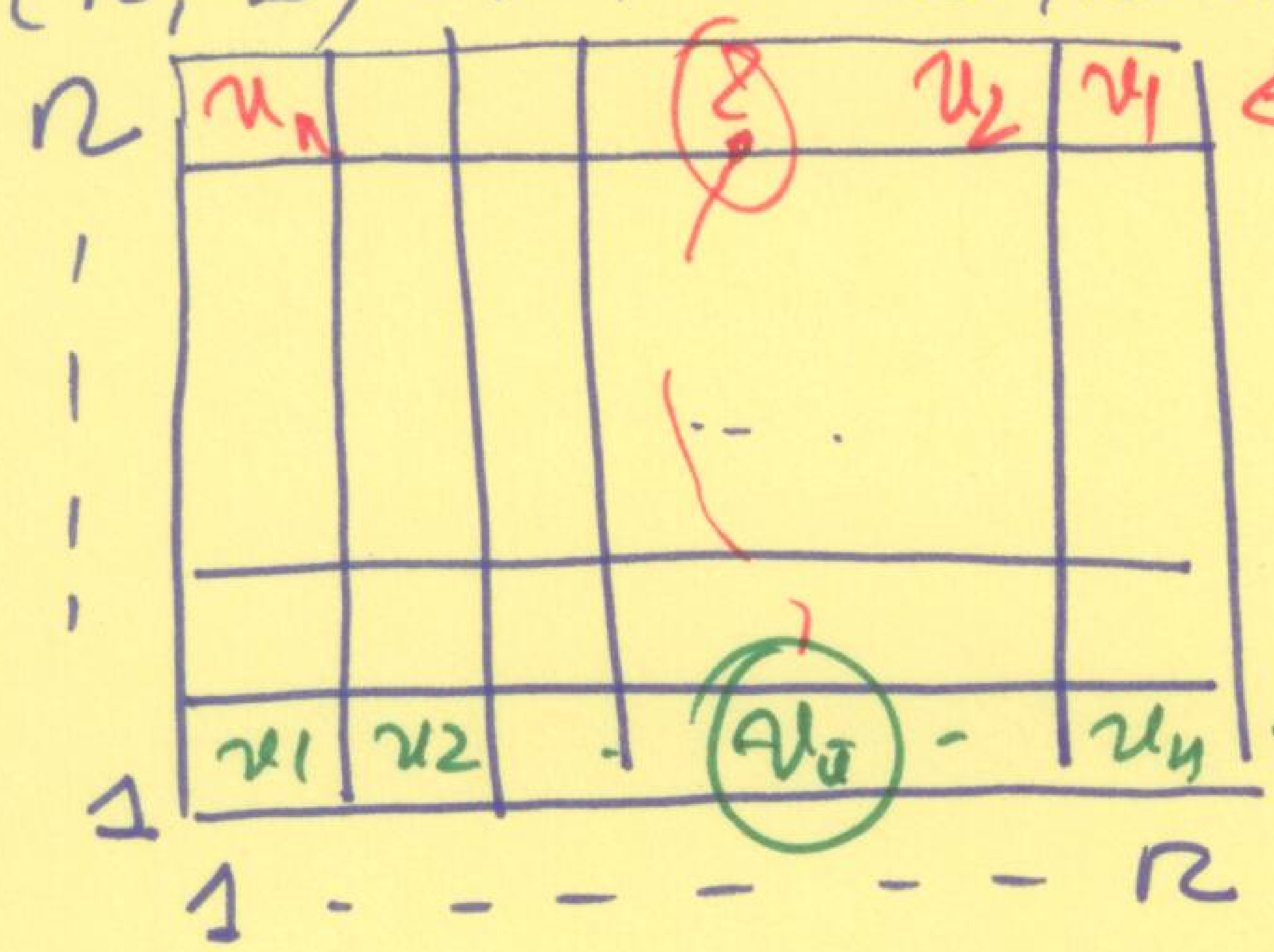
Problems using heuristics.

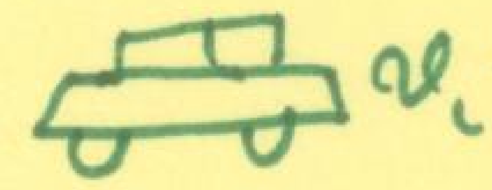
1) Problem 3.27 from Textbook.

n vehicles occupy squares $(1,1)$ through $(n,1)$ → the bottom row of an $n \times n$ grid.

