Power Question 12300

The following ten questions are worth 10 points each [where a question is broken into multiple parts, the point values for each part are indicated]. To receive full credit the presentation must be legible, orderly, clear, and concise. In general, proofs are not required and will earn no points except on questions that ask for proofs. Derivations are not necessary but may be helpful in assigning partial credit if your final answer is not 100% accurate.

Any question that asks for a function or expression in terms of one or more variables requires a closed-form expression to receive full credit; expressions that involve summation notation or recursion are worth half credit. Throughout the Power Question, the variables n and k are assumed to be integers greater than 1.

The pages submitted for credit should be numbered in consecutive order at the top of each page in what your team considers to be proper sequential order. Please write on one side of the answer papers only.

1. (a) How many total digits are in the base-10 written representation of all integers from 1 to 100, inclusive? Do not include leading zeros. [2 points]

   (b) How many total digits are in the base-10 written representation of all integers from 1 to 1000, inclusive? Do not include leading zeros. [3 points]

   (c) How many total digits are in the base-10 written representation of all integers from 1 to 10000, inclusive? Do not include leading zeros. [5 points]

2. (a) How many total zeros are in the base-10 written representation of all integers from 1 to 100, inclusive? Do not include leading zeros. [2 points]

   (b) How many total zeros are in the base-10 written representation of all integers from 1 to 1000, inclusive? Do not include leading zeros. [3 points]

   (c) How many total zeros are in the base-10 written representation of all integers from 1 to 10000, inclusive? Do not include leading zeros. [5 points]

3. How many total digits are in the base-10 written representation of all integers from 1 to 10^n, inclusive? Do not include leading zeros. Express your answer in terms of n.

4. Prove your answer to the previous question.

5. How many total zeros are in the base-10 written representation of all integers from 1 to 10^n, inclusive? Do not include leading zeros. Express your answer in terms of n.

6. For each of the digits 1 through 9, how many total copies of that digit are in the base-10 written representation of all integers from 1 to 10^n, inclusive? Do not include leading zeros. Express your answer(s) in terms of n.

7. Prove your answer to the previous question.

8. (a) How many total zeros are in the base-2 written representation of all integers from 1 to 2^n, inclusive? Do not include leading zeros. Express your answer in terms of n. [5 points]

   (b) How many total ones are in the base-2 written representation of all integers from 1 to 2^n, inclusive? Do not include leading zeros. Express your answer in terms of n. [5 points]

9. For each of the digits 0 through $k-1$, how many total copies of that digit are in the base-k written representation of all integers from 1 to k^n, inclusive? Do not include leading zeros. Express your answer(s) in terms of n and k.

10. Prove or disprove: The number of zeros used to write all integers in base k from 1 to k^n, inclusive, without leading zeros equals the number of total digits used to write all integers in base k from 1 to k^{n-1}, inclusive, without leading zeros. If it helps, you may use the formulas you found in questions 3 or 6 without additional proof; any other formulas you derived in earlier questions must be proven as part of your answer to this question.

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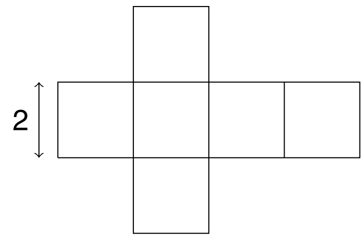
Select E only if you cannot determine a uniquely correct answer between A, B, C, and D.

1. (A) (B) (C) (D) (E)	11. (A) (B) (C) (D) (E)	21. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)	12. (A) (B) (C) (D) (E)	22. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)	13. (A) (B) (C) (D) (E)	23. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)	14. (A) (B) (C) (D) (E)	24. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)	15. (A) (B) (C) (D) (E)	25. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)	16. (A) (B) (C) (D) (E)	26. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)	17. (A) (B) (C) (D) (E)	27. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)	18. (A) (B) (C) (D) (E)	28. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)	19. (A) (B) (C) (D) (E)	29. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)	20. (A) (B) (C) (D) (E)	30. (A) (B) (C) (D) (E)

1. The net shown below is of a 3D figure where all the faces are squares. The side lengths are marked as shown. What is the volume of the 3D figure formed by the net?



