Lecture #11
6/14/11

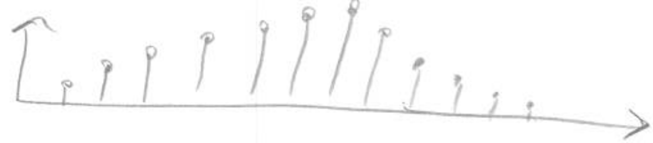
Plan
- review of Normal Curve, CLT
- ch 14 (skip skewness/kurtosis)
(skip ch 14.4)

Admin
- placed in midterm?
- HW due Thurs 3PM mailslot
- Shorter off hrs Thurs 11-noon

Review of Normal Curve

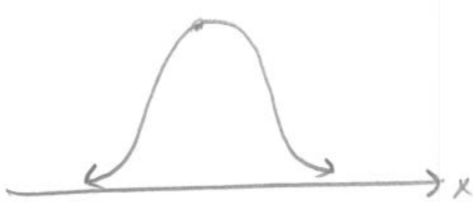
$X_1 \sim$ Die

$S_2 = X_1 + X_2$



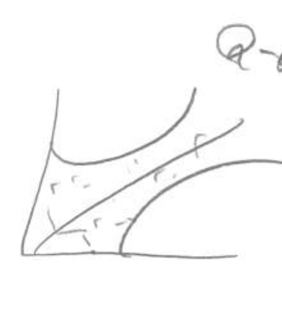
P(extremes) ↓ P(avg values) ↑ w/ # of r.v.'s summed up

The "natural" balance between extremes and means is



Can't integrate this ⇓ use table

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



Once again the Central Limit Thm (CLT) is:

$X_1, X_2, \dots X_n \overset{iid}{\sim}$ (something) and $\underbrace{n \text{ is large}}$, then $S_n = X_1 + \dots + X_n \sim N(n\mu, n\sigma^2)$

$\underline{n \geq 30}$ AND $\overline{X}_n = \frac{X_1 + \dots + X_n}{n} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$

From now on I will drop subscript on $\overline{X}_n$ it will just be $\overline{X}$

$\bar{X}$ is really important since we know its distribution.

We need new terminology:

$$\underbrace{SE}[\bar{X}] \triangleq SD[\bar{X}]$$

std error of the mean or just "std error"

Why? I don't know...

Case study

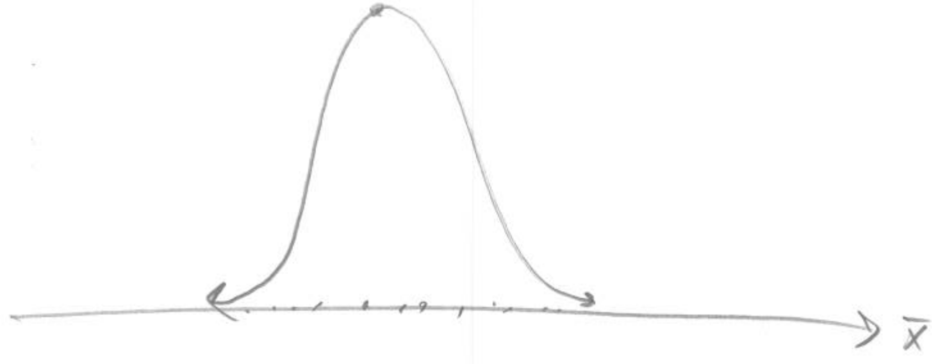
The chips in Ch 14 have a "highly-accelerated life test" score (HALT)

the avg $\mu = 7$, $\sigma = 4$ hence $\bar{X} \sim N\left(7, \left(\frac{4}{\sqrt{20}}\right)^2 = 0.89^2\right)$

we take 20 samples / day

Each daily sample $(\bar{x})$ should resemble a draw from $(\bar{X})$ the r.v.

