

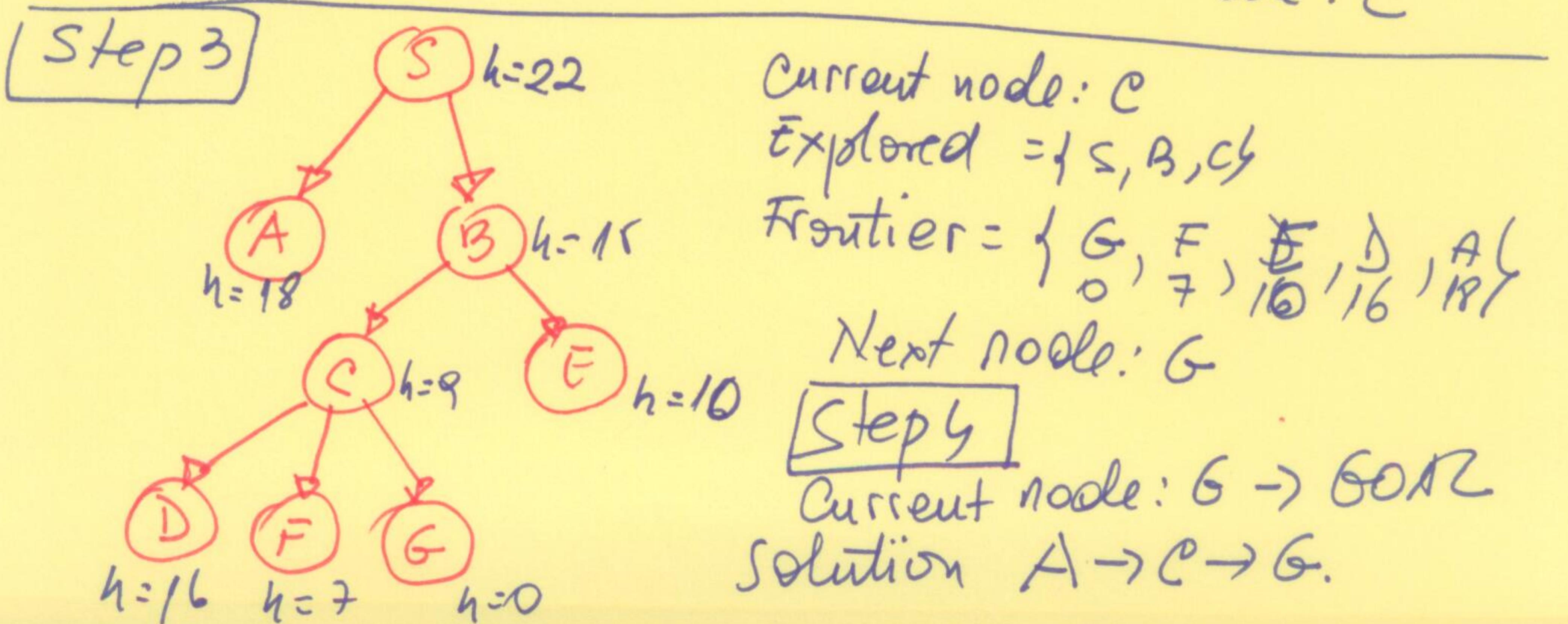
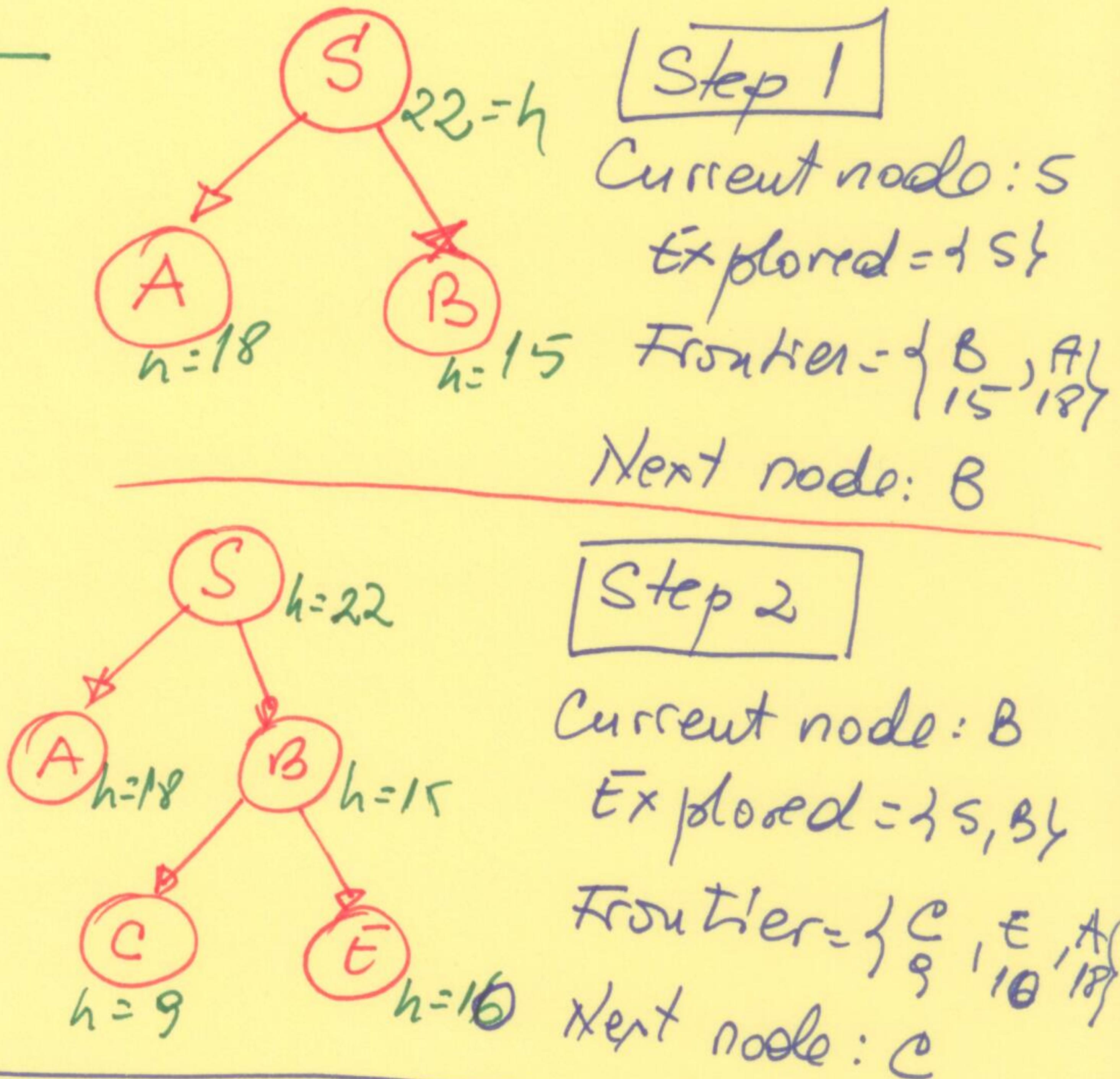
Examples of Informed - search

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Algorithm

1 Greedy - search example.

Nodes	h
A	18
B	15
C	9
D	16
E	10
F	7
G	0
S	22



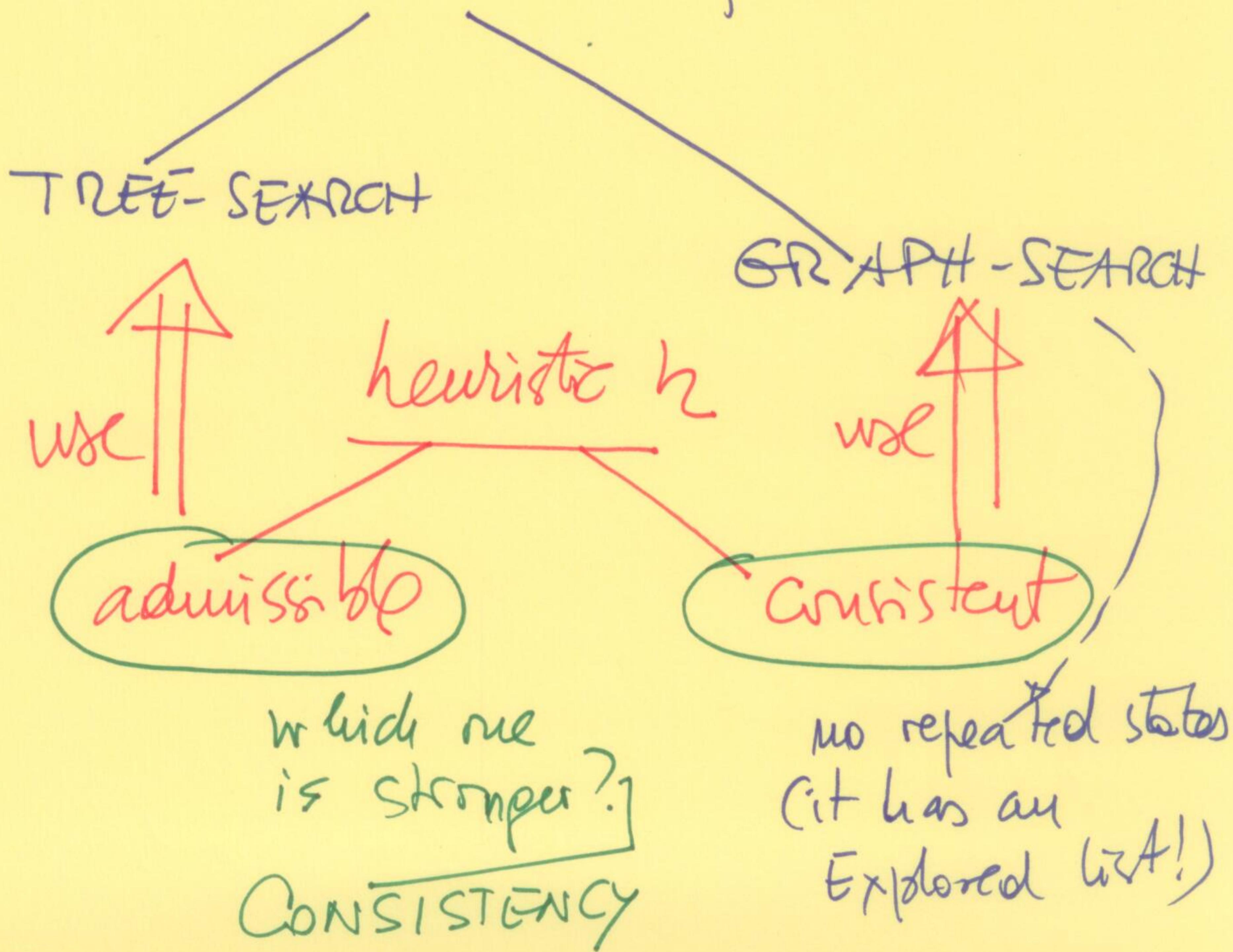
When does SEARCH continue?

