(draw deck of cards)

Lecture #4
5/26/11

Admin Stuff

- HW posted

Difference between $\binom{52}{5}$, $5!$ for $|\Omega|$ in poker hands

Bonferroni Eq

Review

A ♡	A ♢	A ♣	A ♠
2 ♡	2 ♢	2 ♣	2 ♠
⋮	⋮	⋮	⋮
K ♡	K ♢	K ♣	K ♠

notation is the $\mathbb{P}$

$P(A) = \frac{1}{13}$

$P(♡) = \frac{1}{4}$

$P(A)$ given you know it's a ♡?

Shrink $\Omega_1 \subset \Omega$

Is drawing a club ind of drawing a spade?

Odds ratios

Flip a coin $P(H) = \frac{1}{2}$ what are the odds?

Odds are a ratio of how much you bet to how much you win.

In this game, you should win \$1 for every bet you make

hence $\Rightarrow$ odds are $\frac{1}{1}$, "1 to 1", 1:1

Roll a die: If you win you take 6 but you bet 1 so 5,
if you lose, lose 1 $\Rightarrow \frac{1}{5}$ 1:5

Fair game $\frac{1}{6}$: 5, $\frac{5}{6}$: -1 what do we expect (next class) $5\cdot\frac{1}{6} + (-1)\frac{5}{6} = 0$

Usually you do odds-against. I'll bet you 5:1 against X.

$$Odds(E) = \frac{1 - P(E)}{P(E)}$$ "against" In the HW, I mean odds against

$$\Rightarrow P(E) = \frac{1}{Odds(E) + 1}$$

We know about independence.. let's talk about it some more. Ponder this:

P(IBM stock ↑ tom given it rains in Buenos Aires Today)

Does raining in Buenos Aires matter?

What does "matter" mean? Informational Relevance is the Litmus test for independence.

$\Rightarrow$ No.. they're independent

this is the same as

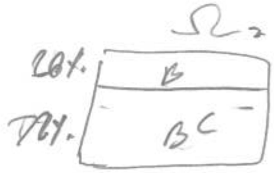
P(IBM stock ↑ tomorrow) in general

Let's formalise this...

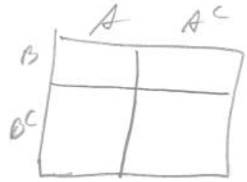
let A be the set of IBM Stock going up

what is this based on? long run freq?

let B be the set of rain in Buenos Aires



As expected, joint space



Now we're looking for

$$P(A \mid B) = P(A)$$

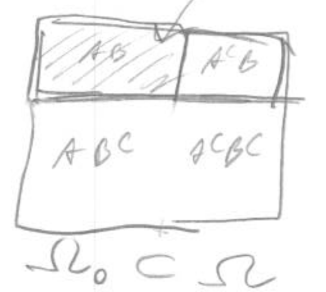
"given" "conditioning on"
shrinking universe

Now look at
the shrunk Ω
Same proportion of
space is belonging to A

Conditional Probability

$P(A|B)$

$= P(AB) \cdot$ scale factor



this region is the smaller universe

~~So is~~ what is the scale factor? Zoom in...

If I zoom in on $\frac{1}{2}$ I have to make everything twice as big

If I zoom in on $\frac{1}{100}$ I have to make everything 100 as big

I'm zooming in by $P(B)$ $\Rightarrow$ I have to make everything $\frac{1}{P(B)}$ times as big

$$\Rightarrow P(A|B) = \underbrace{P(AB)}_{\text{the event we care about}} \cdot \underbrace{\left(\frac{1}{P(B)}\right)}_{\text{scale factor}}$$

$$\Rightarrow \boxed{P(A|B) = \frac{P(AB)}{P(B)}}$$ Bayes Rule

Is this consistent? For independent events,

$P(AB) = P(A) \cdot P(B)$

$P(A|B) = \frac{P(A)\cancel{P(B)}}{\cancel{P(B)}} = P(A)$ just as our intuition said!

Bayes Rule not interesting in the case of A, B independent.

