PH108 : Electricity & Magnetism : Problem Set 11

Only * problems are to be solved in the tut session

1. * A very long solenoid of n turns per unit length carries a current which increases uniformly with time, $i = Kt$.

   (a) Calculate the electric field and magnetic field inside the solenoid at time t (neglect retardation).

   (b) Consider a cylinder of length l and radius equal to that of the solenoid, and coaxial with the solenoid. Find the rate at which energy flows into the volume enclosed by this cylinder and show that it is equal to $\frac{d}{dt}(\frac{1}{2}lLi^2)$, where L is the self-inductance per unit length of the solenoid.

2. It has been proposed that a spacecraft may be propelled by harnessing the pressure of sunlight. Assume that a spacecraft is sufficiently away from earth and is under the sun's gravitational field alone. A very large and fully reflecting sail is oriented at right angles to the sun's rays and attached to the craft. How large must the sail be so that the craft can start sailing away from the sun?
   The sun radiates 10^{26} W and has a mass of 10^{30} kg, the total mass of the spacecraft and sail is 1500 kg.

3. * An infinite wire carries a current up the rotational symmetry axis of a toroidal solenoid with N tightly wound turns and a circular cross section. The inner radius of the toroid is a and the outer radius is b. Find the mutual inductance M between the wire and the solenoid

4. A rectangular coil has length $2L$ and width $2w$. The coil is in the $x-z$ plane, centred at the origin and rotates about the z-axis with uniform angular velocity ω. A uniform magnetic field of B_0 is applied in the y-direction. Determine the emf induced in the coil using motional EMF. Then check your answer using rate of change of flux in a coil

5. * Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0 and frequency ω and phase angle zero, that is travelling (a) in the negative x direction and polarized in z direction, (b) traveling in the direction from the origin to the point (1,1,1) with polarization parallel to the xz plane.

6. * Consider a propagating wave in free space given by

$$\vec{E} = E_0 \frac{\sin\theta}{r} \left[\cos(kr - \omega t) - \frac{\sin(kr - \omega t)}{kr} \right] \hat{\phi} \tag{1}$$

   (a) Calculate the magnetic field $\vec{B}$ and the Poynting vector $\vec{S}$. You would need to use the expansions of $\boldsymbol{\nabla}\times$ in spherical co-ordinates.
   (b) What is the total average power radiated by the source?

7. * A cylinder of radius R and infinite length is made of permanently polarized dielectric. The polarization vector $\vec{P}$ is proportional to radial vector $\vec{r}$ everywhere, $\vec{P}$=a$\vec{r}$ where a is positive constant. The cylinder rotates around its axis with an angular speed ω. This is a non-relativistic problem.

   (a) Calculate electric field $\vec{E}$ at a radius r both inside and outside the cylinder.

   (b) Calculate magnetic field $\vec{B}$ at a radius r both inside and outside the cylinder.

   (c) What is the total electromagnetic energy stored per unit length of the cylinder before it started spinning and while it is spinning? Where did the extra energy come from?

8. Suppose,

$$E(\vec{r},t) = \frac{-1}{4\pi\epsilon_0}\frac{q}{r^2}\Theta(vt-r)\hat{r}, \qquad B(\vec{r},t) = 0$$

a) Show that they satisfy Maxwell's equations.
b) Determine ρ and $\vec{J}$
Here,

$$\begin{aligned}\Theta(x) &= 1\,, \quad if\, x > 0\\ &= 0\,, \quad if\, x \leq 0\end{aligned}$$

and

$$\frac{d\Theta}{dx} = \delta(x)$$