MATH 368/621 Fall 2020 (3 credits)
Course Syllabus

Adam Kapelner, Ph.D.

Queens College, City University of New York

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Instructor	Professor Adam Kapelner
Contact	`kapelner@qc.cuny.edu` and at `#discussions` in our slack workspace
Section A Time / Loc	Monday and Wednesday 3:10-4:25PM on zoom
Section B Time / Loc	Monday and Wednesday 7-8:15PM on zoom
My Office Hours / Loc	Monday and Wednesday 4:25-4:55PM on zoom
TA	Tzipora Horowitz
TA Office Hours / Loc	Monday and Wednesday 6:20-6:50PM on zoom **(no email or slack)**
Course Homepage	https://github.com/kapelner/QC_Math_621_Fall_2020

Course Overview

MATH 368/621 is an introduction to the intermediate concepts and techniques of probability. Moreover it is a course on *distribution theory* and contains derivations from scratch of ≈30 of the most commonly applied and studied probability models, and those models that are found when employing statistical tests. Thus, this class derives much of the theory behind statistics and data science. I plan to follow the curriculum when I taught the class previously and the homepage of the previous course can be found here and here. Lectures and their topics with time estimates are below:

Lec 1 [10min] Review of discrete random variables (rvs), support; [10min] the Bernoulli rv, parameter space, degenerate rv, indicator functions; [10min] old-style probability mass functions (PMFs), new-style PMFs using the indicator function; [10min] cumulative distribution functions (CDFs), survival functions, [10min] vector rvs, joint mass functions (JMFs), independence, identical distributedness, iid; [25min] sum of two Bernoulli rvs, convolution operator, tree diagram for two rvs, conditional probability, marginal probability

Lec 2 [10min] Plotting the PMF of the convolution; [20min] discrete convolution formulas (general, independent rvs, iid rvs, old and new style), convolution support; [5min] visualizing convolutions as passing PMFs "through each other"; [5min] defining combinatorial terms with indicator functions; [35min] Binomial rv with PMF derivation, Pascal's identity, Vandermonde's identity

Lec 3 [20min] Sequences of rv's, derivation of the geometric rv as a "waiting time" or "survival" distribution; [30min] convolution of geometric rvs, the negative binomial rv; [25min] derivation of the Poisson rv, convolution of Poisson rvs

Lec 4 [30min] Strategy to compute $\mathbb{P}(X > Y)$ for two rvs X, Y, the sum reindexing trick, geometric series; [25min] multinomial rv, multichoose notation; [20min] proof that multinomial's marginals are Binomial

Lec 5 [30min] Proof that conditional multinomials are multinomial, indicator functions for undefinedness; [25min] review of expectation, variance, standard deviation, covariance of two rvs, covariance rules; [5min] expectation of vector rvs; [15min] variance-covariance matrix (varcov)

Lec 6 [5min] rules of expectation of vector rvs; [15min] rules of varcov and quadratic forms; [5min] Markowitz optimal portfolio theory application; [25] expectation and varcov of Multinomial rv; [5min] Uniform discrete rv; [15min] The negative of a discrete rv

Lec 7 [30min] derivation of the Skellam rv as the difference of two poissons; [10min] Conditional-on-sum calculation; [30min] Derivation of exponential rv from geometric via the CDF, non-existence of PMF, review of continuous rvs and the PDF

Lec 8 [35min] derivation of convolution formulas for continuous rvs; [35min] continuous uniform rv, convolution of two continuous uniform rvs

Lec 9 [25min] convolution of exponential rvs to derive the Erlang rv; [30min] gamma function, incomplete gamma functions, regularized incomplete gamma function, gamma function properties and identities; [20min] CDF of Erlang, CDF of Poisson

Lec 10 [15min] Poisson Process; [10min] Gamma rv and Extended Negative Binomial rv; [15min] transformations of discrete rvs to derive PMFs; [30min] transformations of continuous rvs to derive CDFs and PDFs; [10min] shift and scale PDF formulas for continuous rvs; [10min] derivation of the Logistic rv

Lec 11 [10min] logistic function and Logistic rv CDF; [35min] quantiles, percentiles and the median, quantile function for continous rvs; [20min] derivation of the Pareto rv PDF, CDF and quantile function; [15min] derivation of the Laplace rv

Lec 12 [10min] error distributions; [15min] derivation of the Weibull rv, memorylessness and the Weibull modulus; [50min] order statistics, CDF/PDF of minimum and maximum, CDF of arbitrary order statistics