A full example:

1,000 adults in worldwide were tracked, 200/1000 which were smokers, 60/1000 had lung cancer, 36/1000 were smokers with lung cancer. Assume $n=1000$ is large enough to estimate the true prob accurately. (An issue that will be studied in depth later)

REAL PROBS!

let A be the event of smoking

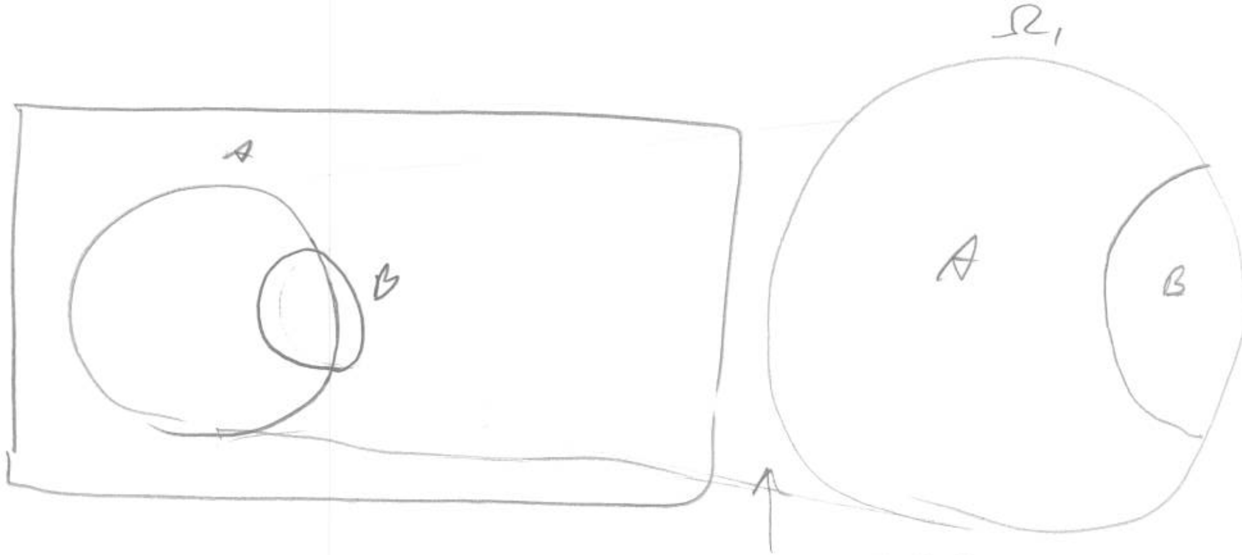
let B be the event of lung cancer

$P(A) = .200, \quad P(B) = .060, \quad P(AB) = .036$

Let's draw Ω to scale .. How?

Dep?

Ind?



Microscope field

What is prob of lung cancer given smoking?

$$P(B|A) = \frac{P(BA)}{P(A)} = \frac{.036}{.2} = .18$$

Would you roll the dice with that probability?

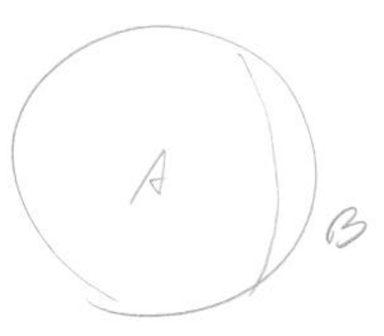
DON'T SMOKE...

What does this area look like? Universe "shrunk" then "expanded" to have area 1 instead of its previous .2

You already smoke ... so A already happened, now how much area does B take?

What if we want prob of smoking given lung cancer?

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{.036}{.06} = 0.60$$



What question does this ask and answer... a person walks in ... you diagnose him with lung cancer, chances are he smoked...

A cool math trick:

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(B|A)\,P(A)}{P(B)} = \frac{.18 \cdot .2}{.06} = .60 \checkmark$$

How about prob you have lung cancer given you did ~~not~~ smoke? Should be LOW.

