

# Mathematical Definitions and Examples

## Generate Random Position

- **p**: Position vector of a point, where the first element represents the x-coordinate and the second element represents the y-coordinate.
- $x_{\min}, x_{\max}$ : Minimum and maximum limits for the x-coordinate.
- $y_{\min}, y_{\max}$ : Minimum and maximum limits for the y-coordinate.
- $\text{rand}()$ : A function that generates a random number between 0 and 1.

A random position **p** within specified limits  $[x_{\min}, x_{\max}]$  and  $[y_{\min}, y_{\max}]$  is defined as:

$$\mathbf{p} = \begin{bmatrix} x_{\min} + (x_{\max} - x_{\min}) \cdot \text{rand}() \\ y_{\min} + (y_{\max} - y_{\min}) \cdot \text{rand}() \end{bmatrix}$$

Example:

$$\mathbf{p} = \begin{bmatrix} 0 + (100 - 0) \cdot 0.5 \\ 0 + (100 - 0) \cdot 0.7 \end{bmatrix} = \begin{bmatrix} 50 \\ 70 \end{bmatrix}$$

## Position Validity

- **p<sub>new</sub>**: The new position being checked for validity.
- **p**: Existing positions against which the new position is being checked.
- $d_{\min}$ : The minimum required distance between any two positions.

A position **p<sub>new</sub>** is valid if it is at least a minimum distance  $d_{\min}$  away from any existing position **p** in a set of positions:

$$\text{valid}(\mathbf{p}_{\text{new}}) = \begin{cases} 1 & \text{if } \|\mathbf{p}_{\text{new}} - \mathbf{p}\| \geq d_{\min} \text{ for all } \mathbf{p} \\ 0 & \text{otherwise} \end{cases}$$

Example: Given  $\mathbf{p}_{\text{new}} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$ ,  $\mathbf{p} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ , and  $d_{\min} = 10$ :

$$\|\mathbf{p}_{\text{new}} - \mathbf{p}\| = \sqrt{(10 - 5)^2 + (10 - 5)^2} = \sqrt{25 + 25} = \sqrt{50} \approx 7.07$$

Thus, **p<sub>new</sub>** is not valid.

## Move Towards Target

- **p**: Current position of an object.
- **t**: Target position to which the object is moving.
- $s$ : Step size, the maximum distance the object can move in one update.

To move a position **p** towards a target **t** with a step size  $s$ , the updated position is:

$$\mathbf{p}_{\text{updated}} = \mathbf{p} + \frac{s}{\|\mathbf{t} - \mathbf{p}\|} \min(s, \|\mathbf{t} - \mathbf{p}\|)(\mathbf{t} - \mathbf{p})$$

Example:  $\mathbf{p} = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$ ,  $\mathbf{t} = \begin{bmatrix} 25 \\ 25 \end{bmatrix}$ ,  $s = 5$ :

$$\mathbf{p}_{\text{updated}} = \begin{bmatrix} 20 \\ 20 \end{bmatrix} + 5 \cdot \frac{\begin{bmatrix} 25 - 20 \\ 25 - 20 \end{bmatrix}}{\sqrt{(25 - 20)^2 + (25 - 20)^2}} = \begin{bmatrix} 20 \\ 20 \end{bmatrix} + \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 25 \\ 25 \end{bmatrix}$$

## Collision Avoidance

- $\mathbf{p}$ : Position of one object.
- $\mathbf{q}$ : Position of another object which might be too close to the first.
- $d_{\min}$ : Minimum allowable distance between any two objects to avoid collision.

To adjust  $\mathbf{p}$  for collision avoidance when too close to another position  $\mathbf{q}$ :

$$\mathbf{p} = \mathbf{p} - 0.5 \times \frac{d_{\min}}{\|\mathbf{q} - \mathbf{p}\|} (\mathbf{q} - \mathbf{p})$$

$$\mathbf{q} = \mathbf{q} + 0.5 \times \frac{d_{\min}}{\|\mathbf{q} - \mathbf{p}\|} (\mathbf{q} - \mathbf{p})$$

Example:  $\mathbf{p} = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$ ,  $\mathbf{q} = \begin{bmatrix} 21 \\ 21 \end{bmatrix}$ ,  $d_{\min} = 5$ :

$$\mathbf{p} = \begin{bmatrix} 20 \\ 20 \end{bmatrix} - 0.5 \cdot \frac{5}{\sqrt{(21-20)^2 + (21-20)^2}} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 19.5 \\ 19.5 \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} 21 \\ 21 \end{bmatrix} + 0.5 \cdot \frac{5}{\sqrt{(21-20)^2 + (21-20)^2}} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 21.5 \\ 21.5 \end{bmatrix}$$

## Check All Reached Targets

- $\mathbf{p}$ : Current position of a robot or moving object.
- $\mathbf{t}$ : Target position that the robot is supposed to reach.
- $\epsilon$ : A small threshold distance within which the position is considered as having reached the target.

All robots have reached their targets if:

$$\text{reached} = \begin{cases} 1 & \text{if } \|\mathbf{p} - \mathbf{t}\| \leq \epsilon \text{ for all } (\mathbf{p}, \mathbf{t}) \\ 0 & \text{otherwise} \end{cases}$$

Example: If  $\mathbf{p} = \begin{bmatrix} 30 \\ 30 \end{bmatrix}$  and  $\mathbf{t} = \begin{bmatrix} 30.4 \\ 30.4 \end{bmatrix}$ , and  $\epsilon = 0.5$ , then:

$$\|\mathbf{p} - \mathbf{t}\| = \sqrt{(30.4 - 30)^2 + (30.4 - 30)^2} = \sqrt{0.16 + 0.16} \approx 0.4$$

Thus, the target is reached.