A discrete random variable (r.v.) X has probability mass function (PMF) $p(x) = P(X = x)$ also noted $X \sim p(x)$ and cumulative distribution function (CDF) $F(x) = P(X \le x)$. The r.v. X has "support" $\text{Supp}[X] := \{x : p(x) > 0,\ x \in \mathbb{R}\}$ and $|\text{Supp}(X)| \le |\mathbb{N}|$ i.e. discrete (finite or countably infinite). The support and PMF are related via $\sum_{x \in \text{Supp}(X)} p(x) = 1$

IMHO, the most fundamental r.v. is the Bernoulli:

$X \sim \text{Bern}(p) = \begin{cases} 1 & \text{w/p } p \\ 0 & \text{w/p } 1-p \end{cases} = p^x (1-p)^{1-x}$ and $\text{Supp}(X) = \{0, 1\}$

p is a "parameter" (thinking knob) which belongs to a "parameter space" $p \in (0, 1)$

$\text{Bern}(0) = \text{Deg}(0) = 0$ w/p 1

$\text{Bern}(1) = \text{Deg}(1) = 1$ w/p 1

Indicator function $\mathbb{1}_A := \begin{cases} 1 & \text{if } A \\ 0 & \text{if } A^c \end{cases}$

$$X \sim \text{Bern}(p) := \underbrace{p^x (1-p)^{1-x} \mathbb{1}_{x \in \underbrace{\{0,1\}}_{\text{Supp}(X)}}}_{p(x)}$$

Now... $\sum_{x \in \mathbb{R}} p(x) = 1 = \sum_{x \in \text{Supp}(X)} p(x)$ old, ← w/out indicator function

$X \sim \text{Deg}(c) = \mathbb{1}_{x=c}$

Consider many r.v.'s $X_1, X_2, \ldots, X_n$, the join mass function (JMF) is:

$$P_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n) := P(X_1 = x_1 \,\&\, X_2 = x_2 \,\&\, \ldots \,\&\, X_n = x_n)$$

If $X_1, X_2, \ldots, X_n \overset{ind}{\sim}$ "are independent" then

$$P_{X_1, \ldots, X_n}(x_1, \ldots, x_n) = \prod_{i=1}^{n} P_{X_i}(x_i) \;\forall \vec{x} \in \mathbb{R}^n$$

$$= \text{if iid} \longrightarrow = \prod_{i=1}^{n} p(x_i)$$

If $X_1 \overset{d}{=} X_2 \overset{d}{=} \ldots \overset{d}{=} X_n$ ($X_1, \ldots, X_n$ are "equal in distribution") then $P_{X_1}(x) = P_{X_2}(x) = \ldots = P_{X_n}(x) \;\forall x \in \mathbb{R}$

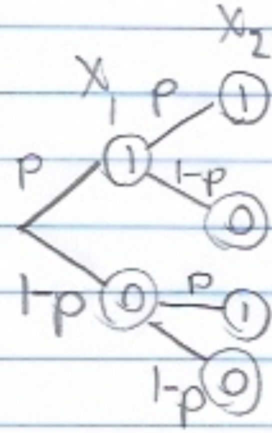
The special case of $X_1, X_2, \ldots, X_n \overset{iid}{\sim}$ means both

$X_1, X_2 \overset{iid}{\sim} \text{Bern}(p)$

$T = X_1 + X_2 \sim ?$

$\text{Supp}(T) = \text{Supp}(X_1) + \text{Supp}(X_2) = \{0, 1, 2\}$

$A + B = \{a + b,\; a \in A,\; b \in B\}$



$\langle x_1, x_2 \rangle$	T	$P_{X_1,X_2}(x_1,x_2)$
$\langle 1,1 \rangle$	2	p^2
$\langle 1,0 \rangle$	1	$p(1-p)$
$\langle 0,1 \rangle$	1	$(1-p)p$
$\langle 0,0 \rangle$	0	$(1-p)^2$

$p_T(t)$

$$\begin{pmatrix} p^2 \\ 2p(1-p) \\ (1-p)^2 \end{pmatrix}$$

$$\sum_{t \in supp(T)} p(t)$$
$$= p^2 + 2p(1-p) + (1-p)^2$$
$$= \cancel{p^2} + \cancel{2p} - \cancel{2p^2} + 1 - \cancel{2p} + \cancel{p^2}$$

$$T \sim \begin{cases} 0 & w/p \ (1-p)^2 \\ 1 & w/p \ 2p(1-p) \\ 2 & w/p \ p^2 \end{cases}$$

$$p_T(t) = \sum_{x_1 \in \mathbb{R}} \sum_{x_2 \in \mathbb{R}} p_{X_1,X_2}(x_1,x_2) \mathbb{1}_{x_1+x_2=t}$$

if X_1, X_2 ind $\rightarrow = \sum_{x_1 \in \mathbb{R}} \sum_{x_2 \in \mathbb{R}} p_{X_1}(x_1) p_{X_2}(x_2) \mathbb{1}_{x_1+x_2=t}$

$$= \sum_{x_1 \in \mathbb{R}} \sum_{x_2 \in \mathbb{R}} p_{X_1}(x_1) p_{X_2}(\underbrace{t-x_1}_{x_2}) \mathbb{1}_{x_2 = t-x_1} \longrightarrow$$

$$\sum_{x\in\mathbb{R}} p_{X_1}(x)\, p_{X_2}(t-x)$$

iid

$\downarrow$

$$= \sum_{x\in\mathbb{R}} p(x)\, p(t-x)$$

$\uparrow$

if ideal distribution

$$\sum_{x\in\mathbb{R}} \left(p^x (1-p)^{1-x} \mathbb{1}_{x\in\{0,1\}}\right)\left(p^{t-x}(1-p)^{\overbrace{1-(t-x)}^{1-t+x}} \mathbb{1}_{t-x\in\{0,1\}}\right)$$

$t=2$

$x=0$

$$= p^t (1-p)^{2-t} \boxed{\sum_{x\in\{0,1\}} \mathbb{1}_{t-x\in\{0,1\}}}$$

$$= \mathbb{1}_{t-0\in\{0,1\}} + \mathbb{1}_{t-1\in\{0,1\}}$$

$$= \mathbb{1}_{t\in\{0,1\}} + \mathbb{1}_{t\in\{1,2\}}$$

$$= \begin{cases} 1 & \text{if } t=0 \\ 2 & \text{if } t=1 \\ 1 & \text{if } t=2 \end{cases} = \binom{2}{t}$$