Sprint Round 12210

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Name ______________________

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1. A box of chocolates has 4 rows containing 6 chocolates each. Anshul takes them all out, eats three of them, and splits the remaining chocolates into 3 equal groups. How many chocolates does each group have?

2. John draws a rectangle with side lengths 7 and 6. What is the numerical difference between the area and perimeter of John's rectangle?

3. Maggie reads two comic books on Monday. Starting on Tuesday, she reads twice as many comic books as she does the previous day. How many comic books does she read from Monday to Friday of the same week?

4. Drake plans on eating $\frac{1}{3}$ of a pizza, but the pizza he ordered is divided into 6 equal slices! How many slices should Drake NOT eat?

5. Elvis rolls a fair six-sided die. What is the probability that the die shows a number that is not prime? Express your answer as a common fraction.

6. A rectangular computer screen has a width of 15 inches and a length of 22 inches. A larger rectangular computer screen has a width of 19 inches and has 90% more area. What is the length of the larger screen in inches?

7. MacKenzie, Jacob, and Amee each have a positive whole number of trading cards. MacKenzie and Jacob together have a total of 17 trading cards. Amee has twice as many trading cards as MacKenzie. Amee and Jacob have a total of 22 trading cards. How many trading cards do all three of them have together?

8. Kaycee is running a lemonade stand, where she sells one lemonade per person. Each lemonade costs \$1 to make, and Kaycee sells lemonade for \$1.20. Kaycee estimates that 500 people will buy lemonade one hot day, but the actual number of people who bought the lemonade was 25% more than Kaycee's estimate. If Kaycee makes exactly as many lemonade as she sells, how much profit, in dollars, does Kaycee make that day?

9. Barbara is doing a 10K race, where she has to run 10 km, and she must finish the race in 1 hour. If Barbara runs the first 8 km at a rate of 5 meters per second, what is the minimum speed, in meters per second, that Barbara needs to run the remainder of the race and finish on time?

10. What is the positive difference between the minimum and maximum area of a rectangle with whole number side lengths and perimeter 44?

11. It is 2:24 AM right now. Isaac's flight arrived yesterday at 11:20 PM. How many minutes have passed since?

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12. Antonio wrote down the number of touchdown passes from the top 10 football players in the area. His results are in the list $[7, 2, 9, 8, 1, 55, 3, 4, 5, 6]$. Antonio realizes that William has the highest number of touchdown passes in the area. The mean of the ten numbers in Antonio's list is a, and the mean of the numbers after removing the number of touchdown passes that William did is b. What is $a - b$?

13. At an intramural competition, Team Power, Team Wisdom, and Team Courage each have 6 people. Brandon is a leader who plans on selecting a number of people at random to do the master trial. If he wants to guarantee that at least one person from each team participates, how many people must Brandon select at random?

14. Cole's dormitory is 6 kilometers from the stadium. Cole takes the bus until he is halfway to the stadium, and for half the remaining distance, Cole walked towards the stadium before taking a water break. After his water break, how far, in *meters*, does Cole need to still walk in order to reach the stadium?

15. One week from Monday to Friday inclusive, Sam does some number of pushups in the martial arts dojo. On Monday and Wednesday, Sam does 8 pushups. On Tuesday, Sam does 12 pushups. On Thursday, Sam does a whole number of pushups, and on Friday, Sam also does a whole number of pushups. After Friday, Sam calculated the median number of pushups he did over the past five days. What is the sum of all possible values of Sam's result?

16. The point $(25, -25)$ is reflected across the line $y = x$ and then rotated 30° counterclockwise about the origin. What is the smallest positive number of degrees counterclockwise does the point then need to be rotated about the origin to end up back where it started?

17. Chris bought 200 candy bars that were \$0.99 each and a car that is worth \$12000. The total cost for the items included the price of the candy bars and car as well as a 7% sales tax. Chris plans on paying with only pennies, which are worth one cent each. How many pennies does Chris need to pay?

18. What is the number of different arrangements of the letters of the word *VIVIDRIA*?

19. The parabola $f(x) = x^2 - x - 6$ intersects the x-axis and y-axis at three different points. What is the area of the triangle formed by these points?