- 1) current node \neq GOAL
- 2) Frontier not empty!

what kind of SEARCH?

\Rightarrow depending on strategy

what search algorithm?



Continue Problem 3.21 from Textbook

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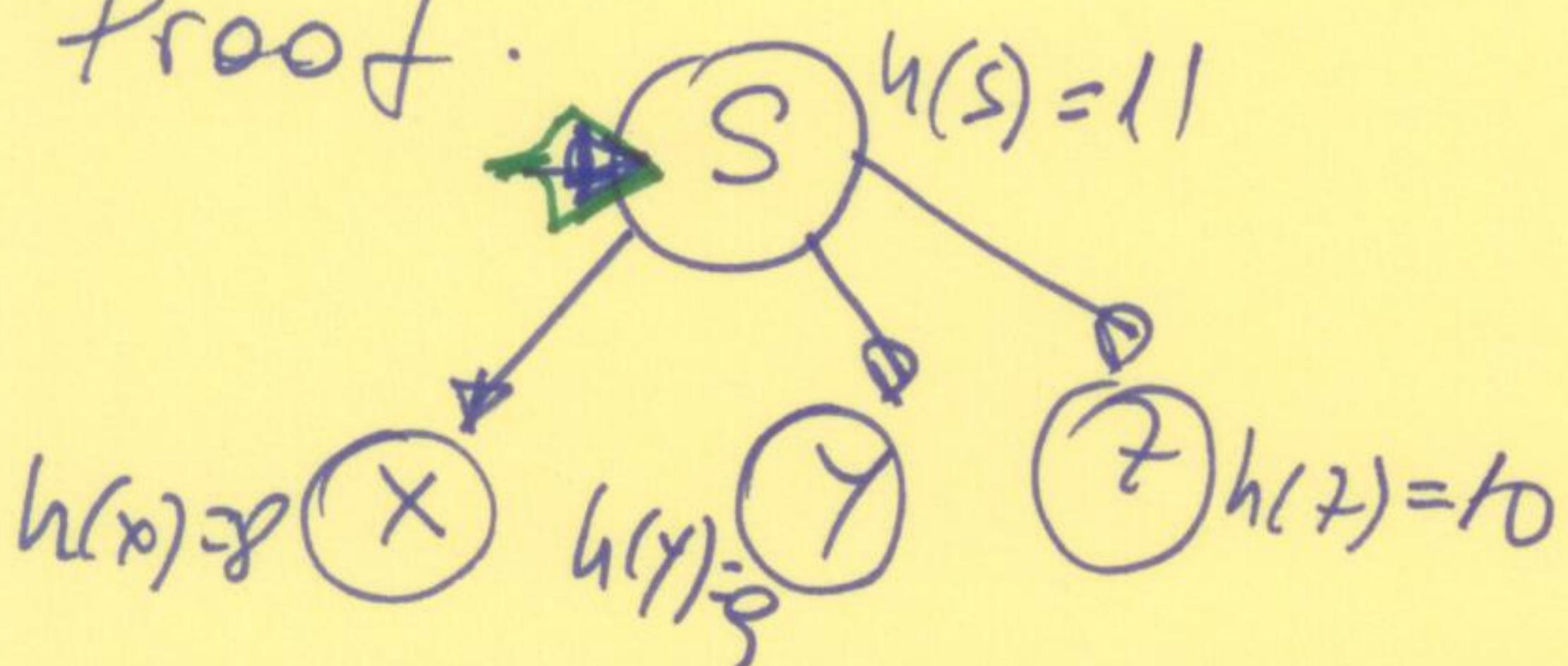
Prove that b) Depth-first search is a special case of greedy best-first search.

Answer. For the search initiating in node S, consider the following heuristic:

Node	S	X	Y	Z	A	B	C	D	E	F	G	K	L
h	11	8	9	10	1	2	3	4	5	6	7		

Then depth search is identical to greedy-first search.

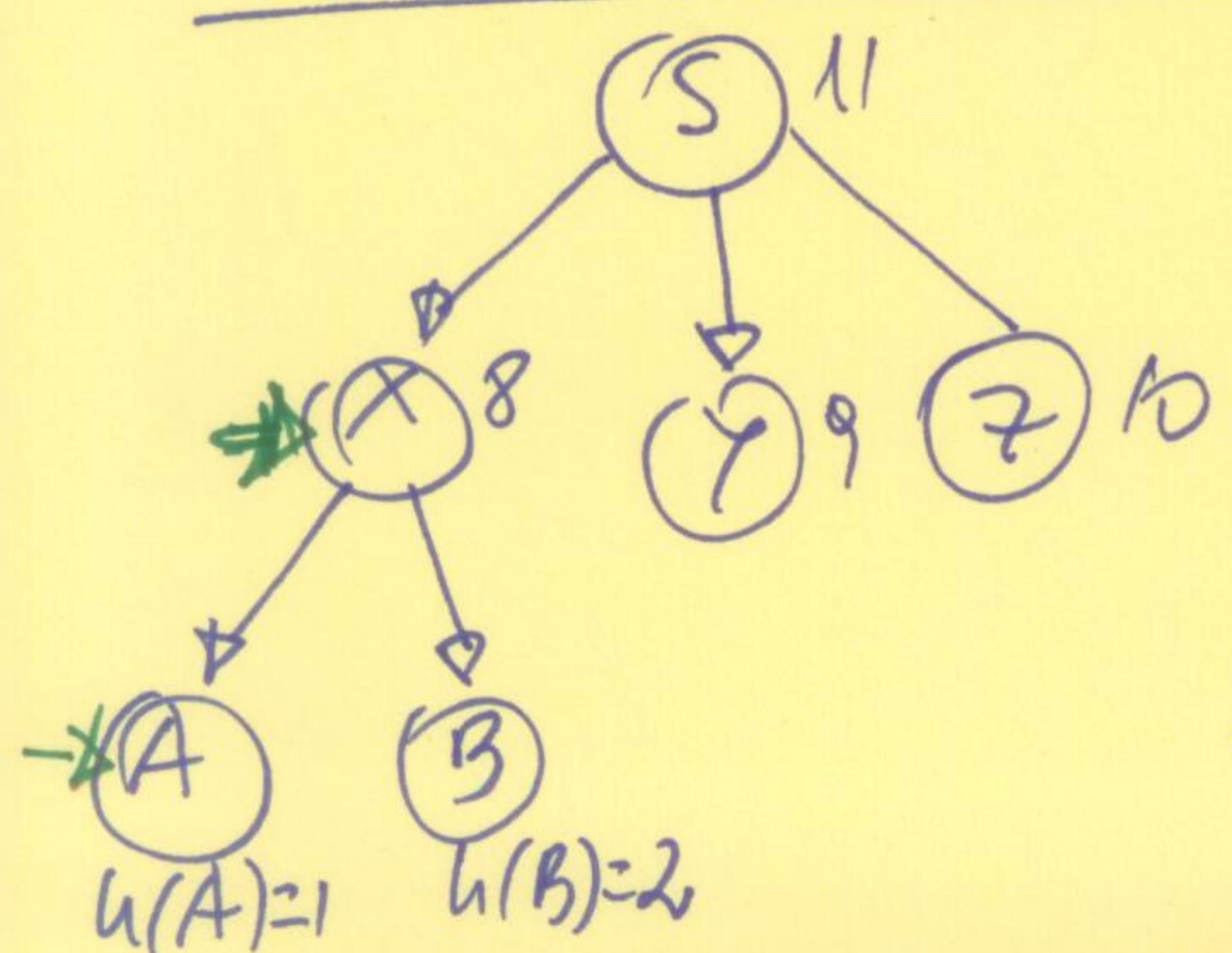
Proof:



