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Let $A \in \mathbb{R}^{m \times n}$, full rank $m \leq n$

$$\vec{X} = A\vec{Z} + \vec{c} \sim N_m(A\vec{0}_n + \vec{c}, AIA^T) = N_m(\vec{c}, \underbrace{AA^T}_{\Sigma})$$

HW $\vec{X} \sim N_n(\vec{\mu}, \Sigma)$ Mahalonobis Distance

$$(\vec{X} - \mu)^T \Sigma^{-1} (\vec{X} - \vec{\mu}) \sim$$

$$\Sigma = AA^T$$

$$\Sigma^{-1} = (A^{-1})^T A^{-1}$$

$$(\vec{X} - \vec{\mu})^T (A^{-1})^T (A^{-1}) (\vec{X} - \vec{\mu})$$

$$= (A^{-1}(\vec{X} - \mu))^T (A^{-1}(\vec{X} - \mu))$$

$$= \vec{Z}^T \vec{Z} \sim \chi^2_n$$

$$(x - \mu) \frac{1}{\sigma} (x - \mu)$$

$$= \left(\frac{x - \mu}{\sigma}\right)^2 = Z^2 \sim \chi^2_1$$

$$\vec{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} \qquad f_{X_2, X_4}(x_2, x_4) = \int_{\mathbb{R}}\int_{\mathbb{R}}\int_{\mathbb{R}} f_{X_1, \ldots, X_5}(x_1, \ldots, x_5)\, dx_1\, dx_3\, dx_5$$

$$\Phi_{\vec{X}}(\vec{t}) = E\left[e^{it_1X_1} e^{it_2X_2} e^{it_3X_3} e^{it_4X_4} e^{it_5X_5}\right]$$

$$\Phi_{\vec{X}} = \left(\begin{bmatrix} 0 \\ t_2 \\ 0 \\ t_4 \\ 0 \end{bmatrix}\right)$$

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Plug in zero for what you don't want.

$= E[e^{it_2X_2} e^{it_4X_4}]$

$= E\left[e^{i[t_2\, t_4]^T \begin{bmatrix} X_2 \\ X_4 \end{bmatrix}}\right] = \phi_{X_2, X_4}(t_2, t_4)$

$\xrightarrow{P_1, P_6} f_{X_2, X_4}(x_2, x_4)$

Let $\vec{X} \sim N_n(\vec{m}, \Sigma)$

What's $X_1 \sim$?

$\Phi_{X_1}(t) = \Phi_{\vec{X}}\left(\begin{bmatrix} t \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right) = e^{i[t, 0, \dots 0]\begin{bmatrix} m_1 \\ \vdots \\ m_n \end{bmatrix} - \frac{1}{2}[t, 0 \dots 0] \Sigma \begin{bmatrix} t \\ 0 \\ \vdots \\ 0 \end{bmatrix}}$

$= e^{itm_1 - \frac{1}{2} - \frac{t^2\sigma^2}{2}} \xrightarrow{P_1} X_1 \sim N(m_1, \sigma_1^2)$

$\longrightarrow X_j \sim N(m_j, \sigma_j^2)$

$t \begin{bmatrix} \sigma_1^2 \\ \sigma_{21} \\ \vdots \\ \sigma_{n1} \end{bmatrix}$