MATH 368/621 Fall 2019 Homework #5 PARTIAL

Professor Adam Kapelner

Due under the door of KY604 11:59PM Friday, November 15, 2019

(this document last updated Thursday 24th October, 2019 at 2:05am)

Instructions and Philosophy

The path to success in this class is to do many problems. Unlike other courses, exclusively doing reading(s) will not help. Coming to lecture is akin to watching workout videos; thinking about and solving problems on your own is the actual "working out." Feel free to "work out" with others; **I want you to work on this in groups.**

Reading is still *required*. For this homework set, read on your own about the Weibull, Frechet, generalized extreme value distributions, order statistics, transformations, the beta distribution.

The problems below are color coded: green problems are considered *easy* and marked "[easy]"; yellow problems are considered *intermediate* and marked "[harder]", red problems are considered *difficult* and marked "[difficult]" and purple problems are extra credit. The *easy* problems are intended to be "giveaways" if you went to class. Do as much as you can of the others; I expect you to at least attempt the *difficult* problems.

This homework is worth 100 points but the point distribution will not be determined until after the due date. See syllabus for the policy on late homework.

Up to 7 points are given as a bonus if the homework is typed using LaTeX. Links to instaling LaTeX and program for compiling LaTeX is found on the syllabus. You are encouraged to use `overleaf.com`. If you are handing in homework this way, read the comments in the code; there are two lines to comment out and you should replace my name with yours and write your section. The easiest way to use overleaf is to copy the raw text from hwxx.tex and preamble.tex into two new overleaf tex files with the same name. If you are asked to make drawings, you can take a picture of your handwritten drawing and insert them as figures or leave space using the "\vspace" command and draw them in after printing or attach them stapled.

The document is available with spaces for you to write your answers. If not using LaTeX, print this document and write in your answers. I do not accept homeworks which are *not* on this printout. Keep this first page printed for your records.

NAME: _____ SECTION: _____ CLASS: 368 | 621

Problem 1

These exercises will give you practice with the Weibull distribution.

(a) [easy] If $X \sim \text{Exp}(1)$ then show that $Y = \frac{1}{\lambda} X^{\frac{1}{k}} \sim \text{Weibull}(k, \lambda)$ where $k, \lambda > 0$.

(b) [harder] Find $\text{Med}[Y]$.

(c) [easy] The parameter k is called the "Weibull modulus" and it is very important in modeling. The three classes of Weibull models are when $k < 1, k = 1, k > 1$. Write a probability statement about each of these cases. Give one example of what each of these cases can potentially model in the real world.

(d) [difficult] [MA] Prove that if $k > 1$ then $\mathbb{P}(Y \geq y + c \mid Y \geq c) < \mathbb{P}(Y \geq y)$ for $c > 0$.

(e) [difficult] If $X \sim \text{Exp}(\lambda)$ then show that $Y = X^\beta \sim$ Weibull where $\beta > 0$. Find the resulting Weibull's parameters in terms of the parameterization we learned in class (i.e. your answer in part a).

(f) [easy] Using Y, the Weibull in terms of the parameterization we learned in class (i.e. your answer in part a), find the PDF of $W = Y + c \sim \text{Weibull}(k, \lambda, c)$ which is known as the "translated Weibull" or "3-parameter Weibull model".

(g) [easy] Using Y, the Weibull in terms of the parameterization we learned in class (i.e. your answer in part a), find the PDF of $V = \frac{1}{Y} \sim \text{Frechet}(k, \lambda)$., the location-zero Frechet distribution.

(h) [harder] Find the CDF of the location-zero Frechet distribution.