Inference in FOL

<u>Examples</u> of finding the Most General Unifier (MGU)

Example 1: Find the MGU of:

$$X = f(g(x, m(y, B), n(B, y)))$$

and

$$Y = f(g(x, z, w))$$

<u>Solution</u>: Observation: Both X and Y are FOL atomic sentences that use the same predicates f and g

Let us look more carefully at X & Y

$$X = f(g(arg^{g}_{1}, arg^{g}_{2}, arg^{g}_{3}))$$

$$Y = f(g(arg^{g'}_{1}, arg^{g'}_{2}, arg^{g'}_{3}))$$

where:

$$\begin{cases} arg^{g}_{1} = x & arg^{g}_{2} = m(y, B) \\ arg^{g}_{3} = n(B, y) & \end{cases}$$

$$\begin{cases} arg^{g'}_{1} = x & arg^{g'}_{2} = z & arg^{g'}_{3} = w \end{cases}$$

Therefore :

$arg_1^{g} = x$ $\quad arg_2^{g} = m(y,B)$ $\quad arg_3^{g} = n(B,y)$

$arg_1^{g'} = x$ $\quad arg_2^{g'} = z$ $\quad arg_3^{g'} = w$

How do we decide on the substitution?

$$\theta = \{ z / m(y,B) ; w / n(B,y) \}$$

How did the UNIFY function work?

X and Y are compounds having the same operators (f and g)

$X.Args = \{ x, m(y,B), n(B,y) \}$

$Y.Args = \{ x, z, w \}$

$\Downarrow$

$UNIFY(x, x) \Rightarrow \theta = \{\}$

$UNIFY(m(y,B), z) \Rightarrow$

$\theta = \{ z / m(y,B) \}$

UNIFY-VAR$(z, m(y,B))\}$

occur-check(z, w)

$UNIFY(n(B,y), w) \Rightarrow$

UNIFY-VAR$(w, n(B,y))\}$

$\theta = \{ z / m(y,B), w / n(B,y) \}$

$$\boxed{MGCS = f(g(x, m(y,B), n(B,y))}$$

$\Downarrow$

$UNIFY(X, Y) = \theta$

Example 2: Find the MGU of:

$$X = C(f(x), y, g(B))$$
$$Y = C(f(x), f(x), x)$$

$$MGU = \{ y/f(x) ; x/g(B) \}$$
$$MGCS = C(f(g(B)), f(g(B)), g(B)\}$$

How did the UNIFY function work?

X and Y are compounds!

$\Rightarrow$ operator f is relying on Arg unificat.

UNIFY-VAR $\Rightarrow$ $y/f(x)$
UNIFY-VAR $\Rightarrow$ $x/g(B)$

$$\left.\begin{array}{l} \text{UNIFY-VAR} \Rightarrow y/f(x) \\ \text{UNIFY-VAR} \Rightarrow x/g(B) \end{array}\right\} MGU$$

Example 3 Find the MGU of:

Mother (Child(x), Jane, Nephew(Cousin(Uncle(Jack))))

and

Mother(Child(John), y, Nephew(Cousin(y)))

x/John ; y/Jane ; y/Uncle(Jack)

occur-check = FAIL

Standardize Apart:

Mother(Child(x), Jane, Nephew(Cousin(Uncle(Jack))))

Mother(Child(John), y, Nephew(Cousin(z)))

MGU = {x/John ; y/Jane ; z/Uncle(Jack)}

Forward Chaining Examples

Example 1

To eat a mouse, a cat should be hungry and chase the mouse instead of fighting dogs, which do not eat mice.

P1: $cat(x) \land mouse(y) \land dog(z) \land chase(x,y) \land \neg eat(z,y) \land \neg fight(x,z) \Rightarrow eat(x,y)$

To chase the mouse, both the cat and the mouse should be in the house and the cat should be healthy.

P2: $cat(x) \land mouse(y) \land in\text{-}house(x) \land in\text{-}house(y) \land healthy(x) \Rightarrow chase(x,y)$

Cats that are friends with dogs do not fight.

