Not covere on Anal

Justify T-test, Omnibus F-fest, partial F-test in Linear Regression.

$$\vec{Y} = \vec{X} \vec{\beta} + \vec{\epsilon}$$
 Typical assumption $\vec{\beta} \vec{\epsilon} \sim N_n(\vec{0}, \sigma^2 I_n) \Rightarrow \vec{z} = \vec{\sigma} \vec{\epsilon} \sim N_n(\vec{0}, I_n)$

Using vector densities, the least square estimate of unknown B is:

$$\hat{\vec{\beta}} = (\vec{x}^T \vec{x})^{-1} \vec{\chi}^T \vec{Y} = (\vec{x}^T \vec{x})^{-1} \vec{\chi}^T (\vec{x} \, \beta + \varepsilon) = (\vec{x}^T \vec{x})^{-1} \vec{x}^T \vec{x} \, \beta + (\vec{x}^T \vec{x})^{-1} \vec{x}^T \varepsilon$$

$$= \hat{\vec{\beta}} + (\vec{x}^T \vec{x})^{-1} \vec{\chi}^T \vec{\sigma} \, \vec{\sigma} \, \vec{\varepsilon}$$

$$\hat{\beta} \sim \mathcal{N}(\hat{\beta}(\sigma(X^{T}X)^{T}X^{T})^{T}(\sigma(X^{T}X)^{T}X^{T}))$$

$$= \mathcal{N}(\hat{\beta}, \sigma^{2}(X^{T}X)^{T})$$

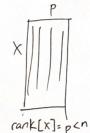
$$= \mathcal{N}(\hat{\beta}, \sigma^{2}(X^{T}X)^{T})$$

$$\frac{\widehat{X}-\mathcal{H}}{\widehat{S}_{n}} \sim N(0,1) \stackrel{=}{\longrightarrow} \widehat{\beta}_{k} - \widehat{\beta}_{k}$$

$$\overline{OV(X^{T}X)^{-1}_{kk}} \sim N(0,1)$$

But we have student's problem, or is unknown.

$$\frac{1}{\sigma^{2}} \underbrace{\vec{\mathcal{E}}^{T}}_{z} \underbrace{\vec{\mathcal{E}}^{T}}_{z} = \frac{1}{\sigma^{2}} \underbrace{\vec{\mathcal{E}}^{T}}_{z} P \underbrace{\vec{\mathcal{E}}^{T}}_{z} + \frac{1}{\sigma^{2}} \underbrace{\vec{\mathcal{E}}^{T}}_{z} (I - P) \underbrace{\vec{\mathcal{E}}}_{z}$$



P= X(XTX) XT orthogonal projection matrix

I-p orthogonal projection anto set of R"

rank[p]=po

rank [I-P] = n-P

$$(X(x^{T}x^{-1})x^{T})(X(x^{T}x)^{-1}x^{T})$$

$$PP = P$$

$$(I-P)(I-P) = P$$