Article

Quantum entanglement tests based on the CHSH inequality

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Abstract: In physics, the CHSH inequality is central to Bell's theorem, which demonstrates that certain consequences of quantum entanglement cannot be reproduced by local hidden-variable theories. Derived by John Clauser, Michael Horne, Abner Shimony, and Richard Holt in a 1969 paper, the CHSH inequality sets a statistical constraint on the "coincidences" observed in a Bell test, which must hold if local hidden-variable theories are valid. Experimental violations of the CHSH inequality provide strong evidence that such theories cannot fully describe nature. Modern quantum experiments routinely violate this inequality, reinforcing the non-local nature of quantum mechanics.

Keywords: Quantum Computing, Quantum Machine Learning.

1. Measurement $\hat{X} \otimes \hat{W}$

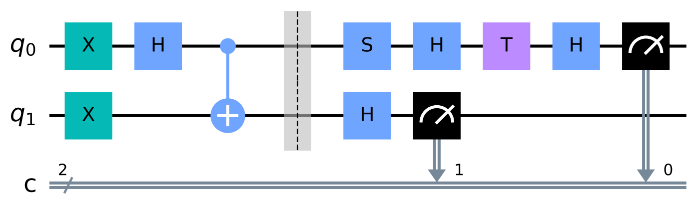

Figure 1. The Circuit.

Table 1. Results of calculations: $\langle P_{\hat{X}}(y) P_{\hat{W}}(x) \rangle = -0.72265625$.

Case No (yx)	$P_{\hat{W}}(x)$	$P_{\hat{X}}(y)$	$P_{\hat{W}}(x) \times P_{\hat{X}}(y)$	p(yx)	$P_{\hat{W}}(x) \times P_{\hat{X}}(y) \times p(yx)$
00	1	1	1	0.078125	0.078125
01	-1	1	-1	0.425781	-0.425781
10	1	-1	-1	0.435547	-0.435547
11	-1	-1	1	0.060547	0.060547

Citation: Title. *Quantum Rep.* **2023**, *1*, 1–5. https://doi.org/

Received:
Revised:
Accepted:
Published:

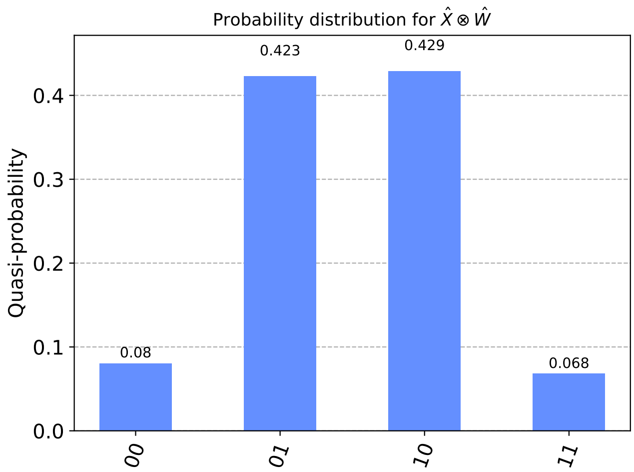

Figure 2. Probability Distribution.

2. Measurement $\hat{X} \otimes \hat{V}$

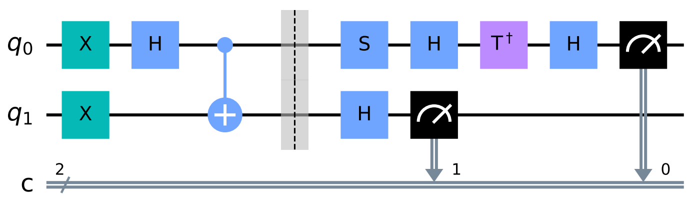

Figure 3. The Circuit.

Table 2. Results of calculations: $\langle P_{\hat{X}}(y) P_{\hat{V}}(x) \rangle = 0.685546875$.

Case No (yx)	$P_{\hat{V}}(x)$	$P_{\hat{X}}(y)$	$P_{\hat{V}}(x) \times \mathrm{P}_{\hat{X}}(y)$	p(yx)	$P_{\hat{V}}(x) \times \mathrm{P}_{\hat{X}}(y) \times p(yx)$
00	1	1	1	0.440430	0.440430
01	-1	1	-1	0.075195	-0.075195
10	1	-1	-1	0.082031	-0.082031
11	-1	-1	1	0.402344	0.402344

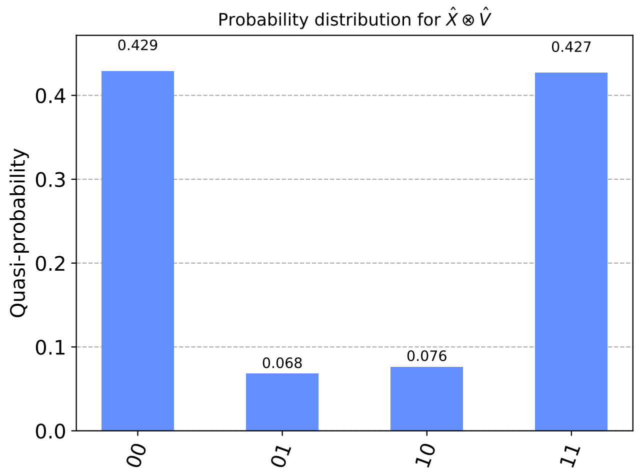

Figure 4. Probability Distribution.

