Let $\vec{X}$ be a vector of random variables such that $\dim[X] = k$.

$$\vec{\mu} = \mathrm{E}[\vec{X}] = \begin{bmatrix} \mathrm{E}[x_1] \\ \dots \\ \mathrm{E}[x_n] \end{bmatrix}$$

$$\varepsilon = \mathrm{Var}[\vec{X}] = \begin{bmatrix} \mathrm{Var}[x_1] & \mathrm{Cov}[x_1, x_2] & \dots & \dots \\ \mathrm{Cov}[x_2, x_1] & \mathrm{Var}[x_2] & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots \\ \dots & \dots & \dots & \mathrm{Var}[x_k] \end{bmatrix}$$

$$= \Big\{ \mathrm{Cov}[x_i, x_i] \text{ for } i = 1, \dots, k,\ j = 1, \dots, k \Big\}$$

$$\varepsilon_0 = \mathrm{Corr}[\vec{X}] = \begin{bmatrix} 1 & & \mathrm{Corr}[x_i, x_j] \\ & 1 & \\ \mathrm{Corr}[x_i, x_j] & & 1 \end{bmatrix}$$

$$= \Big\{ \mathrm{Corr}[x_i, x_j] \text{ for } i = 1, \dots, k,\ j = 1, \dots, k \Big\}$$

Let $T = X_1 + \cdots + X_k = T^T \vec{X} = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \dots \\ x_k \end{bmatrix}$.

$$\mathrm{E}[T] = \sum_{i=1}^{k} \mu_i = T^T \vec{\mu}$$

$$\mathrm{Var}[T] = \mathrm{Var}[T^T \vec{X}] = \sum_{j=1}^{k} \sum_{i=1}^{k} \mathrm{Cov}[X_i, X_j]$$

Let $Y = \vec{c}^T \vec{X}$. Then $\mathrm{E}[Y] = \sum c_i \mu_i = \vec{c}^T \vec{\mu}$. What's $\mathrm{Var}[Y] = \mathrm{Var}[\vec{c}^T \vec{X}]$?
If $A \in \mathbb{R}^{n \times n}$ and $\vec{c} \in \mathbb{R}^n$, what is $\vec{c}^T A \vec{c}$?

$$\vec{c}^T a \vec{c} = \vec{c}^T \begin{bmatrix} c_1 a_{11} + \cdots + c_n a_{1n} \\ c_1 a_{21} + \cdots + c_n a_{2n} \\ \dots \\ c_1 a_{n1} + \cdots + c_n a_{nn} \end{bmatrix}$$

$$= c_1^2 a_{11} + c_1 c_2 a_{12} + \cdots + c_1 c_n a_{1n} +$$
$$c_2 c_1 a_{21} + c_2^2 a_{22} + \cdots + c_2 c_n a_{2n} +$$
$$\dots$$
$$c_n c_1 a_{n1} + c_n c_2 a_{n2} + \cdots + c_n^2 a_{nn}$$
$$= \sum_{j=1}^{n} \sum_{i=1}^{n} c_i c_j a_{ij}$$

Thus what is $\text{Var}[\vec{c}^T\vec{X}]$?

$$\begin{aligned}\text{Var}[\vec{c}^T\vec{X}] - \text{Var}[c_1X_1 + \cdots + c_kX_k] \\ &= \sum_{i=1}^k\sum_{j=1}^k \text{Cov}[c_iX_i, c_jX_j] \\ &= \sum_{i=1}^k\sum_{j=1}^k c_ic_j\text{Cov}[X_i, X_j] \\ &= \vec{c}^T\text{Var}[\vec{X}]\vec{c}\end{aligned}$$

Markovits Optimal Portfolio: Let $X_1, \ldots, X_k$ be random variable models for the returns on k assets. Let $w_1, \ldots, w_k$ be the weights or allocations for each. Note that $T^T\vec{w} = 1$. In addition,

$$\begin{aligned}V &= \vec{w}^T\vec{X} \\ \text{E}[V] &= \vec{w}^T\vec{\mu} \\ \text{Var}[V] &= \vec{w}^T\sum\vec{w}\end{aligned}$$

Given μ_0, minimize $\vec{w}^T\sum\vec{w}$ such that $T^t\vec{w} = 1(\left\{\vec{w} : T^T\vec{w} = 1\right\})$.