Step 1 Current node = S

Frontier = {X, Y, Z} {8, 9, 10}

Next node: X



Step 2 Current node = X

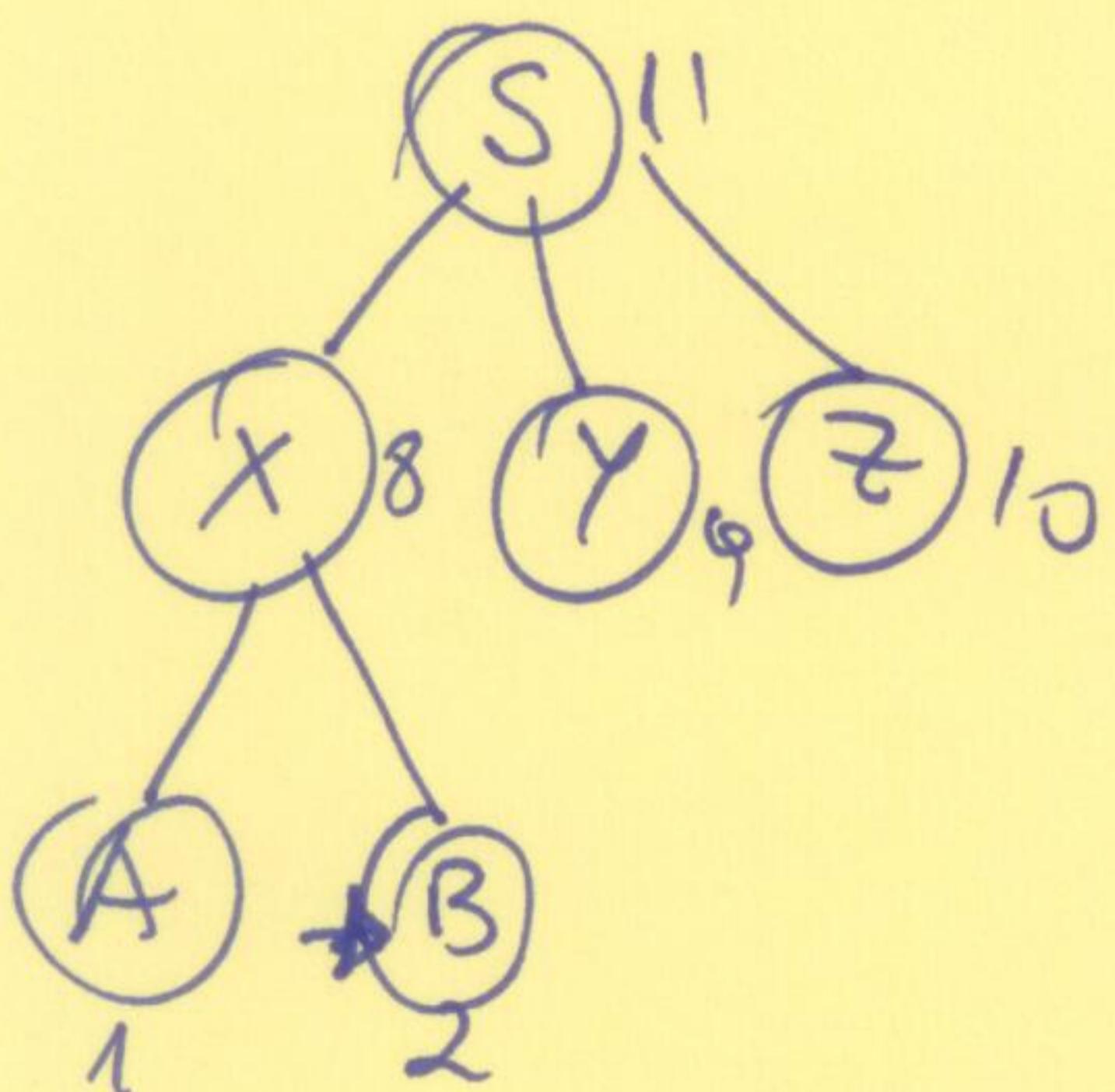
Frontier = {A, B, C} {1, 2, 9, 10}

Next node: A

Step 3: Current node = A

Frontier = {B, C} {2, 9, 10}

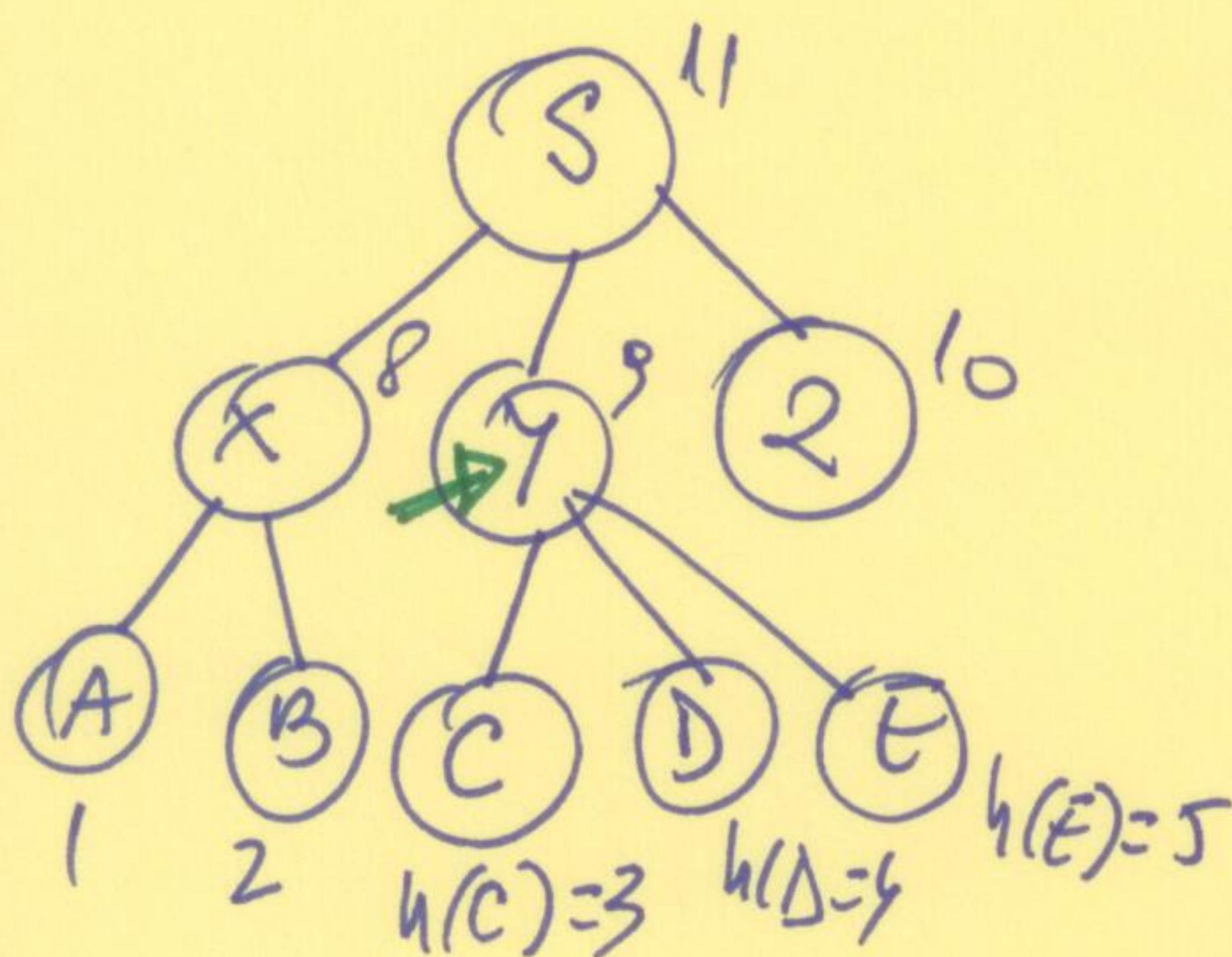
Next node: B.



