$0.3 \cdot 4 \,|_{t \to 7} f$

9/7/17 $\vec{X} \sim \text{multinom}_k(n, \vec{p}) := \binom{n}{x_1, x_2, \ldots x_k} p_1^{x_1} p_2^{x_2}, \ldots p_k^{x_k}$

$\dim(\vec{x}) = k$ } No vector function since multichoose is 0 unless

$$\sum_{i=1}^{k} x_i = n \text{ \& } \forall i, x_i \in \mathbb{N}_0$$

$\text{Supp}(\vec{X}) = \{\vec{x} : \vec{1} \cdot \vec{x} = n \text{ \& } \vec{x} \in \mathbb{N}_0^k\}$.

Param Space $\vec{P} \in \{\vec{e} : \vec{e} \in (0,1)^k \text{ \& } \vec{e} \cdot \vec{1} = 1\}$

P(getting 3 apples, 2 bananas, 5 cantaloupes)

$= \binom{10}{3,2,5} \left(\frac{1}{4}\right)^3 \left(\frac{1}{8}\right)^2 \left(\frac{5}{8}\right)^5$

If $P_A = \frac{1}{4}$, $P_B = \frac{1}{8}$, $P_C = \frac{5}{8}$

$P\left(\vec{X} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}\right) = \binom{10}{3,2,5} \left(\frac{1}{4}\right)^3 \left(\frac{1}{8}\right)^2 \left(\frac{5}{8}\right)^5$

let $k = 2$, $\vec{P} = \begin{bmatrix} p \\ 1-p \end{bmatrix}$

$P(\vec{x}) = P(x_1, x_2) = \text{multinom}\left(n, \begin{bmatrix} p \\ 1-p \end{bmatrix}\right) = \binom{n}{x_1, x_2} p^{x_1} (1-p)^{x_2}$

$\vec{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \neq \text{Bin}(n, p)$ $\forall x_1 \in \text{supp}(X_1), \forall x_1 \in \text{supp}(X_2)$

Are X_1, X_2 iid? = $P_{X_1, X_2}(x_1, x_2) = P_{X_1}(x_1) P_{X_2}(x_2)$

No independent

Bayes Rule

$$\underbrace{P_{X_1|X_2}(x_1|x_2)}_{\text{conditional PMF}} = \frac{P_{X_1,X_2}(x_1,x_2)}{P_{X_2}(x_2)}$$

marginal ~~distr~~

$$\Rightarrow P_{X_1|X_2}(x_1|x_2) = P_{X_1}(x_1)$$

$$\Rightarrow P_{X_2|X_1}(x_2|x_1) = P'_{X_2}(x_2) \text{ bins}$$

PMF $\quad P_{X_1|X_2}(x_1|x_2) = \dfrac{P_{X_1X_2}(x_1,x_2)}{P_{X_2}(x_2)}$

$P_{X_1,X_2}(x_1,x_2)$

$$= \frac{\binom{n}{x_1,x_2} p^{x_1}(1-p)^{x_2}}{?}$$

$$P_{X_2}(x) = P(X_2 = \ldots) = \sum_{x \in \text{supp}(X_1)} P(x_1, x_2)$$

$$= \sum_{x \in \{0,\ldots,n\}} \binom{n}{x_1,x_2} p^{x_1}(1-p)^{x_2}$$

$$= \sum_{x_1 \in \{0,1,\ldots,n\}} \frac{n!}{x_1!\,x_2!} p^{x_1}(1-p)^{x_2} \Big|_{x_1+x_2=n}$$

$$= \frac{n!}{x_2!}(1-p)^{x_2} \sum_{x_1 \in \text{supp}(X_1)} \frac{p^{x_1}}{x!} \Big|_{x_1+x_2=n}$$