(A) 12 (B) 8 (C) 2 (D) 4 (E) Other

2. Kyle's teacher assigns him to do all odd-numbered problems from Problem 1 to Problem 19 inclusive. How many problems must Kyle do for homework?

(A) 10 (B) 19 (C) 20 (D) 8 (E) Other

3. Andrew wants to sign up for a special cup where his monsters can fight other particpants' monsters. The only monsters that can participate in the cup are monsters whose ID number is at most 151 and whose power level is at most 1500. The table below lists the ID number and power level of his six favorite monsters. How many of these monsters can participate in the cup?

Name	**ID Number**	**Power Level**
Blasty	9	1472
Big Ounce	6	1792
Ferno	391	1373
Holmes	25	969
Pirouette	648	1675
Sarpal	131	1550

(A) 4 (B) 5 (C) 3 (D) 1 (E) Other

4. Of the five descriptions below, which describe quadrilaterals that CANNOT exist?
   I. A parallelogram with exactly 3 obtuse angles
   II. A rhombus with exactly 4 right angles
   III. A trapezoid with at least 2 angles with equal measure
   IV. A rectangle with no right angles
   V. A quadrilateral with all angles of different measure

(A) I, IV (B) V (C) I, V (D) IV, V (E) Other

5. A cluster from a specific protein consists of a manganese ion (which has charge $+2$) bound to three neutral (meaning they have have charge 0) histidine residues, a neutral water molecule, and an aspartate residue. Given that the aspartate residue has charge -1, what is the total charge of all the components of the cluster?

(A) -1 (B) 1 (C) -2 (D) 0 (E) Other

6. Chelina is currently at Page 13 of a humanities text but hasn't read the page yet. She starts reading on Monday but has to finish Page 100 by Friday of the same week. She plans to read the same number of pages every day until the day before the due date, but because the discussion is on Friday, she does not plan on reading any pages on Friday. How many pages does Chelina need to read every day in order to reach the goal?

(A) 21 (B) 25 (C) 20 (D) 22 (E) Other

7. Each of the side lengths of a certain quadrilateral is either 5 or 22, and its perimeter is an odd number. What is the perimeter of the quadrilateral?

(A) 39 (B) 37 (C) 69 (D) 45 (E) Other

8. Consider the equation $0.11x + 0.13 = 2$. What is the least positive integer that can be added to the right hand side of the equation so that the solution for x also increases by an integer amount?

(A) 7 (B) 11 (C) 100 (D) 1 (E) Other

9. The side lengths of a rectangle with area 25 are whole numbers, and the diagonals of this rectangle are not perpendicular. What is the perimeter of this rectangle?

(A) 20 (B) 100 (C) 25 (D) 52 (E) Other

10. Given that $f(x) = ax^2 + bx$, $f(1) = 3$, and $f(2) = 4$, find the value of b.

(A) 3 (B) -1 (C) 4 (D) 5 (E) Other

11. Freya is flying 2700 miles from San Francisco (with time zone GMT minus 8 hours) to Boston (time zone GMT minus 5 hours). Her flight takes off from San Francisco at 10:05 PM on Tuesday. If she arrives in Boston after 7:05 AM (Boston time) on Wednesday, she will be jet-lagged. Compute the minimum speed, in miles per hour, that the plane must travel for Freya to avoid being jet-lagged.

(A) 300 (B) 600 (C) 450 (D) 900 (E) Other

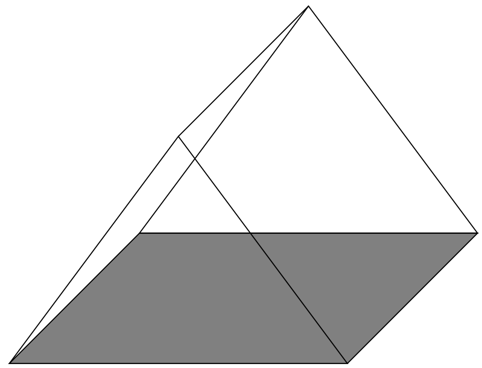
12. An infused drink is made by putting some combination of sliced fruits in a dispenser filled with water. If the choices of fruits to include are peaches, pomelos, and persimmons, how many different kinds of infused drinks can be made? (At least one type of fruit must be used.)