Step 4 Current node = β ~~4~~

Frontier = { γ, δ }

Next node = γ



Step 5 Current node = γ

Frontier = { $\epsilon, \delta, \epsilon, \zeta$ }

Next node = ϵ

Step 6 Current node = ϵ

Frontier = { δ, ϵ, ζ }

Next node = δ

Step 7 Current node = δ

Frontier = { ϵ, ζ }

Next node = ϵ

Step 8 Current node = ϵ

Frontier = { ζ }

Next node : ζ

Step 9 Current node = ζ

Frontier = { κ, λ }

Next node = κ

Step 10 Current node = κ

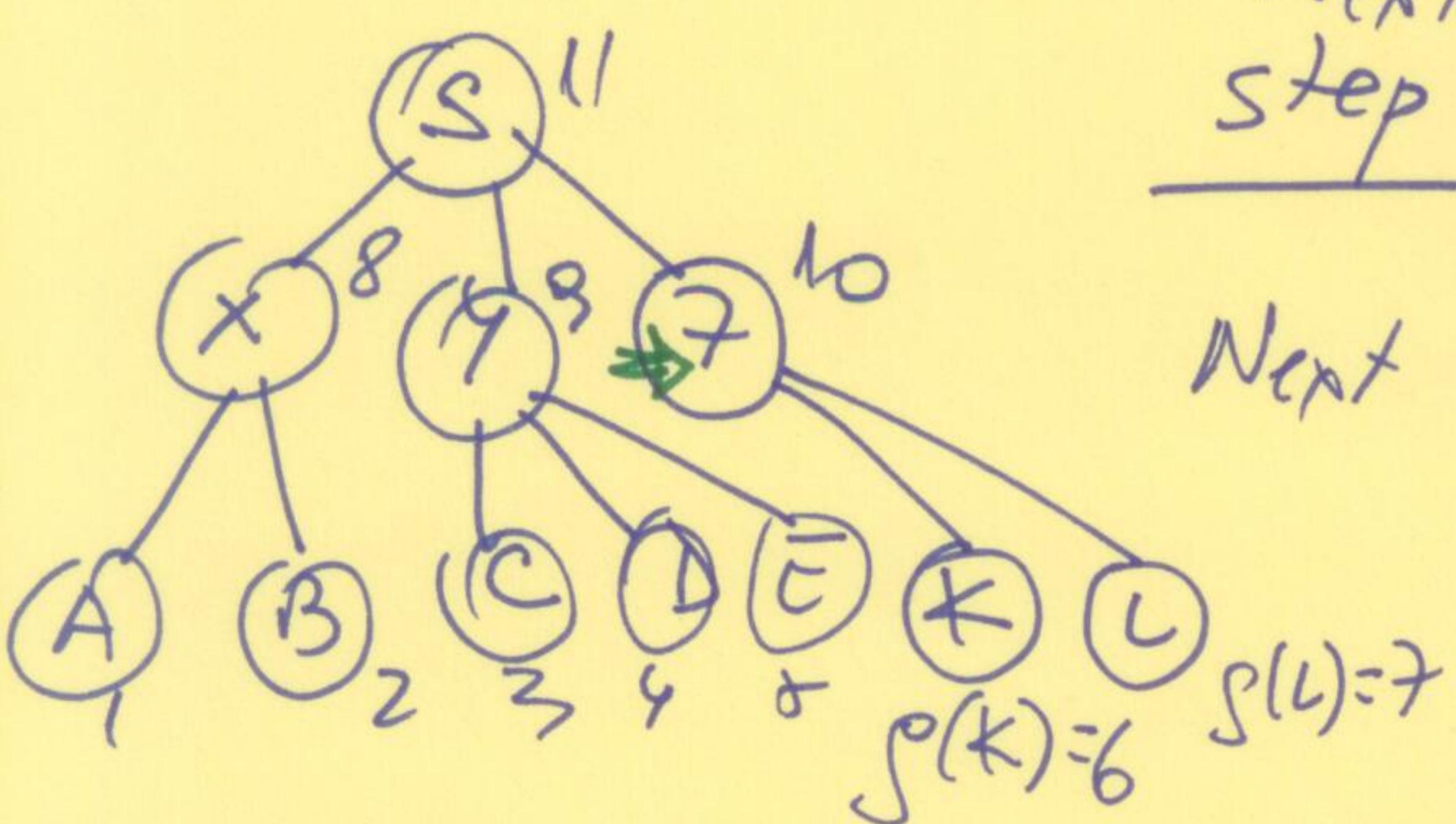
Frontier = { λ }

Next node : λ

Step 11 Current node = λ

Frontier = {}

STOP



~~Prove that c) uniform-cost search
is a special case of A*~~

Answer Uniform-cost search uses only the values of $g(n)$ to order the frontier in ascending order.

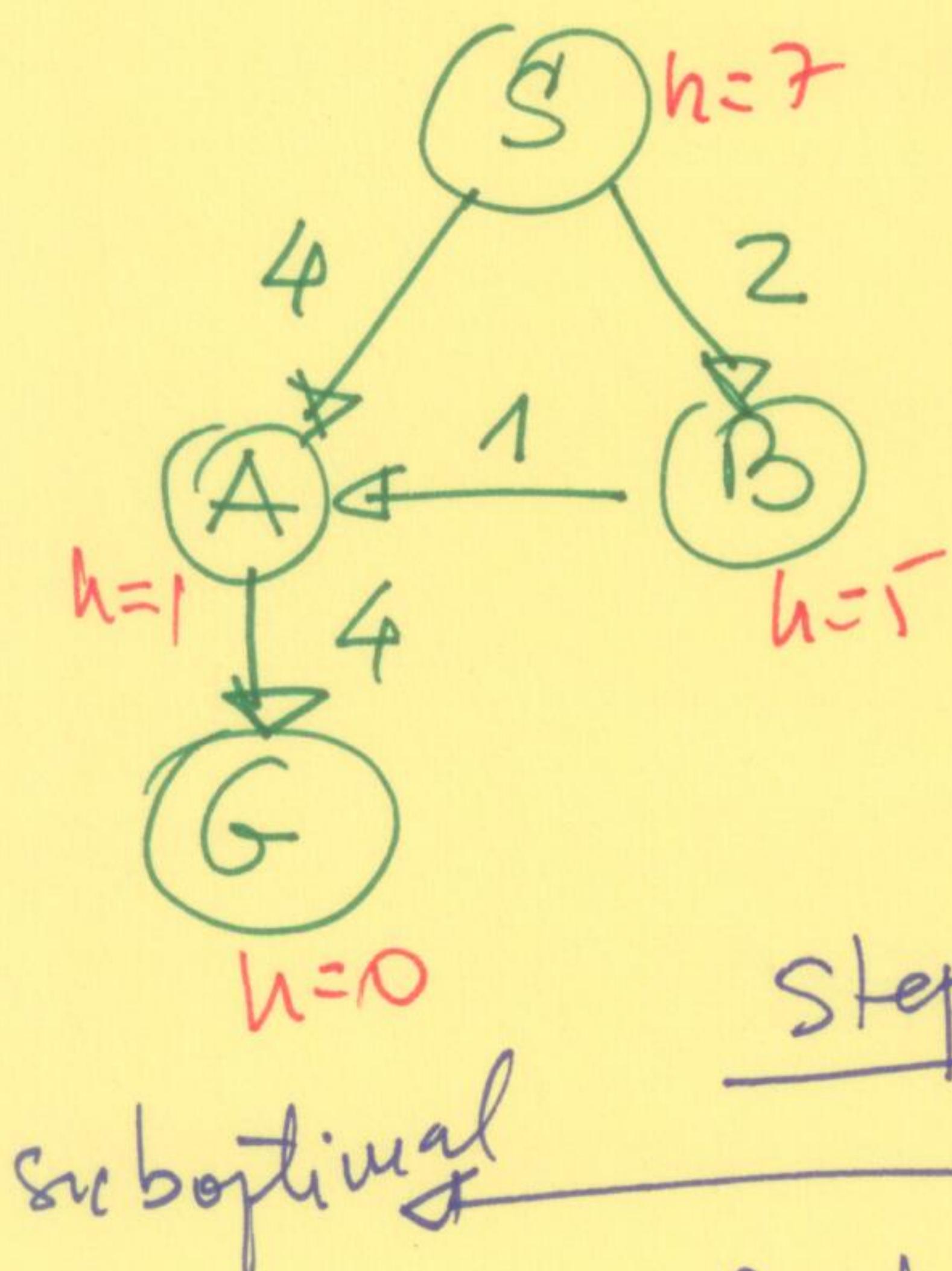
If $h(n)=0$ uniform-cost and A* are identical.

Problem 3.24 from Texbook

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Derive a state space in which A* using GRAPH-SEARCH returns a sub-optimal solution with a heuristic $h(n)$ that is admissible but inconsistent.

Solution



suboptimal

The heuristic is admissible because $h(G) = 0$. Step 1

Successors(S) = {A, B}

$$f(A) = g(A) + h(A) = 4 + 1 = 5$$

$$f(B) = g(B) + h(B) = 2 + 5 = 7$$

Frontier = { $\frac{A}{5}, \frac{B}{7}$ } Explorad = {S}

Next node = A

Step 2

successors(A) = G

Solution: $S \rightarrow A \rightarrow G$

But if I take the path

$S \rightarrow B \rightarrow A \rightarrow G$ cost = 7

Consistency Test

in A: father = S

$$h(S) = 7$$

$$h(A) + c(S, A) = 1 + 4 = 5$$

consistent heuristics $h(n) \leq C(n, n') + h(n')$



(I u B)

father = S $h(S) = ?$

F7

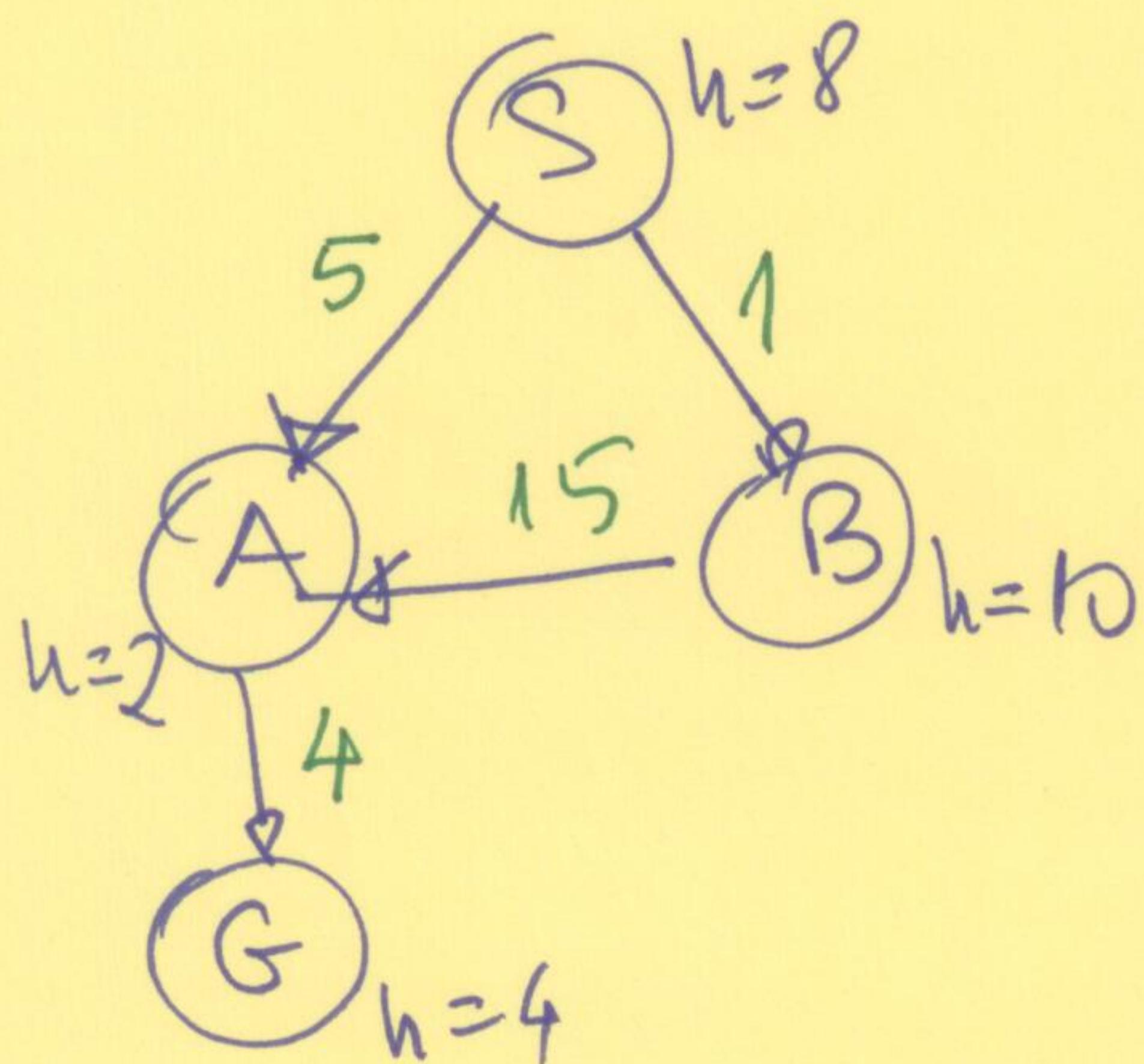
$$h(B) + c(S, B) = 5 + 2 = 7 \quad \checkmark$$

