Examples of INFERENCE in Propositional logic

(A) Resolution

Example 1 The Wumpus World

slide 79

In the KB we have 2 sentences:

$S_1: \neg B_{11}$

$S_2: (B_{11} \Leftrightarrow (P_{12} \vee P_{21})) \wedge \neg B_{11}$

To perform resolution we need to convert both S1, S2 and $\boxed{\alpha = \neg P_{12}}$ into CNF.

observation: S_1 and α are already in CNF

$$S_2 = (\underbrace{B_{11} \Leftrightarrow P_{12} \vee P_{21}}_{S_2^A}) \wedge \underbrace{\neg B_{11}}_{S_2^B = S_1}$$

S_2^B is in CNF. All we need to do is transform S_2^A into CNF.

Steps for converting S_2^A into CNF

Step 1: Biconditional elimination

$$(B_{11} \Rightarrow P_{12} \vee P_{21}) \wedge ((P_{12} \vee P_{21}) \Rightarrow B_{11})$$

Step 2: Eliminate "$\Rightarrow$" $\quad a \Rightarrow b \equiv \neg a \vee b$

$$(\neg B_{11} \vee P_{12} \vee P_{21}) \wedge (\underline{\neg (P_{12} \vee P_{21})} \vee B_{11})$$

Step 3: Push $\neg$ near literals: Use de Morgan

$$(\neg B_{11} \vee P_{12} \vee P_{21}) \wedge (\underline{(\neg P_{12} \wedge \neg P_{21})} \vee B_{11})$$

Step 4: Apply the distribution rules:

$$\underbrace{(\neg B_{11} \vee P_{12} \vee P_{21})}_{S_2'} \wedge \underbrace{(B_{11} \vee \neg P_{12})}_{S_4'} \wedge \underbrace{(B_{11} \vee \neg P_{21})}_{S_3'}$$

Now we have in the KB:

S_1: $\neg B_{11}$

S_2': $\neg B_{11} \vee P_{12} \vee P_{21}$

S_3': $B_{11} \vee \neg P_{21}$

S_4': $B_{11} \vee \neg P_{12}$

We want to infer $\neg \alpha = P_{12}$ (remember $\alpha = \neg P_{12}$ & apply double negation)

How is the KB?

$$\begin{cases} S1: & \neg B_{11} \\ S_2: & \neg B_{11} \vee P_{12} \vee P_{21} \\ S3: & \neg P_{21} \vee B_{11} \\ S_4: & \neg P_{12} \vee B_{11} \end{cases}$$

$\neg \alpha = P_{12}$

How should I perform resolution?

$\Downarrow$

which pairs of sentences should I consider, & which literal?

Possibilities of pairs of sentences:

1) (S_3, S_2) – with P_{21}
2) (S_3, S_2) – with B_{11}
3) (S_2, S_4) – with P_{12}
4) (S_2, S_4) – with B_{11}
5) (S_3, S_1) – with B_{11}
6) $(S_4 \& S_1)$ – with B_{11}

let us apply the RESOLUTION inference rule:

1) $$\frac{S3: \neg P_{21} \vee B_{11}, \quad S2: \neg B_{11} \vee P_{12} \vee P_{21}}{X1: \neg B_{11} \vee P_{12} \vee B_{11}}$$

2) $$\frac{S3: \neg P_{21} \vee B_{11}, \quad S2: \neg B_{11} \vee P_{12} \vee P_{21}}{X2: P_{12} \vee P_{21} \vee \neg P_{21}}$$

3) $$\frac{S2: \neg B_{11} \vee P_{12} \vee P_{21}, \quad S_4: \neg P_{12} \vee B_{11}}{X3: \neg B_{11} \vee P_{21} \vee B_{11}}$$

4) $$\frac{S2: \neg B_{11} \vee P_{12} \vee P_{21}, \quad S_4: \neg P_{12} \vee B_{11}}{X4: P_{12} \vee P_{21} \vee \neg P_{12}}$$

5) $$\frac{S3: \neg P_{21} \vee B_{11}, \quad S_1: \neg B_{11}}{X_5: \neg P_{21}}$$

6) $$\frac{S_4: \neg P_{12} \vee B_{11}, \quad S_1: \neg B_{11}}{X_6: \neg P_{12}}$$

$\Rightarrow$ $$\frac{X_6: \neg P_{12}, \quad \neg\alpha: P_{12}}{NIL}$$

therefore α can be inferred/entailed from KB

What did we learn?