(A) 15 (B) 8 (C) 4 (D) 3 (E) Other

13. For some integers a and b, the polynomial $\left(4x^{13} - 6x^{45}\right)\left(7x^{36} + 10x^{23}\right) + ax^b$ has degree less than 80. Compute $100a + b$.

(A) 3494 (B) 3681 (C) 4281 (D) 1683 (E) Other

14. A camping tent is in the shape of a triangular prism, as shown below. The shaded face is a square with area 28 square meters, and the height of the prism (measured above the square base) is 3 meters. What is the surface area of the entire camping tent (all four "walls"" and the floor), in square meters?



(A) $28+22\sqrt{7}$ (B) $20+16\sqrt{7}$ (C) $28+24\sqrt{7}$ (D) $24+20\sqrt{7}$ (E) Other

15. If $\left\lfloor \frac{500+\frac{1}{n}}{0.05} \right\rfloor - \left\lfloor \frac{500}{0.05} \right\rfloor = 1$, then n must lie in the interval $(a, b]$, where a and b are real numbers. Compute $a+b$.

(A) 23 (B) 30 (C) 15 (D) 25 (E) Other

16. A capsule machine has 9 toys, where six are standard toys while three are rare toys. Ellie picks 5 toys at random from the capsule machine. What is the probability that she gets all the rare toys?

(A) $\frac{5}{9}$ (B) $\frac{1}{3}$ (C) $\frac{5}{42}$ (D) $\frac{1}{126}$ (E) Other

17. Given that $\cos x + \frac{1}{\cos x} = -2$, what is the value of $(\sin x)^{12} + (\cos x)^{18} + (\sin x \cdot \cos x)^{15}$? (It may help to note that $\sin^2 x + \cos^2 x = 1$ for all x.)

(A) 0 (B) -1 (C) -33344 (D) 33344 (E) Other

18. Sally enters two odd numbers into a calculator and computes their sum. She reveals that one of the odd numbers is divisible by 45 and the other number is divisible by 9. What is the largest number that must evenly divide the two odd numbers' sum?

(A) 45 (B) 6 (C) 27 (D) 18 (E) Other

19. Given that $x + \frac{1}{x} = -\frac{5}{2}$, compute the greatest integer less than or equal to the least possible value of $x^7 + \frac{1}{x^9}$.

(A) -511 (B) -513 (C) -128 (D) -129 (E) Other

20. Triangle ABC has $AB = 6$, $BC = 7$, $CA = 8$. Transversal line $\overline{DE}$ is parallel to $\overline{AB}$, with D and E lying on $\overline{BC}$ and $\overline{CA}$ respectively. In addition, $DE = CD + CE - 3$. Compute the difference in perimeters between quadrilateral $ABDE$ and triangle DCE.

(A) 14 (B) 12 (C) 11 (D) 13 (E) Other

21. The quantity $\sqrt{48{,}400{,}000}$ can be written in the form $a\sqrt{b}$, where a and b are positive integers; this form is not necessarily the *simplest* radical form (in that b may contain the square of a prime number in its prime factorization). Compute the number of possibilities for the ordered pair (a, b).

(A) 40 (B) 28 (C) 24 (D) 32 (E) Other

22. Kana's class has some number of students, and she considers picking 0 or 1 student. The number of ways she can do so is less than 2% of the number of ways to pick *any* number of students (without respect to order). What is the smallest possible number of students in Kana's class?

(A) 8 (B) 10 (C) 11 (D) 12 (E) Other

23. Compute the value of $\log_{(9-4\sqrt{5})}(38 + 17\sqrt{5})$.

(A) $-\frac{3}{2}$ (B) $\frac{5}{3}$ (C) $\frac{4}{3}$ (D) $-\frac{5}{2}$ (E) Other

24. In the wave world, there are 7 countries connected by a number of wave roads. Each wave road links two countries, and the electric country is the country with 4 wave roads leading out, the most of all the countries. Georgia found that she can find a route only on wave roads such that she can start at one country, visit all other countries exactly once, and returning to the starting country, while only being at the starting country at the beginning and at the end. What is the least possible number of wave roads in this wave world?