3. Measurement $\hat{Z} \otimes \hat{W}$

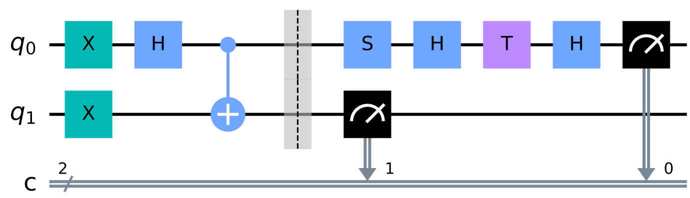

Figure 5. The Circuit.

Table 3. Results of calculations: $\langle P_{\hat{Z}}(y) P_{\hat{W}}(x) \rangle = -0.716796875$.

Case No (yx)	$P_{\hat{W}}(x)$	$P_{\hat{Z}}(y)$	$P_{\hat{W}}(x) \times \mathrm{P}_{\hat{Z}}(y)$	p(yx)	$P_{\hat{W}}(x) \times \mathrm{P}_{\hat{Z}}(y) \times p(yx)$
00	1	1	1	0.068359	0.068359
01	-1	1	-1	0.458008	-0.458008
10	1	-1	-1	0.400391	-0.400391
11	-1	-1	1	0.073242	0.073242

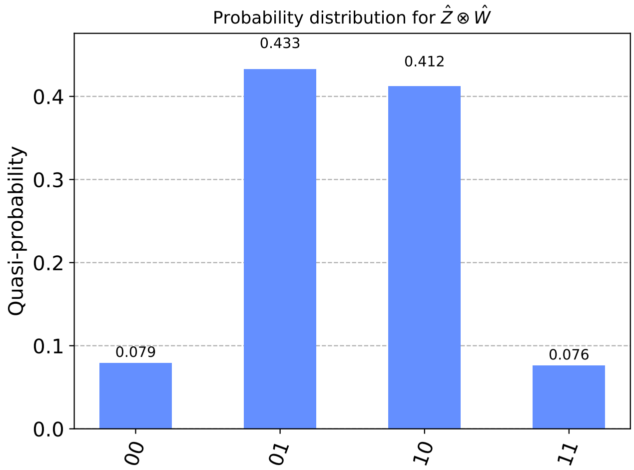

Figure 6. Probability Distribution.

4. Measurement $\hat{Z} \otimes \hat{V}$ 12

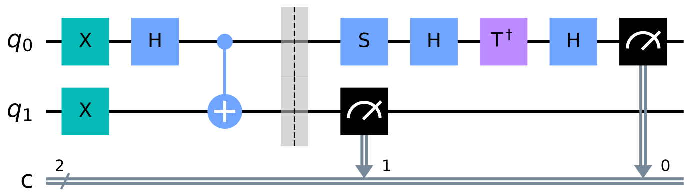

Figure 7. The Circuit.

Table 4. Results of calculations: $\langle P_{\hat{Z}}(y) P_{\hat{V}}(x) \rangle = -0.720703125$.

Case No (yx)	$P_{\hat{V}}(x)$	$P_{\hat{Z}}(y)$	$P_{\hat{V}}(x) \times P_{\hat{Z}}(y)$	p(yx)	$P_{\hat{V}}(x) \times P_{\hat{Z}}(y) \times p(yx)$
00	1	1	1	0.064453	0.064453
01	-1	1	-1	0.449219	-0.449219
10	1	-1	-1	0.411133	-0.411133
11	-1	-1	1	0.075195	0.075195

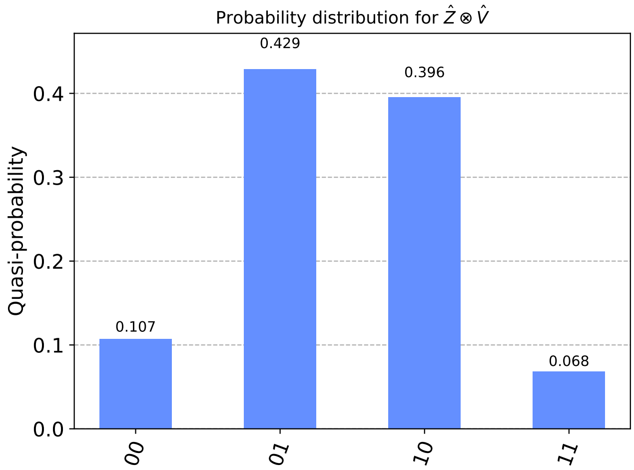

Figure 8. Probability Distribution.

5. Calculation of $\langle S \rangle$

$$\langle S \rangle = \langle \hat{X} \otimes \hat{W} \rangle - \langle \hat{X} \otimes \hat{V} \rangle + \langle \hat{Z} \otimes \hat{W} \rangle + \langle \hat{Z} \otimes \hat{V} \rangle. \tag{1}$$

Table 5. The results were calculated over 5 repetitions.

	$\hat{X} \otimes \hat{W}$	$\hat{X} \otimes \hat{V}$	$\hat{Z} \otimes \hat{W}$	$\hat{Z} \otimes \hat{V}$
0	-0.691406	0.691406	-0.701172	-0.693359
1	-0.703125	0.710938	-0.708984	-0.730469
2	-0.669922	0.712891	-0.714844	-0.644531
3	-0.656250	0.675781	-0.689453	-0.751953
4	-0.732422	0.662109	-0.732422	-0.708984

Finally, the obtained mean value of $\langle S \rangle$ is equal to:

$$\langle S \rangle = -2.8054(0.0396). \tag{2}$$

Standard deviation is reported in parenthesis.