11/27/2019

$$\vec{Z} \sim N_n(\vec{0}, I) = \frac{1}{\sqrt{(2\pi)^n}} e^{-\frac{1}{2}\vec{Z}^T\vec{Z}}$$

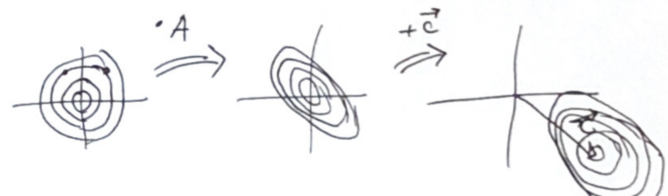
where $\Sigma = AA^T$

Let $A \in \mathbb{R}^{n\times n}$, invertible, $\vec{\mu} \in \mathbb{R}^n$

$$\vec{X} = A\vec{Z} + \vec{\mu} \sim N_n(\vec{\mu}, \Sigma) = \frac{1}{\sqrt{(2\pi)^n \det[\Sigma]}} e^{-\frac{1}{2}(\vec{X}-\vec{\mu})^T \Sigma^{-1} (\vec{X}-\vec{\mu})}$$

$$\phi_{\vec{Z}}(\vec{t}) = e^{-\frac{1}{2}\vec{t}^T\vec{t}}$$

$$\phi_{\vec{X}}(\vec{t}) = e^{i\vec{t}^T\vec{\mu} - \frac{1}{2}\vec{t}^T\Sigma\vec{t}}$$



$B \in \mathbb{R}^{m\times n}$, $\vec{c} \in \mathbb{R}^m$, $m<n$

$$\vec{Y} = B\vec{X} + \vec{c} \overset{(P2)}{\sim} N_m(B\vec{\mu} + \vec{c}, B\Sigma B^T)$$

Let $A \in \mathbb{R}^{m\times n}$, $m<n$, full rank, $\vec{c} \in \mathbb{R}^m$

$$\vec{X} = A\vec{Z} + \vec{c} \sim N_m(A\vec{0}_n + \vec{c}, AIA^T) = N_m(z, \underbrace{AA^T}_{\Sigma})$$

$$\vec{Z} \sim N_n(\vec{0}_n, I_n) \Rightarrow \vec{X} \sim N_n(\vec{\mu}, \sigma^2 I)$$

$$\vec{Z}^T\vec{Z} = \sum z_i^2 = \sum \left(\frac{x_i - \mu}{\sigma}\right)^2 = (\vec{X}-\vec{\mu})^T(\sigma^2 I)^{-1}(\vec{X}-\vec{\mu})$$

$$\sim \chi^2_n$$

$$(\vec{X}-\vec{\mu})^T \Sigma^{-1} (\vec{X}-\vec{\mu}) \sim ?$$

$$= (\vec{X}-\vec{\mu})^T (AA^T)^{-1} (\vec{X}-\vec{\mu})$$

$$= (\vec{X}-\vec{\mu})(A^{-1})^T A^{-1}(\vec{X}-\vec{\mu})$$

$$= (A^{-1}(\vec{X}-\vec{\mu}))^T A^{-1}(\vec{X}-\vec{\mu})$$

$$= \vec{Z}^T\vec{Z} \sim \chi^2_n$$

$$\vec{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} \quad f_{X_2X_4}(x_2, x_4) = \iiint_{\mathbb{R}\mathbb{R}\mathbb{R}} f_{X_1\ldots X_5}(x_1 \ldots x_5)\, dx_1\, dx_3\, dx_5$$

$$\phi_{\vec{X}}(\vec{t}) = E[e^{i\vec{t}\vec{X}}] = E\left[e^{it_1X_1} e^{it_2X_2} e^{it_3X_3} e^{it_4X_4} e^{it_5X_5}\right]$$

$$\phi_X\left(\begin{bmatrix} 0 \\ t_2 \\ 0 \\ t_4 \\ 0 \end{bmatrix}\right) = E[e^{it_2X_2} e^{it_4X_4}] = E\left[e^{i[t_2\, t_4]\begin{bmatrix} X_2 \\ X_4 \end{bmatrix}}\right] = \phi_{X_2,X_4} \overset{\text{inversion, P1}}{\Rightarrow} f_{X_2,X_4}(x_2, x_4)$$

$\vec{X} \sim N_n(\vec{\mu}, \Sigma)$ what is distribution of X_1?

$$\phi_{\vec{X}}(\vec{t}) = e^{i\vec{t}^T\vec{\mu} - \frac{1}{2}\vec{t}^T\Sigma\vec{t}}$$

$$\overbrace{\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & & & \\ \sigma_{n1} & \cdots & & \sigma_n^2 \end{bmatrix}}\begin{bmatrix} t \\ 0 \\ \vdots \\ 0 \end{bmatrix} = t\begin{bmatrix} \sigma_1^2 \\ \sigma_{21} \\ \vdots \\ \sigma_{n1} \end{bmatrix}$$

$$\phi_{\vec{X}}\left(\begin{bmatrix} t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) = e^{\underbrace{i[t\,0\cdots0]\begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix}}_{\mu_1} - \underbrace{\frac{1}{2}[t\,0\cdots0]\,\Sigma\begin{bmatrix} t \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}}_{t^2\sigma_1^2}} = e^{it\mu_1 - \frac{\sigma_1^2 t^2}{2}} \overset{P1}{\Rightarrow} X_1 \sim N(\mu_1, \sigma_1^2)$$

$$X_j \sim N(\mu_j, \sigma_j^2).$$