Lec 13 [15min] derivation of the PDF of arbitrary order statistics; [15min] order statistics of iid Uniform rvs to derive the Beta rv; [15min] kernels of PMFs/PDFs; [15min] proof that convolution of Gamma rvs is a Gamma rv; [15min] the beta function, incomplete beta function, regularized beta function, the Gamma-Beta function identity, the CDF of the Beta rv

Lec 14 [35min] Multivariate transformations of rvs, Jacobian determinants; [10min] PDF of ratio of rv formulas; [10min] PDF of proportion of sum of rv formulas; [10min] proof that the proportion of sum of two Gamma rvs is a Beta rv; [10min] proof that the ratio of two Gamma rvs is a BetaPrime rv

Lec 15 [35min] mixture and compound rvs; [10min] derivation that the Poisson rv compounded with a Gamma rv is a ExtendedNegativeBinomial rv; [15min] derivation that the Binomial rv compounded with a Beta rv is the BetaBinomial rv; [15min] review of complex numbers, proof of Euler's famous formula relating 0, 1, e, i and π.

Lec 16 [20min] basics of Fourier transform / analysis and Fourier inverse transform / synthesis, L1 integrable functions, statement of the Fourier inversion theorem, demo with singing voices; [40min] definition of characteristic functions (chfs) and properties 0, 1, ..., 8, definition of convergence in distribution for Levy's Continuity theorem; [5min] definition of moment generating functions (mgfs) and properties 0, 1, . . . , 4; [10min] derivation of the chf for the Gamma rv and proof that two Gamma rvs convolved is a Gamma rv

Lec 17 [45min] Proof of the central limit theorem (CLT), derivation of the PDF of the standard Normal rv; [15min] derivation of the general Normal rv, its moments, its chf; [10min] proof that the convolution of two independent Normal rvs is a Normal rv; [5min] derivation of the LogNormal rv and an application in portfolio theory

Lec 18 [25min] derivation of the χ^2_n rv and its equivalence to a Gamma rv, derivation that scaled Gamma rvs are Gamma rvs, derivation of the χ_n rv; [20min] derivation of the F rv as the ratio of scaled χ^2_n rvs; [20min] derivation of Student's T rv as the ratio of a Normal rv to a scaled χ rv; [10min] derivation of the standard and general Cauchy rv

Lec 19 [5min] Proof that the expectation of the Cauchy rv doesn't exist; [10min] the physicist's derivation of the Cauchy rv; [15min] derivation that the scaled sample variance is a χ^2_n rv; [15min] quadratic forms embedded in χ^2_n; [15min] Cochran's theorem; [15min] the independent of sample variance and sample average in the case of iid normal rvs

Lec 20 [5min] Derivation of that the t statistic is indeed distributed as Student's T rv; [10min] derivation of the standard MultivariateNormal rv; [20min] derivation of the general MultivariateNormal rv; [10min] multivariate chfs; [10min] chf of the MultivariateNormal rv; [10min] proof that the shifted and scaled MultivariateNormal rv is another MultivariateNormal rv; [10min] Mahalanobis distance

Lec 21 [10min] derivation of multivariate chf property that finds conditional distributions; [55min] derivation of the Markov tail bound and its corrollaries including Chebyshev's tail bound and Chernoff's tail bound; [10min] tail bounds for the Exponential rv

Lec 22 [20min] derivation of the Cauchy-Schwartz inequality and proof that correlation is bounded between -1 and +1; [20min] Jensen's inequality; [25min] convergence in distribution, proofs for PMF and CDF convergences; [10min] Convergence in probability to a constant

Lec 23 [15min] weak weak law of large numbers (WLLN); [40min] convergence in law, proof that convergence in law implies convergence in probability; [10min] law of iterated expectation; [10min] law of total variance

This is more of a typical mathematics theory course than the rest of the data science series. But we will still attempt to keep our eye on developing ideas and concepts for helping to make decisions in the real world. Thus we may make limited use of computation using the `R` statistical language.

Prerequisites

MATH 241 (basic probability), 201 (multivariable calculus) and 231 (linear algebra) or equivalents. I expect a 241 class that covers more or less what I cover in 241. See the course homepage for links under "prerequisite review". The multivariable calculus and linear algebra we will use I will try to review in class.

The 621 Section

You are the students taking this course as part of a masters degree. Thus, there will be *extra* homework problems for you and you will be graded on a separate curve.