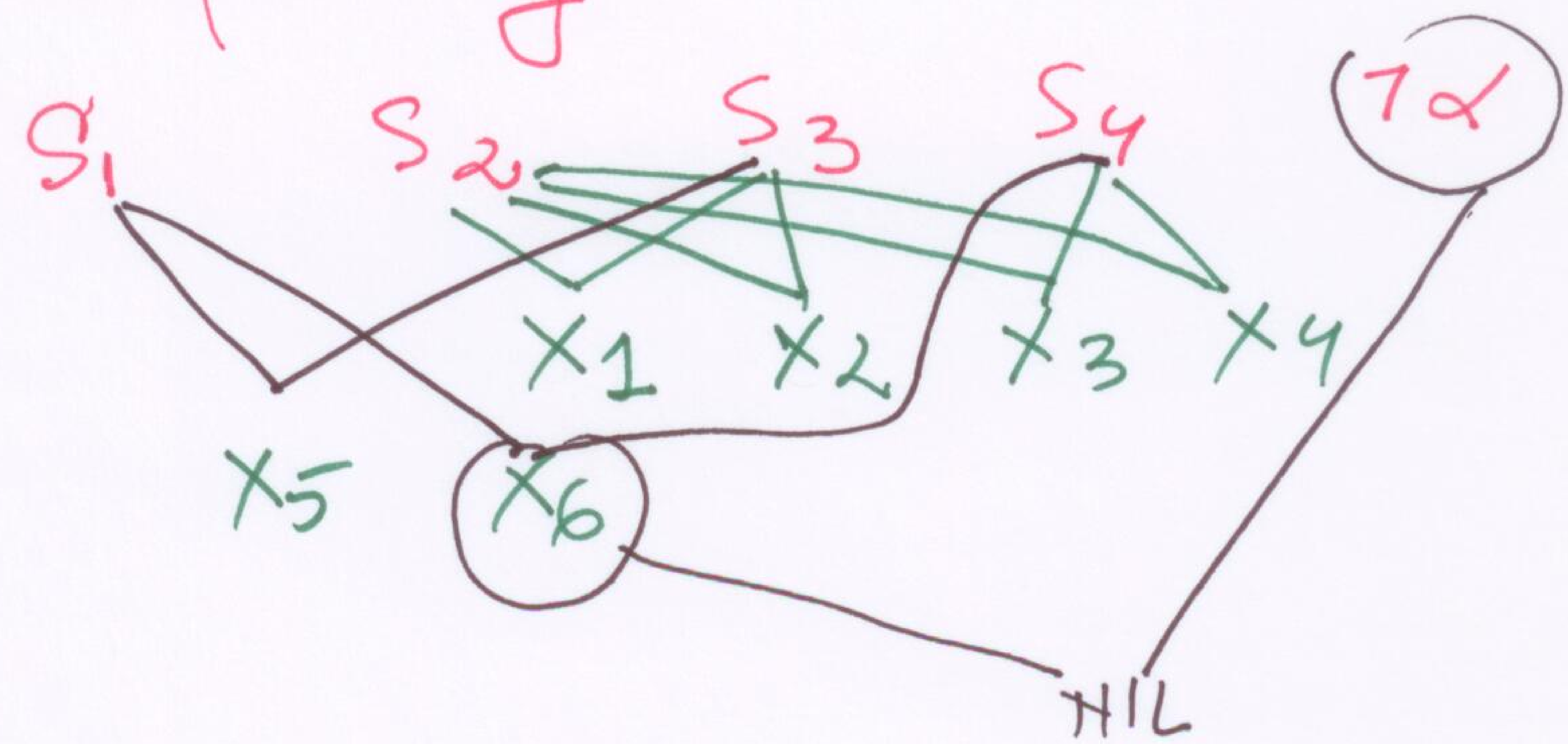
We have to look for a chain of resolutions that lead to $\alpha = \neg P_{12}$

What was the chain?

Resolution $(S_4, S_1) \Rightarrow X_6 : \neg P_{12}$

$(X_6, \neg\alpha) \Rightarrow NIL$

Graphically:



2^{nd} Resolution Example

Suppose the KB is:

$(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)$

Prove α : $Food \wedge Drinks \Rightarrow Party$

Solution

Convert KB and α in CNF

KB: $(\neg Food \vee Party) \vee (\neg Drinks \vee Party)$

$\rightarrow$ S1 : $\neg Food \vee \neg Drinks \vee Party$

α : $\neg Food \vee \neg Drinks \vee Party$

$\neg\alpha$ in CNF becomes

$\neg(\neg Food \vee \neg Drinks \vee Party)$.

$\underbrace{Food}_{S2} \wedge \underbrace{Drinks}_{S3} \wedge \underbrace{\neg Party}_{S4}$

Start apply the RESOLUTION rule:

(S1, S4) with Party

X1 : $\neg Food \vee \neg Drinks$.

(X1, S3) with Drinks

X2 : $\neg Food$

(X2, S2) with Food

NIL

3^{rd} Resolution Example

If you have the KB:

$S_1 : \neg(Hot \lor Rain)$

$S_2 : Rain \land Wet$

$S_3 : Sunny \lor Cloudy$

$S_4 : \neg Hot \Rightarrow Cloudy$

prove α : Wet

Answer : Transform KB and α in CNF

$S_1 : \neg Hot \land \neg Rain$

$S_2 : Rain \land Wet$

$S_3 : Sunny \lor Cloudy$

$S_4 : Hot \lor Cloudy$

$\neg\alpha : \neg Wet$

$C_1 : \neg Hot$

$C_2 : \neg Rain$

$C_3 : Rain$

$C_4 : Wet$

$C_5 : Sunny \lor Cloudy$

$C_6 : Hot \lor Cloudy$ ✓

$$\frac{C4,\ \neg\alpha}{NIL}$$

4th RESOLUTION example

Given the KB:

$S_1: B \wedge C \Rightarrow A$
$S_2: B$
$S_3: D \wedge E \Rightarrow C$
$S_4: E \vee F$
$S_5: D \wedge \neg F$

if the query is: A

Use Resolution to answer the query.

Answer

A: Translate in CNF the KB.

