5/23/11

Lecture #1

Syllabus

- June 8/9 reschedule
- Office Hours
- Test dates
- Calculator

Fridays
June 3, 17, 24
double classes?

Roadmap

I Probability Theory
- elementary set theory
- counting
- axioms
- calculations
- disjoint, independence
- conditional probability

II Random Variables / Distribution Theory
- r.v.'s as functions
- probability mass functions
- expectation, variance
- combining r.v.'s
- covariances
- sums and the Central Limit Theorem

III Real Data
- summary statistics
- boxplots, bar charts, scatterplots

IV Statistical Inference
- confidence intervals
- hypothesis testing proportions and means

V Linear Regression
- association
- correlation / causation
- intro to some 102 topics

Basic Set theory

Almost All of mathematics is built on the foundation of "sets."

Because prob & statistics is a branch of mathematics, we need to at least be familiar with sets.

Invented in 1870's, formalized in early 20th century.

A set A is collection of unique elements and is denoted like so:

$$F = \{\text{Jane}, \text{Mary}, \text{Susan}\}$$

$$M = \{\text{Bob}, \text{Joe}, \text{Max}\}$$

Sometimes they're drawn like cartoons:

Inclusion in a set is denoted with an epsilon-like symbol:

$$\text{Jane} \in F \quad \text{or} \quad \text{Jane} \in \{\text{Jane}, \text{Mary}, \text{Susan}\}$$

Sets can have any # of elements, 0, 1, 2, ..., even infinite elements!

Subsets are denoted with a tilted "U" symbol:

$\{Jane, Mary\} \subset F$, $\{Jane, Mary\} \subset \{Jane, Mary, Susan\}$

We use a $\subset$ symbol to denote "proper subset" when the daughter subset is not equal to the parent subset itself.
We use $\subseteq$ symbol when we allow the subset to be equal to the parent set. This is the subset symbol.

We combine sets by ~~taking~~ unions $\cup$ a "cup" symbol.
Takes all elements and puts it into one set

$\{Jane\} \cup \{Mary, Susan\} = F$

Due to uniqueness:

$\{Jane, Mary\} \cup \{Mary, Susan\} = F$

Set union is like addition, but you have to be careful not to "double-count."

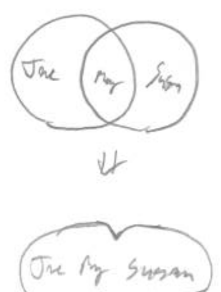
Union is also sometimes denoted "or"

$F \cup M$ is F or M, female, male, or both at once

We can intersect sets by using the $\cap$ symbol — an upside union or "cap" symbol. Intersections only take the common elements;

"Venn diagram"

$$\underbrace{\{\text{Jane}, \text{Mary}\}}_{A_1} \cap \underbrace{\{\text{Susan}, \text{Mary}\}}_{A_2} = \underbrace{\{\text{Mary}\}}_{A}$$



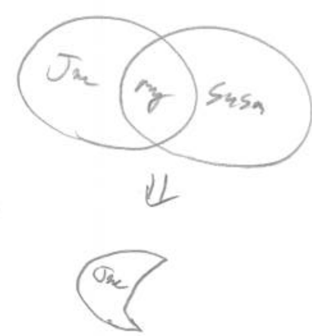
What about this:

$$F \cap M = \{\} \triangleq \emptyset$$

special symbol for "null set"
the set with nothing in it

Set differences are one set minus all the elements in another set that are relevant

$$\{\text{Jane}, \text{Mary}\} \setminus \{\text{Susan}, \text{Mary}\} = \{\text{Jane}\}$$



When two sets have an empty intersection, they're called "disjoint" or "mutually exclusive"

Explain this term

"Powersets" – the set of all sets. All sets which are subsets. For instance: lets find the powerset of these females:

$$\underbrace{2^F}_{\text{notation}} = \{ \emptyset,$$

the null set

{Jane},
{Mary},
{Susan},
{Jane, Mary},
{Jane, Susan},
{Susan, Mary},
{Jane, Mary, Susan}

the whole set

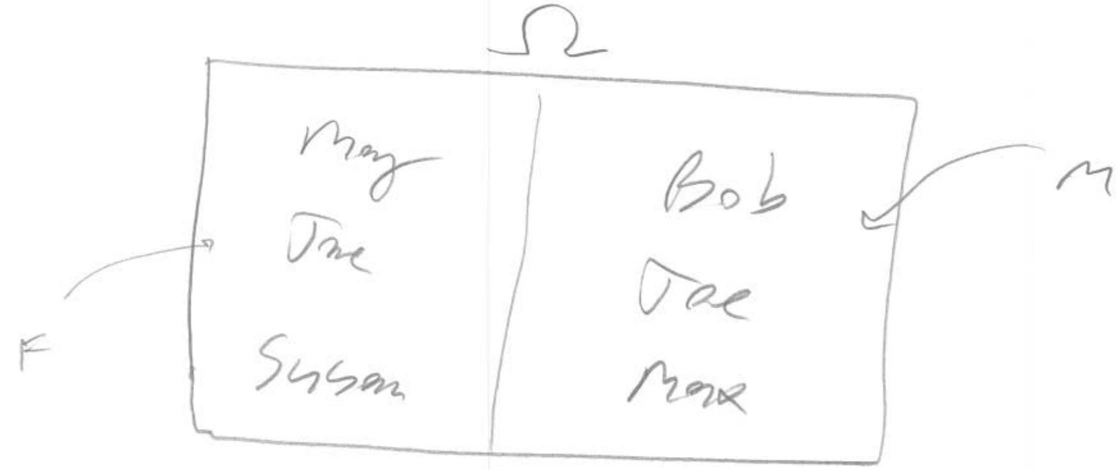
}

Why is this important? These are all the teams females can do! Only 8. You probably have noticed that there are 3 females and $2^3 = 8$. You will get good at this later

The most special set in probability is called Ω (Capital ω last greek letter). This is the "universe", the "universe of discourse", or the "sample space".
In the book p156, they call it "S". Ω is all the elements we're restricted our current investigation to.

In our case, $\Omega = \{\text{Mary, Jane, Susan, Bob, Joe, Max}\}$

We usually draw Ω as a box:



We can then indicate the subsets $F \subset \Omega$, $M \subset \Omega$

Notice how $\Omega = F \cup M$ but $F \cap M = \emptyset$?

This is called "mutually exclusive" and "collectively exhaustive".

Set "size" or "measure". Right now, we will just count the # of elements in the set also called cardinality. We use the $|\cdot|$ symbol (abs val) for this: For instance

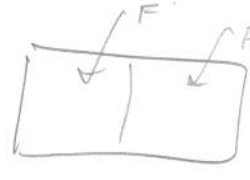
$$|\Omega| = 6, \quad |F| = 3, \quad |M| = 3$$

Already we can see the roots of probability being planted... If I were to pick a random person out of the universe. What is the chance/odds/prob they are female?

$$\underbrace{\frac{|F|}{|\Omega|} = \frac{3}{6} = \frac{1}{2}}_{\text{informal, don't trust this now}}$$

Set complement: everything that's not in the set in the universe

$$F^c = \Omega \setminus F$$



Almost by definition $\Omega = F \cup F^c$, $F \cap F^c = \emptyset$