Tuesd - 09-29-20

Difference formula. $D = X_1 - X_2 \sim$ for (d) = ?

$D = \underbrace{X_1}_{X} + \underbrace{(-X_2)}_{Y} \sim ?$

Pmf of Y = $\boxed{P_Y(y) = \frac{e^{-\lambda} \lambda^{-y}}{(-y)!}}$

Convolution $T = X_1 + X_2 \sim$ Poisson (2λ)

Support $[X] = \{0, 1, 2 \ldots\}$

Support $[Y] = \{\ldots -2, -1, 0\}$

Pmf of the difference.

$P_D(d) = \sum_{x \in Sup(X)} P_X^{old}(x) P_Y^{old}(d-x) \mathbb{1}_{d-x \in Supp[Y]}$

Pmf for Poisson is

$\underbrace{\sum_{x \in \{0,1,2\ldots\}} \frac{e^{-\lambda} \lambda^x}{x!} \frac{e^{-\lambda} \lambda^{-(d-x)}}{(-(d-x))!}}_{\text{all this is a substitution in } P\text{ old}}$ $\mathbb{1}_{d-x \in \{\ldots -1, 0\}}$

$(x-d)$

$(x-d)$

$x \in \{d+1, d+2, \ldots\}$

$x-d \in \{0, 1, \ldots\}$

take $(-)$

$$= e^{-2\lambda} \sum_{x \in \{0,1,\dots\}} \frac{\lambda^x}{x!} \frac{\lambda^{x-d}}{(x-d)!} 1_{x \in \{d, d+1, \dots\}}$$

Let $x' = x - d \Rightarrow x = x' + d$

let $d' = -d = |d|$

$$e^{-2\lambda} \begin{cases} d > 0 \quad \sum_{x \in \{d, d+1, \dots\}} \frac{\lambda^{2x-d}}{x!(x-d)!} = \sum_{x' \in \{0,1,\dots\}} \frac{\lambda^{2(x'+d)-d}}{(x'+d)!(x'+d-d)!} = \sum_{x' \in \{0,1,\dots\}} \frac{\lambda^{2x'+d}}{(x'+d)!x'!} \\ d \leq 0 \quad \sum_{x \in \{0,1,\dots\}} \frac{\lambda^{2x-d}}{x!(x-d)!} = \sum_{x \in \{0,1,\dots\}} \frac{\lambda^{2x+d'}}{x!(x+d')!} = \sum_{x \in \{0,1,\dots\}} \frac{\lambda^{2x+|d|}}{x!(x+|d|)!} \end{cases}$$

formula of Modified Bessel function of the first kind (comes up in diff eq)

$$I_{|d|}(2\lambda) = \sum_{x=0}^{\infty} \frac{\left(\frac{2\lambda}{2}\right)^{2x+|d|}}{x!\,(x+|d|)!}$$

$$\boxed{= e^{-2\lambda} I_{|d|}(2\lambda) 1_{d \in \mathbb{Z}}} = \text{Skellam}(d, \lambda)$$

distribution of

1946

lands

It's used to model pointspreads in sports games, photo noise, etc.