(A) 8 (B) 7 (C) 9 (D) 11 (E) Other

25. In triangle $\triangle XYZ$, all three side lengths are distinct even integers. If the internal bisector of $\angle X$ intersects $\overline{YZ}$ at M and $\frac{YM}{MZ} = \frac{1}{6}$, compute the smallest possible perimeter of $\triangle XYZ$.

(A) 50 (B) 42 (C) 40 (D) 48 (E) Other

26. For how many positive integers $p \leq 1000$ is $p^{600} - 1$ a multiple of 1001?

(A) 720 (B) 600 (C) 360 (D) 840 (E) Other

27. Let x and y be positive real numbers such that $x + xy = 20y - y^2$ and $y + xy = 22x - x^2$. Compute $x + y$.

(A) $21 + \sqrt{2}$ (B) $21 - \sqrt{2}$ (C) $19 - \sqrt{2}$ (D) $20 - \sqrt{2}$ (E) Other

28. Parallelogram $ABCD$ has side lengths $AB = 22$ and $BC = 24$, and diagonal $AC = 28$. Point E is on $\overline{AB}$, and F is the intersection of $\overline{CE}$ and $\overline{BD}$. If $DF = 4BF$, what is the positive difference between the perimeters of $\triangle ACE$ and $\triangle BCE$?

(A) 14 (B) $\frac{25}{2}$ (C) 15 (D) $\frac{29}{2}$ (E) Other

29. If $\log_{10}(x) - 2023 = 0$, and $\log_{10}(x^x) - 2023 = k$, what is the sum of the digits of k?

(A) 18196 (B) 18207 (C) 18218 (D) 18217 (E) Other

30. Captain Joe's space cruiser is confined to the coordinate plane with integer x- and y-coordinates between 0 and 6, inclusive. He may only move between points using a revolutionary technique called *wormholing*, which takes him from one point to another such that his y-coordinate increases. Let N be the number of distinct ways Captain Joe can travel from $(0, 0)$ to $(6, 6)$ using only a series of wormholes. Find the remainder when N is divided by 1000.

(A) 776 (B) 768 (C) 401 (D) 807 (E) Other

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Problems 1 & 2

1. Steve brought 100 students to Kay's Cafe to get hamburgers for lunch on a field trip, which can either be gluten free or not gluten free. The regular hamburgers cost \$5.50, while the gluten free hamburgers cost \$7.50. However, one-fourth of the students have a gluten allergy, so they must get gluten free hamburgers. How many dollars is the positive difference between the minimum and maximum cost of giving lunch to all the participants?

1.

2. An integer m is chosen at random between -3 and 3, inclusive, and an integer b is chosen at random between -2 and 2, inclusive. What is the probability that the x-intercept of the line $y = mx + b$ exists and its x-coordinate is positive? Express your answer as a common fraction.

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Problems 3 & 4

3. A cylinder has radius 13 and height 37. A plane perpendicular to the circular faces of the cylinder slices through the cylinder while passing through the centers of the circular faces. Compute the perimeter of the resulting cross section.

3.

4. Rebecca is doing a math problem where her answer is of the form $\frac{p}{50}$, where $\gcd(p, 50) = 1$ and p is a positive integer less than or equal to 50. She then needs to turn in a "converted" answer of $p + 50$. Compute the sum of all possible final "converted" answers.

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Problems 5 & 6

5. Let m and c be integers chosen at random from the intervals $0 \le m \le 9$ and $0 \le c \le 399$. Compute the probability that a height of m meters and c centimeters is equivalent to a height of at least 8 meters. Express your answer as a common fraction.

5.

6. A plane figure composed of four equilateral triangles joined along 3 of their edges can be folded up along those edges into a regular tetrahedron. Given that the outer perimeter of this figure is 18, what is the square of the volume of the tetrahedron? Express your answer as a common fraction.

6.