$$P(B \mid A^c) = \frac{P(BA^c)}{P(A^c)} = \frac{.024}{0.8} = .03 \quad \text{still happens...}$$

$$P(B) = \underbrace{P(B \cap A) + P(B \cap A^c)}_{\text{disjoint sum}} \Rightarrow P(BA^c) = P(B) - P(BA)$$

$$= 0.06 - 0.036$$

$$= .024$$

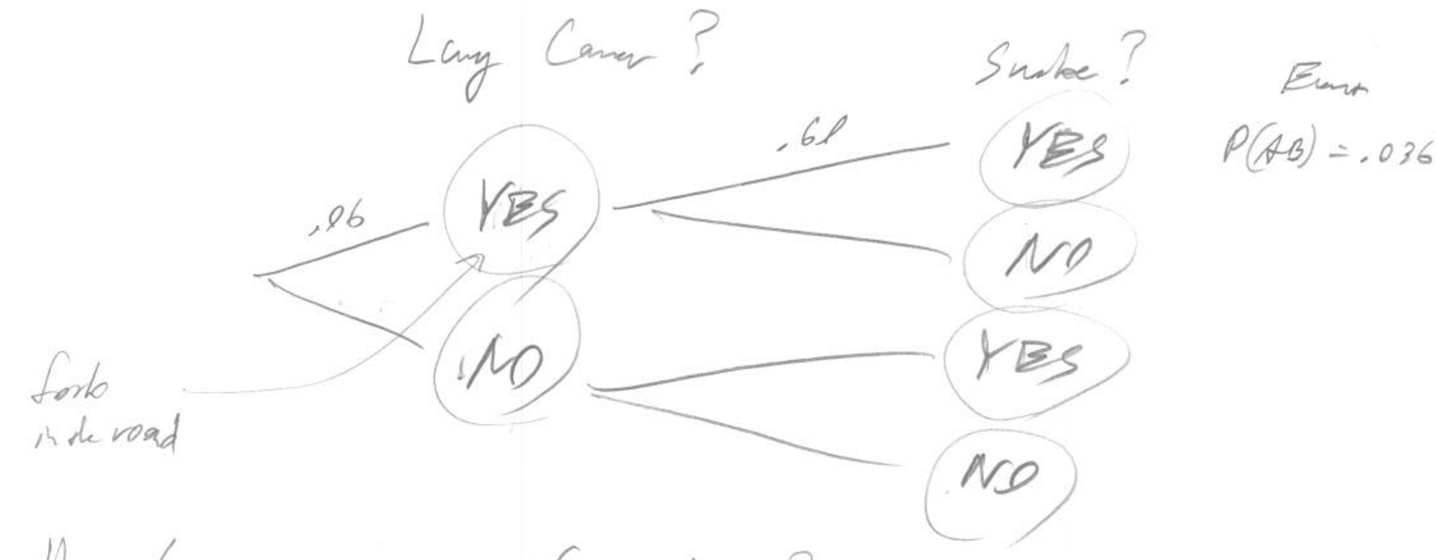
$$A \cap A^c = \phi$$

Increased Risk?

$$\frac{P(B|A)}{P(B|A^c)} = \frac{.18}{.03} = 6 \times \text{ more risk!!}$$

Let's look at this from a different angle..

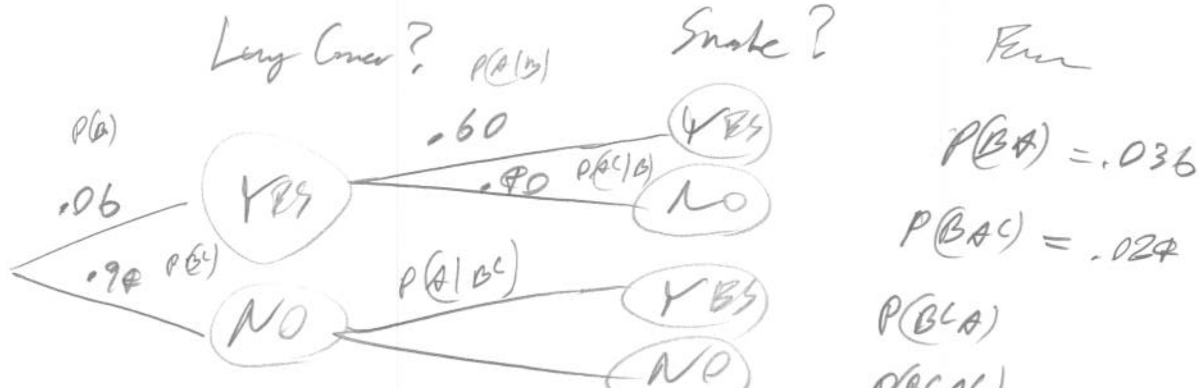
Let's draw trees



How do we get $P(A^c|B)$?