P3: $cat(x) \land dog(z) \land friends(x,z) \Rightarrow \neg fight(x,z)$

Kitty is a cat in the house.

P4: Cat (Kitty)

P5: In-house (Kitty)

Fido is our in house dog.

P6: dog (Fido)

P7: in-house (Fido)

Mickey, the mouse, entered the house.

P8: mouse (Mickey)

P9: in-house (Micky).

Kitty is healthy.

P10: healthy(Kitty)

Fido and Kitty are friends.

P11: friends(Kitty, Fido)

Dogs do not eat mice.

P12: $dog(z) \Rightarrow \neg eat(z, y)$

$\land$ ~~mice~~ mouse(y)

Prove that Kitty eat Micky.

Q: eat(Kitty, Micky).

Forward chaining

a) which sentences have implications?

P1, P2, P3, P12

b) Find sentences from the KB which unify with premises of P1, P2, P3 or P12

Iteration 1: P1 & P4 : θ_1 {x/Kitty}

cat(Kitty) P1 has unsatisfied premises.

P2 & P8 : θ_2 {y/Micky}

mouse(Micky) P2 has unsatisfied premises.

P3 & P6 : θ_3 : {z/Fido)

dog(Fido)

P3 has unsatisfied premises

P12 ⇒ all premises satisfied.

dog(Fido) ∧ mouse(Micky)

$\boxed{\text{Q1: } \neg eat(\text{Fido, Mickey})}$

Iteration 2: • P1 {& ~~θ_1~~ & θ_2 & θ_3 & Q1}

cat(Kitty) ∧ mouse(Mikey) ∧ dog(Fido)
∧ chase(Kitty, Fido)
∧ ¬eat(Fido, Micky)
∧ ¬fight(Kitty, Fido)

P1 has unsatisfied premises.

• P2 & P5 & θ_3
cat(Kitty) ∧ in_house(Kitty)
∧ healthy(Kitty) ∧ In_house(Micky)

P2 has unsatisfied premises

• P3 & P11 cat(Kitty) ∧ dog(Fido) ∧ friends(Kitty, Fido)

all premises satisfied!

P3 has

$\boxed{\text{Q2: } \neg fight(\text{Kitty, Fido})}$

Iteration 3

- P_1 premises:

cat(Kitty) $\wedge$ mouse(Mickey) $\wedge$ dog(Fido)

$\wedge$ chase(Kitty, Fido)

$\wedge$ $\neg$eat(Fido, Mickey) $\wedge$ $\neg$fight(Kitty, Fido)

still has 1 unsatisfied premise!

- P_2 & P_9

cat(Kitty) $\wedge$ in-house(Kitty) $\wedge$ in-house(Mickey)

$\wedge$ healthy(Kitty).

P_2 still has 1 unsatisfied premise!

Iteration 4:

P_1 has unsatisfied premise:
$\neg$chase(Kitty, Fido)

P_2 & P_{10} $\Rightarrow$ P_2 has all premises satisfied!