20. An octagon has two opposite vertices colored white and blue, and all other vertices are colored gray. A move consists of swapping the colors of the white vertex and a vertex adjacent to it by an edge. At most how many moves are required in order to swap the positions of the white and blue vertices?

21. When $\frac{2}{63}$ is written in decimal form, what is the 100th digit to the right of the decimal point?

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22. Jayden encounters moving walkways in an airport while trying to find his gate. If he walks on an operating walkway, it only takes him 25 seconds. If he walks along a non-operating walkway, it takes him 100 seconds to get through. How many seconds does it take Jayden to ride down an operating walkway while just standing on it? Round your answer to the nearest whole number.

23. Zach got a new TV where the screen has a diagonal of length 60 inches. The dimensions of the new TV's screen are $\frac{3}{5}$ and $\frac{4}{5}$ of the diagonal length. The area of the new TV's screen is the same as the area of Zach's old TV screen, which is a square. Then the side length of the screen of Zach's old TV has a length of $a\sqrt{b}$ inches, where b is not divisible by the square of a prime. What is $a + b$?

24. One highway has 900 mile markers in a single-file line on the road such that the distance between two consecutive mile markers on the road is 1 mile. Daniel wants to install the lowest number of call boxes such that each mile marker is at most one mile from a call box. What is the lowest number of call boxes that Daniel needs to install?

25. Compute the product of all distinct negative values of x such that $(x^2 + 7x + 10)(x^2 + 2x - 8) + (x^2 + 3x - 10) = 0$.

26. Anthony has a circular piece of paper that is 30 centimeters in diameter. He draws two lines and then cuts the paper using scissors on the lines so that he has four pieces in total. The average perimeter of the pieces is 45 centimeters. To the nearest whole number, what is the total length of the two cuts, in centimeters?

27. The Greek alphabet has 24 uppercase letters. Elise tries to come up with a name for her college club by writing down all sequences of 3 letters that contain exactly two different Greek uppercase letters. How many different sequences did Elise write?

28. Percy is reading a book with 899 pages on Thanksgiving (that is, the page number of the last page is 899). The prologue is 100 pages long, so he starts reading from page 101. Each page of the book has the page number at the bottom of the page. Whenever he gets to a page where the page number is a multiple of 37, he writes down the sum of its digits on a piece of paper. After he finishes reading the book, what is the sum of all the numbers Percy wrote down?

29. Carson is designing a spaceship capsule, which is in the shape of a frustum. He designs the capsule by taking a right circular cone with height 24 and base diameter 20, then making a cut parallel to the base such that he can take out a right circular cone with height 12. The remaining piece is Carson's frustum, and the surface area of the frustum can be written as $k\pi$ for some positive whole number k. What is the value of k?

30. A function f is defined such that $f(x) = \frac{f(x+1)+f(x-1)}{2}$ for all positive whole numbers $x > 1$. If $f(1) = 1$ and $f(3)$ is a positive whole number less than 6, what is the sum of all possible values of $f(15)$?

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Problems 1 & 2

1. What is the sum of all positive even whole numbers that are less than 32?

1.

2. The side lengths of a rectangle are positive whole numbers. The area of the rectangle is numerically equal to the perimeter of a square with area 12.25. What is the sum of all possible values of the perimeter of the rectangle?

2.

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Problems 3 & 4

3. In college, Evan went to bed at exactly 11 : 55 PM and woke up at exactly 8 : 01 AM the next day. The minimum recommended number of hours to sleep for young adults is 7 hours. How many more minutes did Evan sleep compared to the minimum recommended number of sleep hours for young adults?

3.

4. Becca and Madison are playing a game during their lunch break. Becca flips four fair coins and wins if the number of heads is greater than the number of tails. What is the probability that Becca wins the game? Express your answer as a common fraction.

4.

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Problems 5 & 6

5. Nicole forms a four-digit number that uses each of the digits 2, 4, 7, and 8 exactly once. Out of all possible numbers Nicole can form, what fraction of them are multiples of four? Express your answer as a common fraction.

5.

