Draw deck of cards before 9 am

5/25/11 L1

Lecture #3

Admin

- webcast is up
- audio lectures? Said OKAY... will convert some 3
- first HW is up (partial HW and post tomorrow, due Tues → Wed)
- recommend: mechanical pencil,
  Staedtler erasers,
  white, unlined, paper

Plan

1) more counting
2) probability (some)
3) addition rule

More applications of counting. First, lets do the 4-of a kind again: Count the size of this set

$$\binom{13}{1}\binom{48}{1} = \binom{13}{1} \cdot \binom{12}{1}\cdot\binom{4}{1}$$

Straight flush

$\binom{4}{1}\binom{10}{1}$ does this include royal flushes?

Full House

$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$

Flush

$\binom{4}{1}\binom{13}{5}$ does this include straight flushes?

Two Pair

$$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{44}{1} \rightarrow \binom{11}{1}\binom{4}{1}$$

Another application of the "divide out what you don't want" rule. $\Omega = \{$ space of 3 coin flips $\}$

Ω

HHH	HHT	HTT	TTT
TTH	THH	HTH	THT

what is $A = \{$ get 2H out of 3 flips $\}$
We can use our "lookup table" above
and just enumerate the answer. Unfortunately, if
we had 20 flips $\Rightarrow 2^{20} \approx 1M$, so we need
to leverage our tools from yesterday.

If we want $2H/3 \Rightarrow 2H, 1T \Rightarrow H_1, H_2, T_1$

$$\underset{\substack{1^{st}\\ \text{flip}}}{\underline{3}} \cdot \underset{\substack{2^{nd}\\ \text{flip}}}{\underline{2}} \cdot \underset{\substack{3^{rd}\\ \text{flip}}}{\underline{1}} = 3! = 6$$

But we don't care about the order of the heads - a "head is a head", a "tail is a tail". Hence

$$|A| = \frac{3!}{2!\,1!} = \binom{\overset{\text{flips}}{3}}{\underset{\text{heads}}{2}}$$

How about 6H of 10 flips?

$$\frac{10!}{\underset{\text{don't care head order}}{6!}\;\underset{\text{don't care tail order}}{4!}} = \binom{10}{6} \qquad \text{prob?} \quad \frac{\binom{10}{6}}{\underbrace{2^{10}}_{\text{whole sample space } |\Omega|}}$$

How about k H out of n flips?

$$\text{prob?} \quad \frac{\frac{n!}{k!\,(n-k)!}}{2^n} = \frac{\binom{n}{k}}{2^n}$$

6H of 7 tosses

$$\binom{7}{6} = \frac{7!}{6!\,1!} = \binom{7}{1} = 7$$

$$\Rightarrow \binom{n}{n-1} = \binom{n}{1} = n$$

7H out of 7 tosses $\Rightarrow$ only one way!

$$\binom{7}{7} = \frac{7!}{7!\,0!} = \binom{7}{0}$$

Many experiments it's natural to assume all events are equally likely to occur.

Coin $P(\{H\}) = P(\{T\})$

Die $P(\{1\}) = \ldots = P(\{6\})$

Cards $P(\{K\}) = \ldots = P(\{A\})$

etc...

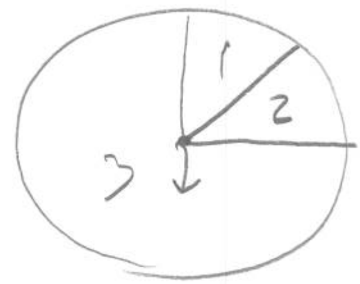
$\Rightarrow$ size of space

$\text{size}(\{1,2\}) = |\{1,2\}| = 2$

works... but doesn't if...

Sometimes $\Rightarrow$ not the case! Sample space is not split into evenly likely events... e.g.

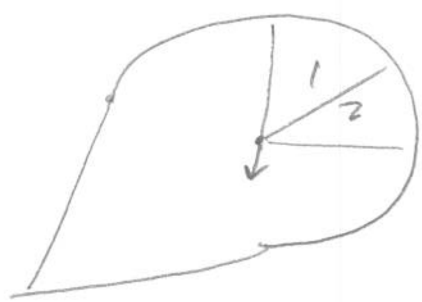
Spinner



Is

$\text{size}(\{3\}) = \text{Area}? = \frac{3}{4}$

But...