$\boxed{\text{Q3: chase(Kitty, Mickey)}}$

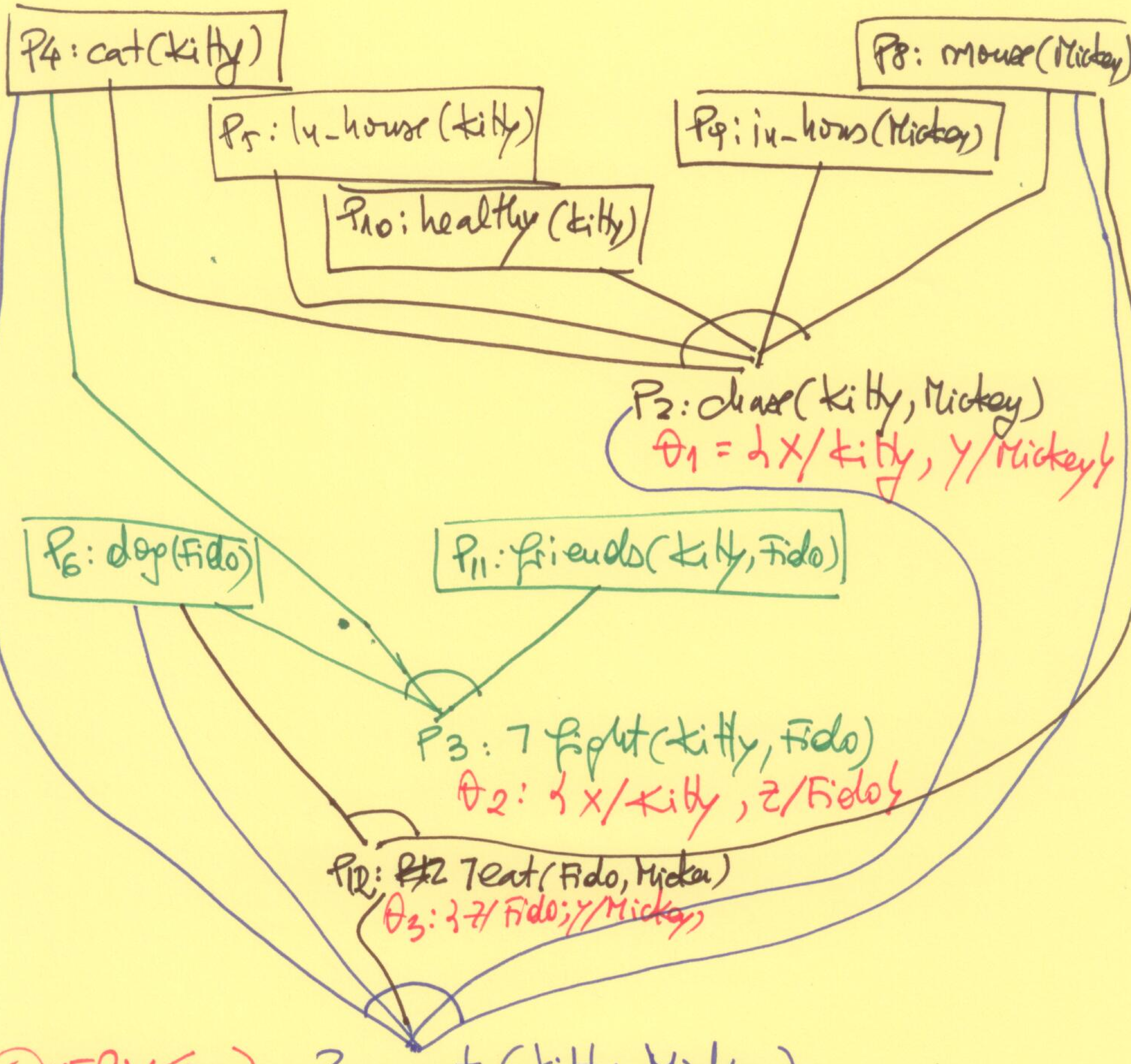
Iteration 5:

P_1 & Q_3 $\rightarrow$ P_1 has all premises satisfied!

$\boxed{\text{Q4: eat(Kitty, Mickey)}}$

Can unify with the query Q

FOL Feedforward Chaining using AND-OR Graphs + Substitutions



QUERY ⟺ P1: eat(kitty, Mickey)

θ_4: {X/kitty; Y/Mickey; ~~Z/Fido~~ Z/Fido}

⇓

can be unified.

2nd Example of FOL Forward Chaining

1. Only athletes that win gold medals at the Olympics are considered olympic champions.
2. The swimmer Michael Phelps won six gold medals at the 2004 summer Olympics.