$S_1': \neg B \vee \neg C \vee A$
$S_2: B$
$S_3': \neg D \vee \neg E \vee C$
$S_4: E \vee F$
$S_5^{(1)}: D$
$S_5^{(2)}: \neg F$

$\neg\alpha: \neg A$

B Select pairs of clauses:

(1) $(S_1' \,\&\, S_2)$ with $B \Rightarrow X1: \neg C \vee A$

(2) $(X_1, \neg\alpha)$ with $A \Rightarrow X2: \neg C$

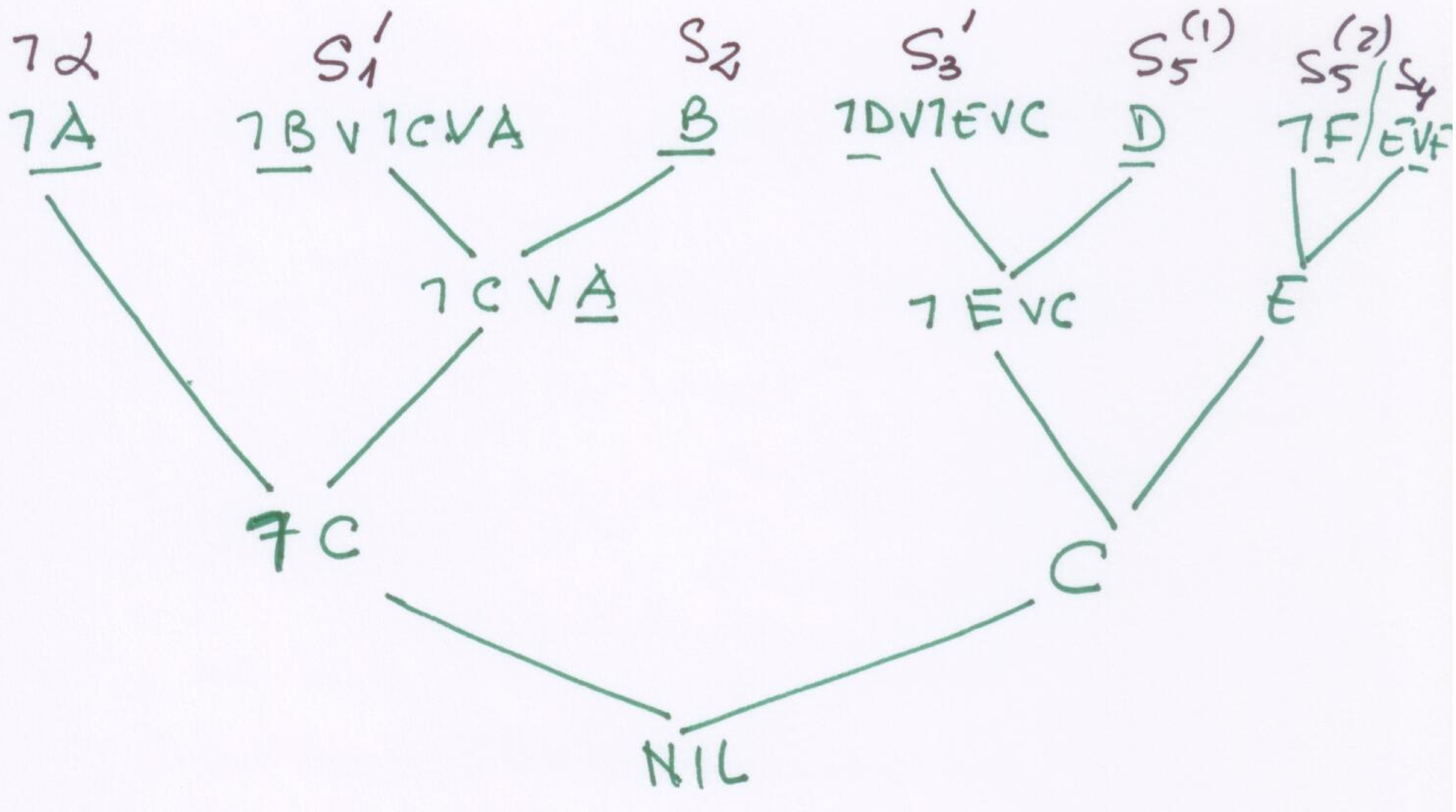
(3) $(S_3', S_5^{(1)})$ with $D \Rightarrow X_3: \neg E \vee C$

(4) $(S_4, S_5^{(2)})$ with $F \Rightarrow X_4 : E$

(5) (X_3, X_4) with $E \Rightarrow X_5 : C$

(6) (X_2, X_5) with $C \Rightarrow NIL = X_6$

Graphically:



$\boxed{B}$ Apply Forward chaining to prove the query Q from: $\boxed{10}$

KB

S1 : $P \Rightarrow Q$

S2 : $L \wedge M \Rightarrow P$

S3 : $B \wedge L \Rightarrow M$

S4 : $A \wedge P \Rightarrow L$

S5 : $A \wedge B \Rightarrow L$

S6 : A

S7 : B

Solution

Agenda: A, B, L, M, P, Q
1 2 3 4 5 6

step1: ~~A~~ step2: ~~B~~ step3: ~~L~~ step4: ~~M~~ step5: ~~P~~ step6: ~~Q~~

Inferred

A B L M P Q {All False

step1: A=T ; step2: B=T ; step3: L=T ; step4: M=T ; step5 : P = T step6 : Q = T

Count	step1	step2	step3	step4	step5
S1 : 1	1	1	1	1	0
S2 : 2	2	2	1	0	0
S3 : 2	2	1	0	0	0
S4 : 2	1	1	1	0	0
S5 : 2	1	0	0	0	0
S6 : 0	0	0	0	0	0
S7 : 0	0	0	0	0	0

Step 1 Based on count=0 in S_6 infer A=TRUE
$\Rightarrow$ count(S_5)=1 ; count (S_4)=1

Step 2 Based on count=0 in S_7, infer B=TRUE
$\Rightarrow$ Count (S_5)=0 count(S_3)=1

Step 3 Based on count(S_5)=0 ; infer L=TRUE
$\Rightarrow$ count(S_3)=0 ; count (S_2)=1

(continued)