Course Materials

Textbook: Introduction to Probability Theory by Hoel, Port & Stone. This book is out of print but you can buy it used on Amazon for ≈$20, a reasonable price (as far as textbooks go). There is no excuse not to have this book. It is *required.* However, I will not ususally be teaching "from the book" — most of the material in the class comes from the lecture notes. The textbook is a way to get "another take" on the material. The textbook covers about only half of the material done in class (yes, sometimes we will be following the textbook page by page). For the other half, you will have to make use of other resources.

Computer Software: We will also be using `R` which is a free, open source statistical programming language and console. You can download it from: `http://cran.mirrors.hoobly.com/`. I do not expect you to do *any* programming. I will be giving you `R` code to run and expect you to interpret the results based on concepts explained during the course.

Calculator: You can use a TI-84, 85, 89 or any calculator which you wish. I strongly suggest you use Wolfram Alpha and its smartphone app.

Announcements

Announcements will be made initially via email. Thus, I will need the email address that you reliably check. The default email is whatever happens to be in CUNYfirst which many of you do not check. (See Homework #0 for more information). Then, we will be shifting to announcements via slack (see page 10). I am known to make many announcements per week on important issues.

Class Meetings and Lectures

This class will meet on Zoom. Meetings are 75 minutes and run from Wednesday, August 26 until Wednesday, December 9 for a total of 28 class meetings. However, only 23 of these will be lectures as two days are reserved for the two midterm exams (in class) and two meetings prior are in-class reviews. The exam schedule is given on page 9. **Zoom policies: your video must always be on and you must have working audio. You can use an appropriate background but it must be a static image. Also, no snap camera or the like. No chatting on zoom.**

Sections A and B

This class runs in two identical sections. You can feel free to come to whichever section fits your schedule.

Lecture Notes Upload

As many previous students have noted, my handwritten notes are useful to me and not to many others. Thus, I will be rewarding students for taking notes, scanning them in and sending them to me. You will be rewarded in two ways: (1) if you do this for more than 10 lectures, you will be given the automatic 5 points (see grading policy on page 10) for your classroom participation grade and (2) you have the option for me to say your name publicly on the course homepage. Make sure you follow these instructions:

- You have *one week only* from the time of the lecture to email me lecture notes.
- There must be *one* file and it must be in PDF format *only*. No jpegs, pngs, etc.
- The file must be named `lecxxkapelner.pdf` where you replace `xx` with two digits corresponding to the lecture number i.e. 01, 02, 09, 10, ..., 23 and you replace `kapelner` with your last name in all lowercase letters. If your file is renamed incorrectly, I will tell you to rename it and send it back.
- The file must be <2MB. No exceptions. I will tell you to shrink the PDF and resend. I recommend grayscale and lowering the camera resolution. There are lots of helpful phone apps (e.g. CamScanner, Android Adobe Scan) and computer apps (e.g. XnView) and websites (`ilovepdf.com`, `pdfcompressor.com`) that can help you here.
- Vertically oriented (you can read it without tilting your head 90 degrees).
- The scan must be clear (not blurry at all).
- Your handwriting must be better than mine.
- You must agree to the MIT license.
- I will accept the first seven only.

Homework

There will be 7–10 homework assignments. Homeworks will be assigned and placed on the course homepage and will usually be due a week later in class. Homework will be **graded** out of 100 with extra credit getting scores possibly > 100. I will be doing the grading. I will grade an *arbitrary subset of the assignment* which is determined after the homework is handed in. But you will still be penalized for leaving questions blank regardless of whichever subset I choose to grade.

During this pandemic, homework must be handed in by emailing it to me as a PDF. You must do one of two things:

- **Print out the homework and handwrite your answers in the alotted space for each question. Then scan your homework as a PDF. There are a ton of good photo-PDF apps for both iPhone and droid.**
- **Open the PDF on your device and use a PDF-editing program to electronically handwrite your answers and save the PDF.**

I will NOT accept homework that is not atop the original rendered homework PDF file. Remember to write your name. There are no regrades during this pandemic semester. Also,

- The file must be named `mathccchwxkapelner.pdf` where you replace `ccc` with the course number *you are registered in* (e.g. 341, 368, 369, 621, 650, etc), the `x` with one digit corresponding to the homework number i.e. 1, 2, 3, ... and you replace `kapelner` with your last name in all lowercase letters. If your file is renamed incorrectly, I will tell you to rename it and send it back.
- The file must be <3MB. No exceptions. I will tell you to shrink the PDF and resend. I recommend grayscale and lowering the camera resolution. There are lots of helpful phone apps (e.g. CamScanner, Android Adobe Scan) and computer apps (e.g. XnView) and websites (`ilovepdf.com`, `pdfcompressor.com`) that can help you here.
- Vertically oriented (you can read it without tilting your head 90 degrees).
- The scan must be clear (not blurry at all).

