$$\vec{z} \sim N_n(\vec{0}, I) = \frac{1}{\sqrt{(2\pi)^n}} e^{-\frac{1}{2}\vec{z}^T\vec{z}}$$

$$\phi_{\vec{z}}(\vec{t}) = e^{-\frac{1}{2}\vec{t}^T\vec{t}}$$

If $A \in \mathbb{R}^{n\times n}$ invertible, $\vec{\mu} \in \mathbb{R}^n$

$$\vec{x} = A\vec{z} + \vec{\mu} \sim N_n(\vec{\mu}, \underset{AA^T}{\underset{\parallel}{\Sigma}}) = \frac{1}{\sqrt{(2\pi)^n \det(\Sigma)}} e^{-\frac{1}{2}(\vec{x}-\vec{\mu})^T\Sigma^{-1}(\vec{x}-\vec{\mu})}$$

$$\phi_{\vec{x}}(\vec{t}) = e^{i\vec{t}^T\vec{\mu} - \frac{1}{2}\vec{t}^T\Sigma\vec{t}}$$

- Let $B \in \mathbb{R}^{m\times n}$, $\vec{c} \in \mathbb{R}^m$, $\vec{Y} = B\vec{X} + \vec{c} \sim N_m(B\vec{\mu} + \vec{c}, B\Sigma B^T)$

Let $A \in \mathbb{R}^{m\times n}$, full rank, $m \le n$, $\vec{c} \in \mathbb{R}^m$

$$\vec{x} = A\vec{z} + \vec{c} \sim N_m(A\vec{0}_n + \vec{c}, AIA^T) = N_m(\vec{c}, \underbrace{AA^T}_{\Sigma})$$

- $$N_1(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\underbrace{(x-\mu)\frac{1}{\sigma^2}(x-\mu)}_{\left(\frac{x-\mu}{\sigma}\right)^2}}$$

z-score.

we generated univariate normal

☆ multinomial r.v. in each vector, bin (dependent)

- Mahalanobis distance (1936):

$$(\vec{x}-\vec{\mu})^T\Sigma^{-1}(\vec{x}-\vec{\mu}) \sim$$

How are they distributed?

$$= (\vec{x}-\vec{\mu})^T(AA^T)^{-1}(\vec{x}-\vec{\mu})$$

for A invertible $(\vec{x}-\vec{\mu})^T(A^{-1})^TA^{-1}(\vec{x}-\vec{\mu})$

$$= (A^{-1}(\vec{x}-\vec{\mu}))^T(A^{-1}(\vec{x}-\vec{\mu}))$$

$$= \vec{z}^T\vec{z} \sim \chi^2_n$$

- $$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_5 \end{bmatrix} \qquad f_{x_2,x_4}(x_2, x_4) = \int_{\mathbb{R}}\int_{\mathbb{R}}\int_{\mathbb{R}} f_{x_1 \ldots x_5}(x_1 \ldots x_5)\, dx_1\, dx_3\, dx_5$$

What's the other way to do this? (easier way)

$$\phi_{\vec{x}}(\vec{t}) = E[e^{i\vec{t}^T\vec{x}}] = E[e^{it_1x_1}e^{it_2x_2}e^{it_3x_3}e^{it_4x_4}e^{it_5x_5}]$$

$$\phi_{\vec{x}}\left(\begin{bmatrix} 0 \\ t_2 \\ 0 \\ t_4 \\ 0 \end{bmatrix}\right) = E[e^{it_2x_2}e^{it_4x_4}] = E[e^{i[t_2\ t_4]\begin{bmatrix} x_2 \\ x_4 \end{bmatrix}}] = \phi_{x_2,x_4}(t_2, t_4)$$

$$\xrightarrow{P_1, P_6} f_{x_2,x_4}$$

$$\phi_{\vec{X}}\left(\begin{bmatrix} t \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) = e^{i[t\ 0 \cdots 0]\begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix} - \frac{1}{2}[t\ 0 \cdots 0]\,\Sigma\begin{bmatrix} t \\ 0 \\ \vdots \\ 0 \end{bmatrix}}$$

$\parallel$

$\phi_{X_1}(t)$

(we're trying to find $X_1 \sim ?$)

$$= e^{it[1\ 0 \cdots 0]\begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix} - \frac{t^2}{2}[1\ 0 \cdots 0]\,\Sigma\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}$$

$$= e^{it\mu_1 - \frac{t^2}{2}\cdot \sigma_1^2}$$

$$\Sigma\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2 & \cdots & \sigma_{2n} \\ & \vdots & & \\ \sigma_{n1} & \cdots & & \sigma_{nn} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 \\ \sigma_{21} \\ \vdots \\ \sigma_{n1} \end{bmatrix}$$

$\Rightarrow X_1 \sim N(\mu_1, \sigma_1^2)$

P1

$\Rightarrow X_j \sim N(\mu_j, \sigma_j^2)$

They are all dependent & each of them is normal just like multibin r.v.