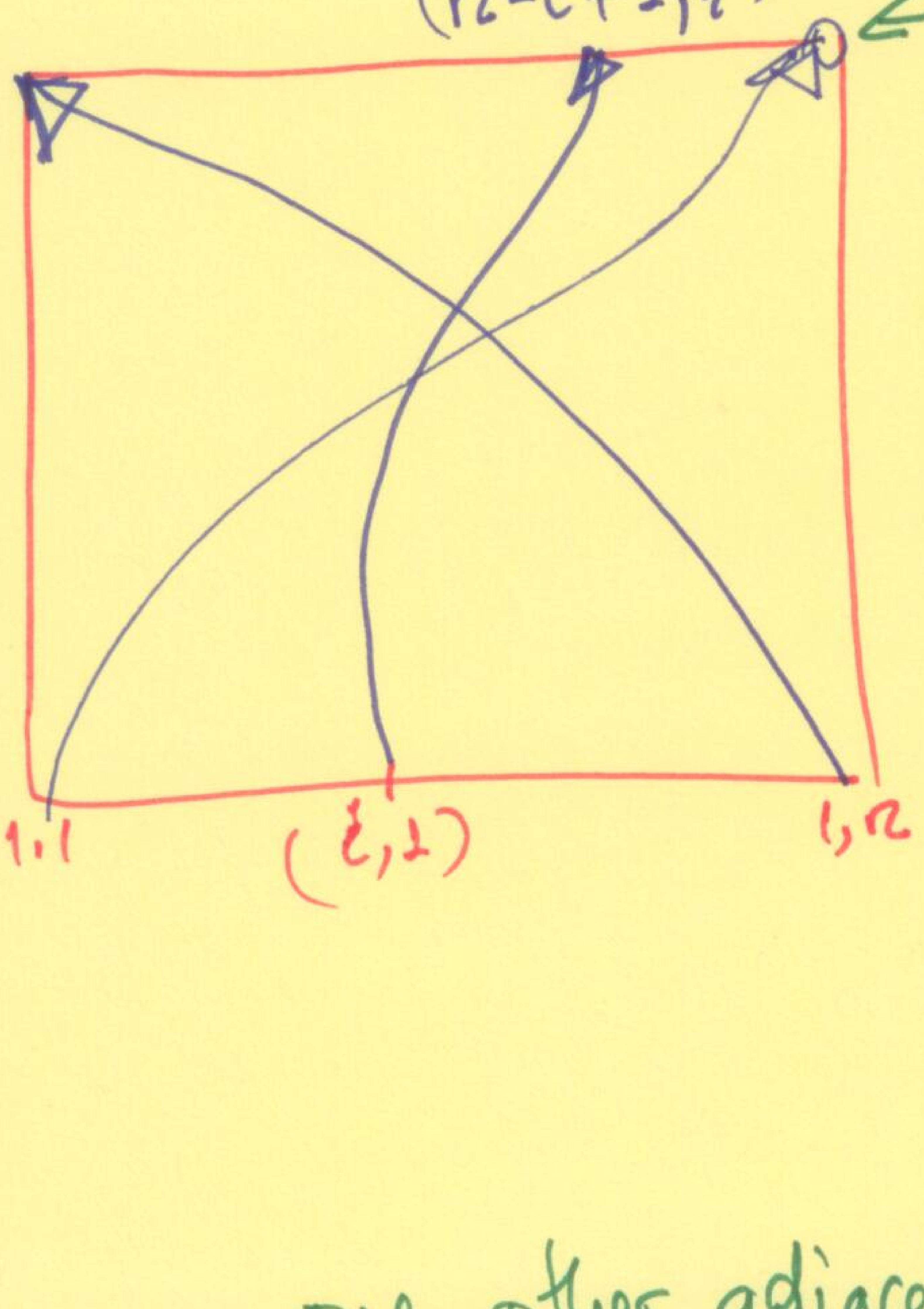


← Goal state

← initial state



$(n-i+1, n)$ ← reverse order!



Actions

At each step, any of the n vehicles can move:

1) one square up
2) one square down
3) one square left
4) one square right
5) <u>stay put</u>. → but if a vehicle stays put, one other adjacent vehicle (but no more than one) can <u>hop</u> over it.

Constraint: Two vehicles cannot ocupy the same square!

a) Calculate the size of the state space as a function of n

There are n vehicles $v_1, \ldots v_n$ in n^2 possible locations.

-If we ignore the fact that only 1 vehicle can be in one position: we shall have $(n^2)^n = n^{2n}$ states

What happens if we impose the constraint?