Step 4 Based on Count(S_3)=0
infer M = TRUE

$\Rightarrow$ Count(S_2)=0 ; count(S_4)=0

Step 5 Based on count(S_2)=0, infer P = TRUE

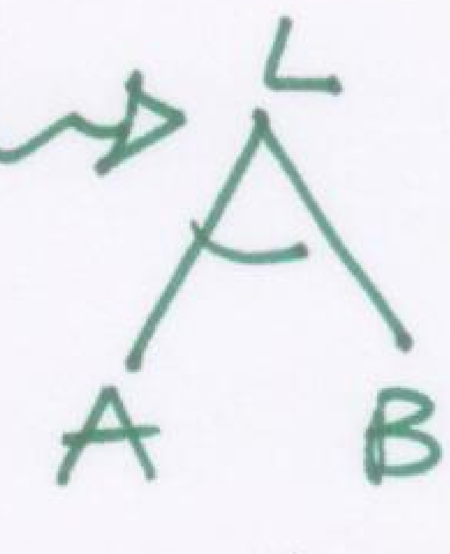
$\Rightarrow$ Count(S_1)=0

Step 6 Based on count(S_1)=0, infer Q = TRUE

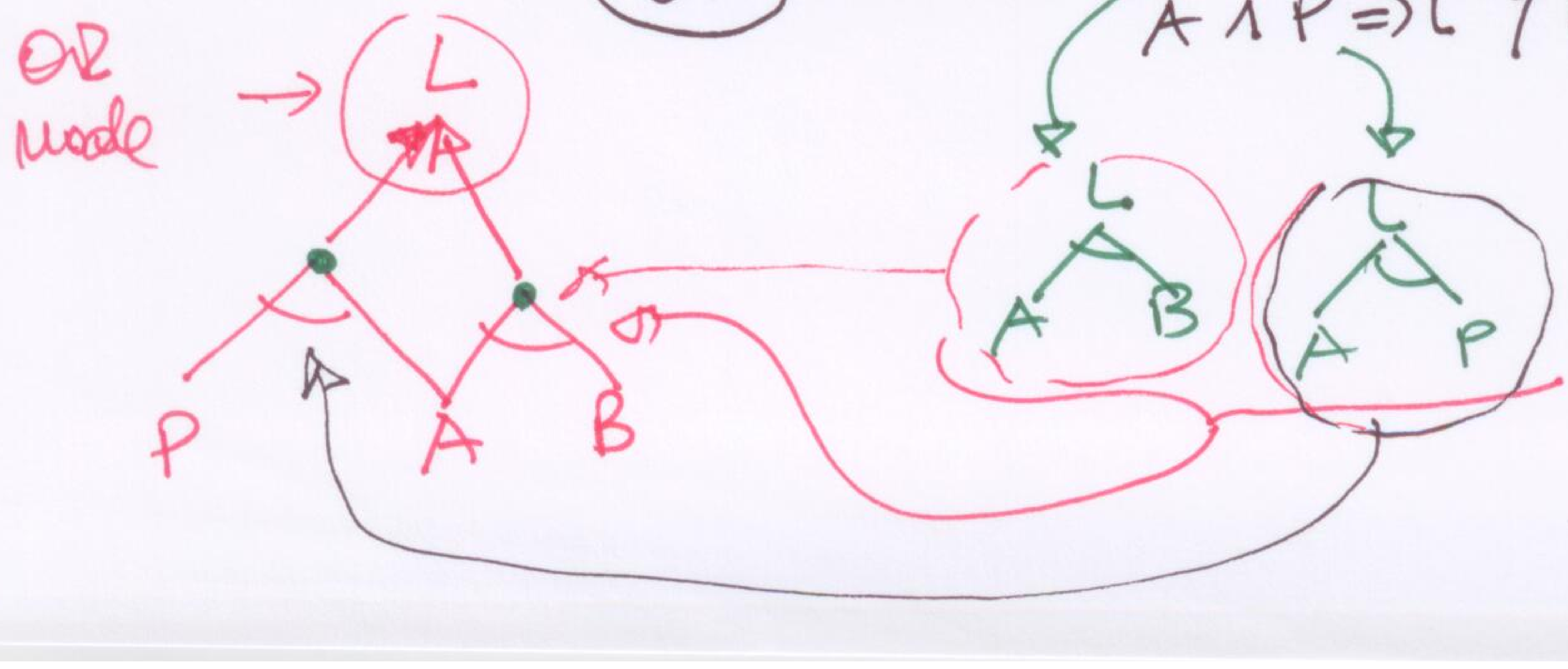
Q is the query!
It was proven to be true!

Use AND-OR graph to visualize the KB
and the forward chaining inference

The IDEA: AND node : $A \wedge B \Rightarrow L$



OR node: $A \wedge B \Rightarrow L$
$A \wedge P \Rightarrow L$ } implicit OR



2nd Example of Forward Chaining

Consider the following KB of Horn Clauses:

- S1: $B \wedge C \Rightarrow A$
- S2: $G \wedge D \Rightarrow B$
- S3: $D \wedge E \Rightarrow C$
- S4: $H \wedge B \Rightarrow G$
- S5: $J \wedge C \Rightarrow E$
- S6: $H \wedge J \Rightarrow D$
- S7: H
- S8: J
- S9: C
- S10: B

