Tue Sept 05, 2017

$X_1\ X_2$ iid Bern(p)

$T = X_1 + X_2 \Rightarrow P_{X_1}(x)\, P_{X_2}(x) = \sum_{x \in \{0,1\}} P_{X_1}(x)\, P_{X_2}(t-x)$

$P(2) = P_{X_1}(0) P_{X_2}(2) + P_{X_1}(1) P_{X_2}(1)$

$2^n = \sum_{i=0} \binom{n}{i}$ set A such that $|A| = n \Rightarrow A = \{w_1, w_2, \dots w_n\}$

$= \{B : B \subseteq A\ \&\ |B| = 0\}$

$\cup \{B : B \subseteq A\ \&\ |B| = 1\}$

$\cup \{B : B \subseteq A\ \&\ |B| = n\}$

$2^A = \bigcup_{i=0}^{n} \{B : B \subseteq A\ \&\ |B| = i\}$

$\sum_{i=0}^{n} |\{B : B \subseteq A\ \&\ |B| = i\}|$

$$E(X) = \sum_{x \in \mathrm{supp}(X)} x\, p(x)$$ prisson dsterefa X

$$E(g(X)) = \sum g(x)\, p(x)$$

let $Y = g(X) = \mathbb{1}_{\ldots}(X)$ such that $X \sim$ Poisson

$$E(Y) = \sum_{x \in \{0,1\}} \mathbb{1}(x) \frac{e^{-\lambda}\lambda^x}{x!} - \mathbb{1}(0)(1)$$

$$= \sum \mathbb{1}(y) \frac{e^{-\lambda}\ldots}{x!}$$

Let $Z = \mathbb{1}_A \sim \mathrm{Bern}(p(A))$

$$E(Z) = E(\mathbb{1}_A) = P(A)$$

$X, Y \overset{iid}{\sim} \mathrm{Geom}(p) \simeq (1-p)^x p$

Geom = # of time you fail before succeed

$$F_X(x) = P(X \le x) = 1 - P(X \geq x)$$

$$= 1 - P(X \geq x+1)$$

$$= 1 - (1-p)^{x+1}$$

$$1 - F_X(x) = P(X > x) = (1-p)^{x+1}$$

$P(X > Y)$ let $Z = \mathbb{1}_{X>Y}$

$$P(X>Y) = E(Z)$$

$$E(Z) = E(g(X,Y)) = \sum_{\text{supp}(Y)} \sum_{\text{supp}(X)} g(x,y) \underbrace{P_{X,Y}(x,y)}_{\text{JMF}}$$

$$\sum_{x\in\{0,1,\dots\}} \sum_{y\in\{0,1,\dots\}} \mathbb{1}_{x>y} \cdot p(1-p)^x p(1-p)^y$$

$$= p^2 \sum_{y\in\{0,1,\dots\}} (1-p)^y \sum_{x\in\{0,1,\dots\}} (1-p)^x \mathbb{1}_{x>y}$$

$$= p^2 \sum_{y\in\{0,1,\dots\}} (1-p)^y \sum_{x=y+1}^{\infty} (1-p)^x$$

Let $x' = x-(y+1) = x-y-1$

$$\Rightarrow x = x' + y + 1$$

$$p^2 \sum_{x\in\{0,1,\dots\}} (1-p)^y \sum_{x'=0} (1-p)^{x'+y+1}$$

Recall if $r\in(0,1) \Rightarrow \sum_{x=0}^{\infty} r^x = \frac{1}{1-r}$

$$= p^2 \sum_{y\in\{0,1,\dots\}} (1-p)^{2y+1} \frac{1}{1-(1-p)}$$

$$= p(1-p)\sum (1-p)^{2y} = p(1-p)\sum_{y=0}^{\infty}\left[(1-p)^2\right]^y$$

$$= p(1-p)\frac{1}{1-(1-p)^2} = \frac{1-p}{2-p}$$

$$\lim_{p\to 0} \frac{1-p}{2-p} = \frac{1}{2}$$

$$P(X=Y) = E(Z) = \sum_{x\in supp(X)} \sum_{y\in supp(Y)} \mathbb{1}_{x=y}\, P_{X,Y}(x,y)$$

