$g: \mathbb{R}^2 \longrightarrow \mathbb{R}^n$ and 1:1, X, Y continuous rv vectors of dim $n s.t Y = g(\overline{X})$. Given frick), find fritis. recall what a multidimensional function is: h=q-1. $Y_1 = g_1(X_1, ..., X_n), \qquad X_1 = b_1(Y_1, ..., Y_n),$ 4. - g. (x,,,,xn), [3h; 5. X, = b, (4,,...,4n), $y_n = g_n(x_1, \dots, x_n), \qquad x_n = h_n(y_1, \dots, y_n)$ Using multivariable calculus, you can show that Fy(g) = fx(h(g)) | Jn(g) | the Bocobian determinant

Let's verily the convolution formula via this general change - of - whole formula.

Y = T = X, + X, ~ [- (+)

Recipe!
(i) Rind a "clever" g so that
(a) We can find an h.

(3) compute 7.
(4) Compute the multidimensional of and
(4) Compute It multidimensional change of variables formula. (5) Integrate out the "nuisance dimension"
(5) Onlage - 1 11 "
Enlegiate out the nuisance dimension
dimension.
$0 \ Y_1 = X_1 + X_2 = g_1(x_1, x_2), \ Y_2 = X_2 - g_2(x_1, x_2)$
(2) $X_1 = Y_1 - X_2 = Y_1 - Y_2 = h_1(Y_1, Y_2), X_2 = Y_2 = h_2(Y_1, Y_2)$
3) In = det [sh. sh. sh.] = det [1 -1] = 1.1 - 1.0 = 1
3) $J_h = det \begin{bmatrix} \frac{\partial h_1}{\partial y_1} & \frac{\partial h_1}{\partial y_2} \\ \frac{\partial h_2}{\partial y_1} & \frac{\partial h_2}{\partial y_2} \end{bmatrix} = det \begin{bmatrix} 1 & -1 \\ -1 & -1 & 0 = 1 \end{bmatrix}$
(h) fy(y) = fx (h(y)) Jh = fx (4,-42,42) 1)
$= \int_{X} (Y_1 - Y_2, Y_2)$
\times
(5) $f_{\gamma}(t) = \int_{\gamma_1} (y_1) = \int_{\gamma_2} \int_{\gamma_3} (y_1 - y_2, y_3) dy = \int_{\gamma_3} \int_{\gamma_4} (t - u, u) du$ exercy exactly the general convolution famula $f_{\gamma_1, \chi_2} = \int_{\gamma_3} \int_{\gamma_4} (t - u) \int_{\gamma_4} \int_{\gamma_5} $
enery exactly the control constitutions
formula
18 X. X. Indea
$= \int_{\Omega} f_{\chi_1}(t-u) f_{\chi_2}(u) du = \int_{\Omega} f(t-u) f(u) du$
 JIK. JIK.
(cold)
Supp[x] fill du. Supp[x] fill du.
f (f -u) 1 1 2 C 2 F 2 f (t) du
J SUPPLXJ 1-11 G SUPPLXJ

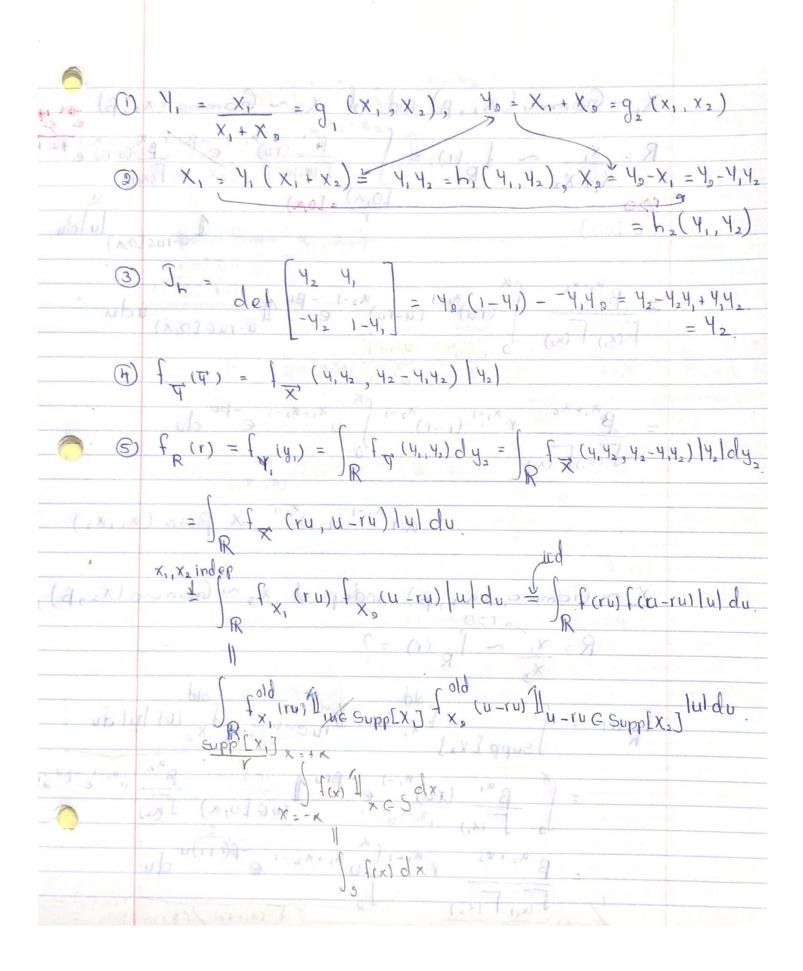
$$R = \frac{x_1}{x_2} \sim R(r) = \frac{1}{2}$$

