

FOL Examples

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Example 1 Translate in FOL:

Students do not like taking exams.

set of objects

(S)

↓
predicate

set of objects
(E)

define: take(a, b): a takes b
like(a, z): a likes z

$\forall s, e \text{ Student}(s) \wedge \text{Exam}(e) \Rightarrow \neg \text{like}(s, \text{take}(s, e))$

Similarly, translate:

children like icecream.

set of objects

a predicate already defined!
CONSTANT

$\forall x \text{ Child}(x) \Rightarrow \text{like}(x, \text{icecream})$

Example 2 Functions vs. Relations

}

unique value.

↓
predicates

before we have seen the predicate
 $\text{like}(x, y)$

if \$ = Sandra

$\text{like}(x, \text{flowers})$

$\text{like}(x, \text{snow})$

But functions refer to a unique value.

Example Translate in FOL:

Mary's grade in AI \Rightarrow it is a unique grade

$\text{AI-Grade}(\text{Mary})$

A-
B+

Example 3 Ground Terms. \rightarrow predicates without arguments

Hot

Tornado

Rain ...

Example 4 Quantifiers of
the same type:

Translate in FOL:

There is a classroom in which

set1: set of objects

Some professor is teaching.

set2: set of objects.

Define the following predicates:

set1 → classroom (α): α is a classroom

set2 → professor (y): y is a professor

teach (y, x): professor y is teaching
in classroom x .

What quantifier? \exists

$\exists x, y \quad \text{classroom}(x) \wedge \text{professor}(y)$
 $\wedge \text{teach}(y, x)$

T4

⑤ All girls love (every) young singer

s₁: set of objects

s₂: set of objects

Define : $\text{girl}(x) : x \text{ is a girl}$

\uparrow

$\text{young-singer}(y) : y \text{ is a young singer}$

\uparrow

s_1

s_2

$\text{loves}(a, b) : a \text{ loves } b.$

What quantifiers? $\forall \exists \nexists \forall$

$\forall g, s \text{ girl}(g) \wedge \text{young-singer}(s) \Rightarrow$
 $\text{loves}(g, s)$

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Example 6 : Mixed quantifiers.

a) Translate in FOL:

Every professor teaches in some classroom.

$$\forall x \exists y \text{ professor}(x) \wedge \text{classroom}(y) \wedge \text{teach}(x, y)$$

b) Translate in FOL:

"There is a professor that teaches in each classroom."

$$\exists x \forall y \text{ professor}(x) \wedge \text{classroom}(y) \Rightarrow \text{teaches}(x, y)$$

Example 7 : Use Equality in FOL
 $=$

a) Translate in FOL:

Only one student will take a grade of A in AI.

definitions:

student(x): x is a student
 grade (x, A_1, A): x takes
 in AI the
 grade A

Quantifiers?

$$\exists x \forall y \text{student}(x) \wedge \text{student}(y) \\ \wedge \text{grade}(x, A_1, A) \wedge \text{grade}(y, A_1, A) \\ \Rightarrow x = y$$

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b) Translate in FOL:

Just one student is left in the
set of objects classroom.
* a constant!

 $\exists x \text{ student}(x) \wedge \text{left}(x, \text{Classroom}) \wedge$
 $\forall y \text{ student}(y) \wedge \text{left}(y, \text{Classroom})$

$$\Rightarrow x = y$$

c) Translate in FOL:

Mary has only two friends.
Mary
set of objects
constant

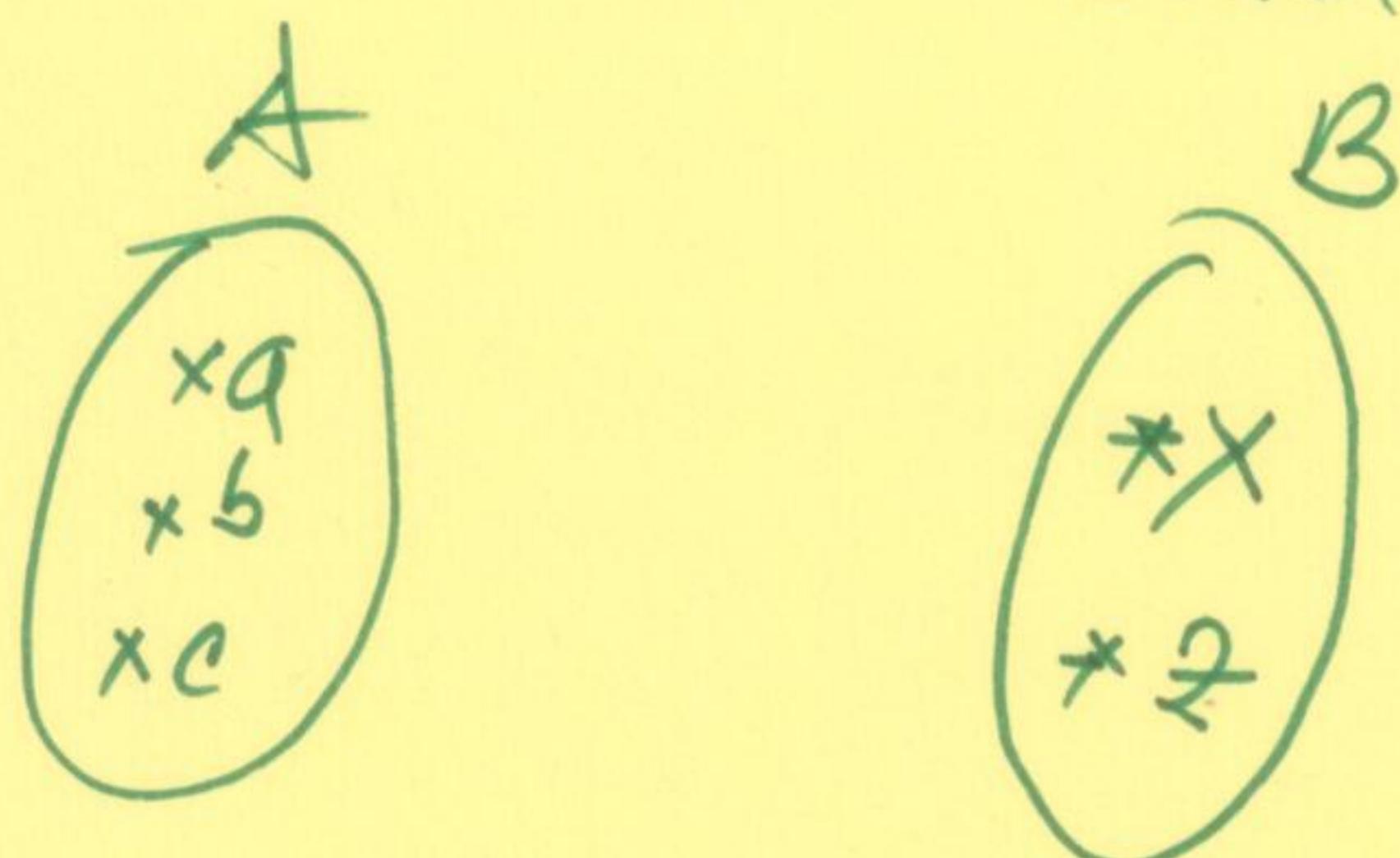
Definitions: friend(a, b): a is a friend of b.

