~~A~~ discrete random variable (rv) $X$ has prob. mass function (PMF)

$P(x) = P(X = x)$ and cumulative distribution function (CDF)

$F(x) = F(X \leq x)$. The rv $X$ has "support" $\text{Supp}[X] = \{x : P(x) > 0, x \in \mathbb{R}\}$.

Since $X$ is discrete, $|\text{supp}(X)| \leq |\mathbb{N}|$.

Supp & PDF are relative $\sum_{x \in \text{Supp}(X)} P(x) = 1$.

The most fundamental discrete r.v is the Bernoulli

$$X \sim \text{Bern}(p) = \begin{cases} 1 & \text{wp } p \\ 0 & \text{wp } 1-p \end{cases}$$

What is $p$? $p$ is parameter.

Parameters have parameter spaces eg

$p \in (0, 1)$. $p \neq 0$ & $p \neq 1$

$$X \sim \text{Deg}(c) = \{c \text{ w.p } 1$$

"degenerate"

$$X \sim \text{Deg}(c) = \underbrace{1_{x=c}}_{\text{indicator function}}$$

$$1_A = \begin{cases} 1 & \text{if } A \\ 0 & \text{if } A^c \end{cases}$$

$X \sim Ben(p) = 1_{x} = p$ if $x = 0 \Rightarrow 0$
$x = 1 \Rightarrow 0$

$X \sim Ben(p) = p^x (1-p)^{1-x}$

If ~~$X_1$~~, $X_2 \sim$ independant
~~there are r.v's~~
"$X_1$ and $X_2$ are independant"
~~by def~~ joint mass functions
$P(x_1, x_2) = P_{X_1}(x_1)\, P_{X_2}(x_2)$

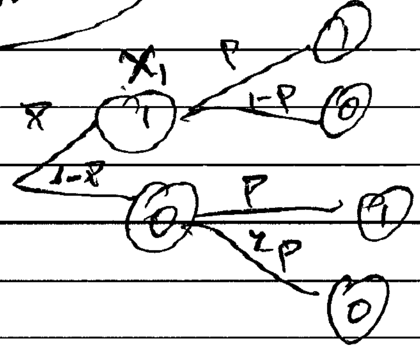
$\forall x_1, x_2$ ie their supports $X_1 = X_2$ (dependent)
~~the r.v's $X_1, X_2$ are equals distribution~~

~~$X_1, X_2 \sim$ independe~~ If $P_{X_1}(x_1) = P_{X_2}(x_2)$
~~the r.v's $X_1, X_2$ are in~~

$X_1, X_2 \sim$ independent
the r.v's $X_1, X_2$ are independent identically dist
Def $X_1, X_2 \overset{ind}{\sim}$ & $X_1 \overset{d}{=} X_2$.

let $T_2 = X_1 + X_2$
where
$X_1, X_2 \overset{iid}{\sim} Bern(p)$
Support$(T_2) = \{0, 1, 2\} = Supp(X_1) + Supp(X_2)$
$A + B = \{a+b : a \in A, b \in B\}$.

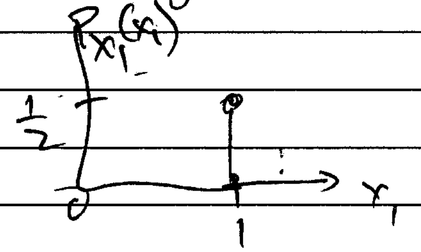
convolution

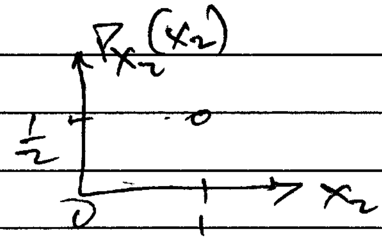



| $P_{X_1,X_2}(x_1,x_2)$ | $T_2$ |
|---|---|
| $p^2$ | 2 |
| $p(1-p)$ | 1 |
| $(1-p)p$ | 1 |
| $(1-p)^2$ | 0 |

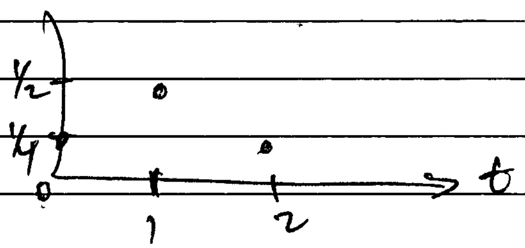
$$T_2 \begin{cases} 2 & \text{w.p. } p^2 \\ 0 & \text{w.p. } (1-p)^2 \\ 1 & \text{w.p. } 2p(1-p) \end{cases}$$