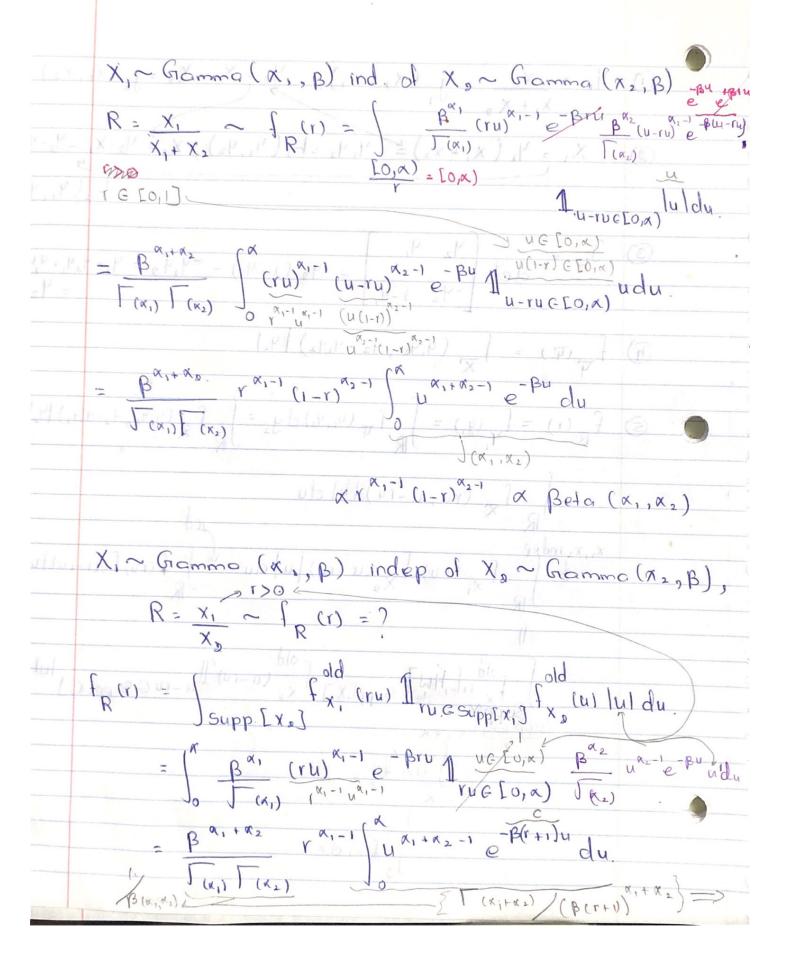
$$V_1 = \frac{x_1}{x_3} = \frac{1}{9}(x_1, x_6) \qquad V_2 = x_3 = \frac{1}{9}(x_1, x_3)$$

$$X_1 = V_1 x_2 = V_1 V_2 = h_1(V_1, V_2) \qquad X_3 = V_2 = h_2(V_1, V_2)$$

$$T_1 \sim \det \begin{bmatrix} V_1 & V_1 \\ 0 & 1 \end{bmatrix} = V_2$$

$$F_1(r_1) = \frac{1}{7}(V_1, V_2) = \frac{1}{7}(V_1, V_1, V_2) = \frac{1}{7}(V_1, V_1, V_2) = \frac{1}{7}(V_1, V_1, V_2) = \frac{1$$





$$\Rightarrow \beta^{\kappa_1 + \kappa_2} (r+1)^{\alpha_1 + \alpha_2}$$

$$= \frac{1}{\beta(\alpha_1, \alpha_2)} \left(\frac{r}{r+1}\right)^{\alpha_1 + \alpha_2} \underline{1}_{r>0}$$

$$\beta \text{ Betc. Prime}(\alpha_1, \alpha_2)$$