 $\exists f_1, f_2 \text{ friend}(f_1, \text{Mary}) \wedge \text{friend}(f_2, \text{Mary})$
 $\wedge \forall f_3 \text{ friend}(f_3, \text{Mary})$

$$\Rightarrow ((f_1 = f_3) \vee (f_2 = f_3))$$

Reminder:

- the difference between relations and functions.



A relation $R \subset \{(a, *), (b, *), (c, *), (a, z), (b, z)\}$

example $R_1 = \{(a, *), (b, *), (b, z)\}$

in FOL: suppose A: Tennis champions.
 $a = \text{Serena Williams}$
 $b = \text{Simona Halep}$
 $c = \text{Rafael Nadal}$.

B: Grand Slam European Tournament.
 $x = \text{Roland Garros}$
 $y = \text{Wimbledon}$

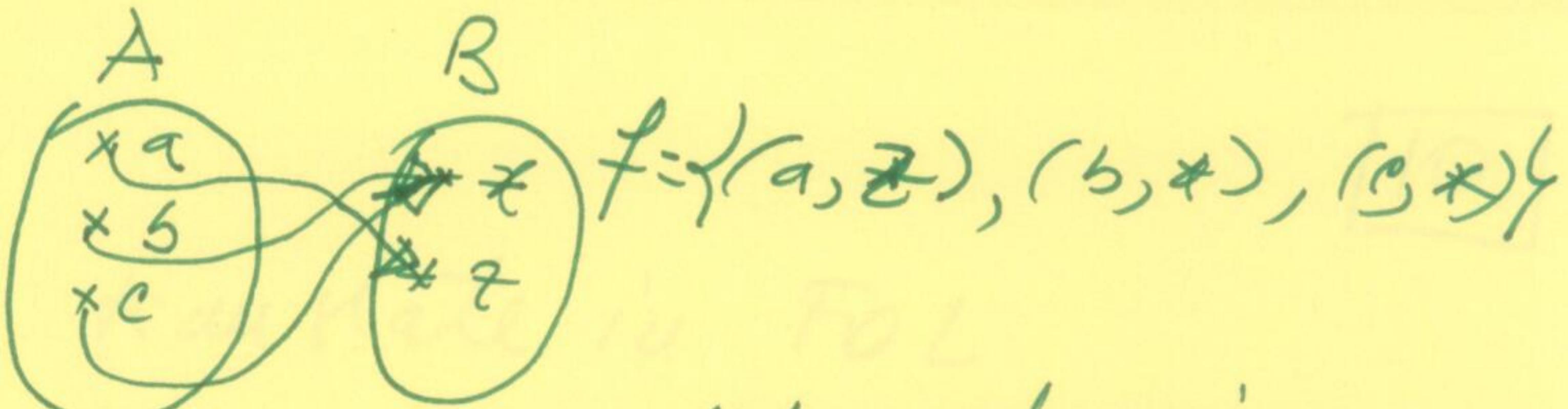
We define a predicate that implements
 a relation between the set of objects A
 and the set of objects B

↓
 won-the-tournament: champion

$\text{champion}(x, y) : x \text{ is a champion of } y$

A function requires that each object from set S_1 is assigned only one object from S_2 .

example



As we have defined the relation champion and the sets A and B:

champion (Serena Williams, Wimbledon)

champion (Simona Halep, Wimbledon)

champion (Simona Halep, Roland Garros)

champion (Rafael Nadal, Roland Garros)

let us define a function:

champion (x, y, \underline{year})
†

champion (Rafael Nadal, Roland Garros, 2023)

champion (Simona Halep, Wimbledon, 2019)

champion (Simona Halep, Roland Garros, 2018)