Lecture - 01 08/26/2020

A discrete random variable (rv) has probability mass function (PMF).

$P(x) := P(X=x)$ and the r.v $X \sim p(x)$ where x is the "realized value"

The cumulative distribution function (CDF) is
$F(x) : P(X \leq x)$
And complementary CDF or "survival function" is.
$S(x) := P(X > x) = 1 - F(x)$.

This rv has "support" given by
$Supp[X] := \{ x : p(x) > 0,\ x \in \mathbb{R} \}$

$|Supp[X]| \leq |\mathbb{N}|$ countably infinite at most.
\# of elements in a set.
Sets this size is called "discrete" sets.

The support and PMF are related by the following identity.

$$\sum_{x \in Supp[X]} p(x) = 1$$

The most "fundamental" rv is the Bernoulli:

$$X \sim Bern(p) := \underbrace{p^x (1-p)^{1-x}}_{p(x)} \text{ with } supp[X] = \{0,1\}$$

$p(7) = p^7 (1-p)^{-6}$ (Not in support)
($\neq$ 0 or 1)

Let's define the "indicator function"

$$\mathbb{1}_A \coloneqq \begin{cases} 1 & \text{if } A \\ 0 & \text{if } A^c \end{cases} \Rightarrow X \sim \text{Bern}(p)$$

$$\coloneqq \underbrace{\overbrace{p^x (1-p)^{1-x}}^{p^{old}(x)} \mathbb{1}_{x \in [0,1]}}_{p(x)}$$

$$\Rightarrow \sum_{x \in \mathbb{R}} p(x) = 1$$

What if $p = 1$

$$X \sim \text{Bern}(1) = 1^x 0^{1-x} \mathbb{1}_{x \in [0,1]} = [1 \text{ with probability } 1] = \mathbb{1}_{x=1}$$

$$\Downarrow$$

$$X \sim \text{Deg}(1) = [1 \text{ wp } 1$$

$$X \sim \text{Deg}(c) \coloneqq \mathbb{1}_{x=c}$$

$$X \sim \text{Bern}(0) = \text{Deg}(0)$$

The convention in this class is that parameter values (p is the parameter of the Bernoulli) that yield degenerate r.v's are not part of the legal "parameter space."

$$p \in (0,1)$$

If we have more than one rv $X_1, X_2, \ldots, X_n$ we can group them together in a column vector:

$$\vec{X} := [X_1, X_2, \ldots X_n]^T$$

and then define the "joint mass function" (JMF) as:

$$P_{\vec{X}}(\vec{x}) = P_{X_1,\ldots,X_n}(x_1,\ldots,x_n) \text{ valid for } \vec{x} \in \mathbb{R}^n \text{ and}$$

$$\sum_{\vec{x} \in \mathbb{R}^n} p(\vec{x}) = 1$$

If $X_1, X_2, \ldots, X_n$ are independent, then

$$P_{\vec{X}}(\vec{x}) = P_{X_1}(x_1) \cdot P_{X_2}(x_2) \cdot \ldots \cdot P_{X_n}(x_n)$$

$$= \prod_{i=1}^{n} P_{X_i}(x_i) \quad \text{"multiplication rule"}$$

If $X_1 \stackrel{d}{=} X_2 \stackrel{d}{=} \ldots \stackrel{d}{=} X_n$ this denotes "equal in distribution" meaning their PMF's are the same. However, his offers no simplification of the JMF unless

$$\forall_x \quad P_{X_1}(x) = P_{X_2}(x) = P_{X_n}(x)$$

$X_1, X_2, \ldots, X_n \stackrel{iid}{\sim}$ that means independent and identically distributed.

$$\Longrightarrow P_{\vec{X}}(\vec{x}) = \prod_{i=1}^{n} P(x_i)$$

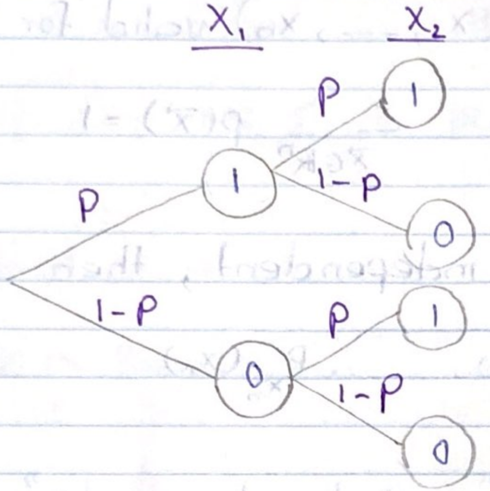
Let $X_1, X_2 \overset{iid}{\sim}$ Bern(p), Let $T_2 = f(X_1, X_2) = X_1 + X_2 \sim P_T(\overline{t})$?

Denoted $P_{T_2}(t) = P_{X_1}(x) * P_{X_2}(x)$

$\uparrow$

Convolution Operation

$\text{Supp}[T_2] = \{0, 1, 2\}$



$P_{X_1, X_2}(x_1, x_2)$

p^2

$p(1-p)$

$(1-p)p$

$(1-p)^2$.