6/16/4
Lecture #13

Admin

HW due noon (mid slot)

Plan

- CI's for samples when σ is unknown
- hypothesis testing for props

Recall: 95% CI's

margin of error

	Proportion	Samples
Real:	$\left[\hat{p} \pm 1.96\sqrt{\frac{p(1-p)}{n}}\right]$	$\left[\bar{X} \pm 1.96\frac{\sigma}{\sqrt{n}}\right]$
Approx:	$\left[\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$	$\left[\bar{X} \pm 1.96\frac{s}{\sqrt{n}}\right]$

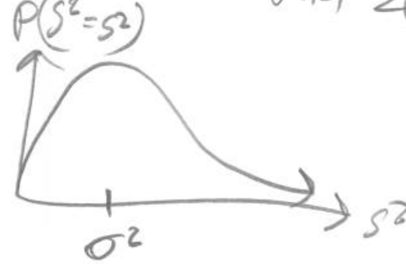
??

Why $\sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ but $s \approx \sigma$ if n is large

the r.v. the r.v.s

remember $s = \sqrt{\frac{1}{n-1}\sum(x_i - \bar{x})^2}$ $\quad S = \sqrt{\frac{1}{n-1}\sum(X_i - \bar{X})^2}$

S^2 has a distribution



s is a draw from its r.v. and has lots of randomness.

Therefore, we have to increase the width of the CI

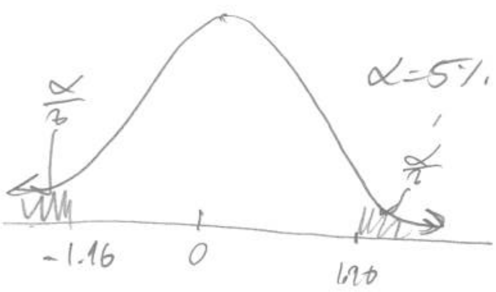
It will be greater than 1.96 but how big...

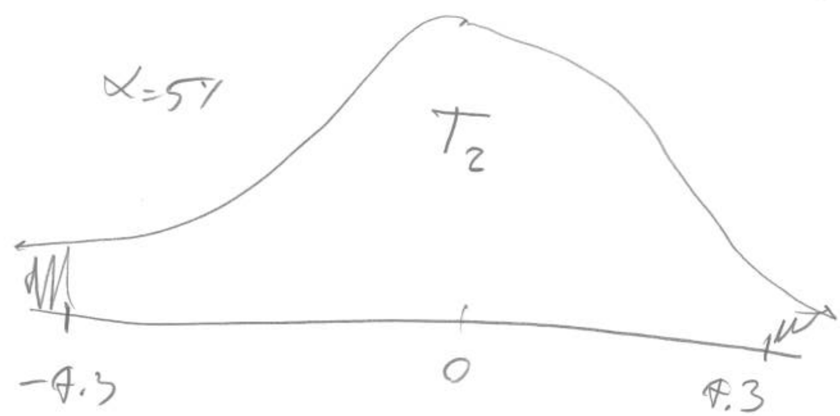
Remember:

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1) \qquad z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$T_{n-1} = \frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} \qquad t_{n-1} = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

the T distribution is just like the Z distr, centered at 0 but has "fatter" tails.





$$\lim_{n \to \infty} T_{n-1} = Z$$ for practical purposes $n \geq 30$ approximate T by Z

An example: Heights of NBA players (qu 6)

$\bar{X} = 77.59$, $s = 4.988$ Find 95% CI for the mean height of all NBA players. Problems? SRS?

$$CI_{\mu,95\%} = \left[\bar{X} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}\right] = \left[\bar{X} \pm t_{2.5\%, 16} \frac{s}{\sqrt{n}}\right]$$

$$= \left[77.59 \pm \underbrace{2.583 \cdot \frac{4.988}{\sqrt{17}}}_{3.12}\right] = [74.47, 80.71]$$

~~Ch 6 Hyp tests~~

You have a coin. You want to know if it's biased towards H. You flip 100x and get 60 #H.

Scen 1	Scen 2	Scen 3
52 H	93 H	60 H
biased?	"	"
prob no		⋮
need Stat 101?	"	

Setup of exp? $n = 100$

What is result of exp? $\hat{p} = 0.6$

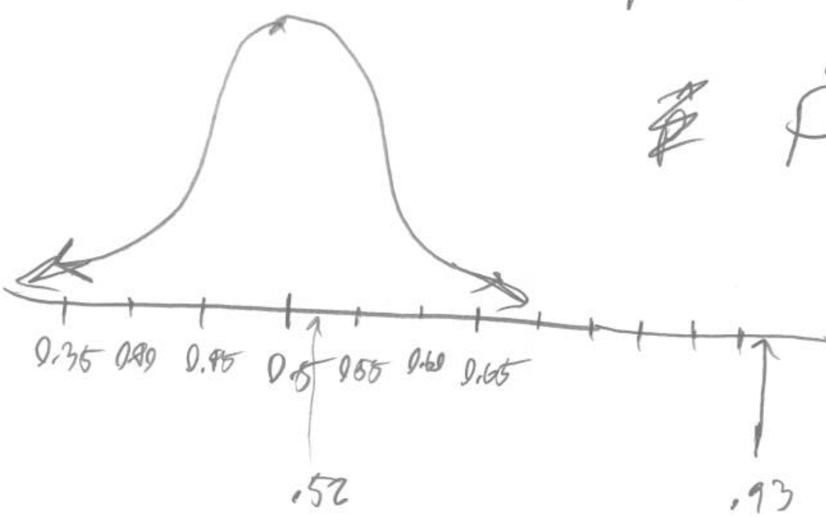
We have no tools... build CI?

$$CI_{p,95\%} = \left[\hat{p} \pm z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] = \left[.6 \pm \underbrace{1.96\sqrt{\frac{.6\cdot .4}{100}}}_{.097}\right] = [.504, .696]$$

does this help us? No...
Need new ~~tools~~ machine
Ch16 Hyp. Tests.