(I u G)

father = * $h(*) = ?$

$$h(G) + c(A, G) = 6 + 4 = 10 \quad \checkmark$$

Another Graph that provides
a sub-optimal solution

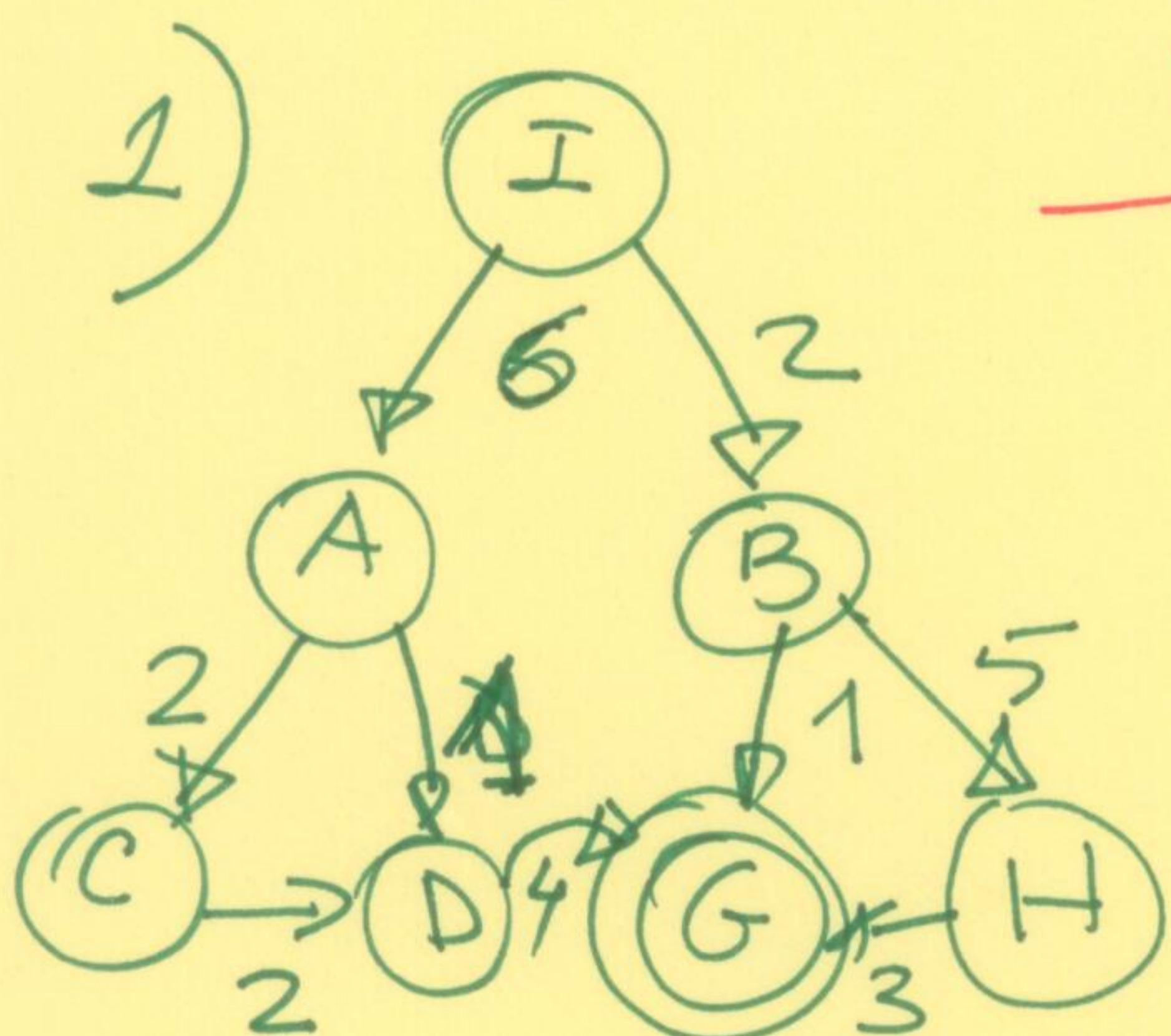


Self-test

why?

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Example of admissible
but not consistent
heuristics.



Node	I	A	B	C	D	G	H
n	10	5	12	8	11	10	12

The heuristic
is admissible!

To check if it is
consistent:

In node A : $\text{father}(A) = I \quad h(I) = 10$
 $h(A) + \text{cost}(I, A) = 5 + 6 = 11 \quad \checkmark$
 $h(D) \leq h(A) + \text{cost}(I, A)$

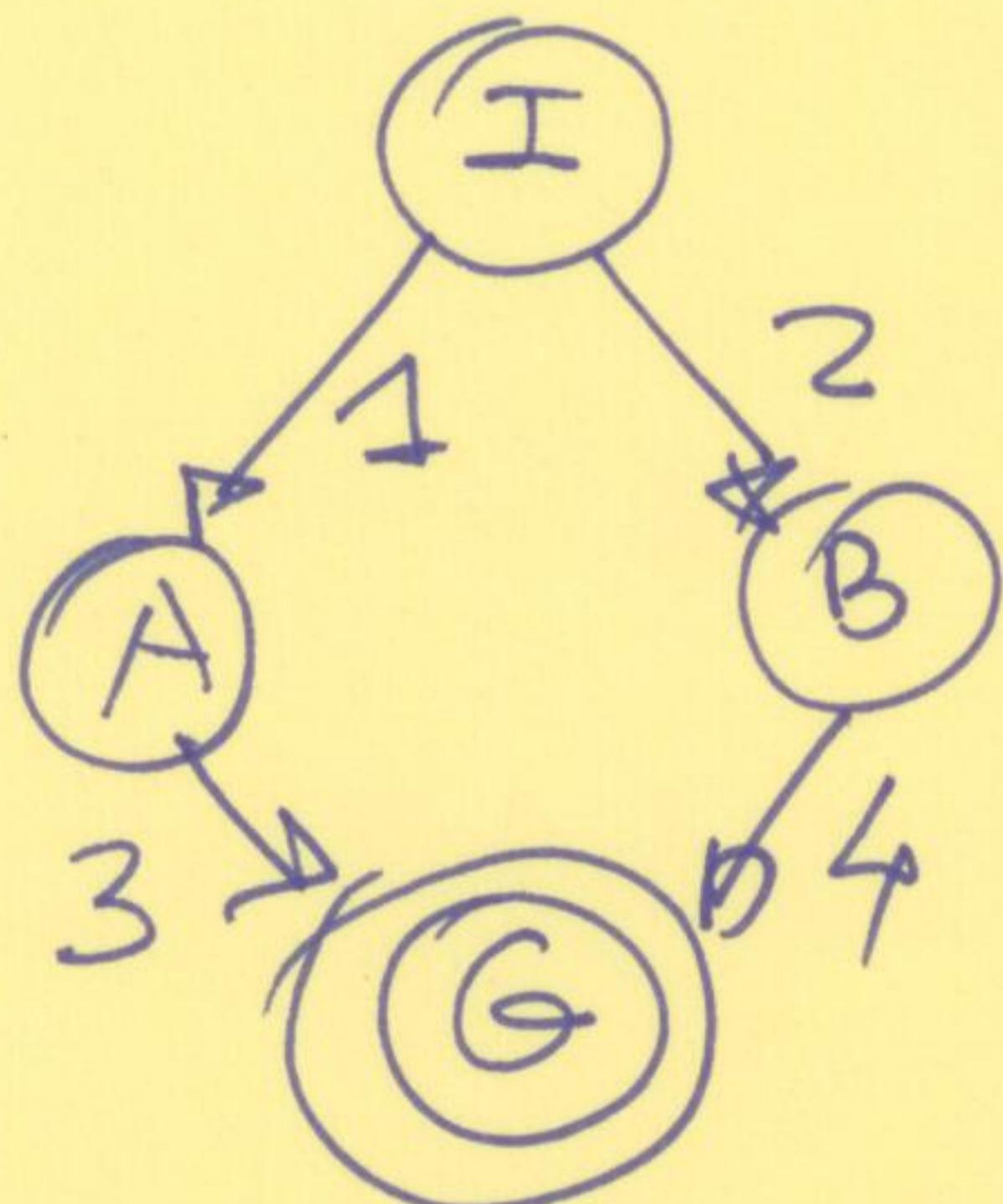
In node B : $\text{father}(B) = I \quad h(I) = 10$
 $h(B) + \text{cost}(I, B) = 12 + 2 = 14 \quad \checkmark$
 $h(C) + \text{cost}(I, C) = 8 + 2 = 10 \quad \checkmark$

In node C : $\text{father}(C) = A \quad h(A) = 5$
 $h(C) + \text{cost}(A, C) = 8 + 2 = 10 \quad \checkmark$