Look at the curve... the angle matters

$\text{size}(\{3\}) = \theta_3 = 270°$

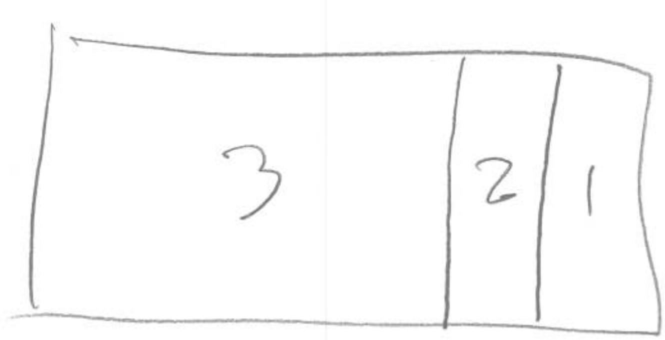
$\text{size}(\{1\}) = \theta_1 = 45°$

$\text{size}(\{2\}) = \theta_2 = 45°$

$\text{size}(\Omega) = ?\ \ 360°$

$$\Rightarrow P(\{3\}) = \frac{270°}{360°} = 3/4 = 0.75$$

What does Ω look like?



The question
prob royal flush
texas hold'em
one player

Which brings us to...

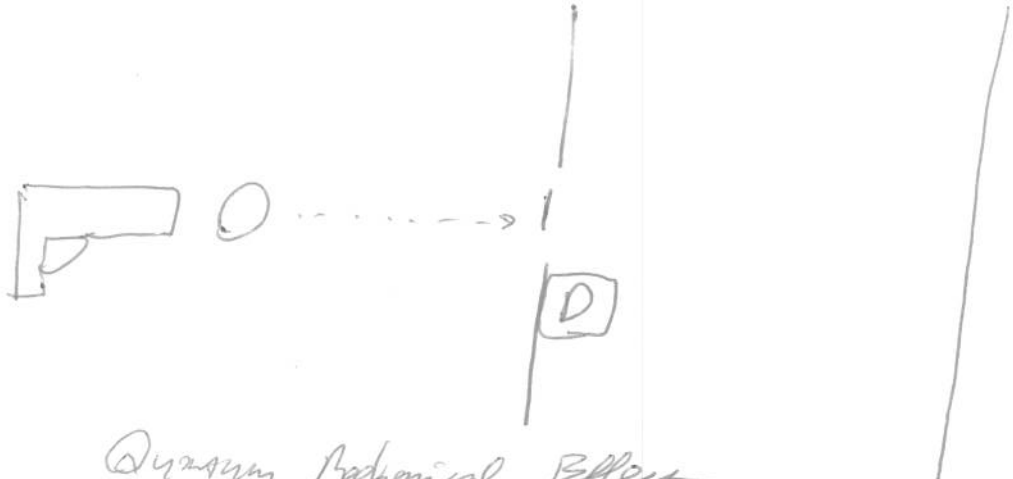
→ Probability

~~But before we get there~~ ~~finally~~... ~~who does~~ we know the prob of flipping a coin H is $\frac{1}{2}$. What does that mean?
$\frac{1}{2}$ chance we get a head. What does chance mean?
Chance means we don't know? Why don't we know? → then we say there's "randomness".

Maybe we don't know the exact speed the coin left our hand and it's rotation, and the atmospheric pressure, and the hardness of the table, and the dirt on the coin itself... but if we knew all that... we'd surely know ~~that~~ the it would turn up exactly heads or tails, maybe? ~~Yes~~.. Maybe.
How about picking a random card. Well,... if I knew exactly how you shuffled... I would know that card.
Is there randomness to the universe? Is it deterministic?

Electron double slit diffraction experiment (1978)

Hay kawa
~1980's



Quantum Mechanical Effect
Wave/Particle Duality — don't know where the electron goes on the film behind it... And if you put a detector near one of the slits... it's no longer random.

Uncertainty principle: momentum, position precision is bilateral. If you know one, you can't know the other...

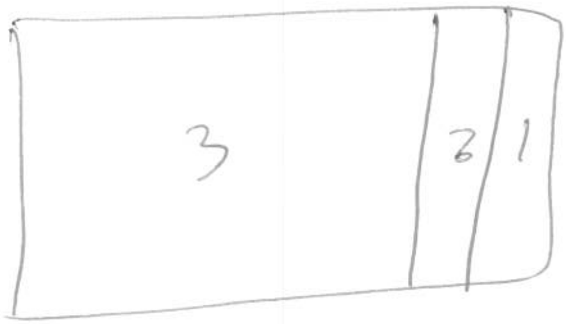
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

So it seems that randomness is sort of "built-in" to the universe at least on the scales of really small particles... Einstein did not like this "God does not play dice with the universe"

[illegible]

Go back to spinner example.. what is Ω look like?