If the coin was not biased: $p=0.5$

$$\hat{P} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right) = N(0.5, 0.05)$$



The "not biased" "status quo" is $p=0.5$. We call this the null hypothesis.

$H_0 : p = 0.5$

The alternative or non-status quo... this is what we are trying to prove is the alternative hypothesis:

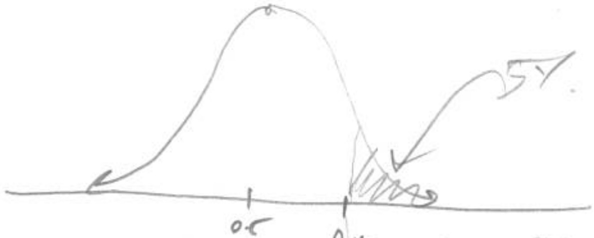
$H_a: p > 0.5$

We assume H_0 is true

We need to establish a "threshold" (cutoff) or "rejection region" for our Null.

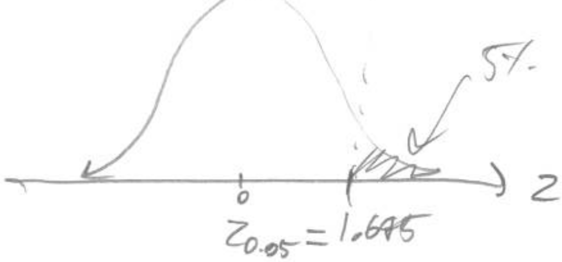
How do we do this? Pick a $\alpha = P(\text{Type I err})$ comfortable with... let's say $\alpha = 0.05$

$H_0: p = 0.5$
$H_a: p < 0.5$



$\hat{p}$

$\hat{p}^* = p + z\,SE[\hat{p}] = 0.5 + 1.65 \cdot 0.05 = .582$



$Z = \frac{\hat{p} - 0.5}{0.05}$

$Z_{0.05} = 1.645$

Are we in the rejection region? Yes $\hat{p} = 0.6 > \hat{p}^* = 0.58$
$\Rightarrow$ Reject H_0

Using $z = \frac{\hat{p} - p}{SE[\hat{p}]} = \frac{0.6 - 0.5}{0.05} = 2 > z_{0.05} \Rightarrow$ Reject H_0

What is the p-value?

$$\text{pval} = P\left(\text{this exper. results } \underline{\underline{\text{or}}} \text{ more extreme} \mid \text{null hypothesis is true}\right)$$

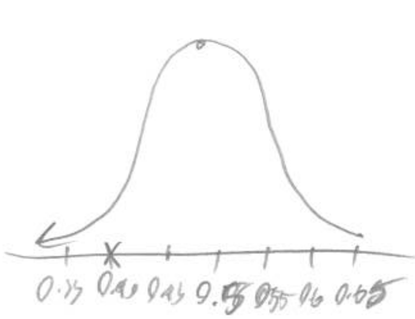
$$= P\left(\hat{p} \geq 0.6 \mid p = 0.5\right) = \int_{0.6}^{\infty} N(0.5, 0.05)\, dx$$

$$= P\left(Z \geq 2\right) = \int_{2}^{\infty} N(0,1)\, dz = .023 < \alpha = 0.05$$

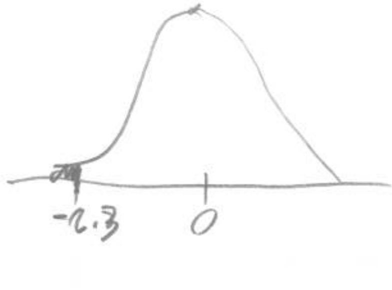
$$\Rightarrow \text{reject}$$

You can do any of these approaches, but I will require you to do the pval approach.

Maybe it's biased towards too few tails. 60H, 40T of 100. Test this at $\alpha = 1\%$.



$$\alpha = 1\% \Rightarrow z = -2.3$$



$$z = \frac{0.4 - 0.5}{0.05} = -2 \not< -2.3$$

fail to reject

$$\text{pval} = P\left(\hat{p} \leq 0.4 \mid p = 0.5\right) = P\left(Z \leq -2\right) = 0.023 > \alpha$$

$\Rightarrow$ fail to R

One-sided, ~~[illegible]~~ < left-sided (less than ...) / right-sided (more than ...)

Two-sided

M&M's ~~[illegible]~~ $\Omega = \begin{cases} \text{Blue w.p. } 24\% \\ \text{Brown w.p. } 13\% \\ \text{Green w.p. } 16\% \\ \text{Orange w.p. } 20\% \\ \text{Red w.p. } 13\% \\ \text{Yellow w.p. } 14\% \end{cases}$

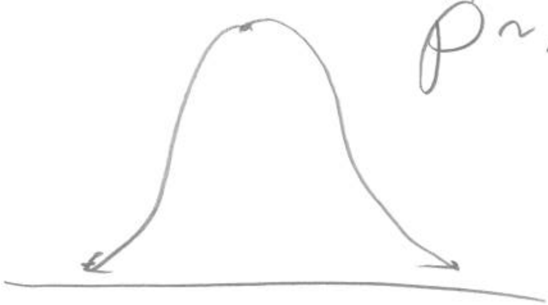
Want to test Blue in 5 packages.

$H_0: p = 0.24$

$H_a: p \neq 0.24$ ← test of just difference

Use $\alpha = 0.05$

$\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right) = N(0.24,$



Then 1-samp Z-tests...