In node D : $\text{father}(D) = A, C \quad h(A) = 5$
 $h(D) + \text{cost}(A, D) = 11 + 1 = 12 \geq h(A) \quad \checkmark$
 $h(D) + \text{cost}(C, D) = 11 + 2 = 13 \geq 8$

In node H : $\text{father}(H) = B \quad h(B) = 12$
 $h(H) + \text{cost}(B, H) = 2 + 5 = 7 \leq 12$
 $\Rightarrow \text{inconsistent}$

2) Another example of admissible
but inconsistent heuristic



Node	I	A	B	G
h	10	5	6	0

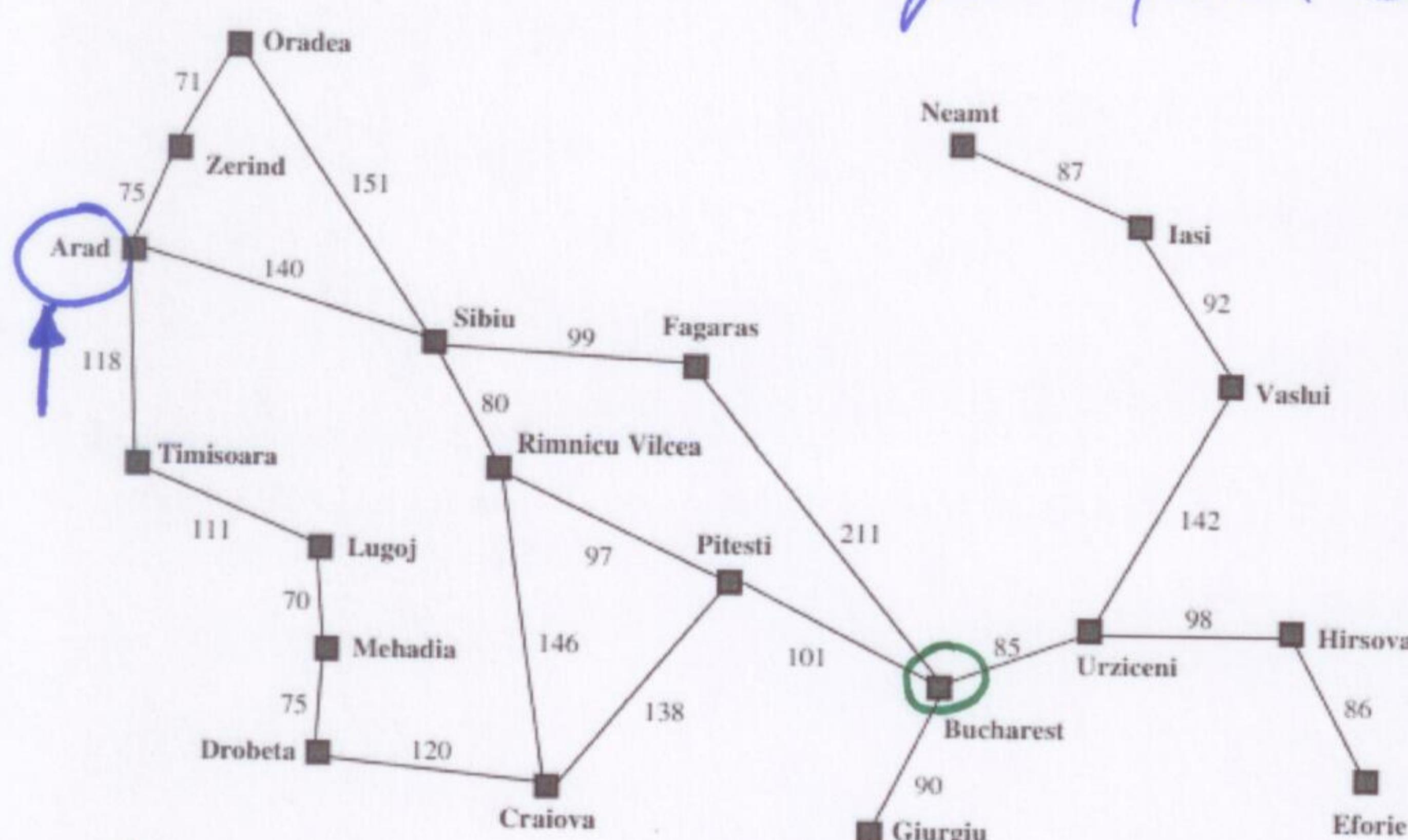
it is
admissible

Check for consistency:

In node A: $\text{father}(A) = I \quad h(I) = 10$
 $h(A) + c(I, A) = 5 + 1 = 6$

inconsistent heuristic!

A* algorithm: Find a solution to get from I=Arad to F=Bucharest 10



Straight-line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Answer.

Step 1 Current node: Arad $h \Rightarrow$

Exploded = {Arad}

children(Arad) = {Sibiu, Timisoara}

$$f(Sibiu) = g(Sibiu) + h(Sibiu) = \frac{75 + 253}{374} = 393$$

$$f(Timisoara) = g(Timisoara) + h(Timisoara) = \frac{118 + 329}{447} = 447$$

$$f(Zerind) = g(Zerind) + h(Zerind) = \frac{75 + 374}{449} = 449$$

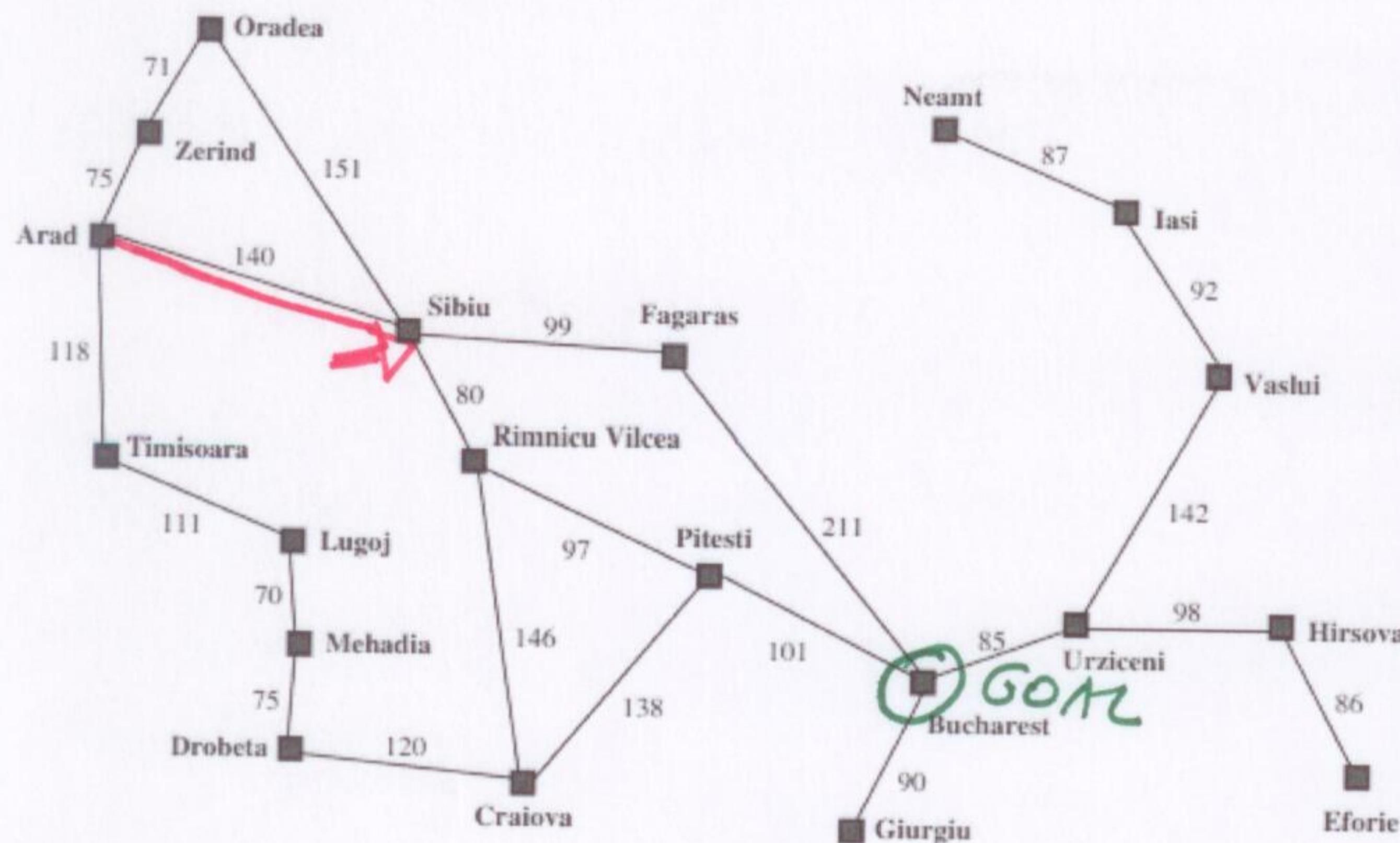
Frontier = {Sibiu, Timisoara, Zerind}

Next node = Sibiu

Note: Heuristic h is consistent \Rightarrow will use GRAPH SEARCH
why? $\{$ consistent \Rightarrow will use GRAPH SEARCH
Exercise $\}$ no repeated nodes! in frontier

A* algorithm: Romanian map (cont)

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Straight-line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
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Step 2 Current node = Sibiu