$v_1 \rightarrow n^2$ states
$v_2 \rightarrow n^2 - 1$ states
$v_3 \rightarrow (n^2 - 2)$ states.
$\vdots$
$v_n \rightarrow (n^2 - n)$ states

N_n states $= n^2(n^2-1)(n^2-2) \ldots (n^2-n)$

b) Calculate the branching factor of the search tree as a function of n.

<u>Answer</u> What actions can the vehicles take?

up

left ← v_1 → right

down

stay put

} 5 actions

$5n$? <u>No</u>

Some assumptions: <u>$n = 2$</u> ⇒ 2 vehicles.

v_1 v_2

5 action 5 actions

$b = 5^2$

$n = 3$ ⇒ 3 vehicles

v_1 v_2 v_3

$b = 53$

⇒ $\boxed{b = 5^n}$

c) Suppose that vehicle v_i is at (x_i, y_i).

Write a non-trivial heuristic h_i for the <u>number of moves</u> it will require to get to its goal location: $(n-i+1, n)$, assuming no other vehicles are on the grid.

<u>Answer</u> $\Rightarrow$ the grid looks a bit like the 8-puzzle with <u>$n=8$</u>.

Let us use a form of Manhattan distance:

$$h(x_i, y_i) = \Big| \underbrace{(n-i+1, \cancel{n})}_{\text{the goal } x} - x_i \Big| + \Big| \underbrace{n}_{\text{the goal } y} - y_i \Big|$$

d) which of the following heuristics are admissible for the problem of moving the vehicles to their destinations? Explain

(i) $\sum_{i=1}^{n} h_i$ remember $h_i = h(x_i, y_i)$

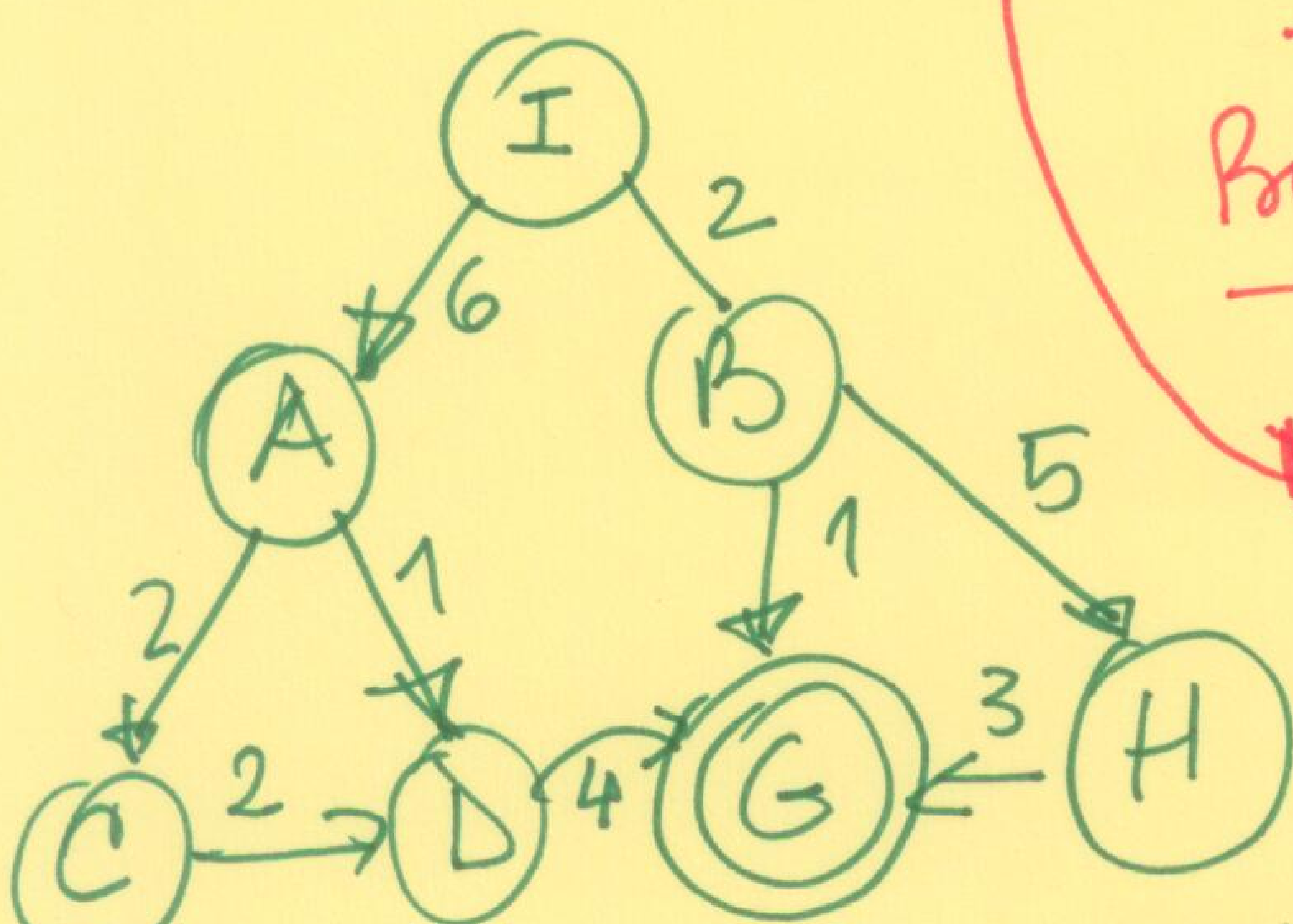
Answer Each h_i assumes only one vehicle v_i. The other vehicles may be more with lower cost! Not admissible.

