$a_1 \mu_1 + \cdots + a_k \mu_k = \vec{a}^T \vec{\mu}$

$\boxed{\text{Wed } 09/16/20}$

Let $A \in \mathbb{R}^{L \times k}$ matrix of Constants $E[A\vec{X}] =$ $\vec{a}_i$ row i of matrix A

$$\begin{bmatrix} E[a_{11}X_1 + a_{12}X_2 + \cdots + a_{1k}X_k] \\ E[a_{21}X_1 + a_{22}X_2 + \cdots + a_{2k}X_k] \\ \vdots \\ E[a_{L1}X_1 + a_{L2}X_2 + \cdots + a_{Lk}X_k] \end{bmatrix} = \begin{bmatrix} E[\vec{a}_1 \cdot \vec{X}] \\ E[\vec{a}_2 \cdot \vec{X}] \\ \vdots \\ E[\vec{a}_L \cdot \vec{X}] \end{bmatrix} = \begin{bmatrix} \vec{a}_1 \vec{\mu} \\ \vec{a}_2 \vec{\mu} \\ \vdots \\ \vec{a}_L \vec{\mu} \end{bmatrix} = A\vec{\mu}$$

$$Var[\vec{a}^T \vec{x}] = Var[a_1 X_1 + \cdots + a_k X_k] = \sum_{i=1}^{k}\sum_{j=1}^{k} Cov[Y_i, Y_j] = \sum\sum Cov[a_i X_i, a_j X_j]$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{k} a_i a_j \sigma_{ij} = \vec{a}^T \underbrace{\Sigma}_{Var[X]} \vec{a}$$ this is called a quadratic form

$(1\times k)(k\times k)(k\times 1) = $ Scaler.

Let $V \in \mathbb{R}^{k\times k}$, $\vec{a} \in \mathbb{R}^{k\times 1}$ $\vec{a}^T V \vec{a} = \vec{a}^T \begin{bmatrix} a_1 v_{11} + \cdots + a_k v_{1k} \\ a_1 v_{21} + \cdots + a_k v_{2k} \\ \vdots \\ a_1 v_{k1} + \cdots + a_k v_{kk} \end{bmatrix} =$

$a_1 a_1 v_{11} + \cdots a_1 a_k v_{1k} + a_2 a_1 v_{21} + \cdots + a_2 a_k v_{2k} + \cdots + a_k a_1 v_{k1} + \cdots + a_k a_k v_{kk} =$

$$\sum_{i=1}^{k}\sum_{j=1}^{k} a_i a_j v_{ij}$$

This is an application in finance. Imagine X_1, ..., X_k are financial assets (e.g. different stocks). Each has mean return mu_i and each pair have covariance sigma_ij. Let w-vector be a vector of weight where each component is the percentage you put into each of these assets. Thus The entries of sum to 1

$F = \vec{w}^T \vec{x}$, $\vec{w}^T \vec{1} = 1$, $E[\vec{x}] = \mu$, $Var[\vec{x}] = \Sigma$.

$E[F] = E[\vec{w}^T \vec{x}] = \vec{w}^T \vec{\mu} = \mu_F$, $Var[F] = Var[\vec{w}^T \vec{x}] = \vec{w}^T \Sigma \vec{w}$

Goal is to pick mu_F and minimize it's variance by Computing the w_vector optimally.