$$\Big|_{x_1 = n-x_2}$$

$$= \frac{n!}{x_2!}(1-p)^{x_2} \frac{p^{n-x_2}}{(n-x_2)!}$$

$$= \text{Bin}(n, 1-p) \Rightarrow X_1 \sim \text{Bin}(n,p)$$

because it's symmetric

$$P_{X_1|X_2}(x_1|x_2) = \frac{P_{X_1,X_2}(x_1,x_2)}{P_{X_2}(x_2)} = \frac{\binom{n}{x_1,x_2} p^{x_1}(1-p)^{x_2}}{\binom{n}{x_2}(1-p)^{x_2}\, p^{n-x_2}}$$

$$= \frac{\frac{\cancel{n!}}{x_1!\,\cancel{x_2!}} p^{x_1}(1-p)^{x_2}\Big|_{x_1+x_2=n}}{\frac{\cancel{n!}}{\cancel{x_2!}(n-x_2)!}(1-p)^{x_2}\, p^{n-x_2}}$$

$$= \frac{(n-x_2)!}{x_1!}\, p^{x_1+x_2-n}\Big|_{x_1+x_2=n}$$

$$P(x_1|x_2) = \begin{cases} 0 & \text{if } x_1 \neq n-x_2 \\ 1 & \text{if } x_1 = n-x_2 \end{cases}$$

$X \sim$ multinom $(n, \vec{p})$ multi$(n,\vec{p})$

$$P_{X_{-j}|X_j}(x_{-j}|x_j) = \frac{\overbrace{P_{X_1,\dots X_k}(x_1,\dots,x_k)}^{\text{multi}(n,\vec{p})}}{\underbrace{P_{X_j}(x_j)}_{\text{Bin}(n,p_j)}}$$

X_{-j}: all x_i s.t. $i \neq j$

$$= \frac{\frac{\cancel{n!}}{x_1!\,x_2!\cdots x_k!} p_1^{x_1}\cdots p_k^{x_k}}{\frac{\cancel{n!}}{\cancel{x_j!}(n-x_j)!}\cancel{p_j^{x_j}}(1-p_j)^{n-x_j}}$$

$$= \frac{(n-x_j)!}{x_1!\cdots x_{j-1}!\,x_{j+1}!\cdots x_k!}\;\frac{p_1^{x_1}\cdots p_{j-1}^{x_{j-1}}\, p_{j+1}^{x_{j+1}}\cdots p_k^{x_k}}{(1-p_j)^{n-x_j}}$$

let $n' = n - x_j'$

~~recall~~ $\sum_{i=1}^{k} x_i' = n \Rightarrow x_1 + x_2 + \cdots + x_{j-1} + x_j' + x_{j+1} + \cdots + x_k = n$

$\Rightarrow n' = x_1 + x \cdots x_{j-1} + x_{j+1} + \cdots + x_k$

let $p_i' = \frac{p^i}{1-p^i} \Rightarrow p^i = p_i'(1-p_j)$

$= \binom{n'}{x_1, \cdots x_{j-1} \cdot x_{j+1}, \cdots x_k} \frac{(p_1'(1-p_j))^{x_1} \cdots (p_j'(1-p_j))^{x_{j-1}} \cdots (p_{j+1}'(1-p_j))^{x_{j+1}} \cdots (p_k(1-p_j))}{(1-p_j)^{n'}}$

$= \binom{n'}{x \cdots} \frac{(1-p_j)^{\overbrace{x_1 \cdots x_j \cdots x_k}^{n'}} p_1'^{x_1} \cdots p_{j-1}'^{x_j} \cdots p_{j+1}'^{x_{j+1}} \cdots p_k'^{x_k}}{(n-p_j)^{n'}}$

$$E(g(x_1, \cdots, x_n)) = \sum_{x \in supp(x_1)} \cdots \sum_{x_n \in supp(x_n)} g(x_1, \cdots, x_n) P(x_1, \cdots, x_n)$$

$$E(x_1 + \cdots x_n) = \underbrace{E(x_1) + \cdots E(x_n)}_{\text{linearity of expectation}} \underset{\text{identically distr}}{=} nN$$

$$E\left(\prod_{i=1}^{n} x_i\right) = \sum \cdots \sum x_1 x_2 \cdots x_n P(x_1 \cdots x_2 \cdots x_n)$$ No independent

