8/26/20

- a discrete rv has a probability mass function (PMF) $p(x) := P(X=x)$ and the rv $X \sim p(X)$ where x is the "realized value"

- the cumulative distribution function (CDF)

$$F(x) := P(X \leq x)$$

- complementary CDF or "survival function" is

$$S(x) := P(X > x) = 1 - F(x)$$

- this rv has "support" given by $Supp[X] := \{x : p(x) > 0, x \in \mathbb{R}\}$

$|Supp[X]| \leq |\mathbb{N}|$ countably infinity at Most

↑ # of elements in set

- sets this size are called "discrete" sets

- support & PMF are related by following identity:

$$\sum_{x \in Supp[X]} p(x) = 1$$

Bernoulli

$$X \sim \text{Bern}(p) := \overbrace{p^x (1-p)^{1-x}}^{p(x) \text{ "OLO"}} \quad \text{with } \text{supp}[X] = \{0,1\}$$

$$p(7) = p^7 (1-p)^{-6}$$

- define "indicator function"

$$\mathbb{1}_A := \begin{cases} 1 \text{ if } A \\ 0 \text{ if } A^c \end{cases} \rightarrow X \sim \text{Bern}(p) := \underbrace{p^x (1-p)^{1-x} \mathbb{1}_{x \in \{0,1\}}}_{p(x)}$$

$$\sum_{x \in \mathbb{R}} p(x) = 1$$

if $p = 1$?

$$X \sim \text{Bern}(1) = 1^x 0^{1-x} \mathbb{1}_{x \in \{0,1\}} = \{1 \text{ w.p. } 1 = \mathbb{1}_{x=1}$$

$\downarrow$

$X \sim \text{Deg}(1) = \{1 \text{ wp } 1$ $\qquad$ $X \sim \text{Deg}(c) := \mathbb{1}_{x=c}$

$$X \sim \text{Bern}(0) = \text{Deg}(0)$$

- in this class p's that yield Degenerate r.v's are not part of legal "parameter space

$$p \in (0,1)$$

- If we have more than one rv $X_1, X_2, \ldots, X_n$ we can group them together in a column vector

$$\vec{X} := [X_1, X_2, \ldots X_n]^T$$

then define "joint mass function" (JMF) as

$$P_{\vec{X}}(\vec{x}) = P_{X_1,\ldots,X_n}(x_1,\ldots,x_n), \text{ valid for } \vec{x}\ \mathbb{R}^n$$

$$\sum_{\vec{x}=\mathbb{R}^n} p(\vec{x}) = 1$$

- if $X_1, \ldots, X_n$ are independent

$$P_{\vec{X}}(\vec{x}) = P_{X_1}(x_1)\, P_{X_2}(x_2) \ldots P_{X_n}(x_n) = \prod_{i=1}^{n} P_{X_1}(x_i)$$

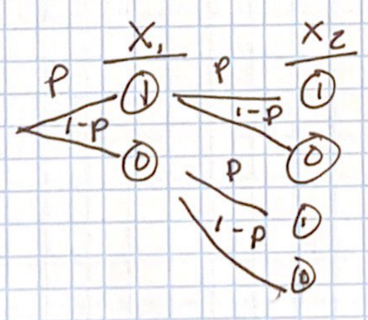
- if $X_1 \stackrel{d}{=} X_2 \stackrel{d}{=} \cdots \stackrel{d}{=} X_n$, then they are equally distributed
  - their PMFs are the same.
  - No simplification of JMF

Let $X_1, X_2 \stackrel{iid}{\sim} \text{Bern}(p)$, $T_2 = f(X_1, X_2) = X_1 + X_2 \sim P_T(t) = ?$

Denoted $P_T(t) = P_{X_1}(x) * P_{X_2}(x)$

$*$ ← convolution operator

Support $[T_2] = \{0, 1, 2\}$



$P_{X_1 X_2}(x_1, x_2)$
p^2
$p(1-p)$
$(1-p)p$
$(1-p)^2$