A discrete random variable (rv) X has probability mass function (PMF) given by $p(x)$: $p(x) := P(X = x)$ and the rv is defined $X \sim p(x)$

when x is the "realized value"

and cumulative distribution function (CDF) denoted $F(x) := P(X = x)$

and complementary CDF also called survival function:

$S(x) := P(X > x) = 1 - F(x)$

The rv has support $\text{Supp}[X] := \{x : p(x) > 0,\ x \in \mathbb{R}\}$

and $|\text{supp}[X]| \leq |\mathbb{N}|$ i.e. finite or at most countably infinite.

sets of this size are called "discrete"

The support and the PMF are related via the following identity:

$$\sum_{x \in \text{supp}[x]} p(x) = 1$$

The most "fundamental" rv is the Bernoulli

$$X \sim \text{Bern}(p) := \underbrace{p^x (1-p)^{1-x}}_{p(x)},\quad \text{Supp}[X] = \{0, 1\}$$

what if $p(7) = p^n (1-p)^{-6} \to ?$

↳ To fix this, we introduce the "indicator function"

$$\mathbb{1}_A := \begin{cases} 1 & \text{if } A \\ 0 & \text{if } A^c \end{cases} \quad \to p^{old}(x)$$

$$X \sim \text{Bern}(p) := \underbrace{\overbrace{p^x (1-p)^{1-x}} \mathbb{1}_{x \in (0,1)}}_{p(x)} \Rightarrow \sum_{x \in \mathbb{R}} p(x) = 1$$

↳ what if $p = 1$?

$$X \sim \text{Bern}(1) = 1^x (1-1)^{1-x} \mathbb{1}_{x \in (0,1)}$$

$$p(0) = 1^0 0^1 = 0,\quad p(1) = 1^1 0^0 = 1$$

$$= \{1 > p,\ 1 = \mathbb{1}_{x=1}$$

This is called a "degenerate" rv. $X \sim \text{Deg}(1) = \{1 \text{ w.p. } 1$

$X \sim \text{Bern}(0) = \text{Deg}(0) = \{0 \text{ w.p. } 1$

Generally, $X \sim \text{Deg}(c) = \{c \text{ w.p. } 1 = \mathbb{1}_{x=c}$

Ex. p is a "parameter" of the Bernoulli rv. What values of p are legal and non-degenerate?

Soln $p \in (0,1)$ ← This is the parameter space of the Bernoulli

If we have more than one rv $X_{-1}, X_{-2}, \ldots, X_{-n}$, we can group them together in a column vector $\vec{X} = [X_1\ X_2\ \ldots\ X_n]^T$

which has a "joint mass function" (JMF) defined as

$$P_{\vec{X}}(\vec{x}) = P_{X_1 X_2 \cdots X_n}(x_1 \cdots x_n) \qquad \sum_{\vec{x} \in \mathbb{R}^n} P_{\vec{X}}(\vec{x}) = 1$$

If $X\_1, \dots, X\_n$ are independent rv's then the JMF can be factored as

$P_{\vec{X}}(\vec{x}) = P_{X_1}(x_1) \cdots P_{X_n}(x_n)$ the "multiplication rule" $= \prod_{i=1}^{n} P_{X_i}(x_i)$

If $X_1 \dots X_n$ are identically distributed denoted $X_1 \overset{d}{=} X_2 \overset{d}{=} \dots \overset{d}{=} X_n$

then $P_{X_1}(x) = P_{X_2}(x) = \dots = P_{X_n}(x) \; \forall x$ but this offers no simplication of the JMF unless...

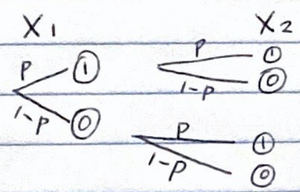
$X_1 \cdots X_n \overset{iid}{\sim}$ denotes "independent and identically distributed"

$$P_{\vec{X}}(\vec{x}) = \prod_{i=1}^{n} \underbrace{P}_{\text{shared PMF}}(x_i)$$

$X_1, X_2 \overset{iid}{\sim} \text{Bern}(p) \qquad T_2 := f(x_1, x_2) = X_1 + X_2 \sim P_{T_2}(t)$

$$P_{T_2}(t) = P_{X_1}(x_1) \underset{\text{convolution operator.}}{*} P_{X_2}(x_2)$$

$\text{Supp}[T_2] = \{0, 1, 2\}$



$P_{X_1, X_2}(x_1, x_2)$
p^2
$p(1-p)$
$(1-p)p$
$(1-p)^2$

$\} \; 1$

$$p^2 + 2p(1-p) + (1-p)^2 = ((p) + (1-p))^2 = 1^2 = 1$$