(ii) $\max(h_1, ---, h_n)$ Answer Not admissible, will overestimate for some vehicles.

(iii) $\min(h_1, h_2, -- h_n)$ Answer YES, it never over-estimates!

let us talk more about admissible heuristics!

We have discussed the consistency of the following example of heuristic on a search graph:



→ found it is not consistent.

But is it admissible?

Node	I	A	B	C	D	G	H
h	10	5	12	8	11	0	2

The definition of an admissible heuristic indicates it should not over-estimate the true-cost

We know that the true-cost of getting to G from G is 0

But what about the other nodes?

We need to know the TRUE-COST of getting to G from the other nodes!

E.g.;	node	I	A	B	C	D	G	H
TC	True Cost	10	6	12	9	11	0	3 → 2
		= 10	→ 5	= 12	→ 8	= 11		

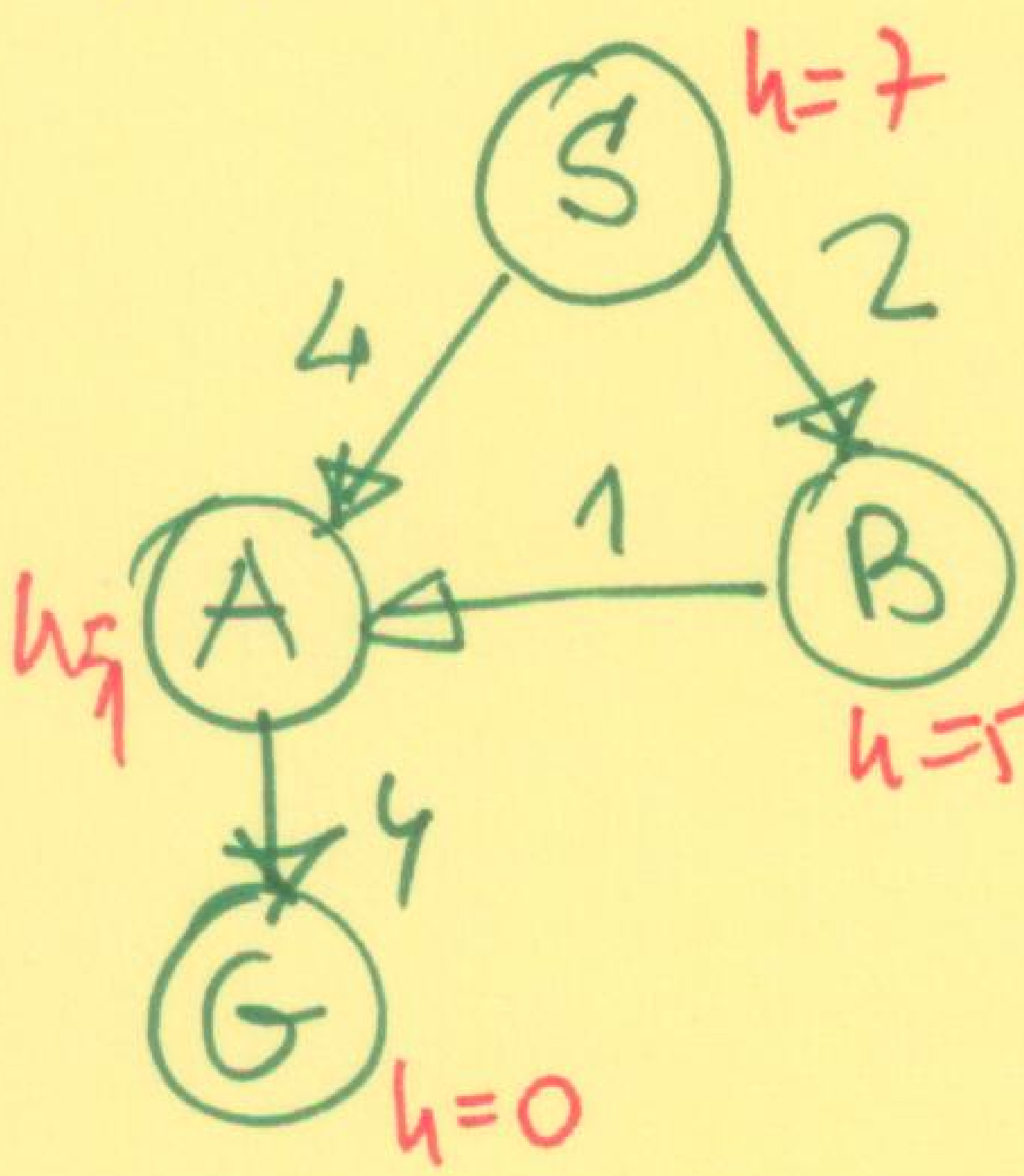
→ It does not overestimate.