This is the most important concept for the next half of the course, let's go over it in detail:

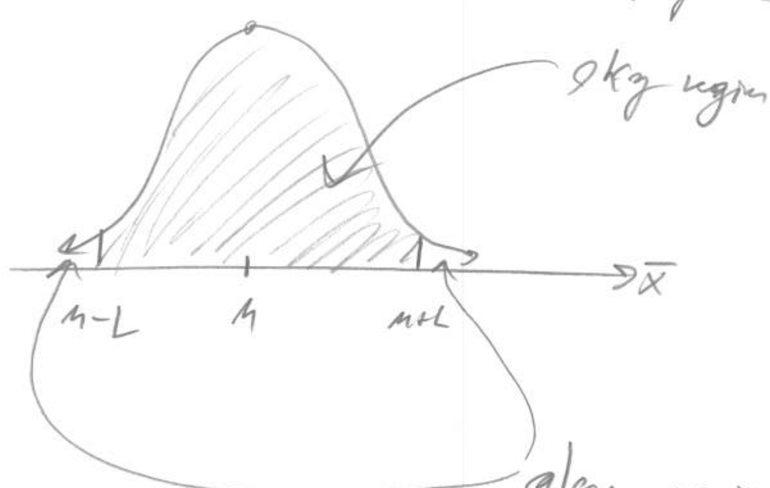


What happens? How does $\bar{X}$ realize?

If $\bar{X}$ is "close to" μ, the machine is working normally (we think). If not, we conclude machine is working abnormally. What is our cutoff?

It's arbitrary: control limits: $\bar{X} \in [\mu - L, \mu + L]$

L is arbitrary



If $\bar{x} \in$ okay region $\Rightarrow$ say okay

If $\bar{x} \in$ alert region $\Rightarrow$ say alert

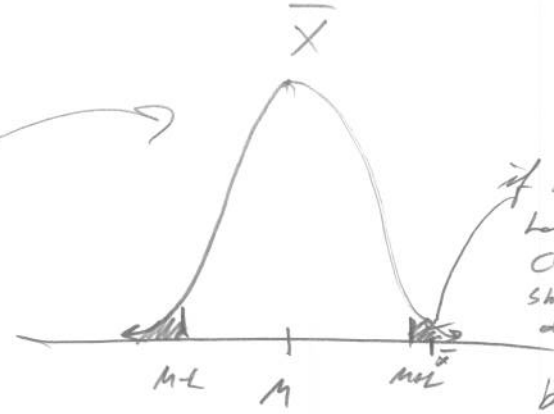
Pretty simple... BUT there are two types of errors you can make. Whenever you make decisions based on random data, you can screw up.

$\bar{x}$ is random since it's a realization of a r.v.

What you think ?

State of the world

	Continue	shutdown
works	✓	Type I error
does not work	Type II error	✓

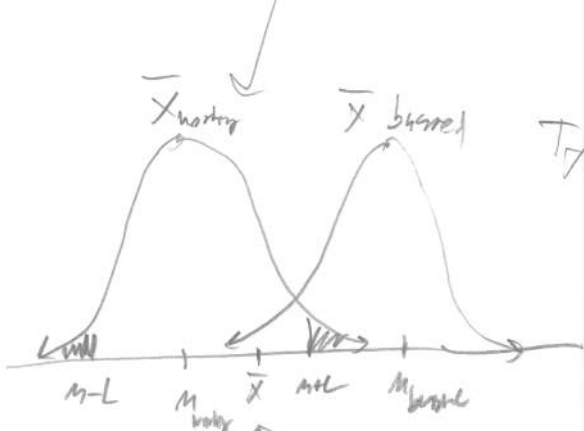


if $\bar{X}$ is here we choose to shut down

but this is normal behavior

$$\int_{\mu+L}^{\infty} f_{\bar{X}}(x)\,dx = \text{same probability right?}$$

$$P(\text{Type I error}) = P(\bar{X} \geq \mu+L) + P(\bar{X} \leq \mu-L)$$



Type II error difficult to understand

$\bar{X}$ is in accept region, but came from busted distribution. BAD

How do we set L? We pick a $P(\text{Type I error}) \triangleq \alpha$

which we denote α. What do you think it should be?