You are encouraged to seek help from me if you have questions. After class and during office hours are good times. **You are highly recommended to work with each other and help each other. You must, however, submit your own solutions, *with your own write-up* and in *your own words.* There can be no collaboration on the actual *writing.* Failure to comply will result in severe penalties.** The university honor code is something I take seriously and I send people to the dean every semester for violations.

Philosophy of Homework

Homework is the *most* important part of this course.[1] Success in Statistics and Mathematics courses comes from experience in working with and thinking about the concepts. It's kind of like weightlifting; you have to lift weights to build muscles. My job as an instructor is to provide assistance through your zone of proximal development. With me, you can grow more than you can alone. To this effect, homework problems are color coded **green** for easy, **yellow** for harder, **red** for challenging and **purple** for extra credit. You need to know how to do all the greens by yourself. If you've been to class and took notes, they are a joke. Yellows and reds: feel free to work with others. Only do extra credits if you have already finished the assignment. The "[Optional]" problems are for extra practice — highly recommended for exam study.

Time Spent on Homework

This is a three credit course. Thus, the amount of work outside of the 2.5hr in-class time per week is 6-9 hours. I will aim for 6hr of homework per week on average. However, doing the homework well is your sole responsibility since by doing the homework you will study and understand the concepts in the lectures.

Late Homework

Late homework will be penalized 10 points per day for a maximum of five days. Do not ask for extensions; just hand in the homework late. After five days, **you can hand it in whenever you want** until the last day of class, Wednesday, December 9. As far as I know, this is one of the most lenient and flexible homework policies in college. I realize things come up. Do not abuse this policy; you will fall far, far behind.

Homework LaTeX Bonus Points

Part of good mathematics is its beautiful presentation. Thus, **there will be a 1–7 point bonus** added to your homework grade for typing up your homework using the LaTeX typesetting system based on the elegance of your presentation. The bonus is arbitrarily determined by me.

I recommend using overleaf to write up your homeworks (make sure you upload both the hw#.tex and the preamble.tex file). This has the advantage of (a) not having to install anything on your computer and not having to maintain your LaTeX installation (b) allowing easy collaboration with others (c) alway having a backup of your work since it's always on the cloud. If you insist to have LaTeX running on your computer, you can download it for Windows here and for MAC here. For editing and producing PDF's, I recommend TeXworks which can be downloaded here. Please use the LaTeX code provided on the course homepage for each homework assignment.

If you are handing in homework this way, read the comments in the code; there are two lines to comment out and you should replace my name with yours and write your section. The easiest way to use overleaf is to copy the raw text from hwxx.tex and preamble.tex into two new overleaf tex files with the same name. If you are asked to make drawings, you can

[1] In one student's observation, I give a "mind-blowing homework" every week.

take a picture of your handwritten drawing and insert them as figures or leave space using the "\vspace" command and draw them in after printing or attach them stapled.

Since this is extra credit, do not ask me for help in setting up your computer with LaTeX in class or in office hours. Also, **never share your LaTeX code with other students** — it is cheating.

Homework Extra Credit

There will be many extra credit questions sprinkled throughout the homeworks (although less for the 621 Masters students). They will be worth a variable number of points arbitrarily assigned based on my perceived difficulty of the exercise. Homework scores in the 140's are not unheard of. They are a good boost to your grade; I once had a student go from a B to an A- based on these bonuses.

Homework #0

The zeroth homework (due Friday 5PM) **constitutes a contract.** To complete it, you must:

(1) Read this course syllabus and the class rules herein carefully as well as the QC Policy on Academic Integrity.

(2) email me at kapelner@qc.cuny.edu from the email address you wish to be contacted at for this course (most commonly this is a gmail address).

(3) The subject of the email is: HW0

(4) The body text of the email is: My name is <Your Full Name as appears in the registrar> and I contractually agree to the course syllabus and the QC Policy on Academic Integrity.