What happens when we do not know the True cost value?

$\Rightarrow$ We need to estimate them!

- e.g. SDL (straight line distance)
- e.g. the discussion from Problem 3.27 d)

Problem 3.24 - example.



We saw that this heuristic is not consistent!

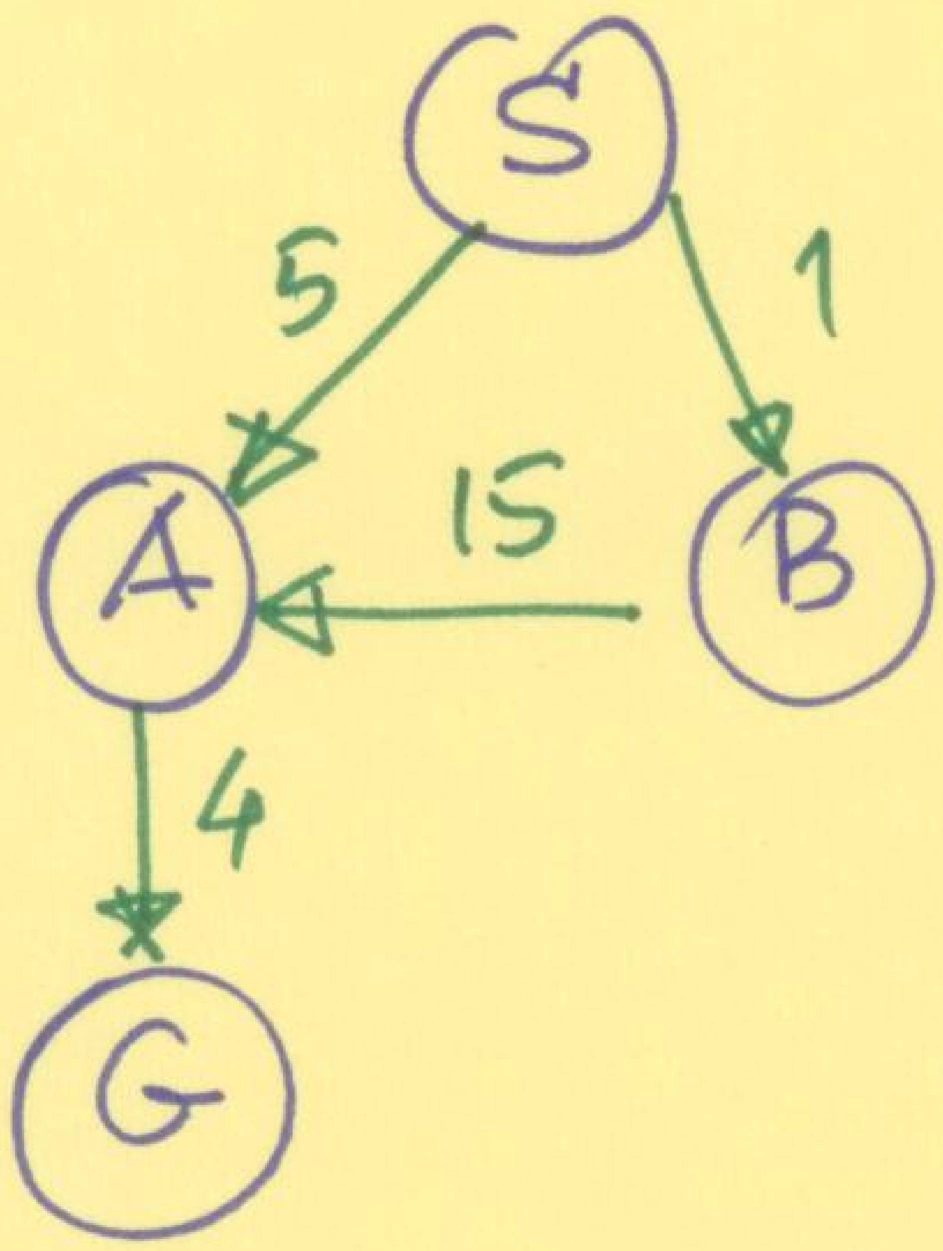
But is is admissible?