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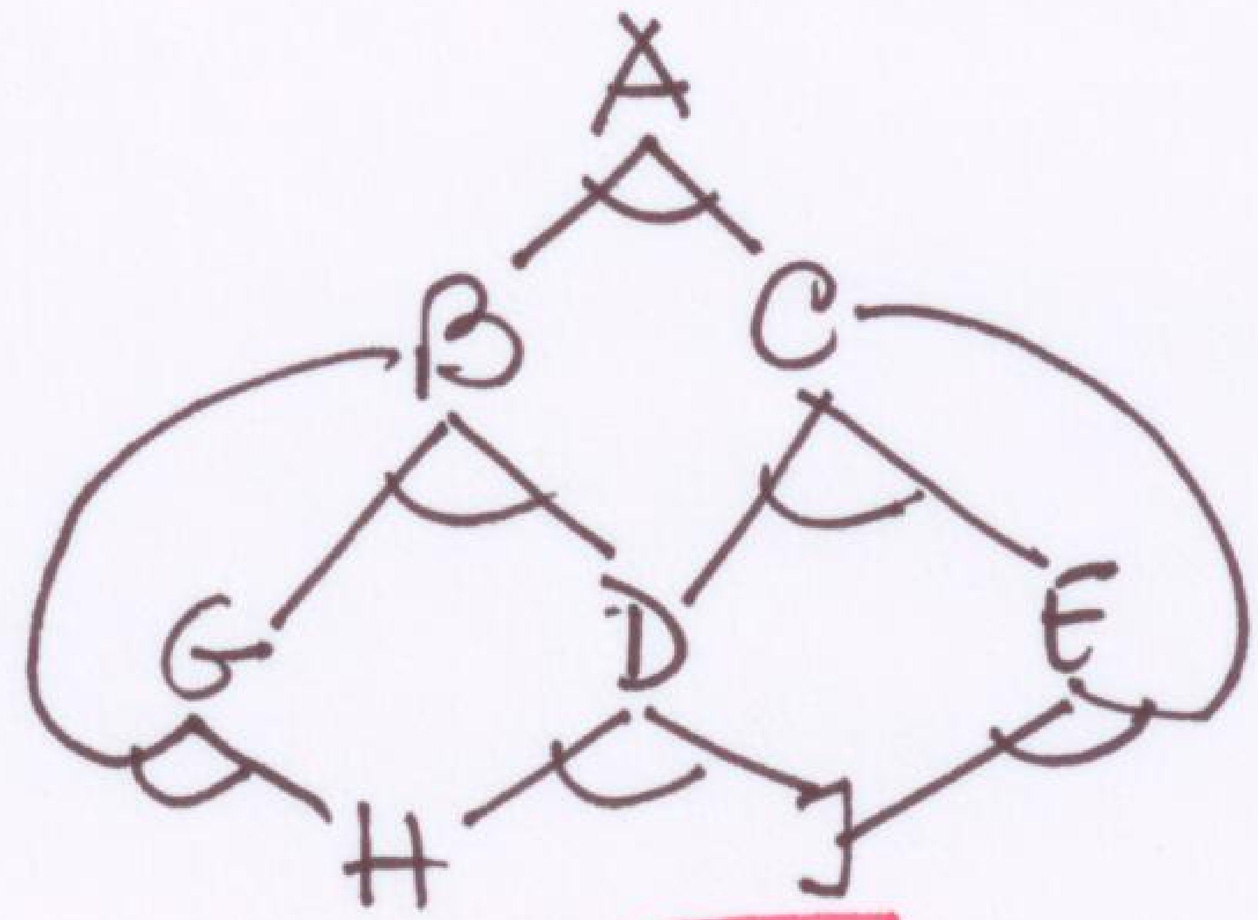
prove the following queries:

Q1: A ; Q2: B ; Q3: C

Solution let us draw the corresponding AND-OR graph:



only AND nodes!

Agenda	H	J	Q3 C	Q2 B	G	D	E	Q1 A
	1	2	3	4	5	6	7	8

step1: ~~C~~ = T

step2: ~~B~~

step3: ~~A~~

step4: ~~H~~

step5: ~~J~~

step6: ~~D~~

step7: ~~E~~

Inferred	H	J	C	B	G	D	E	A

{all FALSE

step1: Infer C=TRUE

step2 Infer B=TRUE

step3 Infer A=TRUE

step4: infer H=TRUE

step5: infer J=TRUE

step6: infer D=TRUE

step7: infer E=TRUE

The count table:

	step1	step2	step3	step4	step5	step6	step7
S1 : 2	1	0	Q1	Q1	Q1	Q1	Q1
S2 : 2	2	2	2	2	2	1	1
S3 : 2	2	2	2	2	2	1	0
S4 : 2	2	1	1	0	0	0	0
S5 : 2	1	1	1	1	0	0	Q5
S6 : 2	2	2	2	1	0	Q4	Q4
S7 : 0	0	0	0	~~QF~~?	0	0	0
S8 : 0	0	0	0	0F²√	0	0	0
S9 : 0	Q3√	Q3	Q3	Q3	Q3	Q3	Q3
S10 : 0	0	Q√2	Q2	Q2	Q2	Q2	Q2

Step 1 chose one of the sentences with <u>count = 0</u> (eventually one that is a query → S9 or S10)

count ⇒ (S9) = 0 ⇒ infer <u>C = TRUE</u>
C is Query 3 ⇒ proven to be TRUE

but also count(S5) = 1 ; count (S1) = 1

Step2 count(S10) = 0 ⇒ infer <u>B = TRUE</u> ; B is Q2
but also count(S1) = 0 ; count(S4) = 1

Step 3 Because count(S1)=0 => Infer A=TRUE
But A = Q 1
You have proven Q1, Q2, Q3

Can you also prove Q4=D and Q_5=E?

step 4 Because count(S_7)=0 => infer H is TRUE
=> count(S_6) = 1 ; count(S_4) = 0

