Dec 16th FINAL EXAM

11/27 EXAM 1:45 pm in - KY 312
6:15 pm - KY 412

Review from last class:

$\vec{Z} \sim N_n(\vec{0}, I) := \frac{1}{\sqrt{(2\pi)^n}} e^{-\frac{1}{2}\vec{Z}^T\vec{Z}}$ $\quad \phi_{\vec{Z}}(\vec{t}) = e^{-\frac{1}{2}\vec{t}^T\vec{t}}$

if $A \in \mathbb{R}^{n \times n}$ invertible $\vec{\mu} \in \mathbb{R}^n \Rightarrow \vec{X} = A\vec{Z} + \vec{\mu} \sim$

$$\vec{X} \sim N_n(\vec{\mu}, \underset{\underset{AA^T}{\shortparallel}}{\Sigma}) := \frac{1}{\sqrt{(2\pi)^n \det[\Sigma]}} e^{\underbrace{-\frac{1}{2}(\vec{X}-\vec{\mu})^T \Sigma^{-1} (\vec{X}-\vec{\mu})}}$$

z-score
(Mahalanobis Distance)

$\phi_{\vec{X}}(\vec{t}) = e^{i\vec{t}^T\vec{\mu} - \frac{1}{2}\vec{t}^T\Sigma\vec{t}}$

Let $B \in \mathbb{R}^{m \times n}$, $c \in \mathbb{R}^m$, $\vec{Y} = B\vec{X} + \vec{c} \sim N_m(B\vec{\mu} + \vec{c}, B\Sigma B^T)$

let $A \in \mathbb{R}^{m \times n}$, full rank, $m \leq n$, $\vec{c} \in \mathbb{R}^m$

new $\vec{X}$:
$\vec{X} = A\vec{Z} + \vec{c}$

if $m > n$ then AA^T would be rank deficient, $\det = 0$, pdf undefin

new $\vec{X} \sim N_m(A\vec{0}_n + \vec{c}, AIA^T) = N_m(\vec{c}, \underbrace{AA^T}_{\Sigma})$

Linear transformation of multivariate normal is multi-variate normal.

$$N_1(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\underbrace{(x-\mu)\frac{1}{\sigma^2}(x-\mu)}_{\left(\frac{x-\mu}{\sigma}\right)^2}}$$

z-score

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Mahalobis Distance

$$(\vec{X}-\vec{\mu})^T \Sigma^{-1} (\vec{X}-\vec{\mu}) \sim$$

$$= (\vec{X}-\vec{\mu})^T (AA^T)^{-1} (\vec{X}-\vec{\mu})$$

$$= (\vec{X}-\vec{\mu})^T (A^{-1})^T A^{-1} (\vec{X}-\vec{\mu})$$

$$= (A^{-1}(\vec{X}-\vec{\mu}))^T (A^{-1}(\vec{X}-\vec{\mu}))$$

$$= \vec{Z}^T \vec{Z} \sim \chi_n^2$$

How far you are away from the mean.

Always Positive = HW Question

max value = mean

$X_1 \sim$?? (what is the component in the vector distributed as?)

Lets say $\vec{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_5 \end{bmatrix}$ AND I WANT THE density of $f_{X_2, X_4}(x_2, x_4)$

given we know $f_{X_1 \dots X_5}(x_1 \dots x_5)$?

use margin.

$$= \iiint_{\mathbb{R}\,\mathbb{R}\,\mathbb{R}} f_{X_1 \dots X_5}(x_1 \dots x_5)\, dx_1\, dx_3\, dx_5$$

Example $\phi_{\vec{X}}(\vec{t}) = E\left[e^{i\vec{t}^T\vec{X}}\right] = E\left[e^{it_1x_1} e^{it_2x_2} e^{it_3x_3} e^{it_4x_4} e^{it_5x_5}\right]$

Examine this.

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$$\phi_{\vec{X}}\left(\begin{bmatrix} 0 \\ t_2 \\ 0 \\ t_4 \\ 0 \end{bmatrix}\right) = E\left[e^{it_2X_2} e^{it_4X_4}\right] = E\left[e^{i[t_2\ t_4]\begin{bmatrix} X_2 \\ X_4 \end{bmatrix}}\right] = \phi_{X_2X_4}(t_2, t_4)$$

now use p1, p6 to get back $f_{X_2,X_4}(x_2, x_4)$.

$$\phi_{\vec{X}}\left(\begin{bmatrix} t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) = \phi_{X_1}(t) = e^{i[t\,0\ldots0]\begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix} - \frac{1}{2}[t\,0\ldots0]\,\Sigma\begin{bmatrix} t \\ 0 \\ \vdots \\ 0 \end{bmatrix}}$$

$$= e^{it[1\,0\ldots0]\begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix} - \frac{t^2}{2}[1\,0\ldots0]\underbrace{\Sigma\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}}$$

$$= e^{it\mu_1 - \frac{t^2\sigma_1^2}{2}}$$

first column

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} \cdots \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \vdots \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn}^2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 \\ \sigma_{21} \\ \vdots \\ \sigma_{n1} \end{bmatrix}$$

(P1) $\Rightarrow X_1 \sim N(\mu_1, \sigma_1^2)$

$\Rightarrow X_j \sim N(\mu_j, \sigma_j^2)$

A Just like multinomial, the entries are all normal distributed but are dependent.

Subset will also be a multi-variate normal of smaller dimension.

END FIRST SCAN.