PH108 : Electricity & Magnetism : Problem Set 4

Only * problems are to be solved in the tut session

1. * Consider a conducting sphere A which is initially uncharged. Another conducting sphere B is given a charge $+Q$, brought into contact with A and then moved far away. The charge on B is then increased to its original value $+Q$ and again brought into contact with A. Show that if this process is repeated many times, the charge on A will tend to the limit $\frac{Qq}{Q-q}$, where q is the charge acquired by A after its first contact with B.

2. *A hemisphere of radius R has $z = 0$ as its equatorial plane and lies entirely in the region $z \geq 0$. The hemisphere has a uniform charge density of ρ. Determine the field at the centre of the hemisphere.

3. * The potential takes the constant value ϕ_0 on the closed surface S which bounds a volume V. The total charge inside V is Q. There is no charge anywhere else. Show that the electrostatic energy contained in the space outside of S is $U_E(out) = \frac{Q\phi_0}{2}$

4. The inside of a grounded spherical metallic shell (inner radius R_1 and outer radius R_2) is filled with space charge of charge density $\rho = a+br$. (a) Find the potential at the center. (b) Calculate the electrostatic energy of the system. (c) Show that the system attains minimum energy configuration when $b = -\frac{14a}{9R_1}$.

5. Show that the field is uniquely determined when the charge density ρ is specified within a bounded region and either the potential ϕ or its normal derivative $\frac{\partial\phi}{\partial n}$ is specified on each boundary.

6. A ring of radius R has a total charge $+Q$ uniformly distributed on it. (a) Calculate the electric field and potential at the center of the ring. (b) Calculate field at a height z above the center. Compare this result with Coulomb's law in large z limit. (c) Consider a charge $-Q$ constrained to slide along the axis of the ring. Show that the charge will execute simple harmonic motion for small displacements perpendicular to the plane of the ring. Calculate time period.

7. Assume that an electron is a small sphere of radius R in which the charge $-\mid e \mid$ is distributed uniformly over its volume. Calculate the total energy of the system as a function of R. Now suppose you equate this energy to m_0c^2 where m_0 is the rest mass of the electron. What value of R do you get? How does it compare with the size of a hydrogen atom?