Monday 7 December 2020

Lecture 23

"Missing First Five minute lecture"

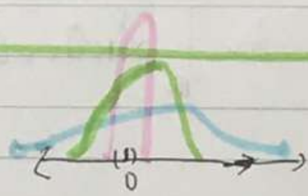
because probabilities are between 0 and 1, if you know the probability is ≤ 0. That means the probability is 0.

$\Rightarrow \lim P(|X_n - \mu| \ge \epsilon) = 0 \Rightarrow X_n \xrightarrow{P} \mu$

e.g. $X_n \sim U(-\frac{1}{n}, \frac{1}{n})$ Prove $X_n \xrightarrow{P} 0$

$E[X_n] = 0 = \mu \ \forall n$, $\sigma_n^2 =$.

e.g. $X_n \sim N(0, \frac{1}{n})$. Prove $X_n \xrightarrow{P} 0$



$E[X_n] = 0 = \mu \ \forall n$ $\sigma_n^2 = \frac{1}{n}$, $\lim \sigma_n^2 = \lim \frac{1}{n} = 0 \Rightarrow X_n \xrightarrow{P} 0$ ✓

Let $X_1, X_2, \dots$ be iid with mean μ and variance $\sigma^2 < \infty$

as the "number" of samples increases, the average cannot "escape" from the mean.

$\bar{X}_n = \frac{1}{n} \sum X_i$, $E[X_n] = \mu \ \forall n$, $Var[\bar{X}_n] = \frac{\sigma^2}{n} = 0$

Prove $\bar{X}_n \xrightarrow{P} \mu$. $\lim Var \bar{X}_n = \lim \frac{\sigma^2}{n} = 0 \Rightarrow \bar{X}_n \xrightarrow{P} \mu$.

This is a very famous thm. It's called the "Weak" "weak" law of large numbers" (WLLN)

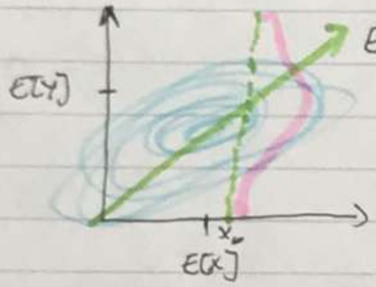
because I assumed finite σ^2 of the $X_1, X_2, \dots$ but it's you don't need it (see HW)

because convergence in probability is actually a weak type of convergence. It turns out you can prove "almost sure" convergence (but we won't discuss this)

$$\lim E[|X_n - 0|^r] = \lim E[X_n^r] = \lim \sum_{x \in \{0, n^2\}} x P_X(x) = \lim 0\cancel{(1-\tfrac{1}{n})} + n^2 \tfrac{1}{n}$$

$$= \lim n = \infty \neq 0 \Rightarrow X_n \overset{L^r}{\not\to} 0$$

Law of Iterated expectation. Consider Two r.v. X, Y with joint $f_{X,Y}(x,y)$



$E[Y|X=x] = E[Y/X]$
Conditional expectation function (CEF).

$$E[Y] = \int_{\mathbb{R}} y f_Y(y)\, dy = \int_{\mathbb{R}} y \int_{\mathbb{R}} f_{X,Y}(x,y)\, dx\, dy = \int_{\mathbb{R}}$$

$$= \int_{\mathbb{R}} f_X(x) \underbrace{\int_{\mathbb{R}} y f_{Y/X}(y,x)\, dy}_{E[Y/X]}\, dx = \int_{\mathbb{R}} E[Y/X] f_X(x)\, dx = E[E[Y/X]]$$

Law of Total Variance

$$Var_Y[Y] = E_Y[Y^2] - E_Y[Y]^2$$

$$= E_X[E_Y[Y^2/X]] - E_X[E_Y[Y/X]]^2$$

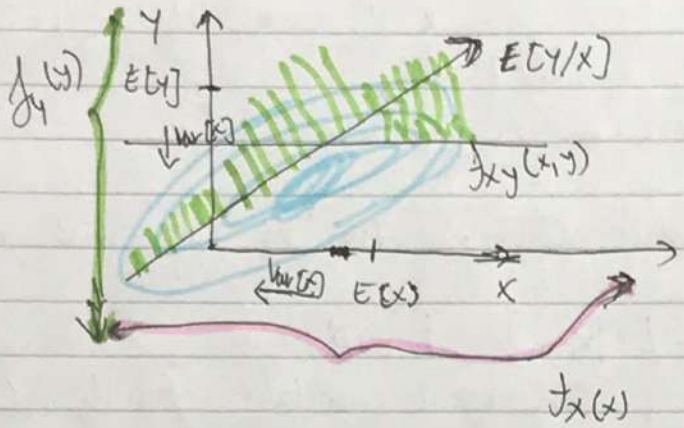
$$= E_X[Var_Y[Y/X] + E_Y[Y/X]^2] - E_X[E_Y[Y/X]]^2$$

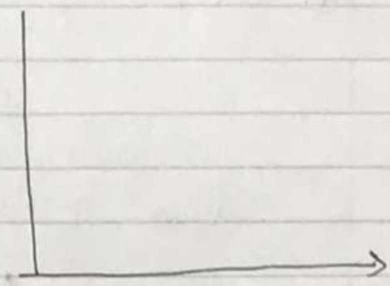
$$= E_X[Var_Y[Y/X]] + E_X[E_Y[Y/X]^2] - E_X[E_Y[Y/X]]^2$$

let $C = E_Y[Y/X]$

$$= E_X[Var_Y[Y/X]] + \underbrace{E_X[C^2] - E_X[C]^2}_{Var_X[C]}$$

$$Var_Y[Y] = \underbrace{E_X[Var_Y[Y/X]]}_{I} + \underbrace{Var_X[E_Y[Y/X]]}_{II}$$ decomposition formula





Var [Y] =]