PH108 : Electricity & Magnetism : Problem Set 9

Only * problems are to be solved in the tut session

1. Calculate the magnetic force of attraction between the northern and southern hemispheres of a spinning charged magnetic shell (with radius R, angular speed ω and surface charge density σ)

2. * What current density would produce the vector potential, $\vec{A} = k\hat{\phi}$ (where k is a constant), in cylindrical coordinates?

3. * A sphere of radius R carries a polarization $\vec{P}(\vec{r}) = k\vec{r}$ where k is a constant and $\vec{r}$ is the vector from the center.

   (a) Calculate the bound charges σ_b and ρ_b

   (b) Find the electric field inside and outside the sphere.

4. A thick spherical shell having inner radius a and outer radius b is made of dielectric material with a "frozen-in" polarization $\vec{P}(\vec{r}) = \frac{k}{r}\hat{r}$ where k is a constant and r is the distance from the center and no free charge is present, then find $\vec{D}$ and $\vec{E}$ for all 3 regions. Refer Figure 1.

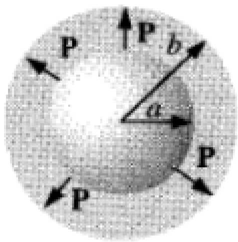

Figure 1

5. In the lecture, we have calculated the total field inside the sphere when a sphere made of homogeneous linear isotropic dielectric (with dielectric constant ϵ_r) material is placed in an otherwise uniform electric field $\vec{E}_0$ by solving Laplace's equation:

$$\vec{E} = \frac{3}{\epsilon_r + 2} E_0$$

   Attempt an alternate (and possibly more intuitive) approach to this, as follows. First find the polarization $\vec{P}_0$ due to $\vec{E}_0$. This polarization generates a field of its own, $\vec{E}_1$ which in turns causes and additional polarization $\vec{P}_1$, which further generates an additional field $\vec{E}_2$ and so on. Show that resultant field inside the sphere $\vec{E} = \vec{E}_0 + \vec{E}_1 + \vec{E}_2 + \ldots$ matches that what we get by solving Laplace's Equation.

6. * Find the electric potential inside and outside a homogeneous linear isotropic dielectric sphere (with dielectric constant ϵ_r and radius R), at the centre of which a pure dipole $\vec{p}$ is imbedded.

7. * A cylinder of radius R and height L is positioned such that the origin is at the center and the z-axis is along the axis of the cylinder. The cylinder carries a frozen polarisation $\vec{P} = P_o\hat{z}$. Calculate $\vec{E}$ and $\vec{D}$ at all points on the z-axis. Are the quantities $\vec{D}$ and $\vec{E}$ proportional to each other inside the material?

8. * A conducting sphere of radius R is half submerged in a linear, homogeneous, semi-infinite liquid dielectric medium of doelectric constant κ. The sphere is at a potential V_0. Assuming there is no bound charge at the liquid-air interface, calculate

(a) the potential at a point outside the sphere,

(b) the electric field, the electric dispacement, and the surface and volume bound charge density in the dielectric,

(c) the total free charge on the conductor.