A discrete random variable (r.v) X has probability mass function (PMF) given by $P(X)$:

$$P(x) := P(X=x) \text{ and the r.v is denoted as } X \sim P(X)$$

and cumulative distribution func (CDF) denoted $F(x)$:

$$F(x) := P(X \leq x)$$

and complementary CDF also called survival func:

$$S(x) := P(X > x) = 1 - F(x)$$

The rv has support

$$Supp[X] := \{x : p(x) > 0, x \in \mathbb{R}\}$$

$$\underbrace{|Supp[X]|}_{\text{number of elem in set}} \leq |\mathbb{N}| \text{ i.e finite / countably infinite}$$

sets of this size are called discrete

The support and the PMF are related via the identity:

$$\sum_{x \in Supp[X]} P(x) = 1$$

The most fundem: r.v is the Bernoulli

$$X \sim Bern(p) := p^x (1-p)^{1-x} \Big\} \text{ PMF or } P(X)$$

$$Supp[X] = \{0,1\}$$

What if $x = 7$?

$$P(X=7) = p^7 (1-p)^{-6} = \text{NOT ZERO}$$

Indicator func:

$$\mathbb{1}_A := \begin{cases} 1 & \text{if } A \\ 0 & \text{if } A^c \end{cases}$$

Lets say:

$$X \sim Bern(p) := p^x (1-p)^{1-x} \mathbb{1}_{x \in \{0,1\}}$$

$$= \sum_{x \in \mathbb{R}} P(x) = 1$$

What if $p=1$?

$$X \sim Bern(1) = 1^x (1-1)^{1-x} \mathbb{1}_{x \in \{0,1\}} = \{ 1$$

This is called a "degenerate" rv $X \sim Deg(1)$

$$X \sim Bern(0) = Deg(0) = \{ 0 \text{ w.p } 1$$

Generally $X \sim Deg(c) = \{ c \text{ w.p } 1 = \mathbb{1}_{x=c}$

p is a parameter of the Bernoulli r.v.
What values of p are legal and non degenerate

$$p \in (0,1) \longrightarrow \text{param space of the Bernoulli}$$

If we have more than one r.v $X_1, X_2, .. X_n$
we can group them together in col vect.

$$\vec{X} = [X_1, X_2, .. X_n]^T$$

which has a joint mass func (JMF) defined as

$$P_{\vec{X}}(\vec{x}) = P_{X_1, .., X_n}(X_1, .., X_n)$$

$$\text{s.t} \quad \sum_{\vec{x} \in \mathbb{R}} P_{\vec{X}}(\vec{x}) = 1$$

If $X_1, ..., X_n$ are indep r.v then the JMF can be factored as

$$P_{\vec{X}}(\vec{x}) = P_{X_1}(X_1) P_{X_2}(X_2) \ldots P_{X_n}(X_n)$$

If $X_1, .. X_n$ are identically distr. denoted:

$$X_1 \overset{d}{=} X_2 \overset{d}{=} \ldots = X_n \text{ then}$$

$$P_{X_1}(X) = P_{X_2}(X) = \ldots = P_{X_n}(X) \quad \forall x$$

but this offers no simplification of the JMF unless

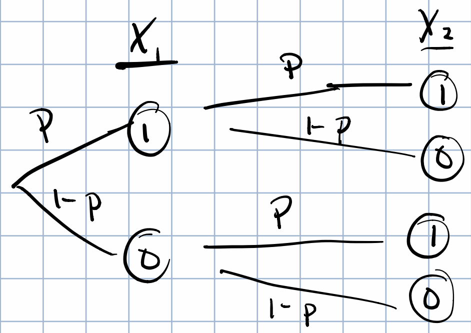
$X_1, .. X_n \overset{iid}{\sim}$ denotes indep and ident distr.

$$P_{\underset{\sim}{X}}(\underset{\sim}{x}) = \prod_{i=1}^{n} P(X_i)$$

Let $X_1, X_2 \overset{iid}{\sim} \text{Bern}(p)$ $\quad T_2 := f(X_1, X_2) = X_1 + X_2$

$$P_{T_2}(t) = \underbrace{P_{X_1}(X_1) * P_{X_2}(X_2)}_{\text{Convolution Operator}}$$

$$\text{Supp}[T_2] = \{0, 1, 2\}$$



$P_{X_1 X_2}(X_1, X_2)$

p^2

$p(1-p)$

$(1-p)\,p$

$(1-p)^2$

Therefore:

$$p^2 + 2p(1-p) + (1-p)^2 = ((p) + (1-p))^2 = 1^2 = 1$$