Let's say $\alpha = 5\%$

This means we have

$$\frac{(\mu+L)-(\mu)}{\sigma} = z \quad \text{use the cum table}$$

$\Rightarrow z = 1.96$

$\Rightarrow L = \sigma z = 1.96 \cdot 0.89 = 1.74$

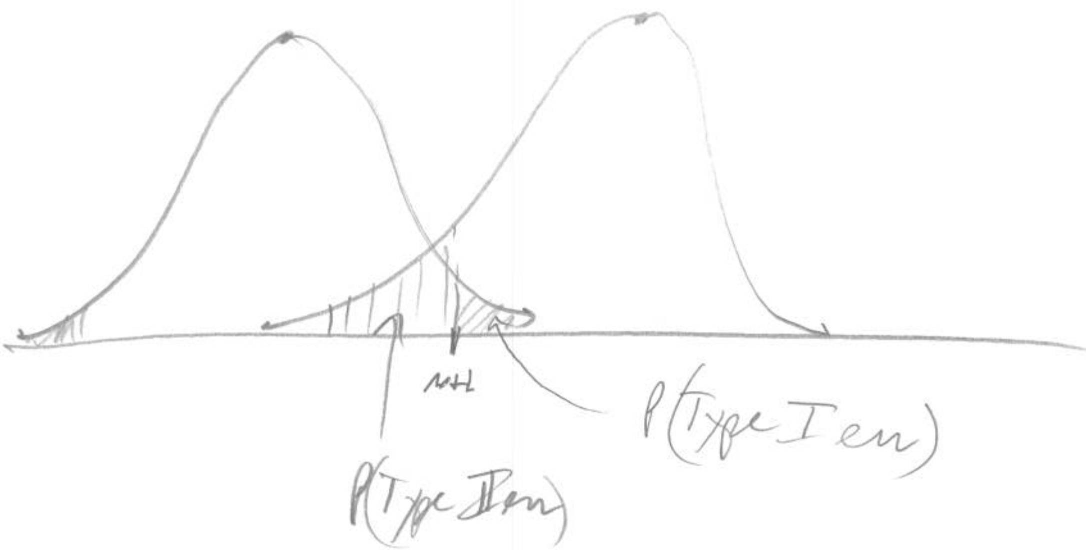
Rule: $\bar{X} \in [5.256, 8.744] \Rightarrow$ ~~accept~~ continue