If $x_1, \cdots, x_n \overset{indep}{\sim}$

then $E\left(\prod_{i=1}^{n} x_i\right) = \sum \cdots \sum (x_1 P(x_1) \cdots) x_n P(x_n) = \prod_{i=1}^{n} E(x_i)$

$$Var(X) = E\left(\big(\underbrace{X-\mu}_{g(x)}\big)^2\right) = E[g(x)]$$

$$= \sum g(x)\, p^x = \sum (x-\mu)^2 p^x$$

$$= \sum x^2 p(x) + \sum -2x\mu\, p(x) + \sum \mu^2 p(x)$$

$$= E(X^2) - 2\mu^2 + \mu^2$$

$$= E(X^2) - \mu^2$$

$Var(X+8) = V(X)$ $\qquad$ $E(X+c) = E(X) + c$

$Var(cX) = c^2 Var(X)$ $\qquad$ $E(3X) = 3E(X)$

$$Var(X_1+X_2) = E\left[\big((X_1+X_2)-(\mu_1+\mu_2)\big)^2\right]$$

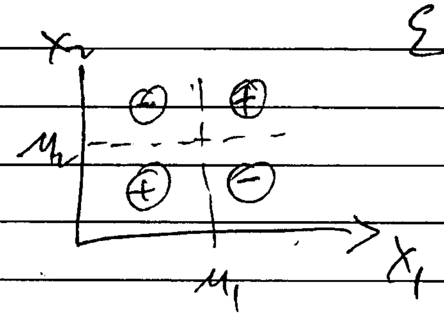
$$Var(X_1) + Var(X_2) + 2\underbrace{\big(E(X_1X_2) - \mu_1\mu_2\big)}_{E((X_1-\mu_1)(X_2-\mu_2))}$$

$Cor[X_1, X_2]$

$$Cor(X_1, X_2) = \frac{Cor(X_1, X_2)}{SE(X_1)\, SE(X_2)} \qquad \in \{-1, 1\}.$$

correlasion (.

$$SE(X) = \sqrt{Var(X)}$$



$Cor[X, X] = Var(X)$

$Cor(a_1 X_1, a_2 X_2) = a_1 a_2 \, Cor(X_1, X_2)$

$Cor(X_1 + c, X_2 + c_2) = Cor(X_1, X_2)$

$Cor(X_2, X_1) = Cor(X_1, X_2)$

$Cor(X + Y, Z) = Cos(X, Z) + Cor(Y, Z)$

$$Cor\left(\sum_{i=1}^{m} X_i, \sum_{j=1}^{n} Y_j\right) = \sum_{i=1}^{m} \sum_{j=1}^{n} Cor(X_i, Y_j)$$

$$Var(X_1 + X_2) = E\left[\left((X_1 + X_2) - (\mu_1 + \mu_2)\right)^2\right]$$

$$= Cor(X_1, X_1) + Cos(X_2, X_2) + Cor(X_1, X_2) + Cor(X_2, X_1)$$

$$= \sum_{i=1}^{2} \sum_{j=1}^{2} Cor(X_i, X_j)$$

$$Var(X_1 + \dots + X_k) = \sum_{i=1}^{k} \sum_{j=1}^{k} Cos(X_i, X_j)$$

If $\vec{X}$ is a vector r.v.

$$E(\vec{X}) = E\left[\begin{pmatrix} X_1 \\ \vdots \\ X_k \end{pmatrix}\right] = \begin{bmatrix} E(X_1) \\ \vdots \\ E(X_k) \end{bmatrix}$$

$cor(X_1, X_2)$

$$Var[\vec{X}] = \begin{bmatrix} Var(X_1) & cor(X_1, X_2) & & \\ cor(X_2, X_1) & Var(X_2) & & \\ & & \ddots & \\ & & & Var(X_k) \end{bmatrix} = \{Cor(X_i, X_j)\} \quad \begin{matrix} i = 1 \dots k \\ j = 1 \dots k \end{matrix}$$