11/17/17 Lecture # 18. X~N(0,1) ind of X2~N(0,1) R=X, SIXI fx,(xr)fx2(x)dx $\int |X| e^{-\frac{1}{2}X^2(r^2+1)} dx$ $(-x)e^{-\frac{1}{2}x^2(r^2+1)}dx + (xe^{-\frac{1}{2}x^2(r^2+1)}dx$ (xe- = x2(r2+1) dx $U = -\frac{1}{2} x^{2} (r^{2} + 1)$ $du = -x(r^2+1)dx$ U2 -0. $\times dx = -du dy$ $\mathbf{z}(r^{2}+1)$ $= \frac{1}{\Pi(r^2+1)} = \text{Cauchy}(0,1)$ Mistelly unknown unknown -in out unknow dist. the variance. mean estimate" >X is the aug. (r.v), "estimator "E[X] = M x+ -- 22 150 # Puspiased S is the sample $S^{2} = \frac{1}{n-1} \sum (X_{i} - \overline{X})^{2}$ varianer-V.

$$E\left[S^{2}\right] = \sigma^{2}$$

$$\int_{0}^{\infty} h \int_{0}^{1} a s e^{i x} ds ds$$

$$X_{L}, \dots, X_{h}, \stackrel{i 2}{\sim} N(M, \sigma^{2})$$

$$\overline{X} \sim N(M, \sigma^{2})$$

$$S^{2} \sim \stackrel{?}{\sim} \left[X_{1} - \overline{X}\right]^{2} + \dots + \left(X_{h} - \overline{X}\right)^{2}$$

$$= \frac{1}{2} \left[X_{1} - \overline{X}\right]^{2} + \dots + \left(X_{h} - \overline{X}\right)^{2}$$

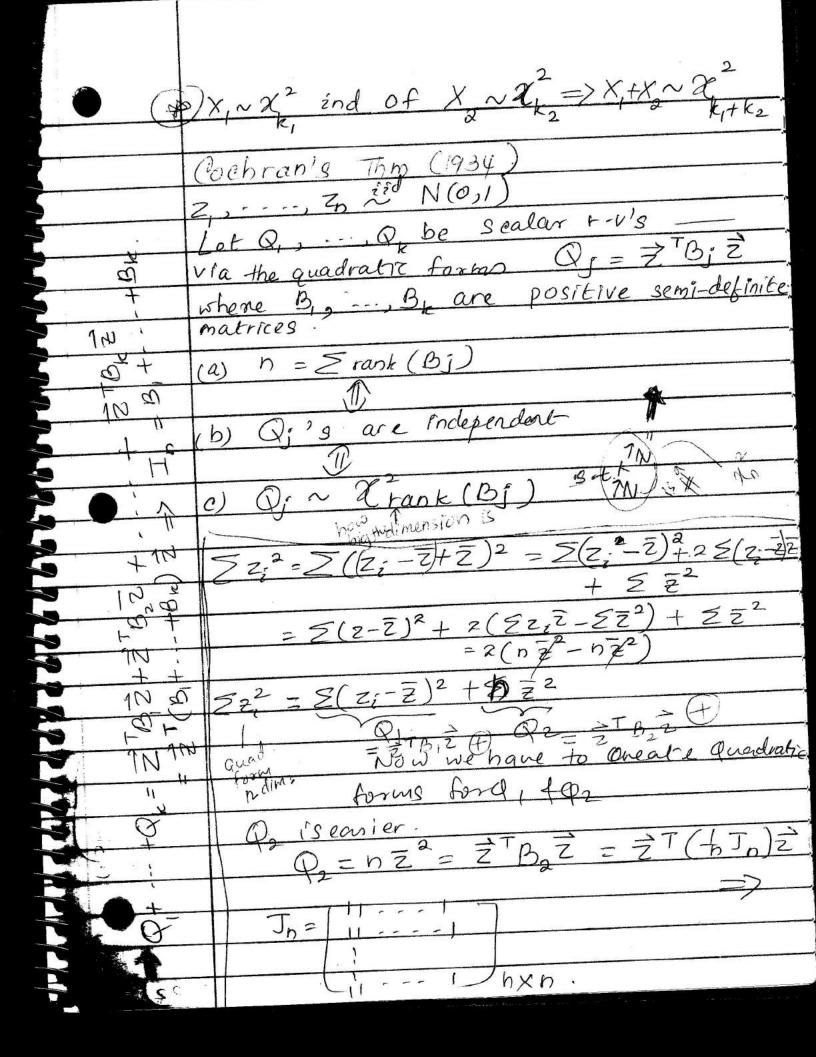
$$= \frac{1}{2} \left[X_{1} - \overline{X}\right]^{2} + \dots + \left(X_{h} - \overline{X}\right)^{2}$$