$\Rightarrow$ what TRUE-COST function would guarantee it is admissible.

Node	S	A	B	G
h	7	1	5	0
TC	$tc(S) \geq 7$	$tc(A) \geq 1$	$tc(B) \geq 5$	0

You also had a self-test:

- for another Graph that provides a sub-optimal solution:



h	S	A	B	G
node	8	2	10	0
TRUE-cost	?	?	?	?

What would make h admissible?

<u>Self test:</u> design a search graph with nodes S, A, B, C, D, G and generate a heuristic that will not be consistent but admissible

- provide the TRUE-COST function in that case!

When I do not need to check if a heuristic is admissible?

Answer When the heuristic is consistent.

How do I check if the heuristic is consistent?

$\Rightarrow$ At <u>every node</u> $\neq$ Initial node the triangle inequality <u>must</u> hold.

$$h(\underbrace{n}_{\text{parent-node}}) \leq c(n, a, \underbrace{n'}_{\text{child node.}}) + h(n')$$

<u>Self test</u>: Generate a search graph with the following nodes and costs of moving to a node:

node	A	B	C
cost	$C(A,B)=3$ $C(A,C)=5$	$C(B,D)=1$ $C(D,E)=4$	$C(C,B)=2$ $C(C,F)=5$ $C(C,G)=3$

~~Cost~~ (cost)

node	D	E	F
cost	$c(D,F)=3$	$c(E,F)=2$	$c(F,G)=1$

with A the initial node and G the goal.

