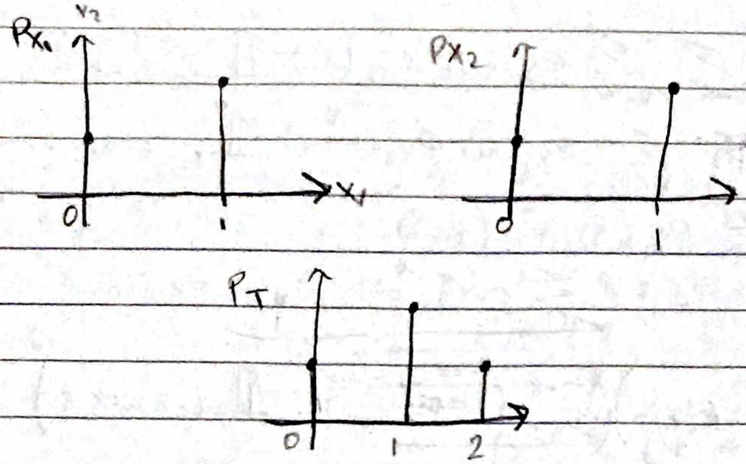
Pascal's Identity:

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$



- $T = X_1 + X_2$ $\qquad X_1, X_2 \overset{iid}{\sim} Bern(p) := \binom{1}{x} p^x (1-p)^{1-x}$

$$\boxed{\mathbb{1}_{x \in [0,1]}}$$

$$p(t) = \sum_{x \in [0,1]} \binom{1}{x} p^x (1-p)^{1-x} \binom{1}{t-x} p^{t-x} (1-p)^{1-(t-x)}$$

$$= p^t (1-p)^{2-t} \sum_{x \in \{0,1\}} \binom{1}{x} \binom{1}{t-x}$$

$$\hookrightarrow \sum_{x \in \{0,1\}} \binom{1}{t-x} = \binom{1}{t} + \binom{1}{t-1} = \binom{2}{t}$$

- $X_1, X_2, X_3 \overset{iid}{\sim} Bern(p)$

$$T = X_1 + X_2 + X_3$$

$$T_3 = X_3 + T_2 \sim \sum_{x \in supp(X_3)} P_{X_1}(x) P_{T_2}(t-x) \underbrace{\mathbb{1}_{t-x \in supp(T_2)}}_{\text{dont need bec } \binom{2}{t-x}}$$

$$= \sum_{x \in \{0,1\}} \left(p^x (1-p)^{1-x}\right) \left(\binom{2}{t-x} p^{t-x} (1-p)^{2+x-t} \mathbb{1}_{t-x \in \{0,1,2\}}\right)$$

$$= p^t (1-p)^{3-t} \sum_{x \in \{0,1\}} \binom{2}{t-x}$$

$$\binom{2}{t} + \binom{2}{t-1} = \binom{3}{t}$$

4 $\quad = \binom{3}{t} p^t (1-p)^{3-t} = Binom(3, p)$