Wed October 21st 2020

$$= \sum_{j=k}^{n} \binom{n}{j} \left(j f(x) F(x)^{j-1} (1-F(x))^{n-j} - (n-j) f(x) F(x)^{j} (1-F(x))^{n-j-1} \right)$$

$$= \sum_{j=k}^{n} \frac{n!}{j!(n-j)!} j f(x) F(x)^{j-1} (1-F(x))^{n-j} - \sum_{j=k} \frac{n!}{j!(n-j)!} f(x) F(x)^{j} (1-F(x))^{n-j-1}$$

reindexing: let $l = j+1 \Rightarrow j = l+1 \Rightarrow j = l-1$

$$= \sum_{j=k}^{n} \frac{n!}{(j-1)!(n-j)!} f(x) F(x)^{j-1} (1-F(x))^{n-j} - \sum_{l=k+1}^{n} \frac{n!}{(l-1)!} f(x) F(x)^{l-1} (1-F(x))^{n-l}$$

$$= \sum_{j=k}^{n} \frac{n!}{(j-1)!(n-j)!} f(x) F(x)^{j-1} (1-F(x))^{n-j} - \sum \frac{n!}{(l-1)!(n-l)!} f(x) F(x)^{l-1} (1-F(x))^{n-l}$$

$$= \frac{n!}{(k-1)!(n-k)!} f(x) F(x)^{k-1} (1-F(x))^{n-k} = f_{X_{(k)}}(x)$$ Done

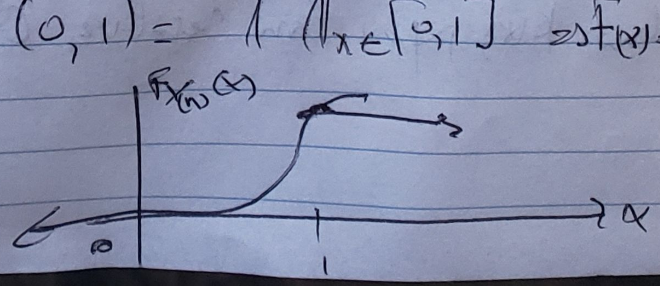
Check min, max

$$f_{X_{(1)}}(x) = \frac{n!}{(1-1)!(n-1)!} f(x) F(x)^{n-1} = n f(x) (1-F(x))^{n-1}$$

$$f_{X_{(n)}}(x) = \frac{n!}{(n-1)!(n-n)!} f(x) F(x)^{n-1} (1-F(x))^{n-n} = n f(x) F(x)^{n-1}$$

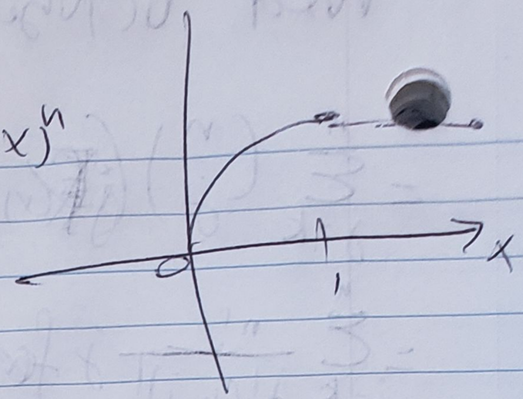
$X_1, \ldots, X_n \overset{iid}{\sim} U(0,1) = 1 \cdot 1_{x \in [0,1]} \Rightarrow F(x) = x$ in the supp.

$$F_{X_{(n)}} = F(x)^n = x^n$$



Wed October 31st 2020

$$F_{X_{(n)}} = F(x)^n = 1 - (1-F(x))^n = 1-(1-x)^n$$



$$f_{X_{(k)}}(x) = \frac{n!}{(k-1)!(n-k)!} x^{k-1} (1-x)^{n-k} \mathbb{1}_{x \in [0,1]}$$

$$\Rightarrow f_{X_n}(x) = n x^{n-1} \mathbb{1}_{x\in[0,1]}, \quad f_{X_{(1)}}(x) = n(1-x)^{n-1}$$

$$= \frac{\Gamma(n+1)}{\Gamma(k)\Gamma(n-k+1)} x^{k-1} (1-x)^{n-k} \mathbb{1}_{x\in[0,1]} = \text{Beta}(k, n-k+1)$$

$X \sim \text{Gamma}(\alpha, \beta)$ indep of $Y \sim \text{Gamma}(\alpha, \beta) \Rightarrow$

seems right

$X + Y \sim \text{Gamma}(\alpha_1 + \alpha_2, \beta)$

The easiest proof of this is to employ "Kernels". what's a Kernel?

$p(x) = c\,k(x)$, $f(x) = c\,k(x)$ Normalizing Constant

kernel

$\Rightarrow p(x) \propto k(x)$, $f(x) \propto k(x)$.

if you know $k(x)$ you can resolve c via the following