6/20/11

~~Lecture #A~~

Admin

HW ~~#A~~ - half will be out this afternoon
- other half out Wed
- will cover material during Thurs.

Plan

- hypothesis testing for numerous samples

Review of CLT

$X_1, \ldots, X_n \overset{iid}{\sim}$ w/ mean μ, satisfying $>0, \sigma$ $\Rightarrow \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{D} Z$

"Convergence in distribution" means as $n \to \infty$, the PDF's become $=$.

Review of shifts and scales of normals

$X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2) \Rightarrow \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \overset{d}{=} Z$

"Equals in distribution" means they have the same PDF right now

Let's go over T distribution again: p.357 Read bottom carefully

$X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2) \Rightarrow \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} \overset{d}{=} T_{n-1}$

the ~~sample~~ population has to be normal. It cannot be just anything with mean μ and σ. HW question about this

Proportions: Populations are Bernoullis and therefore

$$\hat{p} \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$$

(book calls this $\hat{p}$ on p383 ⇒ wrong!)

One parameter

Samples: population could be anything

$$\bar{X} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$$

two params

In the sample case, we ~~kn~~ are stuck estimating two parameters, μ and σ. This "extra" σ forces us to use the T distr. when estimating from sample SD, s.

Let's build this up slowly...

See 16.3 p389 has an ex of a prop. mgmt. comp. company expanding into Denver. In order to be profitable they need to have rents more than \$500. Design an experiment. $\alpha = 1\%$. Take SRS $n=45$, find $\bar{x} = \$647$. The company knows $\sigma = 300$, something we'll challenge in a second. Let's do a hyp. test...

$H_0: \mu = 500$

$H_a: \mu > 500$ ← Only care about this side we are "trying" to reject the null

Form test statistic assuming the Null is true

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \Rightarrow z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\$647 - \$500}{\frac{\$300}{\sqrt{45}}} = 3.28 > z_{0.01} = 2.326$$

$\Rightarrow$ reject H_0

Now find p-value...

$$p_{val} = P(\bar{X} \geq \$647 \mid \mu = 500) = P(Z > 3.28) = .0005 < \alpha = 1\% \Rightarrow \text{reject } H_0$$

The data is inconsistent with the hypothesis. Testing tells us what μ is not, it does not tell us what μ is. Tells us when data contradicts a specific claim about the population!

$\Rightarrow$ The result is said to be "statistically significant" since $p_{val} < \alpha$. It's a stat. sign. difference from H_0. It does not mean that result is important or earth-shattering. the problem

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Now consider the case we do not know σ, but we do know that vars are norm distr. How do we know norm distr? We used a Q-Q plot & perhaps not covered in this class. What do we do now?

Since $n \geq 30 \Rightarrow$ no need for t-test just use z-test

fine.. let's say $n = 29$, $\bar{X} = \$647$, $s = \$299$ ... let's try this...

$H_0: \mu = 500$, $H_a: \mu > 500$

$$T_{n-1} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}, \quad t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{\$647 - \$500}{\frac{\$299}{\sqrt{29}}} = 2.648 > t_{0.01,28} = 2.467$$

⇒ reject H_0

not quite as dramatic

How??

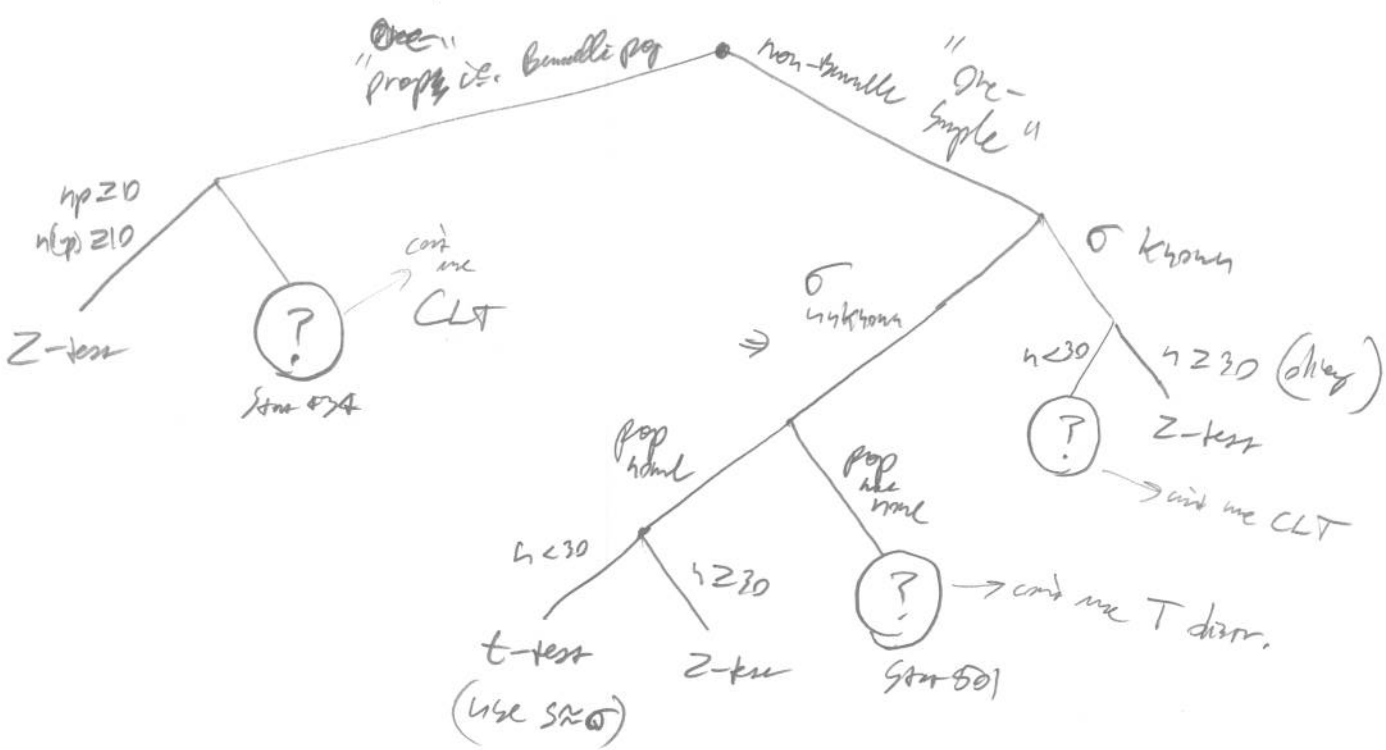
$$\text{pval} = P(\bar{X} \geq \$647 \mid \mu = 500) = P(T_{28} \geq 2.648) \approx 0.5\text{–}1\%$$

