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Random Variables

- A discrete random variable, X has PMF given by $p(x)$
  - $p(x) := P(X = x)$ and the rv is denoted $X \sim p(x)$ where x is the "realized value"
- and CDF denoted $F(x)$:
  - $F(x) := P(X \leq x)$
  - and complementary CDF (also survival function):
  - $S(x) = P(X > x) = 1 - F(x)$

Support

- $Supp[x] := \{x : p(x) > 0, x \in \mathbb{R}\}$
- and $|Supp[x]| \leq |\mathbb{N}|$ i.e finite or at most countably infinite, sets of this size are called "discrete"
- The support and the PMF are related via the following identity:$\sum_{x \in Supp[x]} p(x) = 1$

Bernoulli

- The most fundamental rv
- $X \sim Bern(p) := p^x(1-p)^{(1-x)}$, $Supp[x] = \{0, 1\}$
- $p(x = 7) = p^7(1-p)^{-6} \leftarrow$ it is *out* of the support, but there *will* be a value.

Indicator function

- $\mathbb{1}_{A=} \begin{cases} 1 & \text{if in } A \\ 0 & \text{if in } A^c \end{cases}$
- $x \sim Bern(p) := p^x(1-p)^{1-x}\mathbb{1}_{x \in \{0,1\}}$ therefore $\sum_{x \in \mathbb{R}} p(x) = 1$
- what if p=1: $X \sim Bern(1) = 1^x 0^{1-x}\mathbb{1}_{x \in \{0,1\}} = \{1 \text{ w.p. } 1\} = \mathbb{1}_x$
- This is a degenerate r.v $X \sim Deg(1)$
- In general $X \sim Deg(c) := \mathbb{1}_{x=c}$
- $X \sim Bern(0)$, also degenerate $Deg(0)$
- p is a parameter of the Bernoulli r.v. what values of p that are legal and non-degerate: $p \in (0, 1)$
- This is the parameter space of the Bernoulli.

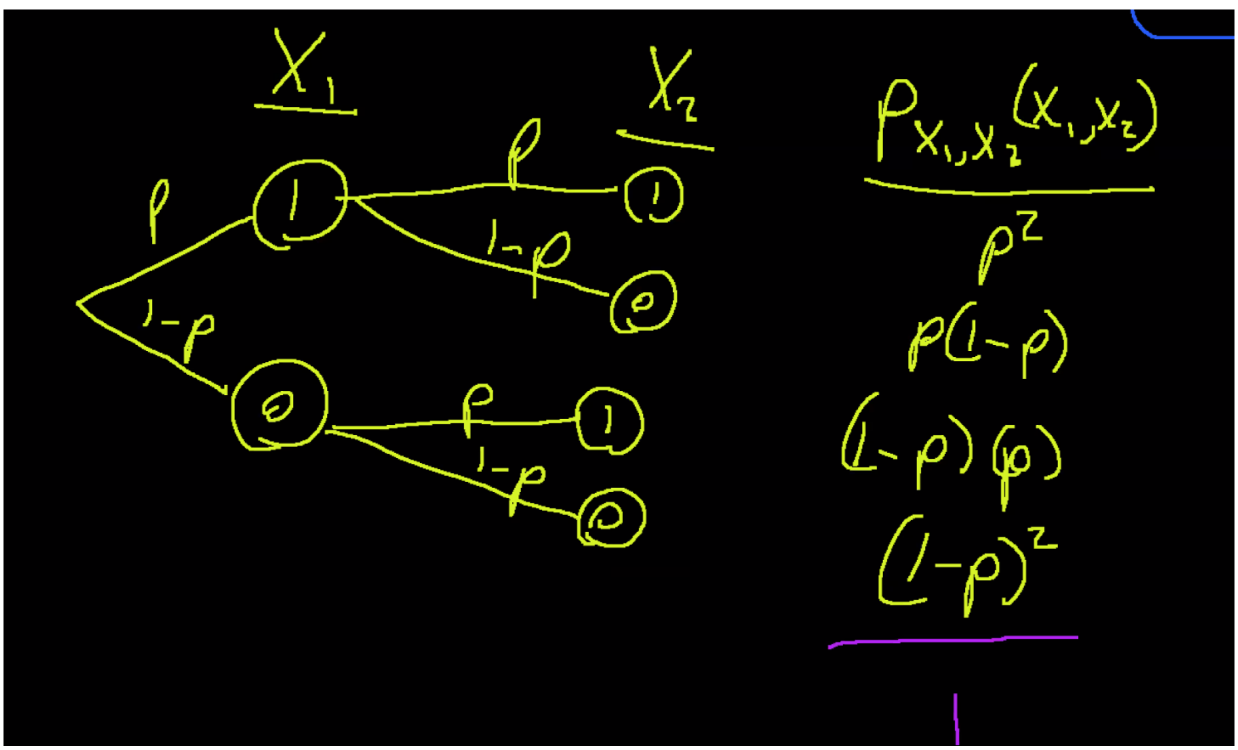
Convolutions

- Using more than one r.v. $x_1, x_2, \ldots, x_n$ we can group them in a column vector $\vec{x} = [x_1.x_2, \ldots, x_n]^t$
- $p_{\vec{X}}\vec{x} = p_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n)$ s.t. $\sum_{\vec{x}\in\mathbb{R}^n} p_{\vec{X}}(\vec{x}) = 1$
- If $x_1, \ldots, x_n$ are independent rv's, then the JMF can be factored $P_{\vec{X}\vec{x}} = P_{X_1}(x_1) * p_{X_2}(x_2) * \ldots * p_{X_n}(x_n)$ the "multiplication rule"
- if $x_1, \ldots, x_n$ are identically distributed, denoted $X_1 \overset{d}{=} X_2 \overset{d}{=} ... \overset{d}{=} X_n$ then $\forall x$ offers no simplification of the JMF unless...
- $X_1, \ldots, \boldsymbol{X}_n \overset{iid}{\sim}$ denotes independent and identically distributed
- $p_{\vec{X}}(\vec{x}) = \prod_i^n p(x_i) \leftarrow$ shared PMF

Topic

- Let $X_1, X_2 \overset{iid}{\sim} Bern(p)$
- Let $T_2 := f(X_1, X_2) = X_1 + X_2 \rightarrow p_T(t) =?$
- $p_{T_2}(t) = p_{x_1}(x_1) \times p_{x_2}(x_2) \leftarrow$ convolution operator
- Supp[T_2]= \0,1,2\

tree diagram:

- 

- $p^2 + 2p(1-p) + (1-p)^2 = ((p) + (1-p))^2 = 1^2 = 1$