}} (t-x+1) \, 1_{x \le t}$$

$$= (1-p)^t p^3 \Big((t+1) \underbrace{\sum_{x \in \{0,1,\dots\}} 1_{x \le t}}_{t+1} - \underbrace{\sum_{x=0}^{t} x}_{\frac{t(t+1)}{2}} \Big)$$

$$= (1-p)^t p^3 \left(\frac{t^2+3t+2}{2} \right).$$

T_3 # of failures until 3 successes

0 1 0 0 1 0 0 0 0 ... $\underset{t+3}{1}$

$$\binom{t+2}{2} = \frac{(t+3)!}{2!\,t!} = \frac{(t+2)(t+1)}{2} = \frac{t^2+3t+2}{2}$$

$T_2 \sim NegBin(2, p)$ "Negative Binomial"

$$P(T_3 = t) = \binom{t+2}{2}(1-p)^t p^3$$

$$X \sim NegBin(k, p) = \binom{x+k-1}{k-1}(1-p)^x p^k$$

$X \sim Bin(n, p)$, $Supp(X) = \{0, 1, \ldots, n\}$.

What if n is really big?
What if p is really small?
But they're related $\lambda = np \Rightarrow p = \frac{\lambda}{n}$

PMF of $n \to \infty$?

$$\lim_{n\to\infty} p(x) = \lim_{n\to\infty} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \lim_{n\to\infty} \binom{n}{x} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \lim \frac{n!}{x!(n-x)!} \frac{\lambda^x}{n^x} \left(1 - \frac{\lambda}{n}\right)^n$$

$$= \lim \frac{n!}{x!(n-x)!} \frac{\lambda^x}{n^x} \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-x}$$

$$= \frac{\lambda^x}{x!} \lim \frac{n!}{(n-x)!\, n^x} \underbrace{\lim \left(1 - \frac{\lambda}{n}\right)^n}_{e^{-\lambda}} \underbrace{\lim \left(1 - \frac{\lambda}{n}\right)^{-x}}_{1} = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\lim \underbrace{\frac{\overbrace{(n)(n-1)\cdots(n-x+1)}^{x}}{(n)(n)(n)}}_{1} \quad x$$

$$X \sim poisson(\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$Supp(X) = \{0, 1, \ldots\}$

Par space $\lambda \in (0, \infty)$

$$X_1, X_2 \overset{iid}{\sim} \text{Poisson}(\lambda)$$

$$T = X_1 + X_2 \sim P_{X_1}^{(x)} \cdot P_{X_2}^{(x)} = \sum_{x \in \text{Supp}\{X_1\}} P_{X_1}^{(x)} P_{X_2}^{(t-x)}$$

$$= \sum_{x \in \{0,\dots\}} \frac{\lambda^x e^{-\lambda}}{x!} \frac{\lambda^{t-x} e^{-\lambda}}{(t-x)!} \underbrace{1_{t-x \in \{0,1,\dots\}}}_{1_{x \le t}}$$

$$= \lambda^t e^{-2\lambda} \sum \frac{1}{x!(t-x)!} 1_{x \le t} \cdot \frac{t!}{t!}$$

$$= \frac{\lambda^t e^{-2\lambda}}{t!} \sum_{x \in \{0,1,2\dots\}} \binom{t}{x} 1_{x \le t}$$

$$= \frac{\lambda^t e^{-2\lambda}}{t!} \sum_{x=0}^{t} \binom{t}{x} = \frac{(2\lambda)^t e^{-2\lambda}}{t!} = \text{Poisson}(2\lambda)$$