If $\vec{X} \sim \text{Multinomial}(n, \vec{p})$,

$$\text{E}[\vec{X}] = \begin{bmatrix}\text{E}[X_1] \\ \ldots \\ \text{E}[X_n]\end{bmatrix} = \begin{bmatrix}np_1 \\ \ldots \\ np_k\end{bmatrix} = n\vec{p}$$

$$\text{Var}[X] = \begin{bmatrix}np_1(1-p_1) & \text{Cov}[X_1, X_2] & \ldots & \ldots \\ & np_2(1-p_2) & \ldots & \\ \vdots & \vdots & \ddots & \vdots \\ \ldots & \ldots & \ldots & np_k(1-p_k)\end{bmatrix}$$

Also

$$\begin{aligned}\text{Cov}[X_i, X_j] &= \text{E}[X_i, X_j] - \mu_i\mu_j \\ &= \sum_{x_i\in\text{Supp}[X_1]}\sum_{x_j\in\text{Supp}[X_2]} x_ix_j \underbrace{\mathbb{P}_{X_iX_j}(X_iX_j)}_{\text{we don't know this yet}} - \mu_i\mu_j\end{aligned}$$

Recall that if $X_1 \sim \text{Binom}(n, p_1), \ldots, X_k \sim \text{Binom}(n, p_k)$, that means that $X_1 = \sum_{i=1}^n X_{i1}$ such that $X_{11}, \ldots, X_{n1} \overset{iid}{\sim} \text{Bern}(p_1)$, all the way through $X_k = \sum_{i=1}^n X_{ik}$ such that $X_{1k}, \ldots, X_{nk} \overset{iid}{\sim} \text{Bern}(p_k)$.
If $\vec{X} \sim \text{Multinomial}(n, \vec{p})$ then $\vec{X} = \sum_{i=1}^n \vec{X}_i$ such that $\vec{X}_1, \vec{X}_2, \ldots, \vec{X}_n \overset{iid}{\sim} \text{Multinomial}(1, \vec{p})$.

Then the covariance of X_i, X_j is as follows

$$\begin{aligned}
\mathrm{Cov}[X_i, X_j] &= \mathrm{Cov}[\sum_{l=1}^{n} X_{li}, \sum_{h=1}^{n} X_{hj}] \\
&= \sum \sum \mathrm{Cov}[X_{li}, X_{hj}] \\
&= \sum_{l=1}^{n} \sum_{h=1}^{n} \mathrm{E}[X_{li}, X_{hj}] - p_i p_j \\
&\text{If } l = h \\
&= \sum_{l=1}^{n} \mathrm{E}[X_{li}, X_{lj}] - p_i p_j \\
&= \sum_{l=1}^{n} -p_i p_j = -n p_i p_j \\
&\text{If } l \neq h \\
&= \mathrm{E}[X_{li}]\mathrm{E}[X_{hj}] \\
&= p_i p_j
\end{aligned}$$

Continuous random variable X have CDF $F(x)$ and PDF $f(x)$ such that

$$f(x) = F'(x)$$

and $\mathrm{Supp}[X] = \Big\{x : f(x) > 0\Big\}$ and $|\mathrm{Supp}| = |\mathbb{R}|$. Note that pmf $P(x) = 0 \forall x$.
Let $X \sim U(a, b) = \frac{1}{b-a}$ where $a, b \in \mathbb{R}$, $b > a$ and $\mathrm{Supp}[x] = [a, b]$.
A standard uniform distribution occurs when $a = 0, b = 1$ forming $X \sum U(0, 1) = 1$. Let $T_2 = X_1 + X_2$ such that $X_1, X_2 \overset{iid}{\sim} U(0, 1)$. Then $\mathrm{Supp}[T_2] = [0, 2]$.
How often does T = 0? That's when $x_1 = 0$, $x_2 = 0$. None. How often does T = 2? That's when $x_1 = 1$, $x_2 = 1$. None. How often does T = 1? That's when $x_1 = 0$ and $x_2 = 1$ or $x_1 = \frac{1}{3}$ and $x_2 = \frac{2}{3}$, and so on.

$$\begin{aligned}
f_T(t) &= \int_{x \in \mathrm{Supp}[X_1]} f_{X_1}(x) f_{X_2}(t - x)\, dx \\
&= \int_0^1 1 \cdot \underbrace{\mathbb{1}_{x \in [0,1]}}_{\text{not needed}} \cdot 1 \cdot \mathbb{1}_{t-x} \in [0, 1]\, dx \\
&= \int_0^1 \mathbb{1}_{x \in [t-1, t]}\, dx \\
&= \int_{\max\{0, t-1\}}^{\min\{1, t\}} dx \\
&= x\Big|_{\max\{0, t-1\}}^{\min\{1, t\}} \\
&= (\min\{1, t\} - \max\{0, t - 1\}) \mathbb{1}_{t \in [0,2]}
\end{aligned}$$

This is the answer for $t \in [0, 2]$. Alternatively,

$$f_{T_2}(t) = \begin{cases} t & \text{if } t < 1 \\ 1 - (t-1) = 2 - t & \text{if } t \geq 1 \end{cases} \mathbb{1}_{t \in [0,2]}$$