$$Z = \mathbb{1}_{X=Y} = \sum_{y\in\{0,1,\dots\}} \sum_{x\in\{0,1,\dots\}} p(1-p)^x p(1-p)^y \mathbb{1}$$

$$= p^2 \sum_{y \in \{0,1,\dots\}} (1-p)^y \sum (1-p)^x \mathbb{1}_{x=y}$$

$$= p^2 \sum_{y \in \{0,1,\dots\}} (1-p)^y \sum_{x \in \{0,1,\dots\}} (1-p)^x$$

$$= p^2 \sum [(1-p)^2]^y = \frac{p^2}{1-(1-p)^2} = \frac{p}{2-p}$$

$$1 = P(X > Y) + P(X < Y) + P(X = Y)$$

$$1 = \frac{1-p}{2-p} + \frac{1-p}{2-p} + \frac{p}{2-p}$$

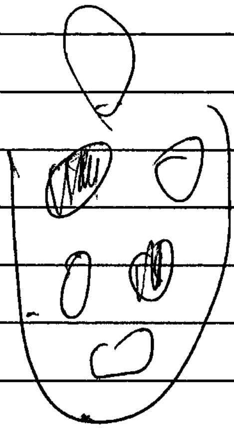
X, Y iid $Bin(n, p)$

$$P(X > Y) = \sum_{y \in \{0,1,\dots\}} P(X=y)(1 - F(y))$$

$$P(X = Y) = \sum_{y \in \{0,1,\dots\}} \sum_{x \in \{0,1,\dots\}} \mathbb{1}_{x=y} \underbrace{\binom{n}{x} p^x (1-p)^{n-x}}_{P_X(x)} \underbrace{\binom{n}{y} p^y (1-p)^{n-y}}_{P_Y(y)}$$

$$= \sum_{y=0}^{n} \binom{n}{x}^2 p^{2y} (1-p)^{2(n-y)}$$

$$P(X > Y) = \frac{1 - P(X=Y)}{2}$$



P_1 = prob. of apples drawn

P_2 = prob. of bananas drawn

P_3 = Prob. of cantaloupes drawn

$P_1 + P_2 + P_3 = 1$ let $\vec{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$P(\vec{X} = \vec{x}) = P(X_1 = x_1, X_2 = x_2, X_3 = x_3)$$

$$P(X_1 = x_1, X_2 = x_2, X_3 = x_3) = \frac{n!}{x_1! \cdot x_2!} p^{x_1} (1-p)^{x_2}$$

$$P(X_1 = x_1) = \binom{n}{x_1} p^{x_1} (1-p)^{n-x_1} \mathbb{1}_{x \in \{0,1,\dots\}}$$

$$P(\vec{X} = \vec{x}) = \frac{n!}{x_1! \cdot x_2! \cdot x_3!} p_1^{x_1} p_2^{x_2} p_3^{x_3} \mathbb{1}_{x_1+x_2+x_3=n}$$

Multichoose $= \binom{n}{x_1, x_2, x_3}$

$\vec{x} \sim \text{Multinorm}(n, \vec{p}) = p(\vec{x})$

$= \binom{n}{x_1, x_2, \dots, x_k} p_1^{x_1} p_2^{x_2} p_3^{x_3}$

$= \text{Supp}(\vec{x}) = \{ \vec{x} = n \quad \vec{x} \in \mathbb{N}_0^k \}$

$\vec{x} = \sum_{i=1}^{k} x_i = \{, x_1, x_2, \dots x_k \in \mathbb{N}_0 \}$

$n \in \mathbb{N} \quad p \in \{ \vec{p} : \vec{p} \cdot \vec{1} = 1 \}$

$\vec{e} \in (0, 1)^k \{$