Q. Is Michael Phelps an olympic champion?

Encoding in FOL

1. athlete(a) ∧ win(a, Gold-Medal, o) ∧ olympics(o) => Olympic-Champion(a)

2. swimmer(MF)
3. win(MF, Gold-Medal, 2004) ∧ Olympics(2004)

Q: Olympic-champion (MF)

We also need 4. swimmer(x) => athlete(x)

→ encoding world knowledge (commonsense knowledge)

Solution:

Transform 3 into 3.a. win(MF, Gold-Medal, 2004)
3.b. Olympics(2004)

$\boxed{\text{2. swimmer(MF)}}$ $\boxed{\text{3.a win(MF, Gold-Medal, 2004}}$ $\boxed{\text{3.b Olympics(2004)}}$

$\boxed{\text{4. athlete(MF)}}$

θ1 {x/MF}

Olympic-Champion(MF) ⇐ Q

θ2 {a/MF, o/2004}

FOL Forward Chaining Example #3

Qual-level Problem!

Suppose your KB contains the following Horn clauses:

1. Descendant (child(x), x)
2. Descendant(x, y) ∧ Descendant(y, z) => Descendant(x, z)

Consider the Forward-Chaining algorithm, that on the j-th iteration terminates if the KB contains a sentence that unifies with the query, else it adds to the KB every atomic sentence that can be inferred from the sentences already in the KB after iteration j-1.

For each of the following queries, say whether the Forward-Chaining algorithm will give an answer, and if so, write down the answer:

Q1: Descendant (child (y), John)

Answer : YES : θ: {Y/X ; X/John}
→ Descendant (child (John), John))

Q2 : Descendant (child (child (y)), John)

YES ; After 1st Iteration : X/John
2nd Iteration, it stops is John = child(y)

Q3 : Descendant (child (Mary), child (child (Mary))

Q3: Descendant(Child(Mary), Child(Child(Mary)))

Answer: $\theta: \{x/Mary\}$ using sentence 1 ⇒ FC does not terminate

FOL Backward Chaining Examples

Let us consider <u>again</u> the Colonel West example.

<u>KB</u>

P1: American(x) ∧ Weapon(y) ∧ Nation(z) ∧ Hostile(z) ∧ Sells(x, y, z) ⇒ Criminal(x)

P2 a. Owns(Nono, M_1)
b. Missile(M_1)
c. Nation(Nono)

P3. Missile(x) ∧ Owns(Nono, x) ⇒ Sells(West, x, Nono)

P4: ∀x Missile(x) ⇒ Weapons(x)

P5: ∀x Enemy(x, America) ⇒ Hostile(x)

commonsense knowledge

P6: American(West)

P7: Enemy(Nono, America)

Query: Criminal(West)

The FOL Backward Chaining Algorithm

FOL-BC-ASK(KB, goals, θ)

goals: a list of conjuncts forming a query

θ: current substitution

goals: Criminal(West)? ~~or not~~ ← first(GOALS)

GOAL 1

g' ← a) where do I have in KB an axiom with the conclusion <u>unifiable</u> to first(GOALS)?

→ P1

b) what is the substitution that would enable the <u>unification</u>?

θ(x/West)

c) Are the preconditions of P1 standardized-apart?

~~YES~~ ~~NO~~ YES

⇒ new-goals ← {Weapon(y), Nation(z), Hostile(z), Sells(West, y, z)}

answers ← FOL-BC-ASK(KB, new-goals, compose(θ', θ)) ∪ answers.

goals ← new-goals.

GOAL 2

g' ← Weapon(y)

a) where do I have in KB an axiom with conclusion Weapon(y)?

→ P4

b) what substitution would enable the unification of g' and P4?

$\theta' = \{y/x\}$

c) Are preconditions of P1 standardized apart?

YES

⇒ new-goals ← {Missile(x), Nation(z), Hostile(z), Sells(West, y, z}

$\theta \leftarrow \theta \cup \theta'$

answers ← FOL-BC-ASK(KB, new-goals, compose(θ', θ)) ∪ answers

↓

goals ← new-goals

$\boxed{\text{GOAL 3}}$ $g' \leftarrow$ Missile(x)

a) where do I have in KB an axiom with conclusion Missile(x)?

