A discrete random variable (rv) has probability mass function (PMF)

$p(x) := \mathbb{P}(X = x)$ and the r.v $X \sim p(x)$ where x is the "realized value" X, x

The cumulative distribution function (CDF) is

$F(x) := \mathbb{P}(X \le x)$

and complementary CDF or "survival function" is

$S(x) := \mathbb{P}(X > x) = 1 - F(x)$

This rv has "support" given by Supp[X] $:= \{x : p(x) > 0, x \in \mathbb{R}\}$

$|\text{Supp}[X]| \le |\mathbb{N}|$ countably infinite at most.

$\uparrow$ # elements in a set

Sets this size are called "discrete" sets.

The support and the PMF are related by the following identity:

$$\sum_{x \in \text{Supp}[X]} p(x) = 1$$

The most "fundamental" rv is the Bernoulli:

$X \sim \text{Bern}(p) := \underbrace{p^x (1-p)^{1-x}}_{p(x)}$ with $\underbrace{\text{Supp}[X] = \{0, 1\}}$

$p(7) = p^7 (1-p)^{-6}$

Let's define the "indicator function"

$\mathbb{1}_A := \begin{cases} 1 & \text{if } A \\ 0 & \text{if } A^c \end{cases} \Rightarrow X \sim \text{Bern}(p) := \underbrace{\overbrace{p^x (1-p)^{1-x}}^{p^{old}(x)} \mathbb{1}_{x \in \{0,1\}}}_{p(x)}$

$\Rightarrow \sum_{x \in \mathbb{R}} p(x) = 1$

What if p = 1?

$X \sim \text{Bern}(1) = 1^x 0^{1-x} \mathbb{1}_{x \in \{0,1\}} = \{1 \text{ w.p. } 1 = \mathbb{1}_{x=1}$

$\Downarrow$

$X \sim \text{Deg}(1) = \{1 \text{ wp } 1$ $\quad X \sim \text{Deg}(c) := \mathbb{1}_{x=c}$

$X \sim \text{Bern}(0) = \text{Deg}(0)$

The convention in this class is that parameter values (p is the parameter of the Bernoulli) that yield degenerate rv's are not part of the legal "parameter space".

$p \in (0, 1)$

If we have more than one rv X_1, X_2, ..., X_n we can group them together in a column vector:

$$\vec{X} := [X_1 \ X_2 \ \dots \ X_n]^T$$

and then define the "joint mass function" (JMF) as

$p_{\vec{X}}(\vec{x}) = p_{X_1, \dots, X_n}(x_1, \dots, x_n)$ valid for $\vec{x} \in \mathbb{R}^n$ and $\sum_{\vec{x} \in \mathbb{R}^n} p(\vec{x}) = 1$

If X_1, X_2, ..., X_n are independent, then

$p_{\vec{X}}(\vec{x}) = p_{X_1}(x_1) \, p_{X_2}(x_2) \cdot \ldots \cdot p_{X_n}(x_n) = \prod_{i=1}^n p_{X_i}(x_i)$ "multiplication rule"

If $X_1 \overset{d}{=} X_2 \overset{d}{=} \dots \overset{d}{=} X_n$

$\Updownarrow$

$\forall x \quad p_{X_1}(x) = p_{X_2}(x) = \dots = p_{X_n}(x)$

this denotes "equal in distribution" meaning their PMF's are the same. However, his offers no simplification of the JMF unless...

$X_1, X_2, \dots, X_n \overset{iid}{\sim}$

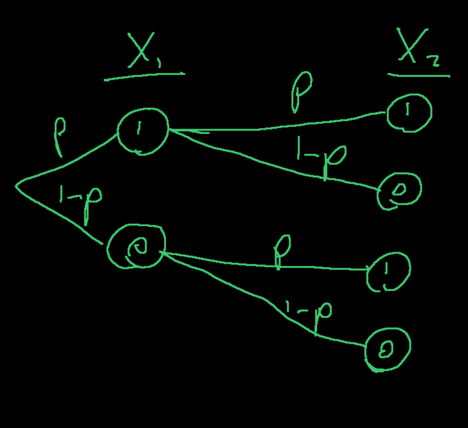
that means independent and identically distributed

$\Rightarrow p_{\vec{X}}(\vec{x}) = \prod_{i=1}^n p(x_i)$

Let $X_1, X_2 \overset{iid}{\sim} \text{Bern}(p)$, Let $T_2 = f(X_1, X_2) = X_1 + X_2 \sim p_T(t)$ $\overset{?}{=}$

Denoted $p_{T_2}(t) = p_{X_1}(x) * p_{X_2}(x)$ $\leftarrow$ Convolution Operator

$\text{Supp}[T_2] = \{0, 1, 2\}$



$p_{X_1, X_2}(x_1, x_2)$

p^2

$p(1-p)$

$(1-p)p$

$(1-p)^2$