Dependent $X_1 + X_2 = n$

$\Rightarrow \vec{X} \sim \text{Multi}(n, \vec{p}) = \binom{n}{x_1, x_2} p_1^{x_1} p_2^{x_2}$ multinomial

rv of dim = 2

Since X_1, X_2 are dependent, we cannot factor

JMF bag of fruit now has cantaloupes.

You draw cantaloupes with probability P_3

and X_3 is the count of cantaloupes

$$\vec{X} \sim \text{Multi}(n, \vec{p}) = \binom{n}{x_1, x_2, x_3} p_1^{x_1} p_2^{x_2} p_3^{x_3} =$$

$$\frac{n!}{x_1!\, x_2!\, x_3!} p_1^{x_1} p_2^{x_2} p_3^{x_3} \mathbb{1}_{x_1 + x_2 + x_3 = n}$$

In general, If there are K types of fruit (# categories) then the general Multinomial rv of dim K is:

$$\vec{X} \sim \text{Multi}(n, \vec{p}) = \binom{n}{x_1, x_2, x_K} \prod_{k=1}^{K} p_k^{x_k}$$

Parameter space: $n \in \mathbb{N}$, $\vec{p} \in \{ \vec{r} : \vec{p} \cdot \vec{1} = 1, r_1 \in (0,1), \ldots r_K \in (0,1) \}$

Support: $\text{Supp}[\vec{X}] = \{ \vec{x} : \vec{x} \cdot \vec{1} = n, x_1 \in \{0, 1, \ldots, n\}, \ldots, x_K \in \{0, 1, \ldots, n\} \}$

$\vec{X} \sim \text{Multi}\left(n, \begin{bmatrix} p \\ 1-p \end{bmatrix}\right)$ K=2 $= \binom{n}{x_1, x_2} p^{x_1} (1-p)^{x_2}$

$$\underbrace{P(X_1 = x_1 \mid X_2 = x_2)}_{\text{Deg}(n - x_2)} \overset{?}{=} \underbrace{P(X_1 = x_1)}_{\text{Bin}(n,p)}$$

Dependent.

Conditional pmf.

Marginal pmf of X_2

$$p_{X_1 | X_2}(x_1, x_2) = \frac{p_{X_1, X_2}(x_1, x_2)}{p_{X_2}(x_2)}$$

Let's show $X_2 \sim \text{Bin}(n, p_2)$

JMF

$$p_{X_2}(x_2) = P(X_2 = x_2) = \sum_{x_1 \in \mathbb{R}} p_{X_1, X_2}(x_1, x_2)$$

this is called "Marginizing out X_1"

$$= \sum_{x_1 \in \mathbb{R}} \binom{n}{x_1, x_2} p^{x_1} (1-p)^{x_2}$$

$$= \sum_{x_1 \in \mathbb{R}} \frac{n!}{x_1!\, x_2!} p^{x_1} (1-p)^{x_2} \mathbb{1}_{x_1 + x_2 = n} \mathbb{1}_{x_1 \in \{0, \ldots, n\}} \mathbb{1}_{x_2}$$

$$= \frac{n!}{x_2!} (1-p)^{x_2} \mathbb{1}_{x_2 \in \{0, 1, \ldots, n\}} \sum_{x_1 \in \{0, \ldots, n\}} \frac{p^{x_1} \mathbb{1}_{x_1}}{x_1!}$$

Wed Sept 09.|20

Let $X, Y \overset{iid}{\sim} \text{geom}(p)$ $P(X>Y) = ?$

$P(X>Y) = P(Y>X)$

$P(X>Y) + P(Y>X) + P(X=Y) = 1$

$\Rightarrow 2P(X>Y) + P(X=Y) = 1$

$\Rightarrow P(X>Y) = 1 - \frac{P(X=Y)}{2} < \frac{1}{2}$

Since $P(X=Y) > 0$.

$$P(X>Y) = \sum_{x \in \mathbb{R}} \sum_{y \in \mathbb{R}} P_{X,Y}(x,y) \mathbb{1}_{x>y}$$

$$= \sum_{x \in \mathbb{R}} P_X(x) \sum_{y \in \mathbb{R}} P_Y(y) \mathbb{1}_{x>y}$$

$$= \sum_{y \in \{0,1,\dots\}} p(1-p)^y \sum_{x \in \{0,1,\dots\}} p(1-p)^x \underbrace{\mathbb{1}_{x>y}}_{x \geq y+1}$$

$$= p^2 \sum (1-p)^y \sum (1-p)^x$$

let $x' = x-(y+1) \Rightarrow x = x'+y+1$

$$= p^2 \sum_{y \in \{0,1,\dots\}} (1-p)^y \sum_{x' \in \{0,1,\dots\}} (1-p)^{x'} (1-p)^1$$

$$p^2(1-p) \sum_{y \in \{0,1,2,\dots\}} (1-p)^{2y} \sum_{x' \in \{0,1,\dots\}} (1-p)^{x'}$$

Geometric Series

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$$

$$= p^2(1-p) \sum_{y \in \{0,1,\dots\}} (1-p)^{2y} \left(\frac{1}{p}\right)$$

$$= p(1-p) \boxed{\sum_{y \in \{0,1,\dots\}} \left((1-p)^2\right)^y} = \frac{p(1-p)}{p(2-p)} = \frac{1}{2}$$

#

n has to accounting

$X_1 = $ # apples, $X_2 = $ # bananas

$X_1 \sim \text{bin}(n,p)$, $X_2 \sim \text{Bin}(n, p_2)$

Are $X_1$ and $X_2$ independent?

Since $X_1 + X_2 = n \Rightarrow$ ~~$X_1$~~ $X_1, X_2$ dependent

$$\vec{X} \sim P_{\vec{X}}(\vec{x}) = P_{\vec{X}}(x_1, x_2) =$$

$$\frac{n!}{x_1!\, x_2!} p_1^{x_1} p_2^{x_2} \underbrace{\mathbb{1}_{x_1+x_2=n}}\ \underbrace{\mathbb{1}_{x_1 \in \{0,1,\dots n\}}}\ \underbrace{\mathbb{1}_{x_2 \in \{0,1,2\}}}$$

$\binom{n}{x_1, x_2}$ multichoose

Notation