$= k\lambda^k y^{k-1} e^{-(\lambda y)^k} \mathbb{1}_{y \geq 0} = k\lambda(\lambda y)^{k-1} e^{-(\lambda y)^k} \mathbb{1}_{y \geq 0} = \text{Weib}(k, \lambda)$

Note Weibull$(1, \lambda) = (1)\lambda (1) y^{(1)-1} \mathbb{1}_{y \geq 0} = \lambda e^{-\lambda y} \mathbb{1}_{y \geq 0} = \text{Exp}(\lambda)$

$k$ is really cool... this is the main property:

$k=1$ $P(Y \geq y+c \mid Y \geq c) = P(Y \geq y)$. e.g. $y=3$, $c=14$

Memorylessness. $P(Y \geq 17 \mid Y \geq 14) = P(Y \geq 3)$

$k > 1$ $P(Y \geq y+c \mid Y \geq c) < P(Y \geq y) \Rightarrow$ survival less likely as time goes on

$k < 1$ $P(Y \geq y+c \mid Y \geq c) > P(Y > y) \Rightarrow$ survival more likely as time goes on.

prove this in HW.

order statistics (p160 in the book)

Let $X_1, X_2, \ldots X_n$ be a collection of continuous rv's and let

$X(1), X(2), \ldots X(n)$ be their "order statistics" defined as:

$X_{(1)} = \min\{X_1, \ldots X_n\}$

$X_{(n)} = \max\{X_1, \ldots X_n\}$

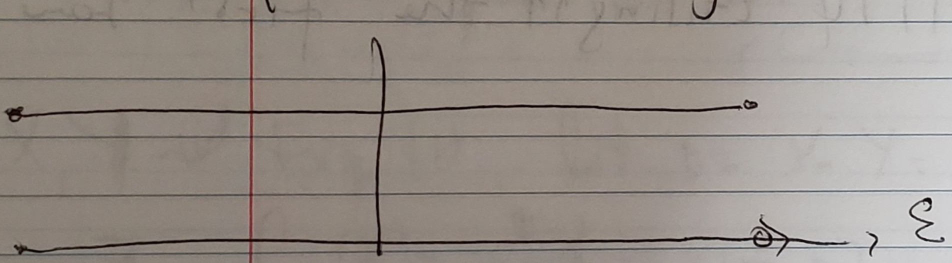
$X_k = k^{th}$ Largest $\{X_1, \ldots X_n\}$

$R = X(n) - X(j)$ "Range"

We want to find the CDF and pdf of the order statistics. We'll start by looking at the CDF of

# Monday October 19th 2020

1774: First Law of errors". Imagine you're trying to measure something a const quantity $\nu$, but your measurements have random error, epsilon, so your measurement M is a rv looking like: $M = \nu + \text{epsilon}$. So what is a good model for the error (epsilon)? It makes sense for $E(\text{epsilon}) = 0$, $\text{Med}[\text{epsilon}] = 0$ and symmetric. How about: $\sim U(-1, 1)$



It also makes sense for larger errors (in magnitude) to be less probable than smaller errors $\Rightarrow \forall \varepsilon > 0,\ f'(\varepsilon) < 0$

$$\forall \varepsilon > 0 \quad f''(\varepsilon) = f'(\varepsilon) \underset{\text{solve}}{\Rightarrow} f(\varepsilon) = Ce^{-d\varepsilon} \Rightarrow \text{Laplace}(0,1)$$

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$X \sim \text{Exp}(1) = e^{-x} \mathbb{1}_{x \geq 0}$. Let $Y = \frac{1}{\lambda} X^{\frac{1}{k}} = g(x)$ s.t. $\lambda, k > 0$

$Y \sim f_Y(y) = ?$ Inverse function $\lambda y = x^{\frac{1}{k}} \Rightarrow x = \lambda^k y^k = g^{-1}(y)$

$$\left|\frac{d}{dy}\left[g^{-1}(y)\right]\right| = \left|\frac{d}{dy}\left[\lambda^k y^k\right]\right| = \left|k\lambda^k y^{k-1}\right| = k\lambda^k y^{k-1}$$

$$f_Y(y) = f_X(g^{-1}(y)) \left|\frac{d}{dy}\left[g^{-1}(y)\right]\right| = e^{-(\lambda y)^k} \mathbb{1}_{\lambda^k y^k \geq 0} \cdot k\lambda^k y^{k-1}$$