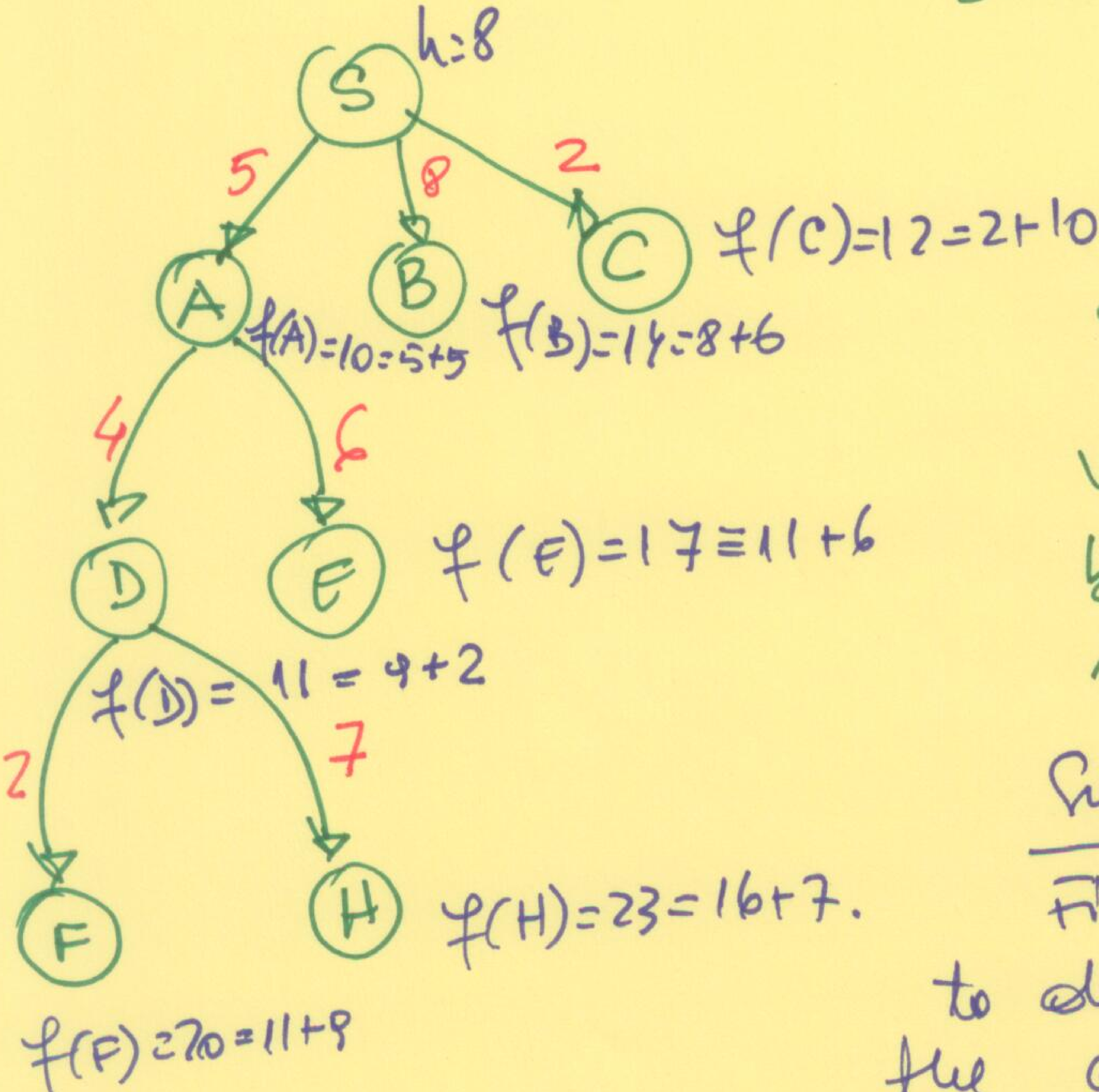
a)* Propose a consistent heuristic

b)* Propose an inconsistent heuristic and provide a TRUE-COST function that would allow it to be admissible.

c)* Propose an inconsistent and inadmissible heuristic for the <u>same</u> TRUE COST function you generated in b)

Self-Test Problem

You use A* on the attached search tree.



Question 1

What node will be expanded next by A*?

Suggested answer

First, you need to discover what is the current node?

Start from the initial node = S

step 1 current node = S
Explored = {S}
Frontier = {A 10, C 12, B 14} → next node = A

step 2 current node = A — current path S → A
Explored = {S, A}
Frontier = {D 11, C 12, B 14, E 17} → next node = D

step 3 current node = D — current path S → A → D
Explored = {S, A, D}
Frontier = {C 12, B 14, E 17, F 20, H 23} → next node = $\boxed{C}$

Question 2

Is the heuristic consistent on the nodes visible?