$= 0.657\%$ using TI-89

Using CDF table to estimate

What if rent was not norm distr? You can't do anything... Stat 501 Rosenbaum non-parametric distr-free tests...

Our decision tree for tests:



This was a right-sided t-test. Left-sided and two-sided are for you to do at home.

Let's build a CI for Denver cars:

$$CI_{\mu,99\%} = \left[\bar{x} \pm t_{0.01,28} \cdot \frac{s}{\sqrt{n}}\right] = \left[\$649 \pm 2.640 \cdot \frac{\$299}{\sqrt{29}}\right] = \left[\$499.98, \$798.02\right]$$

Wait a minute $\$500 \in CI$ but we rejected the test!! We would not reject acc'd to CI.

CI's $\Longleftrightarrow$ Hyp. testing only when hyp. test is <u>2-sided</u>.

CI's are "less powerful" than hyp. tests.

What is power? $\text{Power} \triangleq P(\text{rejecting } H_0 \mid \text{when } H_0 \text{ is } \underline{\text{not}} \text{ true})$

$= P(\text{finding an effect})$

		what we say	
		H_0	H_a
State of the world	H_0	✓	Type I
	H_a	Type II	✓

explain this problem

→ Prob of this box is power $= 1 - P(\text{Type II error})$

Type II is are very difficult to peg...

A beer manufacturer makes 40 oz bottles. The nozzle has $\sigma = 0.2$ oz. The machine is known to break a valve and starts spewing out 40.1 oz instead of 40 oz. If we take a SRS of 30 bottles $\rightarrow \bar{x} = 40.05$ from production, what is the prob we find out if the valve is broken? $\alpha = 5\%$.

Let's go ahead and run a hyp test and worry about the question later.

$H_0: \mu = 40$ oz

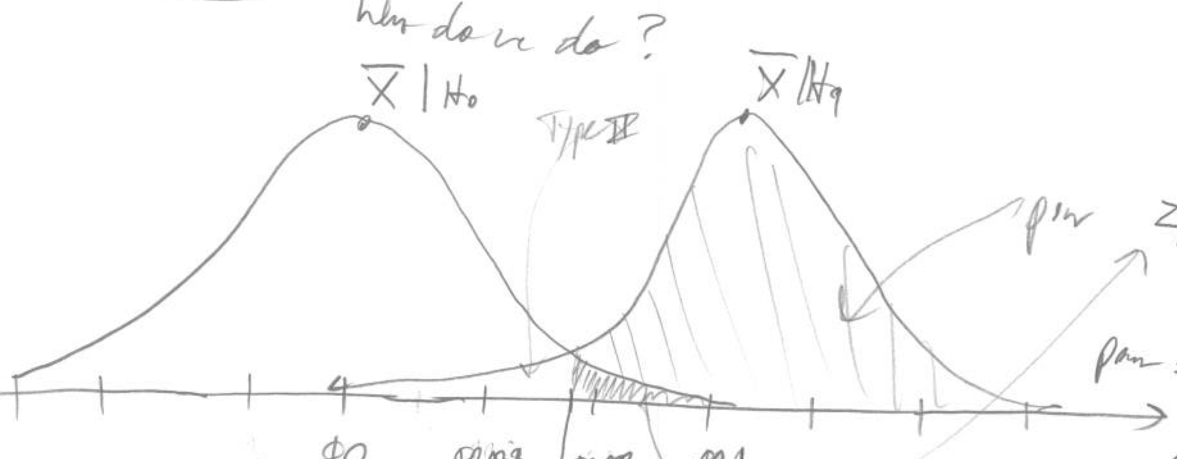
$H_a: \mu = 40.1$ oz ← Simple H_a different than before

Since $n \geq 30$ and σ known $\Rightarrow$ z-test:

$$z = \frac{40.05 - 40}{\frac{0.2}{\sqrt{30}}} = 1.369 < 1.645 \Rightarrow \text{fail to reject}$$

Could be here...

What do we do?



$$z_{H_a} = \frac{40.06 - 40.1}{\frac{2}{\sqrt{30}}} = -1.095$$

$$pow = P(Z > z_{H_a} \mid H_a)$$

$\approx 54\%$ LOW!!

$\bar{X}^* = \mu + z\sigma = 40 + 1.645 \cdot \frac{0.2}{\sqrt{30}} = 40.06$

Set power: 80%.

What n do we need for that [illegible]