Math 621

Go over lecture 1 and 2 on syllabus

Exam dates:
- 9/23
- 11/4
- 12/14

HW 20%
C.P. 5%
Exam 1 20%
Exam 2 20%
Final 35%

- A discrete random variable (rv) has a prob. mass f(x) (pmf).

  $p(x) := P(X=x)$ and the r.v. $X \sim p(x)$ where x is the "realized value."

  X, x ← state of the random variable.

  ↑ random variable

- The cumm. distributive f(x) (cdf) is

  $F(x) := P(X \leq x)$

  and the complementary (cdf) or "survival function" is $S(x) := P(X > x) = 1 - F(x)$.

  - This rv has "support" given by $\text{Supp}[X] := \{x : p(x) > 0, x \in \mathbb{R}\}$. (stuff that could be seen)

$|\text{Supp}[X]| \leq |\mathbb{N}|$ countingly infinite at most

↑ # of elts. in a set

Sets this size are called "discrete" sets.

The support and the pmf are related by the following identity:

$$\sum_{x \in Supp[X]} p(x) = 1$$

The most "fundamental" rv is the Bernoulli:

$$X \sim Bern(p) := \underbrace{p^x (1-p)^{1-x}}_{p(x)} \text{ with } supp[X] = \{0,1\}$$

$$p(7) = p^7 (1-p)^{-6}$$

Let's define the "indicator function"

$$\mathbb{1}_A := \begin{cases} 1 \text{ if } A \\ 0 \text{ if } A^c \end{cases} \Rightarrow X \sim Bern(p) := \underbrace{\overbrace{p^x (1-p)^{1-x}}^{p^{old}(x)} \mathbb{1}_{x \in \{0,1\}}}_{p(x)}$$

$$\Rightarrow \sum_{x \in \mathbb{R}} p(x) = 1.$$

- What if $p = 1$?

If $p = 1$, $X \sim Bern(1) = 1^x 0^{1-x} \mathbb{1}_{x \in \{0,1\}} = \{1 \text{ with probability } 1\}$

$$\{1 \text{ with probability } 1\} = \mathbb{1}_{x=1}$$

$$\Downarrow$$

$$X \sim Deg(1) = \{1 \text{ w/probability } 1\}$$

$$X \sim Deg(c) := \mathbb{1}_{x=c}.$$

- What if $X \sim Bern(0) = Deg(0)$.

The convention in this class is that parameter values (p is the parameter of the Bernoulli) that yield degenerate rv's are not part of the legal "parameter space".

PART 2

$p \in (0,1)$

If we have more than rv $X_1, X_2, \ldots, X_n$ we can group them together in a column vector:

$$\vec{X} := [X_1 \; X_2 \; \ldots \; X_n]^T$$

and then define the "joint mass function" (jmf) as:

$$p_{\vec{X}}(\vec{x}) = p_{X_1, X_2, \ldots, X_n}(x_1, \ldots, x_n) \quad \text{valid for } \vec{x} \in \mathbb{R}^n$$

and if $\sum_{\vec{x} \in \mathbb{R}^n} p(\vec{x}) = 1$

If $X_1, X_2, X_3, \ldots, X_n$ ~~that~~ are independent then,

$$p(\vec{x})(\vec{x}) = p_{X_1}(x_1)\, p_{X_2}(x_2) \times \ldots \times p(x_n)(x_n) = \prod_{i=1}^{n} p_{X_i}(x_i)$$

"the multiplication rule"

If $X_1 \overset{d}{=} X_2 \overset{d}{=} \ldots \overset{d}{=} X_n$ this means "equally distributed" meaning their pmf are the same. However, this offers no simplification of the jmf unless...

$$p_{X_1}(x) = p_{X_2}(x_2) = \ldots = p_{X_n}(x_n) \quad \forall x$$

$X_1, X_2, X_3, \ldots, X_n$ iid that means indep. & identically distributed

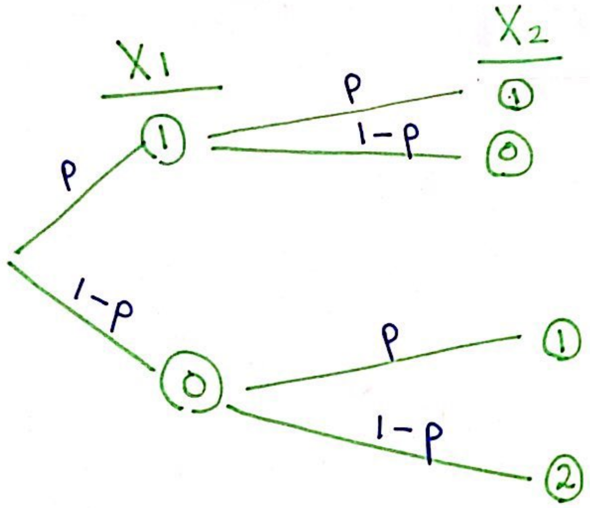
$$\Rightarrow p(\vec{x})(\vec{x}) = \prod_{i=1}^{n} p(x_i)$$

Let $X_1, X_2 \overset{iid}{\sim}$ Bern(p) and let $T_2 = f(x_1, x_2) = X_1 + X_2 \sim \; ? = P_T(t)$

denoted $P_{T_2}(t) = P_{X_1}(x) * P_{X_2}(x)$

"called the convolusion operator"

$Supp[T_2] = \{0, 1, 2\}$



$p_{X_1, X_2}(x_1, x_2)$

p^2

$p(1-p)$

$(1-p)p$

$(1-p)^2$