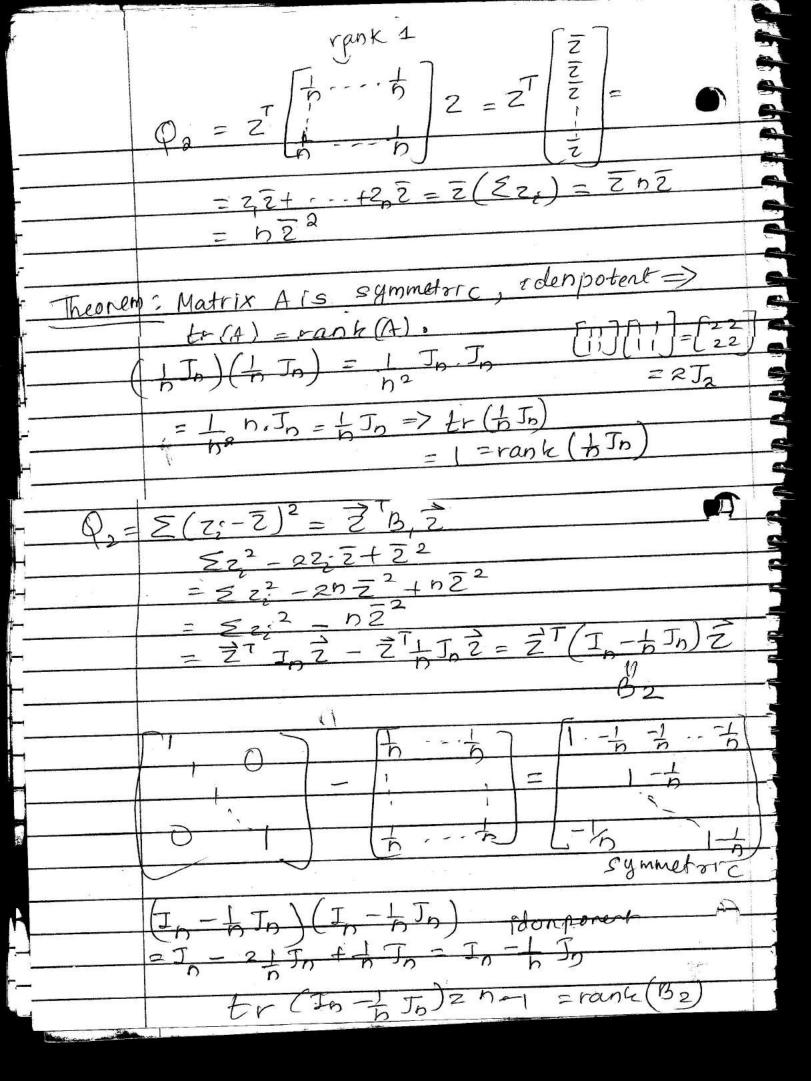
$$= \frac{1}{2} \left[X_{1} - \overline{X}\right]^{2} + 2 \left(X_{1} - \overline{X}\right)^{2} \sim \chi_{h}^{2}$$

$$= \frac{1}{2} \left(X_{1} - \overline{X}\right)^{2} + 2 \left(X_{1} - \overline{X}\right) \left(\overline{X} - M\right) + (\overline{X} - M)^{2}$$

$$= \frac{1}{2} \left(X_{1} - \overline{X}\right)^{2} + 2 \left(X_{1} - \overline{X}\right) \left(\overline{X} - M\right) + n \left(\overline{X} - M\right)^{2}$$

$$= \frac{1}{2} \left(X_{1} - \overline{X}\right)^{2} + 2 \left(n_{\overline{X}}^{2} - n_{\overline{X}} + n_{\overline{X}}$$





Def Matrix A 18 pos. semi. def. if \v v , Proven 5 $\frac{n(\bar{x}-\mu)^2}{\sigma^2} = \pm (\hat{x}-\hat{\mu})^T (\pm J_n) \pm (\bar{x}-\hat{\mu})$ $\frac{h(x-\mu)^2}{\sigma^2}$ and of $\frac{(h-1)s^2}{\sigma^2}$ => x, 52 are independent. Fisher proved thin 1935. Gleary, 1936 proved this is unique to the $\frac{(h-1)S^2}{h-1} \sim \frac{\chi^2}{h-1} = \frac{S^2 n \sigma^2 \chi^2}{h-1} = \frac{Gam(\frac{h-1}{2})^{\frac{1}{2}}}{h-1}$

Z test 4 $\sim N(0,1)$ Student 1908