6. Claudia has five flower beds that each have a *distinct* positive whole number of flowers. One flower bed has 19 flowers, and the rest each have at least 29 and at most 36 flowers. Among all five flower beds, the median number of flowers is 30, and the range is 17 flowers. What is the smallest possible average number of flowers among the five flower beds?

6.

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Problems 7 & 8

7. A line that is perpendicular to the line $y = 2x+3$ is drawn. The lines intersect at a point with a y-coordinate of 3. What is the sum of the coordinates of the y-intercept of the line that was drawn?

7.

8. Let x be the value when $\frac{n}{60}$ is rounded to the nearest hundredth. How many positive whole numbers $n \leq 2021$ are there such that $x < \frac{n}{60}$? For example, if $n = 40$, then $\frac{n}{60} = 0.\overline{6}$, so $x = 0.67$, which is greater than $\frac{n}{60}$.

8.

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Team Round
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School or Team

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1. Sienne is planning on sowing a rectangular quilt from patches. All of her patches are squares with the same side length, and she has just enough patches for the quilt to be exactly 10 patches long by 4 patches wide. Of the patches she has, 10 of them are red, and the rest are blue. How many blue patches does Sienne have?

2. Alex plans on working 5 total hours from Monday to Friday for mathleague.org. He worked for 0.25 hours on Monday, 2.65 hours on Tuesday, 0.4 hours on Wednesday, and 0.5 hours on Thursday. How long in hours should Alex work on Friday? Express your answer as a decimal to the nearest tenth.

3. The product of two positive fractions is $\frac{1}{25}$. One of the fractions is 36 times the other fraction. What is the positive difference between the two fractions? Express your answer as a common fraction.

4. The *half-life* of an object is the amount of time it takes for that object's mass to decrease by half its size. Collin finds a rock that has a half-life of 1 year. To the nearest whole number, how many years will it take for the rock to reach 0.8% of its original mass?

5. Suppose x and y are positive whole numbers such that $\frac{x}{5} + \frac{y}{7} = \frac{74}{35}$ and $\frac{5}{x} + \frac{7}{y} = \frac{74}{35}$. What is the value of $x + y$?

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6. A list of four positive whole numbers is created. For each pair of numbers in the list, the larger of the two numbers is written down. If the six numbers that are written down are $1, 2, 2, 5, 5$, and 5, then what is the sum of the four numbers in the original list?

7. A vehicle shipping container is shaped like a rectangular prism with dimensions 2 meters by 2 meters by 10 meters. Phil transports safari pod vehicles in boxes that are cubes with side length 2 meters. However, Phil discovers a shrink function in each safari pod vehicle, and to store a shrunk vehicle, he only needs a box with 12.5% of the dimensions of the original box. How many more shrunk safari pod vehicles can Phil store in the shipping container compared to when they are their original size?

8. Andrea, Belinda, Chelina, Diana, and Evelyn are standing in a line for a group photo. Andrea is not next to Diana and Chelina is not next to Evelyn. How many different ways can the five people be standing in a line?

9. The decibel system is a measure of sound. An $X+10$ decibel sound is 10 times as loud as an X decibel sound. For any positive whole number n, there is a fixed number k such that a $X+n$ decibel sound is always k times as loud as an X decibel sound. Phoenix, a lawyer, usually speaks at a volume of 62 decibels, but in court, he speaks with a volume of 87 decibels. How many times as loud is he in court compared to his normal voice? Round your answer to the nearest integer.

10. Hilda and Marianne are playing a cooperative game of Ones, which is played with colored cards with numbers. In the beginning, a green 5 is in the middle. Hilda has a green 2, a green 8, a yellow 2, a yellow 8, and a blue 6; Marianne has a green 3, a yellow 7, a blue 2, and a blue 8. Starting with Hilda, they take turns placing down cards in the middle. A turn consists of placing a card on top of the card in the middle that has either the same color or the same number; if this isn't possible, the game ends. At the end of the game, both Hilda and Marianne have no cards left. Compute the sum of the numbers on all possible cards that can be the last card placed down.

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Countdown Round 12210

1. In the following geometric sequence, what is the product of a and b? $4, a, b, 36$

2. What is the fifth triangular number?

3. The measures of the angles of a triangle are in the ratio 2 : 3 : 4. What is the degree measure of the smallest angle in the triangle?