Imagine $p = \frac{1}{2}$




$P_T(t)$




$P_{T_2}(t)$

$$P(T_2 = t) = \sum_{x \in \operatorname{supp}(X_1)} P_{X_1}(x)\, P_{X_2}(t-x)$$

$$T_2 = X_1 + X_2 = \sum_{x\in\{0,1\}} \left(p^x(1-p)^{1-x}\right)\left(p^{t-x}(1-p)^{1-t+x}\right)$$

$$= p^t \sum (1-p)^{2-t} = p^t(1-p)^{2-t}\sum 1 = 2p^t(1-p)^{2-t}$$

$$p(t) = P(T_2 = t) = \sum_{x\in supp(x)} p_{X_1}(x)\, p_{X_2}(t-x)$$

$$= \sum_{x\in\{0,1\}} p^x(1-p)^{1-x}\,1_{x\in\{0,1\}}\, p^{t-x}(1-p)^{1-t+x}\,1_{t-x\in\{0,1\}}$$

$$= p^t(1-p)^{2-t} \sum_{x\in\{0,1\}} 1_{x\in\{0,1\}} 1_{t-x\in\{0,1\}}$$

$$= p^t(1-p)^{2-t}\Big(\underbrace{1_{0\in\{0,1\}}}_{1} 1_{t-0\in\{0,1\}} + \underbrace{1_{1\in\{0,1\}}}_{1} 1_{t-1\in\{0,1\}}\Big)$$

$$\underbrace{\left(1_{t\in\{0,1\}} + 1_{t-1\in\{0,1\}}\right)}_{\binom{2}{t}}$$

$$p(0) = (1-p)^2, \quad p(2) = p^2$$

$$\cancel{p(1) = 2p(1-p)}$$

$$p(1) = 2p(1-p)$$

$$\Rightarrow p(t) = \binom{2}{t} p^t (1-p)^{2-t}$$

$X \sim \text{Bern}(p) = \text{Bin}(1, p) = \binom{1}{x} p^x (1-p)^{1-x}$

$\stackrel{?}{=} p^x (1-p)^{1-x} 1_{x \in \{0,1\}}$.

$\binom{n}{k}$ only valid if $k \leq n$

o/w $0$.

$$p(T_2 = t) \neq \sum_{x \in \text{supp}(X_1)} p_{X_1}^{(x)} p_{X_2}^{(t-x)} = \sum_{x \in \{0,1\}} \binom{1}{x} p^x (1-p)^{1-x} \binom{1}{t-x} p^{t-x} (1-p)^{1-t+x}$$

$$= \underline{p^t (1-p)^{2-t}} \sum \binom{1}{x}\binom{1}{t-x} = \binom{1}{0} \cdot \binom{1}{t} + \binom{1}{1}\binom{1}{t-1}$$

$$= \binom{2}{t} p^t (1-p)^{2-t} \qquad \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}.$$

$$p^{(t)} = P(T_2 = t) = \underline{p_{X_1}^{(x)} * p_{X_2}^{(x)} = \sum_{x \in \text{supp}(X_1)} p_{X_1}^{(t)} p_{X_2}^{(t-x)}}.$$

~~independent~~

Convolution of two independent PMF's

$$T_3 = X_1 + X_2 + X_3 = X_3 + T_2 = p_{X_3}^{(x)} * p_{T_2}^{(x)}$$

$$X_1, X_2, X_3 \stackrel{ind}{\sim} \text{Bern}(p) \qquad = \sum p_{X_3}^{(x)} p_{T_2}^{(t-x)}$$

$$= \sum_{x \in \{0,1\}} \binom{1}{x} p^x (1-p)^{1-x} \binom{2}{t-x} p^{t-x} (1-p)^{2-t+x} = p^t (1-p)^{3-t} \sum_{x \in \{0,1\}} \binom{1}{x}\binom{2}{t-x}$$

$$= p^t(1-p)^{3-t}\left(\binom{2}{t}+\binom{2}{t-1}\right)$$

$$= \binom{3}{t}p^t(1-p)^{3-t}$$

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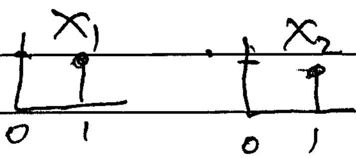
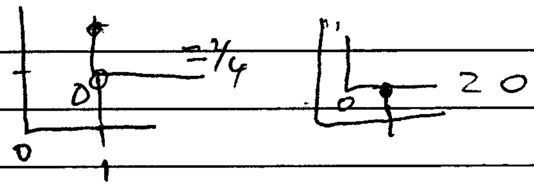
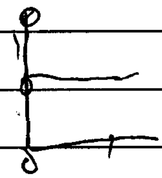
$X_1, X_2 \overset{iid}{\sim} Bern(\frac{1}{2})$




$T_2 = X_1 + X_2$



$$T = X_1 + X_2 \sim p_{X_1}(x) * p_{X_2}(x) = \sum_{x \in Supp(X_1)} p_{X_1}(x)\, p_{X_2}(t-x)$$

$$= \sum_{x \in \{0,1\}} p^x (1-p)^{1-x}\, p^{t-x}(1-p)^{1-t+x}$$

$$= \sum_{x \in \{0,1\}} p^t (1-p)^{2-t} = p^t(1-p)^{2-t} \underbrace{\sum 1}_{2} = 2p^t(1-p)^{2-t}$$

$X_1, X_2 \overset{iid}{\sim} Bin(n, p)$

$$Y = X_1 + X_2 \sim p_{X_1}(x) * p_{X_2}(x) = \sum_{x \in Supp(X_1)} p_{X_1}(x)\, p_{X_2}(y-x)$$

$$= \sum_{\substack{x=0 \\ x \in \{0,\dots,n\}}}^{n} \binom{n}{x} p^x (1-p)^{n-x} \underbrace{\mathbb{1}_{x \in \{0,\dots,n\}}}_{\text{not needed}} \binom{n}{y-x} p^{y-x} (1-p)^{n-y+x} \underbrace{\mathbb{1}_{y-x \in \{0,\dots,n\}}}_{\text{not needed}}$$

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