Explored = {Arad, Sibiu}

Current Path: Arad \rightarrow Sibiu

children (Sibiu) = {Arad, Fagaras,

Oradea, Rimnicu Vilcea} is in Explored

$$f(Fagaras) = g(Fagaras) + h(Fagaras) = \frac{415}{140+99=239 \quad 174}$$

$$f(Oradea) = g(Oradea) + h(Oradea) = \frac{671}{140+151=291 \quad 380}$$

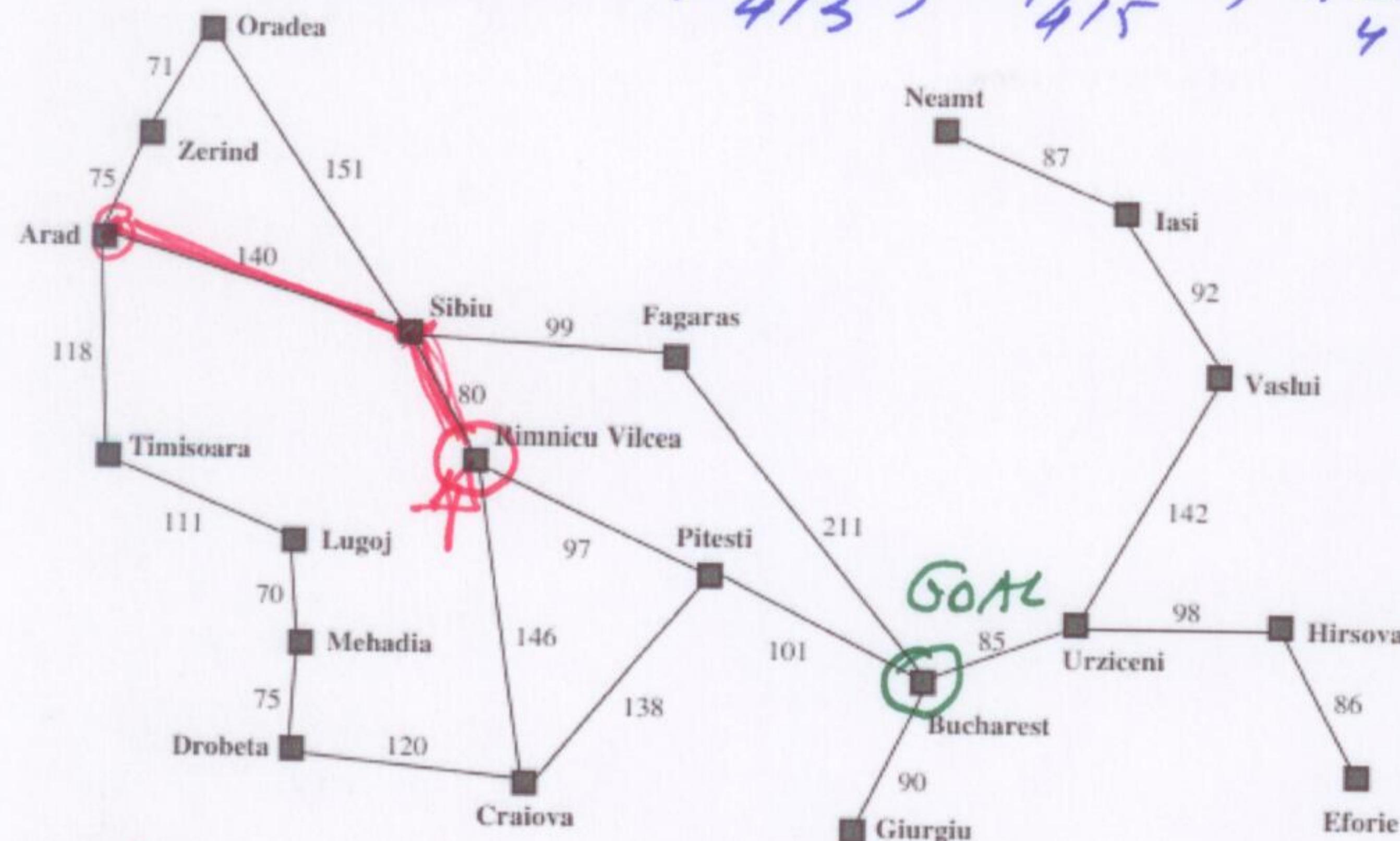
$$f(Rimnicu Vilcea) = g(Rimnicu Vilcea) + h(Rim Vil) = \frac{413}{140+80=220 \quad 193}$$

Frontier (at step 1) = {Sibiu, Timisoara, Zerind}

Frontier = {Rimnicu Vilcea, Fagaras, Timisoara, Zerind, Oradea}

Next node = RV

A* algorithm: Romanian map (cont) 12
 Frontier = { $\frac{R}{413}$, $\frac{F}{415}$, $\frac{T}{447}$, $\frac{Z}{449}$, $\frac{O}{671}$ } $\frac{G}{671}$



Straight-line distance to Bucharest

Arad	366
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Zerind	374

Step 3 Current node: Rimnicu Vilcea
 Explored = {Arad, Sibiu, RV}

Current Path: Arad \rightarrow Sibiu \rightarrow RV

Children (RV) = {Craiova, Pitesti, Sibiu}

Sibiu
 ↑ in Explored.

$$f(Craiova) = g(Craiova) + h(Craiova) = \frac{366}{160}$$

$$140 + 80 + 146 = 366$$

$$f(Pitesti) = g(Pitesti) + h(Pitesti) = \frac{415}{98}$$

$$140 + 80 + 97 = 317$$

Frontier = { $\frac{F}{415}$, $\frac{P}{415}$, $\frac{T}{447}$, $\frac{Z}{449}$, $\frac{O}{671}$ }

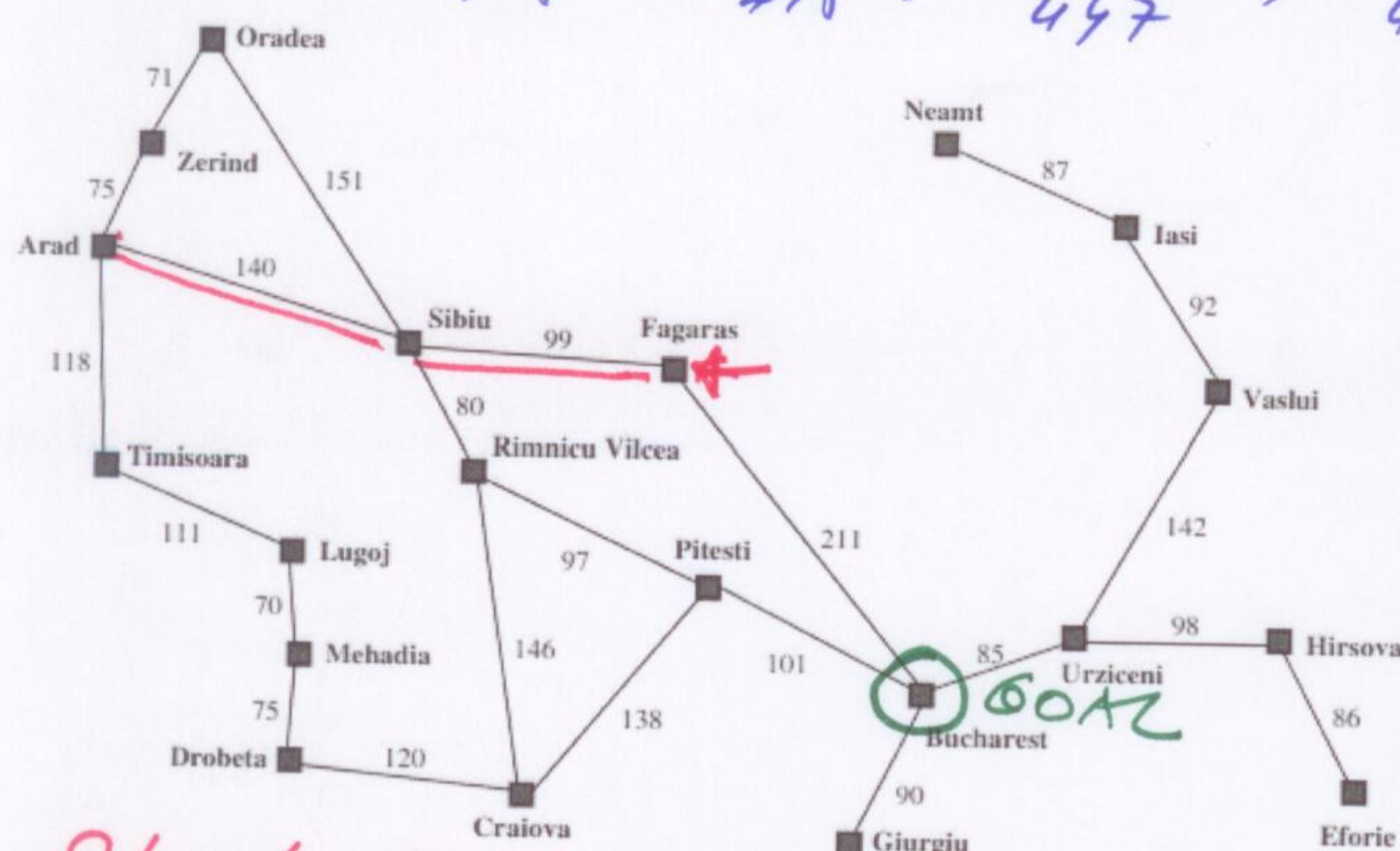
Craiova, Oradea
 526, 671

Next node: Fagaras

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~~A* algorithm: Romanian Map (cont)~~

~~Frontier: Fagaras, Pitesti, Timisoara, Zerind, Craiova, Drobota, 418, 415, 447, 449, 526, 671, 0, 198095~~



Straight-line distance to Bucharest

Arad	366
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Craiova	160
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Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
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Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Step 4 Current node: Fagaras

explored = { Arad, Sibiu, RV,

Current Path: Arad \rightarrow Sibiu \rightarrow Fagaras

Children (Fagaras) = { Sibiu, Bucharest }
in explored!