Ω



Drawn to scale

$$P(\{3\}) = \frac{\text{size}(\{3\})}{\text{size}(\Omega)} = 3/4 = 0.75$$

$$P(\{1, 2\}) = \frac{\text{size}(\{1, 2\})}{\text{size}(\Omega)} = 1/4$$

Probability is the size of sample set relative to Ω,

If it's relative to Ω ... then

How do we know

measure each of the definitions.

I'll do two.. ① the division of Ω represents the propensity of the spinner to ~~[illegible]~~ yield 1, 2, 3

② Long run frequency... Imagine throwing a dart at the board randomly

?

Axiom 1 — $P(\Omega) = 1$

Axiom 2 — $0 \le P(E) \le 1$

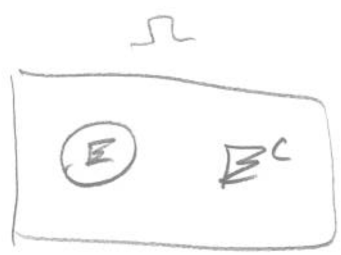
Axiom 3 — $P(A_1 \cup A_2 \cancel{\cup}) = P(A_1) + P(A_2)$ disjoint?

explain notation

Propensity [91%, 100%] $\boxed{\begin{smallmatrix}1&2&3\\4&5&6\end{smallmatrix}}$ $P\left(\bigcup_{i=1}^{4} A_i\right) = \sum_{i=1}^{4} P(A_i)$ If $A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$, ⋮

Chance 1 in 7, 2 in 5 ~~[illegible]~~ odds = $\boxed{\text{p157 - p158 S\&F}}$

What can we prove from these axioms?



Are E, E^c disjoint

Axiom 3 $\Rightarrow$ $\underbrace{P(E \cup E^c)}_{\underbrace{P(\Omega)}_{1}} = P(E) + P(E^c)$

Axiom 1

$\Rightarrow \quad 1 = P(E) + P(E^c)$

$\Rightarrow \quad P(E) = 1 - P(E^c), \quad P(E^c) = 1 - P(E)$

Complement Rule ... SUPER USEFUL

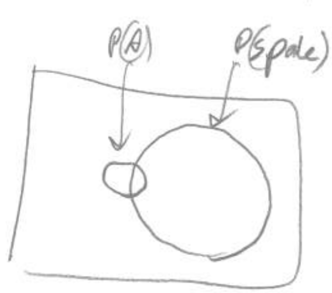
What's the prob of getting ... or fewer 6H of 7 flips that's 1H, 2H, 3H, 4H, 5H, 6H

or it's $1 - P(7T \text{ of } 7) = \boxed{1 - \frac{1}{2^7}}$

What about adding probs of overlapping sets?

Deck of cards

$P(A \text{ or a spade}) = P(A) - P(\text{Spade}) - P(A, \text{spade})$



Need $P(A \setminus \text{spade}) + P(A, \text{spade}) + P(\text{Spade} \setminus A)$

How do these work for disjoint?

Independence "informational irrelevance"

2 coin flips

$P(1^{st} \text{ flip } H) = \frac{1}{2}$

Does my first flip affect my second flip? No

$P(2^{nd} \text{ flip } H) = \frac{1}{2}$

$$P(1^{st} \text{ flip } H \cap 2^{nd} \text{ flip } H) = P(1^{st} \text{ flip } H)\, P(2^{nd} \text{ flip } H) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

1st flip

2nd flip	H_1	T_1
H_2	$H_1 H_2$	$T_1 H_2$
T_2	$H_1 T_2$	$T_1 T_2$

Ω

explain conditioning

How about 10 coin flips?

$$P(1^{st} \text{ flip } H, 2^{nd} \text{ flip } H, \dots, 10^{th} \text{ flip } H) = (P(H))^{10}$$

Mult. Rule for Independence:

$$P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1) \cdot \ldots \cdot P(A_n) = \prod_{i=1}^{n} P(A_i)$$

Sample w/ replacement vs. without replacement

Decks of cards