Wed, 28th October 2020

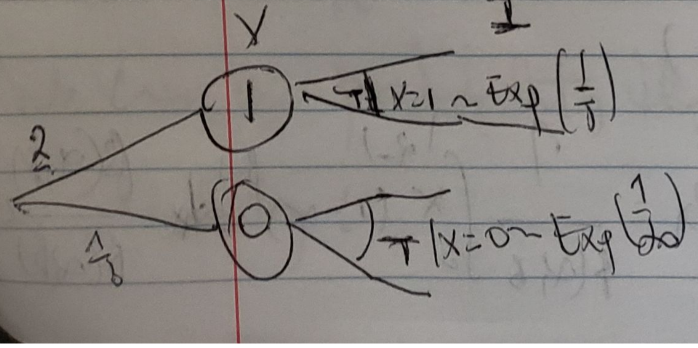
Mixture and Compound distribution

e.g. 1/3 of the time, you get bad internet traffic and your down speed are $T \sim Exp(1/20)$ i.e $E(T) = 20$ and 2/3 of the time you have good internet traffic and your download speeds are $T \sim Exp(1/5)$ i.e $E(T) = 5$. What is the distribution of T "overall"?

Let $X \sim Bern(2/3)$, a rv modeling traffic. if $X = 1$ then we have good traffic and if $X = 0$, we have bad traffic. So now we have $T|X=1 \sim Exp(1/5)$ and $T|X=0 \sim Exp(1/20)$.

Now we essentially use marginalization to get T "unconditional" (meaning overall).

first let's draw a tree:



Marginalization

$$h(x) = \int_{\mathbb{R}} h(x,y)\,dy$$

or

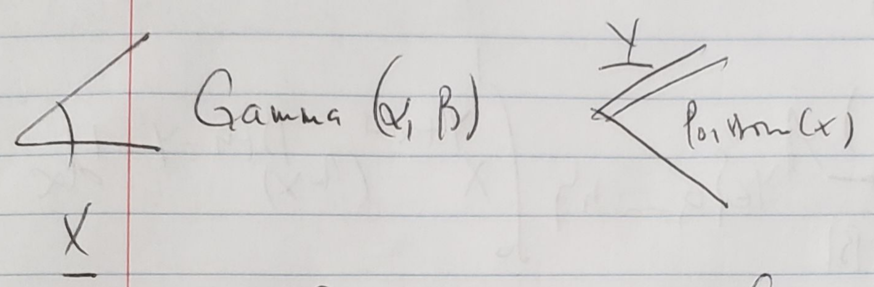
$$h(x) = \sum_y h(x,y)$$

$$F_T(t) = \sum f_{T,X}(t,x) = \sum_{x \in Supp[X]} f_{T|X}(t,x)\, p_X(x) = \sum_{x \in \{0,1\}} f_{T|X}(t,x)\, p_X(x)$$

$$= f_{T|X}(t,0)\, p_X(0) + f_{T|X}(t,1)\, p_X(1) = \frac{1}{20}t\,\frac{1}{3} + \frac{1}{5}e^{-\frac{1}{5}t}\,\frac{2}{3}$$

This was a our first "mixture model"

Pr 6-15? Let $Y|X=x \sim \text{Poisson}(x)$ and $X \sim (\alpha, \beta)$ $Y \sim$?



$$p_Y(y) = \int_{Supp[X]} p_{Y,X}(y,x)\, dx = \int_{Supp[X]} p_{Y|X}(y,x)\, dx = \int_0^\infty \frac{x^y e^{-x}}{y!} \mathbb{1}_{y \in \{0,1,\ldots\}}$$

$$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}\, dx = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{y!} \mathbb{1}_{y \in \mathbb{N}_0} \int_0^\infty x^{y+\alpha-1} e^{-(\beta+1)x}\, dx$$

$$= \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{y!} \frac{\Gamma(y+\alpha)}{(\beta+1)^{y+\alpha}} \mathbb{1}_{y \in \mathbb{N}_0} \overset{HW}{=} \ldots \propto \text{ExtNegBin}\left(\alpha, \frac{\beta}{1+\beta}\right)$$

This is a "more flexible" count distribution than the poisson.