Monday 10/05/20

$$\boxed{T_3 = X_1 + X_2 + X_3 \sim f_{T_3}(t) = ?}$$

$$= \int f^{old}_{T_2}(x) f^{old}_{X_3}(t-x) \mathbb{1}_{t-x \in supp[X_3]} dx$$

$$= \int_0^\infty x\lambda^2 e^{-\lambda x} \underbrace{\lambda e^{-\lambda(t-x)}}_{e^{-\lambda t} e^{\lambda x}} \mathbb{1}_{t-x \in [0,\infty)} dx$$

$$= \lambda^3 e^{-\lambda t} \int_0^\infty x \mathbb{1}_{x \le t} dx = \lambda^3 e^{-\lambda t} \int_0^t x dx \, \mathbb{1}_{t \in [0,\infty)}$$

$$\boxed{= \frac{t^2}{2} \lambda^3 e^{-\lambda t} \mathbb{1}_{t \in [0,\infty)} = \text{Erlang}(3, \lambda)}$$

$$T_4 = X_1 + X_2 + X_3 + X_4 = T_3 + X_4 \sim f_{T_4}(t) = ?$$

$$f_{T_4}(t) = \int_{supp[T_3]} f^{old}_{T_3}(x) f^{old}_{X_4}(t-x) \mathbb{1}_{t-x \in supp[X_4]} dx$$

$$= \int_0^\infty \frac{x^2}{2} \lambda^3 e^{-\lambda x} \lambda e^{-\lambda(t-x)} \mathbb{1}_{t-x \in (0,\infty)} dx$$

$= \frac{1}{2} \lambda^4 e^{-\lambda t} \int_0^\infty x^2 \mathbb{1}_{x \le t} \, dx$

$$\boxed{= \frac{1}{2 \cdot 3} t^3 \lambda^4 e^{-\lambda t} \mathbb{1}_{t \in [0,\infty)} = \text{Erlang}(4, \lambda)}$$

$$T_k = \sum_{i=1}^{k} X_i \sim \text{Erlang}(k, \lambda) = \frac{t^{k-1} \lambda^k e^{-\lambda t}}{(k-1)!} \mathbb{1}_{t \in [0,\infty)}$$

$\text{Supp}[T_k] = [0, \infty)$, $\lambda \in (0, \infty)$ $k \in \mathbb{N}$

$\text{Exp}(\lambda) \xRightarrow{\text{odd}} \text{Erlang}(k, \lambda)$

$\Updownarrow$ conceptually analogous

$\text{Geom}(p) \xRightarrow{\text{odd}} \text{NegBin}(k, p)$

Let's do some pure math. want to define the gamma family of function. Beginning with the gamma function for x non neg:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, dt = \underbrace{\int_0^a t^{x-1} e^{-t}}_{\gamma(x,a)} \underbrace{\int_a^\infty t^{x-1} e^{-t} dt}_{\Gamma(x,a)}$$

Lower Incomplete gamma functn

Upper " " " "