$$f(Bucharest) = g(Bucharest) + h(Bucharest)$$

$$140 + 99 + 211 = \underline{450}$$

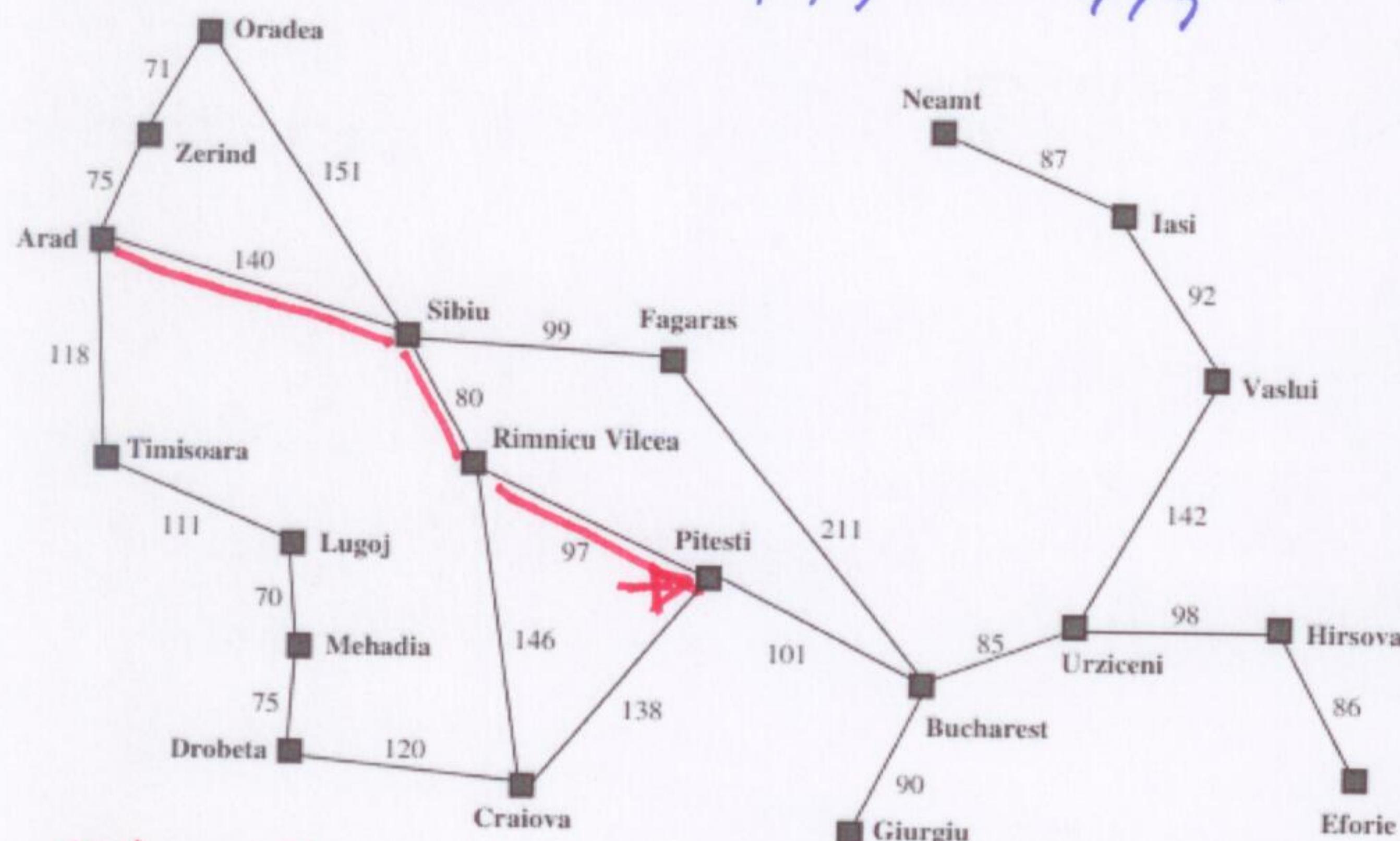
Frontier = { Pitesti, Timisoara, Zerind, 415, 447, 449, 450, 526, 671 }

Bucharest, Craiova, Drobota }

Next node? Pitesti

Note: Even if the GOM node is in the Frontier \Rightarrow we first expand the first node from the FRONTIER.

A* algorithm: Romanian Map (Cont) 14
 Frontier = {Pitesti 418, Timisoara 447, Zerind 449, Bucharest 450, Craiova 526, Oradea 671}



Straight-line distance to Bucharest

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Craiova	160
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Rimnicu Vilcea	193
Sibiu	253
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Vaslui	199
Zerind	374

Step 5 Current node: Pitesti
 explored = {Arad, Sibiu, DV, Fagaras, Pitesti}
 Current Path: Arad → Sibiu → DV → Pitesti
 Path cost: $140 + 80 + 97 = 317$
 children(Pitesti) = {Craiova, RV, Bucharest} → in explored!

$f(Bucharest) = f(Pitesti)$

$$317 + 101 = \underline{418}$$

This is a shorter path to the goal!!!

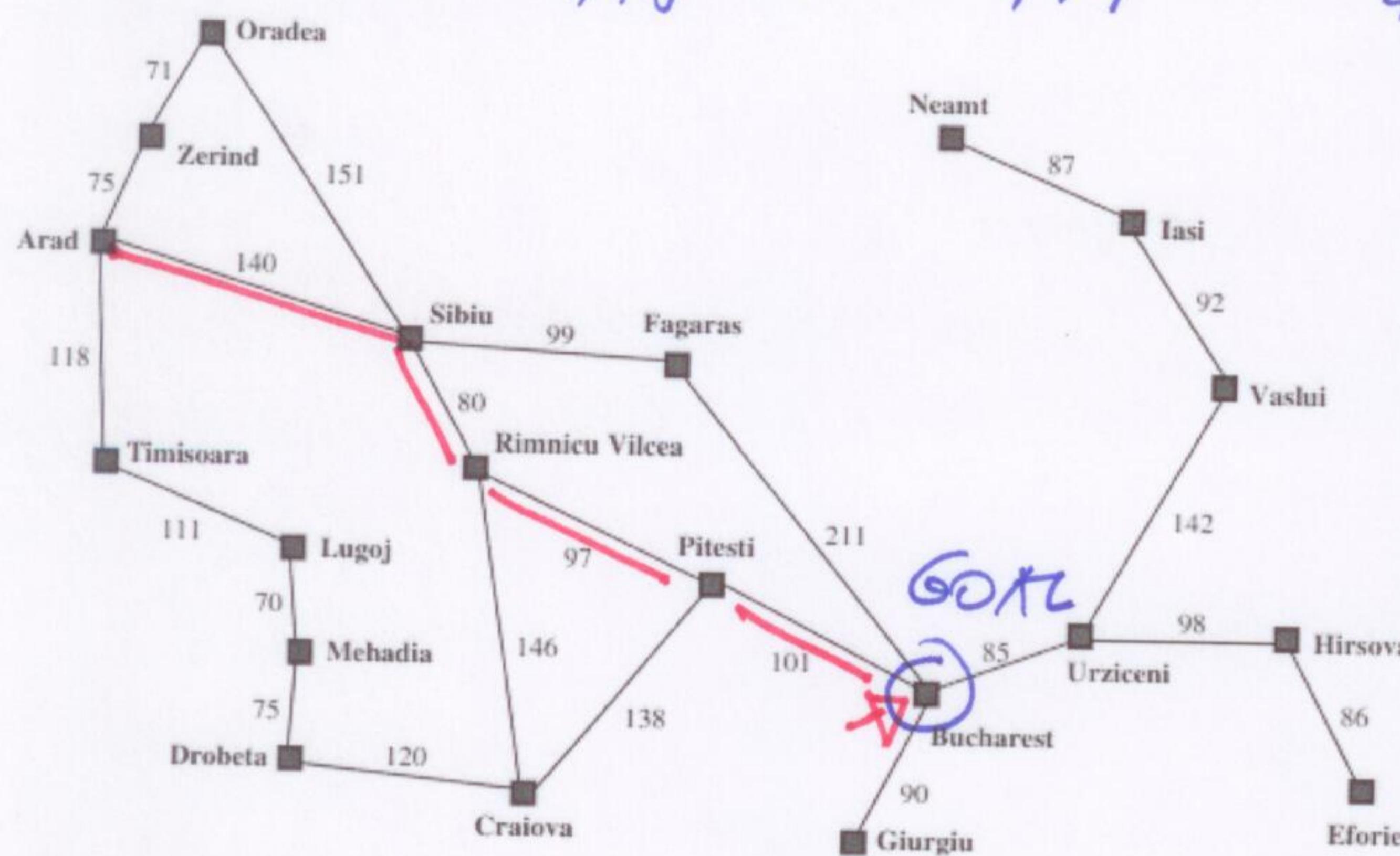
Frontier = {~~Pitesti~~ 418, Bucharest, Timisoara, Zerind 449, Craiova 526, Oradea 671}

Next Node: Bucharest

A* algorithm: Romanian Map (Cont)

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Frontier of Bucharest, Timisoara, Zerind, Craiova, Oradea, Drobota, 418, 447, 449, 526, 621



Straight-line distance to Bucharest

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Craiova	160
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Zerind	374

Step 6 Current node: Bucharest

explored = Arad, Sibiu, RV,
Fagaras, Pitesti, Bucharest,

Current Path: Arad \rightarrow Sibiu \rightarrow RV \rightarrow
 \rightarrow Pitesti \rightarrow Bucharest

Path cost: 418

Bucharest is the GOAL

Solution: Arad $\xrightarrow{140}$ Sibiu $\xrightarrow{80}$ RV $\xrightarrow{97}$ Pitesti $\xrightarrow{101}$ Bucharest

Path cost = Solution cost = 418