There's a fork in the road ... $P(A^c|B) = 1 - P(A|B) = 1 - .6 = .4$

Let's continue building this tree



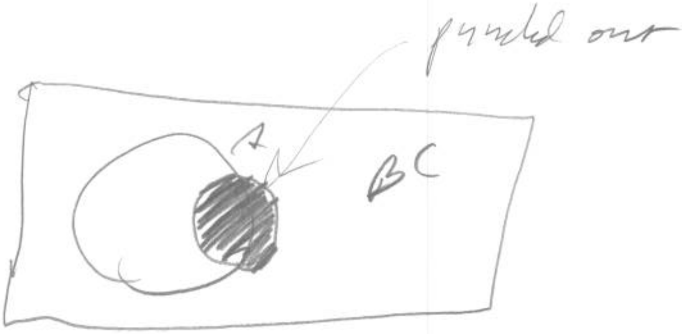
Full Bayes Rule ... p 180

Use Bayes Rule 3X!

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(A|B)\,P(B)}{\underbrace{P(AB) + P(AB^c)}_{\text{disjoint}}} = \frac{P(A|B)\,P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)}$$

$$\Rightarrow P(A|B^c) = \frac{1}{P(B^c)}\left(\frac{P(A|B)\,P(B)}{P(B|A)} - P(A|B)P(B)\right) = \frac{1}{.94}\left(\frac{.6\cdot .06}{.18} - .6\cdot .06\right) = \frac{1}{.94}(.164) = .174$$

What does this space look like?



$.174 \approx .2$ which makes sense

Complete the tree on previous page

.174 .164

.826 .776

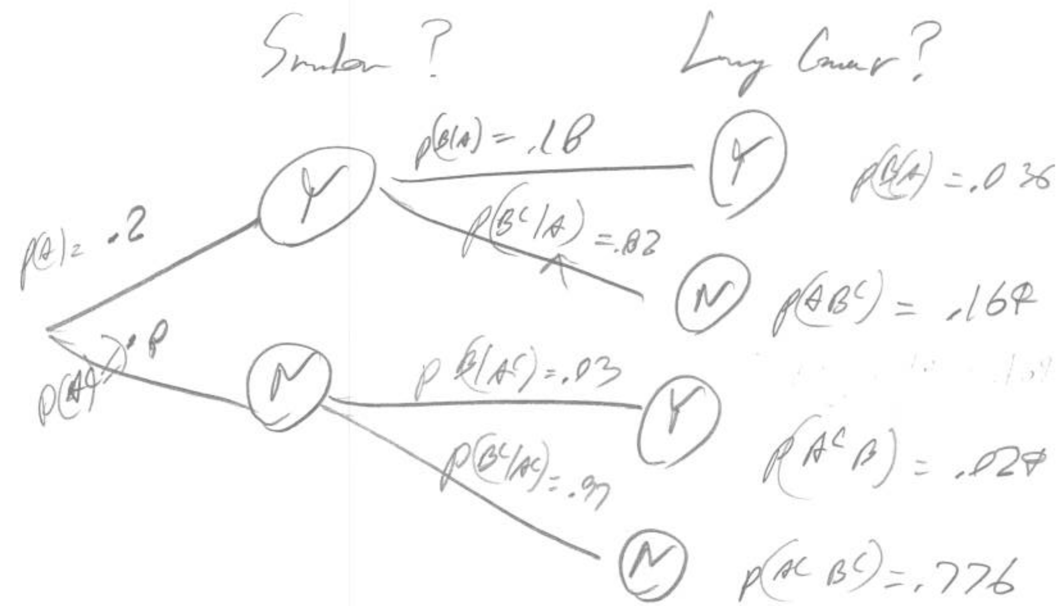
downstream

.036
.024
.164
.776 ← most people don't even get near smoking or lung cancer ⇒ good!

1

Now let's build the "inverse" tree

Still stuff to explore...



Count: How many conditional prob's there are?

$$\underset{\text{spot 1}}{\underline{4}} \cdot \underset{\text{spot 2}}{\underline{2}} = 8$$

Roadmap

- Sets
- Counting sets
- prob axioms
  - independence / dependence
- Cond probability / prob trees

week 1

- Random variables
- Expectation / Variance
- Distribution Theory
- Covariance
- Adding r.v.'s
- DATA!

week 2