8/26/2020
Lecture 1

A Discrete random Variable (r.v) has probability mass function (PMF)
$P(x) = P(X=x)$ and the r.v $X \sim P(X)$ where x is the "realized value". X, x

The cumulative distribution function (CDF) is
$F(x) := P(X \leq x)$

and Complementary CDF or "Survival function" is
$$S(x) := P(X > x) = 1 - F(x)$$

This rv has "support" given by
$$Supp[X] := \{x : P(x) > 0,\ x \in \mathbb{R}\}$$

$|Supp[X]| \leq |\mathbb{N}|$ countably infinite at most.
↑
number of elements in a set

Sets this size are called "discrete" sets.
The Support and the PMF are related by the following identity:
$$\sum_{x \in Supp[X]} P(x) = 1$$

The most "fundamental" rv is the Bernoulli:
$X \sim Bern(P) := \underbrace{P^x (1-P)^{1-x}}_{P(x)}$ with $Supp[X] = \{0, 1\}$

Example:
$P(7) = P^7 (1-P)^{-6}$ ← can't be done as it's not in the $Supp[X]$.

Not A

Let's define the "indicator function"

$$\mathbb{1}_A := \begin{cases} 1 & \text{if } A \\ 0 & \text{if } A^c \end{cases} \Rightarrow X \sim \text{Bern}(p) := \overbrace{p^x (1-p)^{1-x}}^{p^{old}(x)} \underbrace{\mathbb{1}_{x \in \{0,1\}}}_{}$$

$p(x)$

$$\Rightarrow \sum_{x \in \mathbb{R}} p(x) = 1$$

w.p. = with probability

what if $p = 1$?

$$X \sim \text{Bern}(1) = 1^x \, 0^{1-x} \, \mathbb{1}_{x \in \{0,1\}} = \{ 1 \text{ w.p. } 1$$

$$\Downarrow \qquad = \mathbb{1}_{x=1}$$

$$X \sim \text{Deg}(1) = \{ 1 \text{ wp } 1 \quad X \sim \text{Deg}(c) : \mathbb{1}_{x=c}$$

If $X \sim \text{Bern}(0) = \text{Deg}(0)$

The convention in this class is that parameter values (p is the parameter of the Bernoulli) that yield degenerate rv's are not part of the legal "parameter space".

$$p \in (0, 1)$$

If we have more than one rv $X_1, X_2, \dots, X_n$, we can group them together in a column vector.

$$\vec{X} := [X_1, X_2, \dots X_n]^T$$

and then define the "joint mass function" (JMF) as $p_{\vec{X}}(\vec{x}) = p_{X_1, \dots, X_n}(x_1, \dots x_n)$ valid for $\vec{x} \in \mathbb{R}^n$

and $\sum_{\vec{x} \in \mathbb{R}^n} p(\vec{x}) = 1$

If $X_1, X_2 \dots X_n$ are independent, then

$$P_{\vec{X}}(\vec{x}) = P_{X_1}(x_1)\, P_{X_2}(x_2) \cdot \dots \cdot P_{X_n}(x_n) = \prod_{i=1}^{n} P_{X_i}(x_i)$$

multiplication rule

If $X_1 \overset{d}{=} X_2 \overset{d}{=} \dots \overset{d}{=} X_n$, this denotes "equal distribution" meaning their PMF are the same. However, this offers no simplication of the JMF unless....

$$\forall x \quad P_{(X_1)}(x) = P_{(X_2)}(x) = P_{X_n}(x)$$

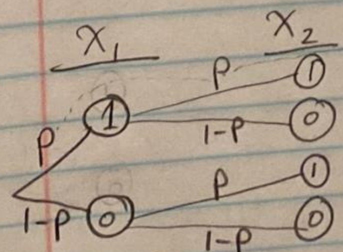
$X_1, X_2, \dots X_n \overset{iid}{\sim}$ that means independent and identically distributed.

$$\Rightarrow P_{\vec{X}}(\vec{x}) = \prod_{i=1}^{n} P(x_i)$$

Let $X_1, X_2 \overset{iid}{\sim} \text{Bern}(p)$, Let $T_2 = f(X_1, X_2) = X_1 + X_2 \sim P_{T}(t)$?

Denoted $P_{T_2}(t) = P_{X_1}(x) * P_{X_2}(x)$ ← Convolution Operator.

$\text{Supp}[T_2] = \{0, 1, 2\}$



$P_{X_1, X_2}(x_1, x_2)$

p^2

$p(1-p)$

$(1-p)p$

$(1-p)^2$