$\bar{X} \notin [\;''\quad ''\;] \Rightarrow$ shutdown

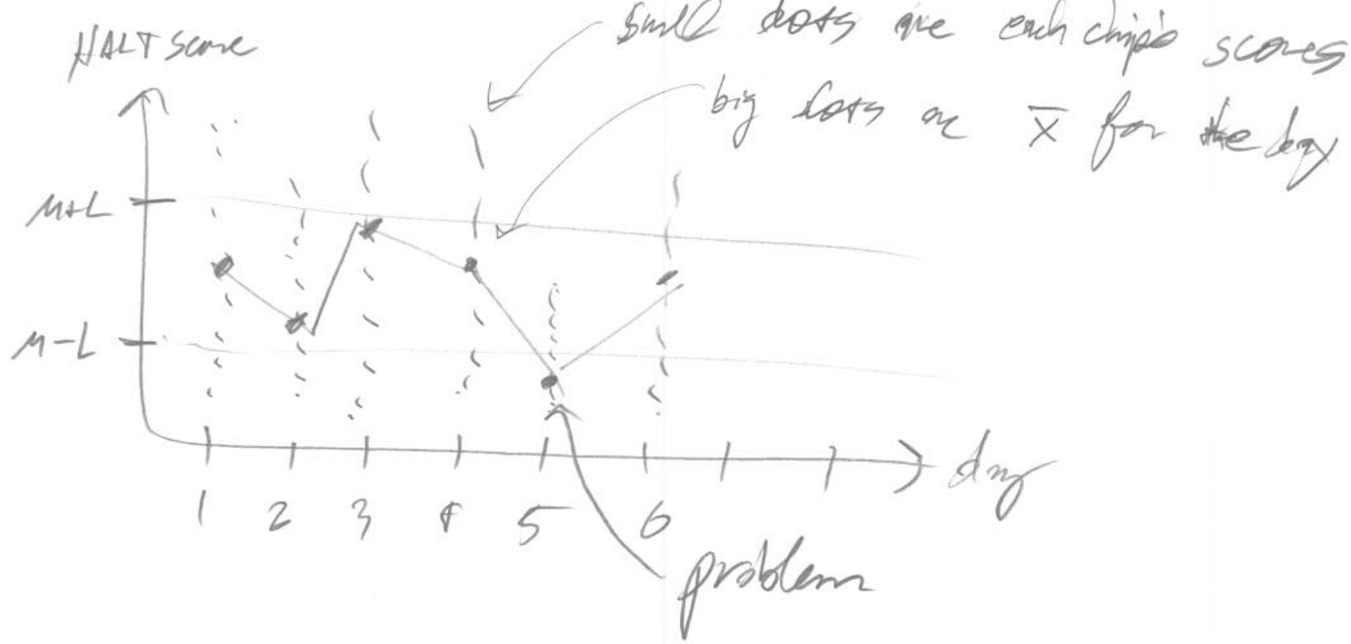
P(Type II error) is impossible to compute unless you know the distribution of $\bar{X}_{busted}$ and who knows what that looks like?

We do know this:

If you push L higher P(Type I error) ↓, but P(Type II error) ↑ why



If I push L up ... $\alpha \downarrow$ but P(Type II err) $\uparrow$
How to balance... very hard!!!!
What's a "control chart"?



Subtle problem. If $\alpha = 0.05$ there's a 5% chance of a type I error per day. What is the chance of a type I error over these 6 days?

$$P\left(\begin{matrix}\text{1 or more} \\ \text{type I errors}\end{matrix}\right) = 1 - \binom{6}{0} \alpha^0 (1-\alpha)^6 = 1 - .95^6 = \underline{\underline{.265}}$$

that's really high! Maybe we should set α lower?

Let's set the "family-wise" Type I error rate to 0.05

$$\Rightarrow \; 0.05 = P(\text{1 or more type I errors}) = 1 - (1-\alpha^*)^6$$

$$\Rightarrow \; .95 = (1-\alpha^*)^6 \Rightarrow .991 = 1 - \alpha^* \Rightarrow \underline{\underline{\alpha^* = 0.009}}$$

This is called an α-correction.

S-charts track "S", the sample std dev
R-charts track "R", the sample range = max − min
not covered

Ch15

Review of CLT

$X_1, \ldots X_n \overset{iid}{\sim}$ someting, n is large $\Rightarrow$ $\overline{X} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$

why? because sum..

Review of Bernoulli / Binomial

$X_1, \ldots, X_n \overset{iid}{\sim} \text{Bernoulli}(p) \Rightarrow \underbrace{S_n = X_1 + \ldots + X_n}_{\text{that's a sum}} \sim \text{Binomial}(n, p)$

does this mean $X_1 + \ldots + X_n \sim N\left(E[S], SD[S]^2\right)$

Yes... you'll see on the HW that Binomial $\propto$ Normal

Remember $X_1, \ldots, X_n$ are 0/1 r.v.'s

so S measures how many yes's or successes

$\Rightarrow \hat{p} \triangleq \overline{X} = \frac{S}{n}$ measure "proportion" of successes

therefore we have special notation (p.370)

