$F^{-1}(q) \geq x \Rightarrow \underbrace{x = F^{-1}(q)}_{\text{quantile function}} = Q[X, q)$

$X \sim Exp(\lambda) = \lambda e^{-\lambda x} \Rightarrow F(x) = 1 - e^{-\lambda x}$. Find the q mile function, $F^{-1}(q) =$

$q = 1 - e^{-\lambda x} \Rightarrow 1 - q = e^{-\lambda y}$

$\Rightarrow \ln(1-q) = -\lambda x \Rightarrow x = -\frac{1}{\lambda} \ln(1-q)$

$\Rightarrow x = \frac{1}{\lambda} \ln\left(\frac{1}{1-q}\right) = F^{-1}(q)$

$X \sim Exp(1) \Rightarrow Med[X] = F^{-1}(0.5) = \ln 2$

$Q[X, Q] = \ln(5)$.

It's actually rare to have a quantile function in closed form since it's rare to even have a cdf in closed form e.g.

$X \sim Erlang(k, \lambda)$, $F(x) = P(k, \lambda x)$. $Q[X, q)$ can be found by solving for x in the following equation: $q = P(k, \lambda x)$

$X \sim Exp(\lambda)$, $Y = k e^{x = g(x)}$, $k \geq 0$. Find $f_Y(y)$

$\frac{Y}{k} = e^{x} \Rightarrow = x \ln\left(\frac{Y}{k}\right) = \ln(y) - \ln(k) = g^{-1}(y)$

Monday 10/14/20 Lect 11.

$X \sim$ Logistic $(0,1) \approx N(0,1)$ Except it has thicker tails

stand Logistic $E(X) = 0$, $SD[X] = \frac{\pi}{\sqrt{3}}$

$Y = \mu + \sigma X \sim$ Logistic $(\mu, \sigma) := f_Y(y) = \frac{1}{\sigma} \frac{e^{\frac{y-\mu}{\sigma}}}{\left(e^{\frac{y-\mu}{\sigma}} + 1\right)^2}$

$\mu \in \mathbb{R}, \sigma > 0$

why is this called the "logistic distribution"?

THEre's a function called "the logistic function" and it has 3 parameters: L (maximum value), k (steepness), mu (center).

$$\ell(x) := \frac{L}{1+e^{-k(x-\mu)}} \underset{\uparrow}{=} \frac{1}{1+e^{-x}} = \frac{e^x}{1+e^x}$$ the standard Logistic function

if $L=1, k=1, \mu=0$

$Y \sim$ Logistic $(0,1) = \frac{e^y}{(1+e^y)^2}$

$$F_Y(y) = P(Y \le y) = \int_{-\infty}^{y} \frac{e^t}{(1+e^t)^2} dt = \int_1^{1+e^y} \frac{u-1}{u^2} \frac{1}{u-1} du = \left[-\frac{1}{u}\right]_1^{1+e^y} = $$

let $= n-1$

let $u = 1+e^t \Rightarrow \frac{du}{dt} = e^t \Rightarrow dt = e^{-t} du = \frac{1}{u-1} du$, $t = -\infty \Rightarrow u = 1$

$t = y \Rightarrow u = 1+e^y$