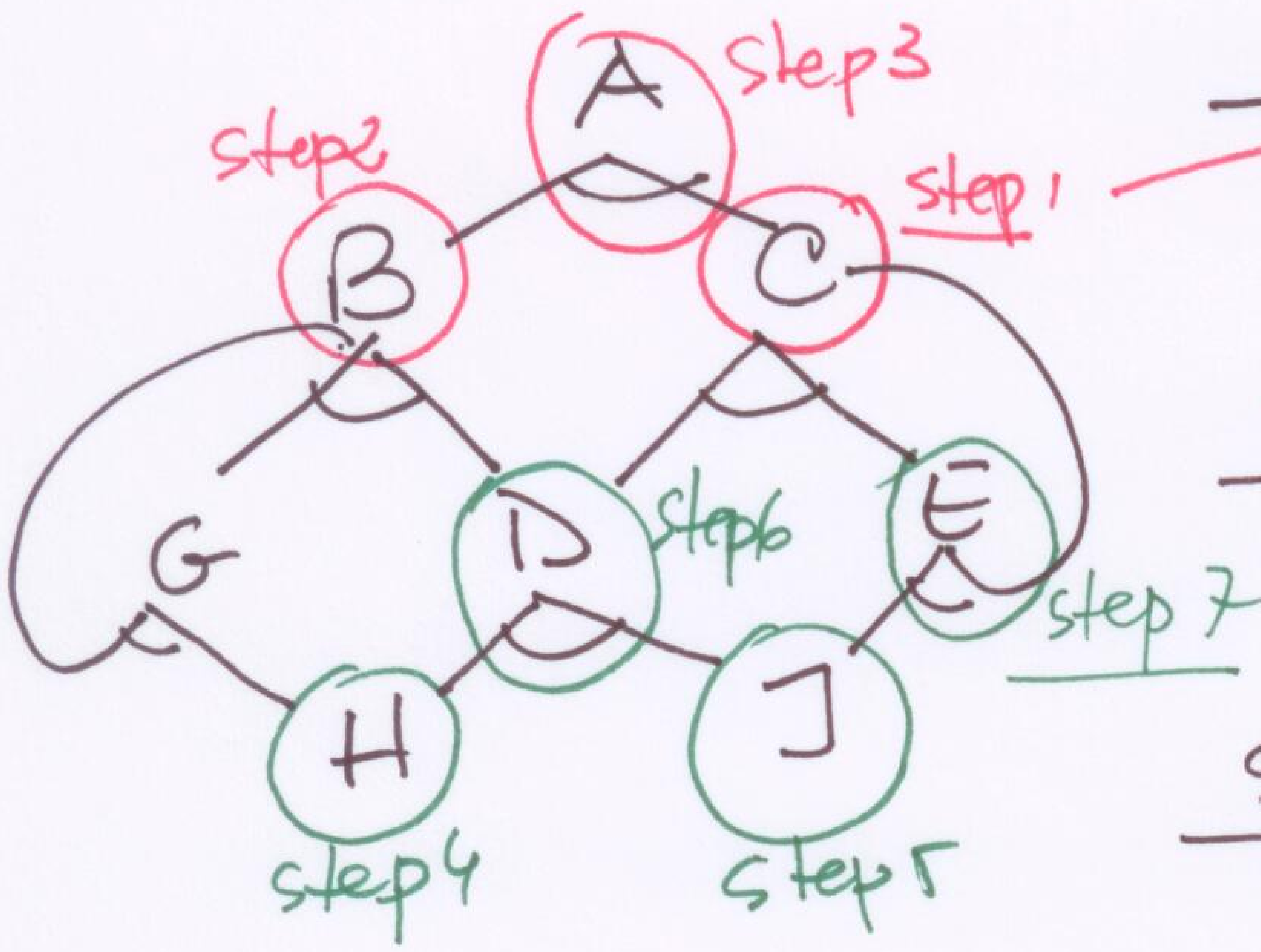
step 5 Because count(S8)= 0, infer J is TRUE
=> count(S_6)= 0 ; count(S_5)=0

step 6 Because count(S_6)=0, infer D is TRUE
But D = Q4 ✓
also => count(S2)=1 ; count(S3)=1

Step 7 Because count(S5)=0 ; infer E is TRUE
But E = Q5 ✓
also => count(S3)=0

Because we did not prove yet G, count of (S2)=1
But count(S4)=0 => infer G=0
=> count(S2)=0.

let us use the AND-OR Graph.



Step1
Because of S9:
Q3 is TRUE

step2
Because of S10:
Q2 is TRUE

Step3
Because of S1
Q1 is TRUE

If you want to prove Q4=D and Q5=E

Step 4: because S7 => H is True.

step5 because S8 => J is TRUE

step6 because S6 => D is TRUE => Q4 is TRUE

step7 because S5 => E is TRUE

(c) Apply ~~Backward Chaining~~ to prove Q from KB

S1: $P \Rightarrow Q$
S2: $L \wedge M \Rightarrow P$
S3: $B \wedge L \Rightarrow M$
S4: $A \wedge P \Rightarrow L$
S5: $A \wedge B \Rightarrow L$
S6: A
S7: B

Solution

Agenda Q, P, M, L, B, A
1 2 3 4 5 6

step 1: $\cancel{Q}$; $\cancel{P}$; step 2: $\cancel{L}$, $\cancel{M}$ step 3: $\cancel{B}$
step 4: $\cancel{A}$

Inferred

Q P M L B A } all False

step 1: Q=T; P=T; Step 2: L=T; M=T ; step 3: B=T
step 4: A=T

Count	step 1	step 2
S1: 1	0	0
S2: 2	0 ~~1~~	0
S3: 2	2	0
S4: 2	2	0
S5: 2	2	0
S6: 0	0	0
S7: 0	0	0