Suggested Answer: check the triangle inequality at nodes A, B, C, D, E, F, H

→ at node A: $h(S) = 8 < h(A) + c(S,A) = 5+5 = 10$

B : $h(S) = 8 < h(B) + c(S,B) = 8+6 = 14$

C : $h(S) = 8 < h(C) + c(S,C) = 10 + 2 = 12$

D : $h(A) = 5 < h(D) + c(A,D) = 2+4 = 6$

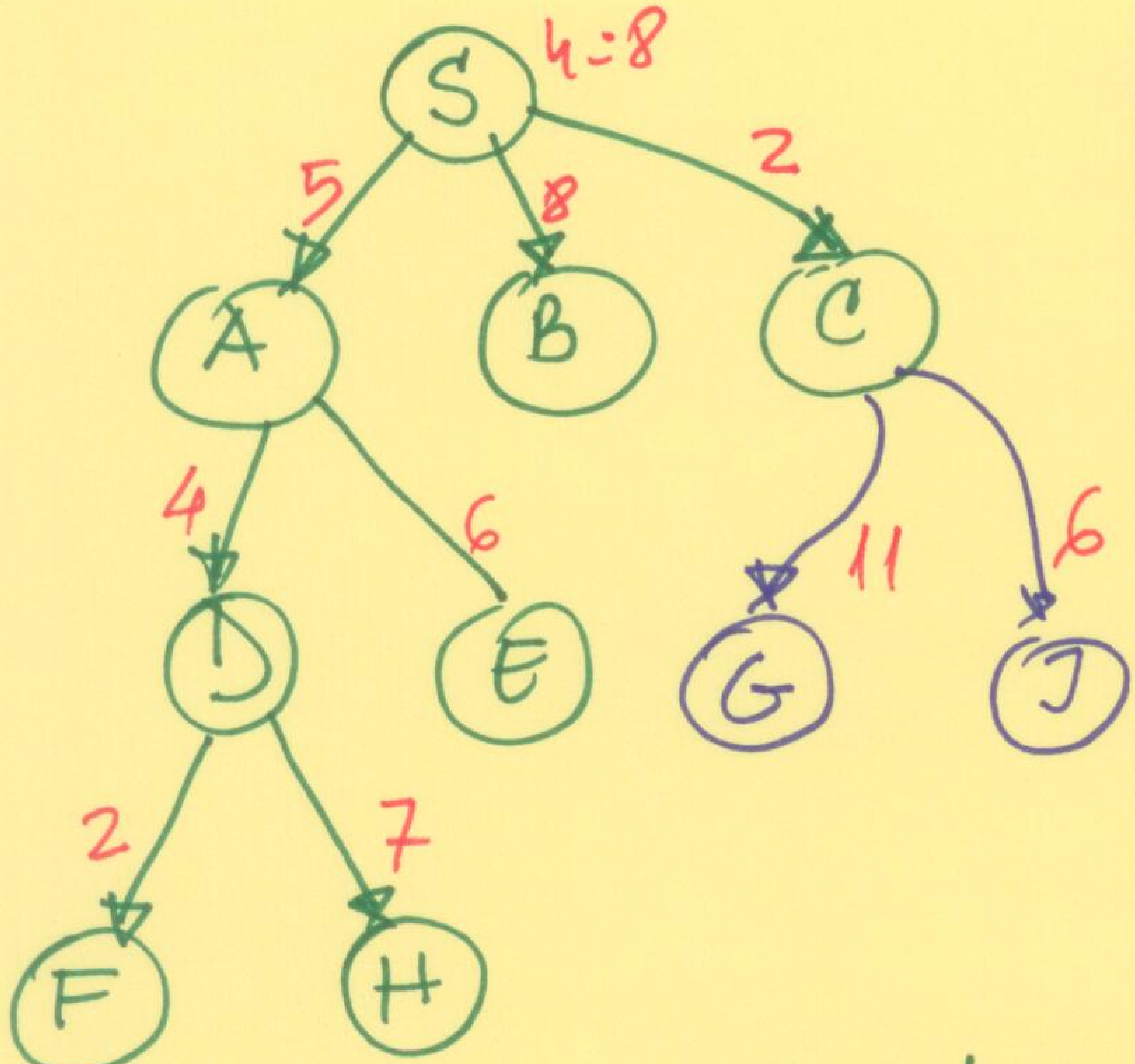
⋮

Question 3

Should I use TREE-SEARCH or GRAPH-SEARCH with A* to find the optimal solution?

Question 4

If after I get in the current node = C the tree search looks like:



$f(A) = 5+5 = 10$

$f(B) = 8+6 = 14$

$f(C) = 2+10 = 12$

$f(D) = \cancel{11+8}\ 9+2 = 11$

$f(E) = 11+6 = 17$

$f(F) = 11+9 = 20$

$f(H) = 16+7 = 23$

$f(G) = 11+0 = 11$

$f(J) = 8+4 = 12$

how do I continue?

Suggested answer

From Question 1, we had before going to C the Frontier = $\{\underset{12}{C}, \underset{14}{B}, \underset{17}{E}, \underset{20}{F}, \underset{23}{H}\}$

Now in Step 4 current node = C, Explored = {S, A, C}

current path S → C

the Frontier = $\{\underset{11}{G}, \underset{12}{J}, \underset{14}{B}, \underset{17}{E}, \underset{20}{F}, \underset{23}{H}\}$

$\Longrightarrow$ next node = G

Step 5 current node = G a goal!

Solution: S → C → G

Solution cost = 2+11 = 13

How do I use A^*?

At each <u>STEP</u>

a) <u>Current Node</u>

b) <u>Current Path</u> (+ cost)

c) Explored list

d) Frontier

1) children Current Node
2) f(each child)
3) Insert them in Frontier

e) Next Node.

If $\boxed{\text{GOAL}}$: PATH (S to GOAL)
+ COST (PATH)

HMW#1 Problem 2