⇒ P2.b.

b) what substitution will enable the unification of g' and P2.b?

$\theta' = \{x/M1\}$

c) Are there preconditions to P2.b ⇒ No

Nothing to standardize apart

But we need to standardize x into y

⇒ new-goals ← {Nation(z), Hostile(z), Sells(West, M1, z}

← REST(new-goals)

$\theta \leftarrow \{x/West; y/M1\}$

answers ← FOL-BC-ASK(KB, new-goals, compose(θ', θ)) ∪ answers

↓

goals ← new-goals

GOAL-4 $g' \leftarrow$ Nation(z)

a) • where do I have in KB an axiom with the conclusion unifiable to g'? ⇒ P2.c

b) • What substitution will enable the unification of g' with P2.C?

$\theta' = \{z/Nono\}$

c) Are there preconditions to P2.C? → No!

⇒ new-goals ← {Hostile(Nono), Sells(West, M1, Nono)}

$\theta = \{x/West, y/M1, z/Nono\}$

answers ← FOL-BC-ASK(KB, new-goals, compose(θ, θ')) ∪ answers

goals ← new-goals

GOAL-5 $g' \leftarrow$ Hostile(Nono)

a) • where do I have in KB an axiom with the conclusion Hostile(Nono)? ⇒ P5

b) • what substitution will enable the unification of g' with P5?

$\theta' = \{x/Nono\}$

c) • Are there pre-conditions to P5 that need to be standardized apart?

Enemy(Nono, America)

⇒ new-goals ← {Enemy(Nono, America), Sells(West, M1, Nono)} answers ← FOL-BC-Ask(KB, new-goals, compose(θ,θ')) ∪ answers

goals ← new-goals

GOAL-6 $g' \leftarrow$ Enemy(Nono, America)

a) • where do I find in KB an axiom with conclusion Enemy(x,y)?

⇒ P7

b) • what substitution will enable the unification of g' with P7? $\theta' = \{\}$

c) • All standardized apart!

⇒ new-goals ← {Sells(West, M1, Nono)}

answers ← FOL-BC-Ask(KB, new-goals, composed(θ',θ)) ∪ answers

goals ← new-goals

GOAL-7 $g' \leftarrow$ Sells(West, M1, Nono)}

a) • where do I find in KB an axiom with conclusion Sells(a,b,c)?

⇒ P3

b) • what substitution will enable the unification of g' with P3?

$\theta' = \{\}$

c) all standardized apart.

⇒ new-goals = {}

answers ← FOL-BC-Ask(KB, {}, compose(θ, θ')} ∪ answers

$\theta = \{x/West; y/M1; z/Nono\}$ ⇒ return $\underline{\theta}$ PROVEN

2nd Example of FOL-BC Proof Tree

let us revisit Example 1 used for FOL Forward-chaining (pages 5-9)

KB

P1: $cat(x) \wedge mouse(y) \wedge dog(z) \wedge chase(x,y) \wedge \neg eat(z,y) \wedge \neg fight(x,z) \Rightarrow eat(x,y)$

P2: $cat(x) \wedge mouse(y) \wedge in\_house(x) \wedge in\_house(y) \wedge healthy(x) \Rightarrow chase(x,y)$

P3: $cat(x) \wedge dog(z) \wedge friends(x,z) \Rightarrow \neg fight(x,z)$

P4: cat(Kitty)

P5: in_house(Kitty)

P6: dog(Fido)

P7: in_house(Fido)

P8: mouse(Mickey)

P9: in_house(Mickey)