4. Simplify: $-1+2-3+4-5+6+\cdots+2012$.

5. An isosceles right triangle has legs of length 5. What would the area of the circle circumscribing this triangle be? Express your answer as a common fraction in terms of π.

6. Melinda places 11 coins into a stack, where each coin is equally likely to face up or down. What is the expected number of pairs of touching faces that show the same side (heads or tails)?

7. A fair 6-sided die is rolled three times. Find the probability that the values of the rolls are strictly increasing. Express your answer as a common fraction.

8. Niah is volunteering at a summer camp that has 40 students. The students can choose to rest or either play dodgeball or knockout (but not both). There are 2 more students who choose dodgeball than knockout. If 16 students choose to play knockout, how many students choose to rest?

9. What is the area of the quadrilateral with vertices at $(3,5)$, $(8,3)$, $(6,-2)$, and $(1,0)$?

10. What is the base-16 number $4A_{16}$ in base 10? (The base 16 digits in order are $0,1,2,3,\ldots,9,A,B,C,D,E,F$.)

11. What is the greatest common divisor of 198 and 165?

12. If x, y, and z are positive integers such that $xyz = 120$, what is the least possible value of $x+y+z$?

13. A train travels 50 miles per hour for the first 40 miles, then slows down to 40 miles per hour for the next 50 miles. What is the average speed of the train in miles per hour? Round your answer to the nearest whole number.

14. In $\triangle ABC$, $AB = 5$, $BC = 6$, and $\measuredangle B = 120^\circ$. Find AC^2.

15. A circle fits inside a 4×6 rectangle. Find the largest possible area of the circle in terms of π.

16. When a whole number is divided by 23, the beginning of the decimal expansion of the result is $0.478\ldots$. What is the value of this whole number?

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17. A fair six-sided die is rolled 6 times. What is the expected value of the sum of the 6 rolls?

18. How many 2-digit integers are relatively prime to 98?

19. Let x be a positive integer. Given that $x = 13y$ and $x - y = 564$, what is x?

20. Evaluate: $3 + 6 + 9 + \cdots + 33$.

21. Euler's theorem states that any convex three-dimensional polyhedron satisfies the equation $V - E + F = 2$, where V, E, and F are the numbers of vertices, edges, and faces of the polyhedron, respectively. An dodecahedron has 20 vertices and 12 faces. How many edges does it have?

22. Find the 11th term in the arithmetic sequence: $9, -3, -15, \ldots$.

23. Two positive integers have a GCD of 2 and LCM of 240. When one number is divided by the other, the result is less than 1. What is the greatest possible value of the result? Express your answer as a common fraction.

24. If a random integer between 1 and 100, inclusive, is chosen, what is the probability that it is composite? Express your answer as a common fraction.

25. For what value of k is there no solution to the equations $4x + 5y = 20$ and $-16x + ky = 60$?

26. Three of the faces of a rectangular prism have areas 20, 24, and 30. What is half of its volume?

27. Three mechanics can fix two cars in five days. Assuming all mechanics work at the same rate, how many cars can five mechanics fix in six days?

28. Find the area of a triangle with side lengths 10, 12, and 14. Express your answer in simplest radical form.

29. Elaine chooses 10 distinct positive integers from $\{1, 2, \ldots, 19\}$. Surprisingly, she does not choose any two numbers that differ by exactly one. How many possible values are there for the sum of the 10 numbers chosen?

30. If $a\#b = a^b - b^a$, what is $5\#7$?

31. How many positive integer factors does $2^3 4^5 6^7$ have?

32. What is the radius of a circle whose area is numerically equivalent to twice its circumference?

33. Michael has a collection of 15 Russian coins, which are either 2 kopeyka coins or 5 kopeyka coins. The coins are worth in total 48 kopeykas. What percent of the coins are 2 kopeyka coins?

34. What is the greatest common divisor of 978 and 1020?

35. Suppose x and y^2 are inversely proportional. If y is multiplied by $\frac{1}{2}$, by what value is x multiplied by?

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36. In Japan, there are coins worth 1 yen, coins worth 5 yen, and coins worth 10 yen. How many different combinations involving any number (or none) of these three coin types are worth 25 yen? Two combinations are considered identical if they consist of the same number of coins for all three coin types.

