let $x_1 = x$

$x_2 = v - x \rightarrow v = x_2 + x \Rightarrow dx_2 = dy$

$\rightarrow \quad x_2 = -\infty \rightarrow v = -\infty$

$\rightarrow x_2 = t - x \Rightarrow x = t$

$$f_T(t) = \frac{d}{dt}\left[\quad \right]$$

Leibnitz's Rule

$$\frac{d}{dx}\left[\int_{a(t)}^{b(t)} g(x,y)\,dy\right] = g(x, b(x))\,b'(x) + g(x, g(x))\,g'(x) +$$

$$\int \frac{\partial}{\partial x}\left[g(x,y)\right]dy$$

if the outer derivative is a third variable then

$$\frac{d}{dt}\left[\int_{a(t)}^{b(t)} g(x,y)\,dy\right] = g(x, b(t))\,b'(t) + g(x, a(t))\,g'(t) +$$

$$\int_{a(t)} \frac{\partial}{\partial t}\left[g(x,y)\right]dy$$

$$\frac{d}{dt}\left[\int_{t}^{b(t)} g(x,y)\,dy\right] = g(x, b(t))\,b'(t) + g(x,c)\,\frac{d}{dt}[c]$$

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$\vec{X}$ Continuous rv $\quad P(\vec{X} \in A) = \int \cdots \int_A f_{\vec{X}}(\vec{x})\, dx_1 \cdots dx_k$

Convolution.

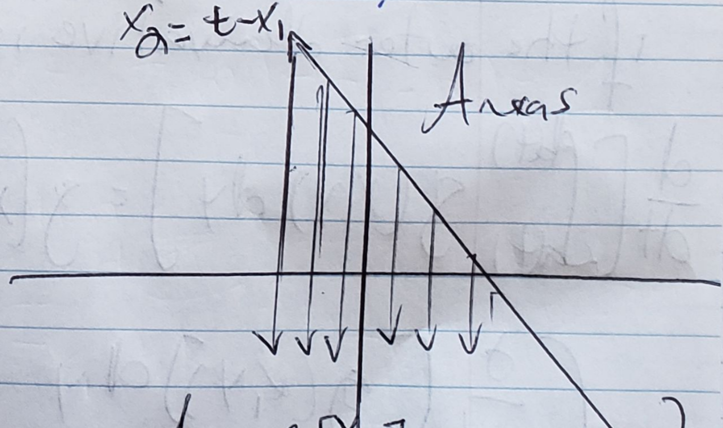
let $T = X_1 + X_2 \sim f_T(t) = ?$

$t = x_1 + x_2$

$x_2 = t - x_1$

Note $f_T(t) = F'(t)$ CDF method.

Usually it is difficult to find the CDF of continuous rv's, so this is not the usual method. The usual method is to use the convolution formula (which we will now derive)



$F_T(t) = P(T \leq t) = P(\vec{X} \in A_t) =$

$\iint_{A_t} f_{X_1, X_2}(x_1, x_2)\, dx_1 dx_2 =$

$\int_{-\infty}^{\infty} \int_{-\infty}^{t - x_1} f_{X_1 X_2}(x_1, x_2)\, dx_2\, dx_1 =$

$A_t := \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_2 \leq t - x_1 \right\}$

$\int_{\mathbb{R}} \int_{-\infty}^{t} f_{X_1 X_2}(x, v - x)\, dx\, dv = \int_{-\infty}^{t} \left(\int_{\mathbb{R}} X_1 X_2 (x, v - x)\, dx \right) dv$