I will email you back a password you can use to check the gradesly, the course grading site once the site is up (which should be a couple weeks into the semester).

This assignment is due Friday, August 28 at 5PM and will receive a grade of 0 or 100 with the usual 10 point penalty for lateness. If you took one of my classes before, I do not store your personal email address! You still have to do Homework #0.

Examinations

Examinations are solely based on homeworks! If you can do all the green and yellow problems on the homeworks, the exams should not present any challenge. I will *never* give you exam problems on concepts which you have not seen at home on one of the weekly homework assignments. There will be three exams and the schedule is below.

Exam Schedule

See course homepage.

Exam Materials

I allow you to bring any calculator you wish but it cannot be your phone. The only other items allowed are pencil and eraser. **No food, only drinks.** I do not recommend using pen but it if you must...

I also allow "cheat sheets" on examinations. For both midterms, you are allowed to bring one 8.5" $\times$ 11" sheet of paper (front and back). **Two sheets single sided are not allowed.** On this paper you can write anything you would like which you believe will help you on the exam. For the final, you are allowed to bring three 8.5" $\times$ 11" sheet of paper (front and back). **Six sheets single sided are not allowed.**

On zoom, the camera must be on your hands at all times. You may have to practice this before the exam.

TO-DO : We need to schedule the same time for midterms and finals for both sections. Otherwise I will make up two tests. And the two tests will differ in difficulty. So one section of the two (at random) will get unfairly penalized. I don't think you want to take that chance... so that's an incentive to get this worked out. As a bonus you may get more office hours too.

Cheating on Exams

If I catch you cheating, you can either take a zero on the exam, or you can roll the dice before the University Honor Council who may choose to suspend you.

Missing Exams

There are no make-up exams. If you miss the exam, you get a zero. If you are sick, I need documentation of your visit to a hospital or doctor. Expect me to call the doctor or hospital to verify the legitimacy of your note. If you need to leave the country for an emergency, I will expect proper documentation as well.

Missing the Final

Automatic WU grade. You can get an F by coming and "taking" the final.

Special Services

If you are a student who takes exams at the special services center, I need to see your blue slip one week before the exam to make proper arrangements with the center.

Class Participation (and attendance)

I will be taking attendance (sometimes formally and sometimes informally) during the class. Attendance counts towards the class participation portion of your grade in equal part with how often you ask and answer questions during the lecture.

The Use of Slack as a Learning Management System

This class has a slack workspace (see page 1). As the course homepage is updated, you will hear about it in slack. You will also find the video recordings of lectures there. (If there are multiple sections of the class, only one section's lectures will be recorded). You can feel free to discuss things with your fellow students there. If you are asking me a question, you must do so in the `#discussions` channel so other students can see the Q&A. Slack is a wildly successful $17.2B company because businesses use it. Pretend you are working at one of these businesses: no posting about random stuff; keep things professional!

Slack will be setup about a week after class begins and you will get an email with instructions about how to sign up.

Grading and Grading Policy

Your course grade will be calculated based on the percentages as follows:

Homework	20%
Class participation	5%
Midterm Examination I	20%
Midterm Examination II	20%
Final Examination	35%

The semester is split into three periods (1) from the beginning until midterm I (2) from midterm I to midterm II (3) from midterm II until the final. The material in each of the periods is tested evenly; thus, it counts the same towards your grade. Since there is 75% of the grade allotted to exams, there is 25% allotted to each period. Thus, the final is upweighted towards the material covered in the third period. In summary, the final will have 5/35 points $\approx$ 14% for the first period's material, 5/35 points $\approx$ 14% for the second period's material and 25/35 points $\approx$ 71% for the last period's material. A good strategy for the final is to just study the material after Midterm II and minimal studying for the previous material.

The Grade Distribution

As this is a small and advanced class, the class is curved and the curve will be quite generous. If you do your homework and demonstrate understanding on the exams, you should expect to be rewarded with an A or a B. $\leq$C's are for those who "dropped out" somewhere mid-semester or who cannot demonstrate basic understanding. To give an idea, of the students who finished the course last time I taught it, there were 43% A's and 29% B's but I am under **no obligation** to repeat this curve.

Checking your grade and class standing

You can always check your grades in real-time using the grading site. You will enter in your QC ID number and the password I will provide to you after homework 0.

Auditing

Auditors are welcome in both sections. They are encouraged to do all homework assignments. I will even grade them. Note that the university does not allow auditors to take examinations.