37. If $a+b=4$ and $a^2+b^2=10$, what is a^4+b^4?

38. If the radius of a sphere is quadrupled, then its surface area is multiplied by what?

39. If $2x^2-25x+9=0$, what is the product of the values for x? Express your answer as a common fraction.

40. For all real x, $f(3x)=x^2-3x+1$. Find $f(12)$.

41. How many 3-digit integers have their tens digit greater than their hundreds digit and their units digit greater than their tens digit?

42. Evaluate: 111111^2.

43. What is the greatest integer less than $\sqrt[3]{-129}$?

44. Simplify: $\frac{286^5}{143^5}$.

45. What is the largest prime divisor of 15015?

46. Lucas is buying boxes of chicken nuggets, where each box either has 5 nuggets or 8 nuggets. What is the maximum number of nuggets that Lucas couldn't get in exact amount from buying some number of boxes that can be either size?

47. What is the positive difference between the sum of the first 40 positive integers and the sum of the first 40 positive odd integers?

48. What is the surface area of a regular octahedron with a side length of 2? Express your answer in simplest radical form.

49. The side length of a regular hexagon is 6. What is the positive difference between the areas of the circumscribed and inscribed circles of the hexagon? Express your answer in terms of π.

50. Riley is picking 3 people from a soccer team of 11 people to have a role of defender. If the order Riley assigns defenders does not matter, how many ways can Riley assign three defenders?

51. Simplify: 72×11.

52. What is the value of $-20+15\cdot 17+5$?

53. What is the sum of all integers that are one more than a one-digit prime?

54. What is the greatest common factor of 125 and 80?

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55. Let x, y, and z satisfy the equation $x^2 + 9y = 9z$. If $x = 6$ and $y = 5$, what is z?

56. Find 6^3.

57. If $\frac{x}{y} = 25$ and $x = 125$, then what is y?

58. The area of a 12-sided regular polygon inscribed in a circle with radius r is given by $A = 3r^2$. If a circle has a circumference of 24π, what is the area of a 12-sided regular polygon inscribed in it?

59. Simplify: $100,000 - 34,933$.

60. How many of the following numbers are rational: 0.6666666, $\sqrt{2/9}$, $\sqrt{-9}$, $\sqrt{7.84}$?

61. The side lengths of a rectangle are both integers. If the perimeter of the rectangle is 18, what is the smallest possible value for the area of the rectangle?

62. If 1% of 10% of a number is 66, then what is half of the number?

63. If Eugene drove at 84 miles per hour for 25 minutes, how many miles did he travel?

64. Evaluate: $13^3 - 8^3$.

65. What is the greatest 3-digit number that is exactly divisible by 15, 25, and 30?

66. The ratio between two consecutive even integers is $\frac{11}{12}$. Find the product of the integers.

67. What is the positive difference between the largest 3-digit number and the smallest 3-digit number?

68. If the lengths of two sides of a right triangle are 8 and 10 units, what is the largest possible length of the third side? Express your answer in simplest radical form.

69. 70% of a number is 490. What is 110% of that number?

70. Express in scientific notation: $\frac{3\times10^5}{8\times10^8}$.

71. What is the least perfect square greater than 4000?

72. Three fair six-sided dice are rolled. What is the probability that the sum of those three numbers is odd? Express your answer as a common fraction.

73. Find the sum of the tens and millions digits of $101 \times 202 \times 303$.

74. $\sqrt{2000}$ lies between two consecutive integers. Find the product of these integers.

75. Find the range of the set $\{99, 33, 8, 78, 13, 38, 28, 48, 9, 30\}$.

76. What is 20% of 60% of 500?

77. Simplify: $41 \cdot 72$.

78. Compute: $\frac{5.3}{1.5} + \frac{3.7}{1.5}$.

79. What is the product of the greatest common divisor and the least common multiple of 16 and 30?

80. How many times does $\frac{1}{49}$ go into $\frac{2}{7}$?

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