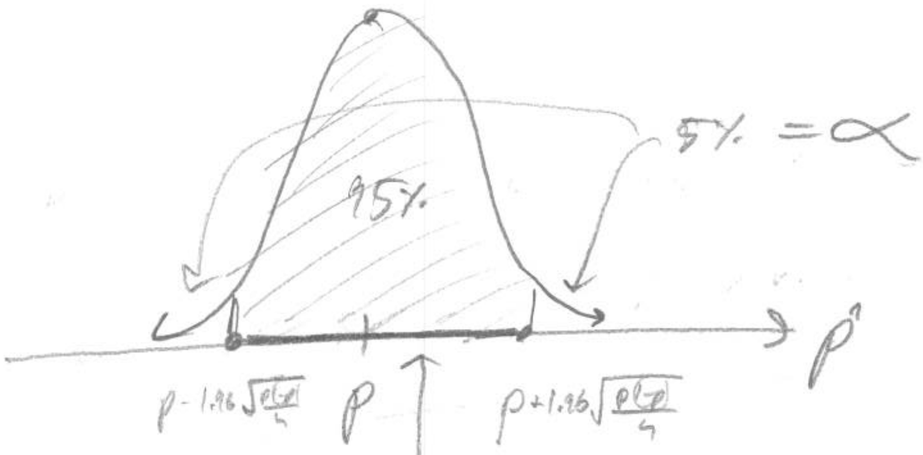
$\hat{p}$ means "sample proportion"

and it estimates p the true prob of success

so $\overline{X} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right) \Rightarrow \hat{p} \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$ When?

$SE[\hat{p}]$

$n\hat{p} > 10$, $n(1-\hat{p}) > 10$



there is a 95% chance $\hat{p}$ will be in this interval

Special notation: since there is only 2.5% left in the right tail we call $z_{0.025} \triangleq 1.96$

No new math at all — just subtleties in concepts

$$P\left(\hat{p} \in \left[p - z_{0.025} \cdot SE[\hat{p}],\ p + z_{0.025} \cdot SE[\hat{p}]\right]\right) = 95\%$$

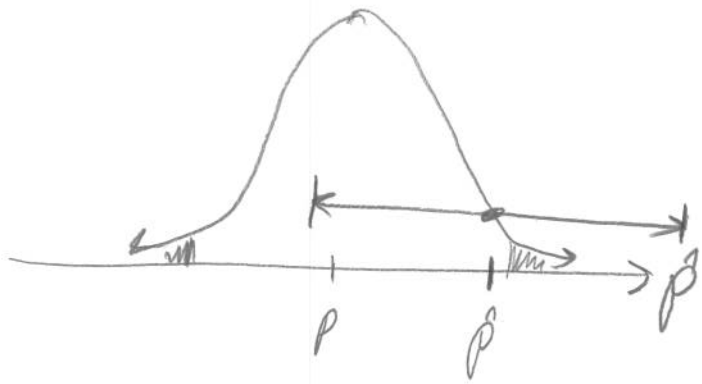
Generally...

"plus or minus" convenient notation

$$P\left(\hat{p} \in \left[p \pm z_{\frac{\alpha}{2}} \cdot SE[\hat{p}]\right]\right) = 1 - \alpha$$

Now... what if we do the following...

Use that width we got, but center it around $\hat{p}$.



p. 358

$\left[\hat{p} \pm z_{0.025} \cdot SE[\hat{p}]\right]$ is called a confidence interval (CI)

Since its width represents 95%, we call this "a 95% CI for p".

Slight problem $SE[\hat{p}] = \sqrt{\frac{\boxed{p(1-p)}}{n}}$ ← usually we don't know p

$\Rightarrow SE[\hat{p}] \approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

book uses "$se[\hat{p}]$" but I don't want to get lost in notation