Step 1 We ~~know~~ suppose / assume Q = TRUE, infer P = TRUE
$\Rightarrow$ count (S2) = 0 ~~, infer~~

Step 2 We know P = TRUE, infer L = TRUE, M = TRUE
$\Rightarrow$ count (S4) = 0 ; count (S3) = 0 ; count (S5) = 0

(continue)

Step 3 Because count $(S_3)=0$; L=TRUE
B=TRUE → a fact from KB

Step 4 Because count$(S_5)=0$; B=TRUE
A=TRUE → a fact from KB

No more literals in Agenda
all litterals have been inferred!
Q is TRUE

2nd Example of Backward Chaining

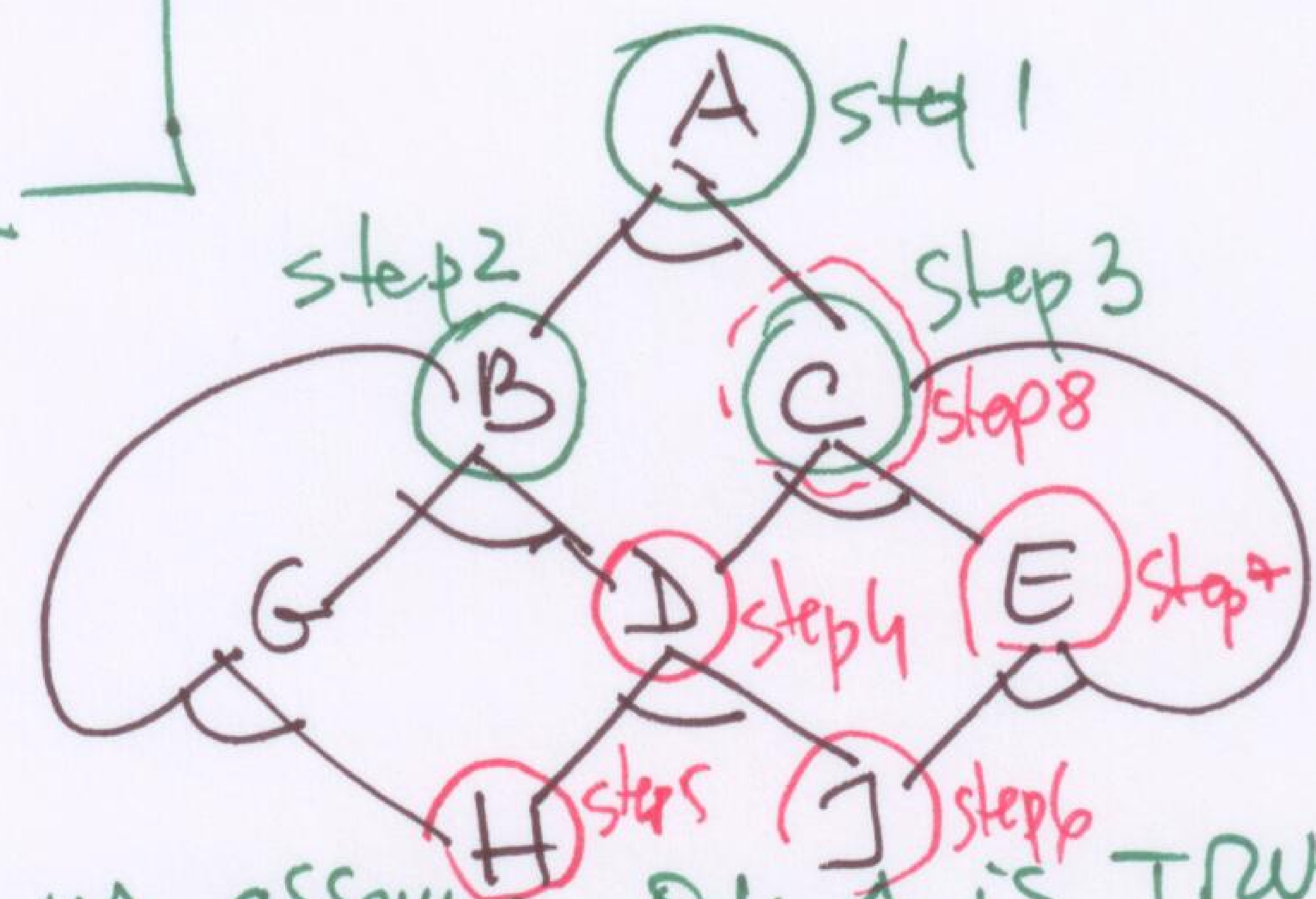
Consider the following KB of Horn Clauses

S1: $B \wedge C \Rightarrow A$
S2: $G \wedge D \Rightarrow B$
S3: $D \wedge E \Rightarrow C$
S4: $H \wedge B = G$
S5: $J \wedge C \Rightarrow E$
S6: $H \wedge J \Rightarrow D$
S7: H
S8: J
S9: C
S10: B

Prove the following queries:
Q1: A ; Q2: B Q3: C

Also prove:
Q4: D and Q5 ~~Q5~~ E

Solution let us draw the corresponding AND-OR graph



Step 1: let us assume Q1: A is TRUE according to S1, B & C are TRUE

Step 2 : B is true due to S10 in KB
$\Rightarrow$ proven Q2

Step 3 C is true due to S9 in KB
$\Rightarrow$ proven Q3
$\Rightarrow$ therefore we also prove Q1

(continued)

Step4 assume Q4: D is true.
according to S6, $H \wedge J$ must be true.

