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Q.1 Find the value of $T(2)$ for the reccurrence relation $T(n) = 3T(n-1) + 12n$, given that $T(0) = 5$

Solution:-

given

$$T(n) = 3T(n-1) + 12n,$$
$$T(0) = 5$$

Now

$n=1$

$$T(1) = 3T(1-1) + 12(1)$$
$$= 3T(0) + 12$$
$$= 3 \times 5 + 12$$
$$= 15 + 12$$
$$= 27$$

$n=2$

$$T(2) = 3T(2-1) + 12(2)$$
$$= 3T(1) + 24$$
$$= 3 \times 27 + 24$$
$$= 81 + 24$$
$$= 105$$

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Q2 Given the reccurance relation, solve it using substitution method.

a) $T(n) = T(n-1) + c$

solution:- $T(n) = T(n-1) + c$

$T(n-1) = T(n-2) + c$

$\therefore T(n) = T(n-2) + c + c$

$= T(n-2) + 2c$

Now $n = n-2$

$T(n-2) = T(n-3) + c$

$T(n) = T(n-3) + 3c$

$\vdots$

$T(n) = T(n-k) + kc$

We will stop when $k = n$ reaching base case $T(n)$

$\therefore T(n) = T(0) + nc$

$\therefore$ reccurance relation is

$T(n) = T(0) + nc$

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b) $T(n) = 2T(n/2) + n$

Solution:- $T(n) = 2T(n/2) + n$

now,
$n = n/2$

$\therefore T(n/2) = 2T\left(\frac{n/2}{2}\right) + n/2$

$= 2T\left(\frac{n}{4}\right) + n/2$

$\therefore T(n) = 4T\left(\frac{n}{4}\right) + 2n$

now,
$n = \frac{n}{4}$

$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + \frac{n}{4}$

$\therefore T(n) = 8T\left(\frac{n}{8}\right) + 3n$

$\vdots$

$T(n) = 2^K T\left(\frac{2n}{2^K}\right) + Kn$

$T(1) = C$

$\therefore T(n) = 2^{\log_2 n} T(1) + n \log_2 n$

$T(n) = n\,T(1) + n\log_2 n$

$T(n) = cn + n\log_2 n$

$\therefore$ closed form solution is

$$T(n) = n\log_2 n + cn$$

e) $T(n) = 2T(n/2) + c$

Solution:-

$T(n) = 2T(n/2) + c$

now $n = n/2$

$T(n/2) = 2T\left(\frac{n}{4}\right) + c$

$\therefore T(n) = 4T\left(\frac{n}{4}\right) + 2c$

now $n = n/4$

$T\left(n/4\right) = 2T\left(\frac{n}{8}\right) + c$

$\therefore T(n) = 8T\left(\frac{n}{8}\right) + 3c$

general pattern

$\therefore$
$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + (2^k - 1) c$$

$$T(1) = c_0$$

Substituting general pattern

$$T(n) = 2^{\log_2 n} T(1) + \left(2^{\log_2 n} - 1\right) c$$

$$\therefore \quad T(n) = n T(1) + (n-1) c$$

$$T(n) = nc + (n-1) c$$

$$T(n) = 2nc - c$$

$\therefore$ Closed form solution is

$$T(n) = 2nc - c$$

d. $T(n) = T(n/2) + c$

Solution :- $T(n) = T(n/2) + c$

now $n = n/2$

$$T(n/2) = T\left(\frac{n}{4}\right) + c$$

$$\therefore T(n) = T(n/4) + 2c$$

now, $n = n/4$

$$T(n/4) = T\left(\frac{n}{8}\right) + c$$

$$\therefore T(n) = T\left(\frac{n}{8}\right) + 3c$$

$\vdots \quad \vdots$

$$T(n) = T\left(\frac{n}{2^k}\right) + kc$$

$$T(1) = c_0 \quad \ldots \left(\frac{n}{2^k} = 1,\ k = \log_2 n\right)$$

substituting $k = \log_2 n$

$$T(n) = T(1) + (\log_2 n)c$$

Q.3 Given a recurrence relation, solve it using recursive tree approach.

a). $T(n) = 2T(n-1) + 1$

→ Solution:- $T(n) = 2T(n-1) + 1$

now, $n = n-1$

$$T(n-1) = 2T(n-2) + 1$$

$$T(n) = 4T(n-2) + 2 \quad \text{— ①}$$

now, $n = n-2$

$$T(n-2) = 2T(n-3) + 1$$

$$\therefore T(n) = 8T(n-3) + 7 \quad \text{— ②}$$

∴ General pattern → $T(n) = 2^{k} T(n-k) + (2^{k} - 1)$

So, when $n-k = 1$

$$k = n-1 \Rightarrow T(1)$$

$$\therefore T(1) = c$$

$$\therefore T(n) = 2^{n-1} T(1) + (2^{n-1} - 1)$$

$$\therefore T(n) = 2^{n} - 1 + 2^{n-1} - 1$$

$$\boxed{T(n) = O(2^{n})}$$

h) $T(n) = 2T(n/2) + n$

Solution :- $T(n) = 2T(n/2) + n$

now, $n = n/2$

$$\therefore T(n/2) = 2T\left(n/4\right) + n/2$$

$$\therefore T(n) = 4T\left(\frac{n}{4}\right) + 2n$$

now $\therefore n = n/4$

$$\therefore T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + \frac{n}{4}$$

$$\therefore T(n) = 8T\left(\frac{n}{8}\right) + 3n$$

Generalizing

$$\text{At level } k = 2^k \times \frac{n}{2^k} = n$$

$$T(n) = n \times n \times n \times \cdots \times n$$

$$\therefore T(n) = n \log_2 n$$

$$\boxed{\therefore T(n) = O(n \log n)}$$