PH108 : Electricity & Magnetism : Problem Set 7

Only * problems are to be solved in the tut session

1. Consider a thin spherical shell (thickness $\rightarrow 0$) of radius R with a surface charge density;

$$\sigma(\theta) = \sigma_0(cos\theta + cos^2\theta)$$

Using solutions of Laplace's equation, find the potential $V(r,\theta)$ everywhere, both for r > R and r < R.

2. *In the following system (see figure), the inner conducting sphere of radius a carries charge Q and the outer sphere of radius b is grounded. The distance between the centres is c which is a small quantity.

   (a) Show that to the first order in c, the equation describing the outer sphere, using the centre of inner sphere as origin, is r(θ)=b+c.$cos\theta$.

   (b) If the potential between two spheres contains only $l = 0$ and $l = 1$ angular components, determine it to first order in c.

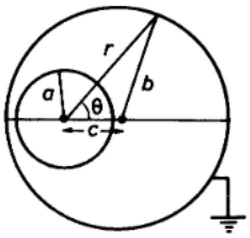

Figure 1: Orientation of the spheres in Q.7

3. *Static charges are distributed along the x-axis (one-dimensional) in the interval $-a \leq x' \leq a$. The charge density is :

$$\begin{array}{ccc} \rho(x') & for & |x'| \leq a \\ 0 & for & |x'| > a \end{array}$$

- Write down the multipole expansion for the electrostatic potential $\phi(x)$ at a point x on the axis in terms of $\rho(x')$, valid for $x > a$.
- For each charge configuration given in Fig. 2, find (a) total charge $Q = \int \rho dx'$, (b) dipole moment $P = \int x' \rho dx'$.

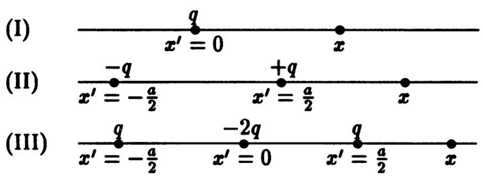

Figure 2: Charge distributions for Q.4

4. A circular disc of radius R lies in the $z = 0$ plane, centred at the origin. It has the following charge density frozen on it;

$$\sigma(r', \phi) = \sigma_0 \, r' \, cos(\phi)$$

(a) What is the monopole moment of the configuration?

(b) Calculate the dipole contribution to the potential due to the configuration at $(0, 0, z)$ using the expression in polar form.

(c) Now calculate the the cartesian components of the dipole moment of the configuration. Use this to calculate the dipole contribution at $(0, 0, z)$. Verify your answer with the expression obtained in (b)

5. If the total amount of charge (monopole) contained in a distribution is zero, show that the dipole moment is independent of the choice of the origin.

6. * Find the dipole moment of:

(a) A ring with charge per unit length $\lambda = \lambda_0 \cos\phi$ where ϕ is the angular variable in cylindrical coordinates.

(b) a sphere with charge per unit areas $\sigma = \sigma_0 \cos\theta$ where θ is the polar angle measured from the positive z-axis.

7. * An electric dipole of moment $P = (P_x, 0, 0)$ is located at the point $(x_0, y_0, 0)$ where $x_0 > 0$ and $y_0 > 0$. The planes $x = 0$ and $y = 0$ are conducting plates with a tiny gap at the origin. The potential of the plate at $x = 0$ is maintained at V_0 and the plate at $y = 0$ is grounded. The dipole is sufficiently weak so that you can ignore the charges induced on the plates.

(a) Based on Fig 3, deduce a simple expression for the electrostatic potential $\phi(x, y)$.

(b) Calculate the force on the dipole.

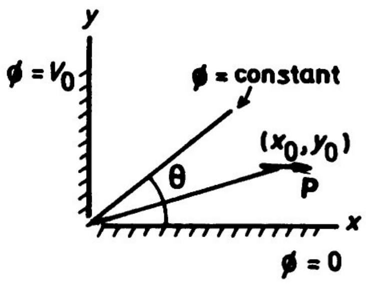

Figure 3: Plates at an angle θ.