Step5 due to S7, in KB, we know H is TRUE

Step6 due to S8, in KB, we know J is tru
therefore D is true => prove Q4

Step7 assume Q5: E is true.
according to S5, $J \wedge C$ must be true.

Step8 due to S9 in KB, we know C is true
we already know J is true from step6

Alternatively: we know C is true from step 3
we know J is true from step6

=> prove Q5 is also true

3rd Example of Backward Chaining

=> Exam, Qual level!

Given the following KB, prove Query = A by using backward chaining.

KB

S1: $B \wedge C \Rightarrow A$
S2: B
S3: $D \wedge E \Rightarrow C$
S4: $E \vee F$
S5: $D \wedge \neg F$

Solution

Observation: Not all sentences in the KB are Horn clauses!

S4: $E \vee F$ is not a Horn clause!

S5: $D \wedge \neg F$ is not a Horn clause!

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It shall be trickier to represent the KB as an AND-OR graph!

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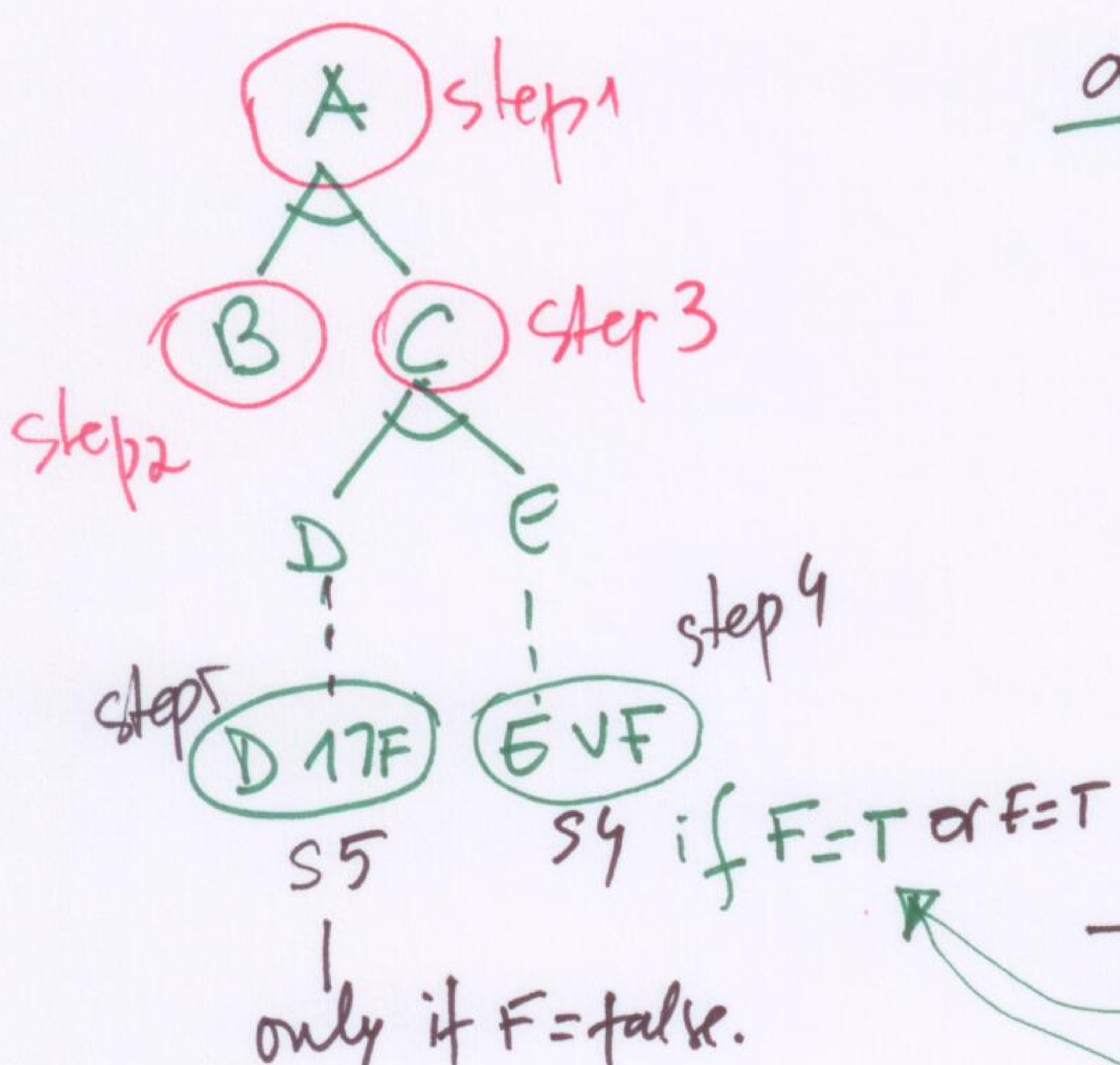
Instead build a TRUTH-TABLE for S_4, S_5 (it allows us to do model checking)



E	F	S4	S5
F	F	F	T (if D=T)
F	T	T	F
T	F	T	T (if D=T)
T	T	T	F

let us perform backward chaining.

Step 1 Suppose A is TRUE
from S1 => B = TRUE must be ; C = TRUE must be

Step 2 from S2 we have proof that B is true.

Step 3 Suppose C = TRUE from S3 =>
D = True, and E = TRUE

Step 4 If E = TRUE, and ~~F = True~~, regardless of F
given S4, we know that ~~it~~ E V F is true. ✓

Step 5 if D = TRUE, ~~or~~ and F = ~~TRUE~~ False.
D shall be true

therefore the assumption that
A is TRUE
was correct