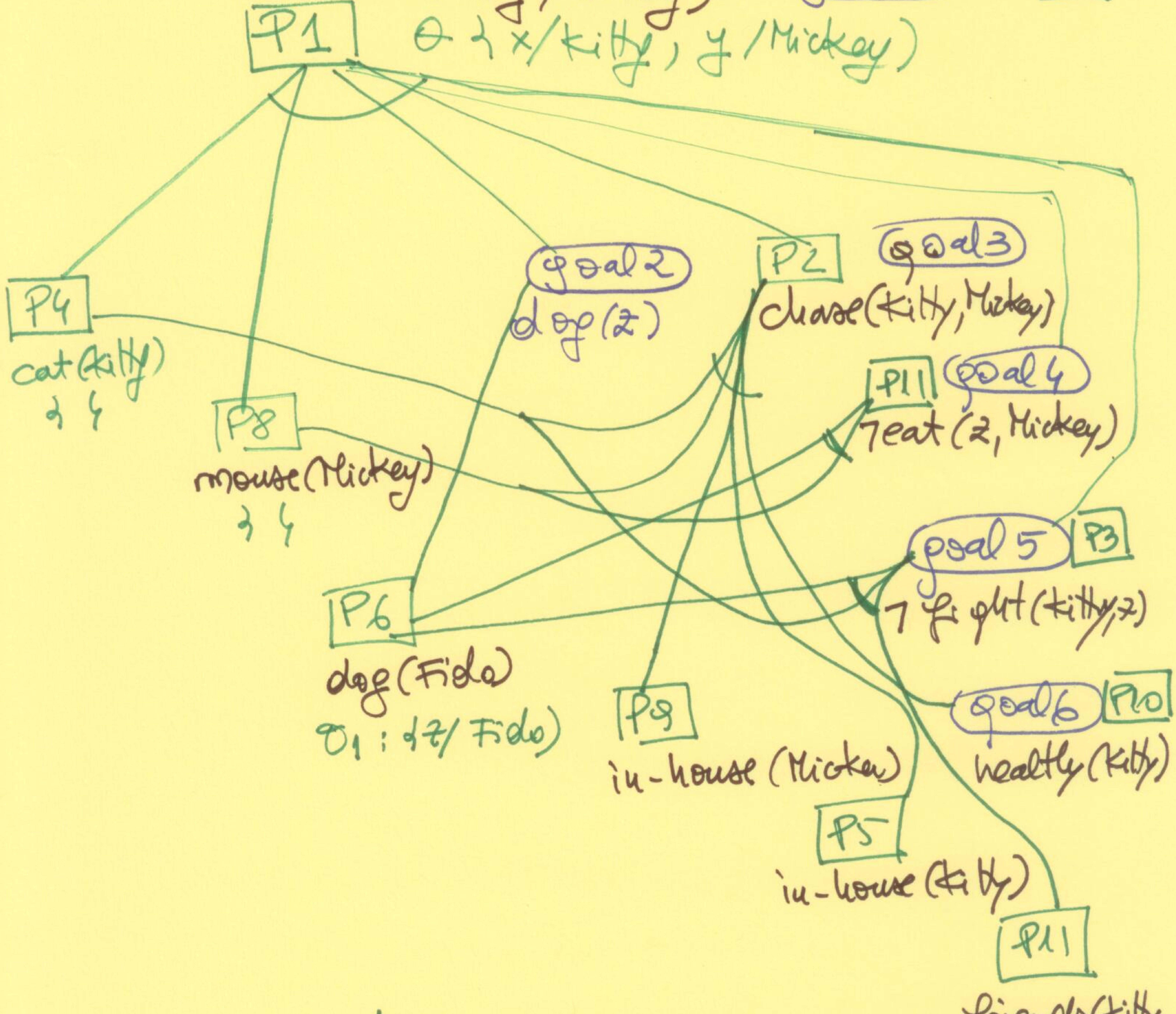
P10: healthy(Kitty)

P11: friends(Kitty, Fido)

P12: $dog(z) \wedge mouse(y) \Rightarrow \neg eat(z,y)$

query: eat(Kitty, Mickey)

Q: Eat(Kitty, Mickey) goal 1



$\theta = \theta \cup \theta' = \{x/Kitty,\ y/Mickey,\ z/Fido\}$