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Problems 7 & 8

7. Let a and b be rational numbers such that $4^a 9^b = 648$ and $(6^a)^b = 216$. Compute $9^a 4^b$.

7.

8. A spider rests at $(0,0)$ on the two-dimensional Cartesian plane. Each second, it moves either 1 unit in the positive x-direction or 1 unit in the positive y-direction. There are two barriers of infinite length along the lines with equations $y = x - 2.9$ and $y = x + 2.9$, which the spider cannot cross. The spider moves until it lands on the point $(45, 45)$, after which it stops. Let A be the number of unique paths the spider could take. Compute the remainder when A is divided by 100.

8.

Team Round
12300

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1.	2.	3.	4.	5.
6.	7.	8.	9.	10.

1. Abigail draws line segment $\overline{AB}$. She then draws a circle with radius AB and center A and a circle with radius AB and center B, and then she marks the two intersections of the circles as points C and D. She then draws $\overline{CD}$ and marks the intersection of $\overline{AB}$ and $\overline{CD}$ as point X. Compute the degree measure of $\angle AXC$.

2. Cindy thinks of a number. She writes down that number, the result when the number is rounded up to the nearest ten, and the result when the number is rounded up to the nearest hundred. Finally, she adds up the three written numbers and gets a result of 1737. What number was Cindy thinking of?

3. Let $f(x) = ax^2 + bx$ for some positive integer constants a and b. If $f(20) - f(23) = 20a - 23b$, compute $\frac{a}{b}$. Express your answer as a common fraction.

4. At a party with 20 people, 5 of them are introverted and the other 15 of them are extroverted. Each of the 15 extroverted people shakes hands exactly once with every other person. However, the introverted people do not take part in any additional handshakes. In total, how many handshakes take place?

5. Let n be a positive integer such that the sum of the digits of $1 + 2 + 3 + \cdots + n$ is a multiple of 9. Compute the sum of the nine smallest values of n.

6. Points X, Y, and Z lie in the coordinate plane with y-coordinates of 20, 19, and 10, respectively. Given that $\overline{XY} \perp \overline{YZ}$ and the difference between the x-coordinates of X is Y is 2, compute the area of $\triangle XYZ$. Express your answer as a common fraction.

7. In a soccer game, the teams do an unconventional penalty shoot-out, where each team has one of their players make 5 attempts to score goals. So far, Riley has scored one goal out of two attempts, while his opponent's team has scored 3 goals out of 5 attempts. In each of the remaining attempts, the probability that Riley scores a goal is $\frac{2}{3}$, and these attempts are independent from each other. Compute the probability that Riley scores at least as many goals as the opposing team over 5 attempts. Express your answer as a common fraction.

8. The polynomial $(x^2+2x+2)(x^2-2x+2)(x^2+k)$ has 6 complex roots. When these roots are plotted in the complex plane, the points create a convex hexagon with area 14. Compute the sum of all possible real values of k.

9. Let x be a real number such that $\left(\cot\left(\frac{\pi}{2} - x\right)\right)^2 = 1$. Compute the number of possible values of x with an absolute value less than 100.

10. Define $d(n)$ to be equal to the number of positive divisors of an integer n. Let S be the set of all positive integers n such that the product of $d(n-2)$, $d(n)$, and $d(n+3)$ is equal to 36. Compute the sum of the six smallest elements of S.

Relay Round 12300

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Relay 1

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Answer 1-1		Answer 1-2		Answer 1-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

Relay Round 12300

(1-1) Let $T = 10$. Let K be the number of prime numbers p such that p is a factor of $(T+1)!$, but p^2 is not. Find the remainder when $K+11$ is divided by 100.

(1-2) Let $T = TNYWR$. Let x be the positive solution to the equation $x = \sqrt{(T+2)\sqrt{(T+3)\sqrt{(T+2)\sqrt{(T+3)x}}}}$. Let $K = x^3$. Find the remainder when $K+14$ is divided by 100.

(1-3) Let $T = TNYWR$. Let K be the number of integers in the domain of the function $f(x) = \sqrt{-2x} + \frac{1}{\sqrt{T-x^2}}$. Find the remainder when $K+13$ is divided by 100.

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Relay 2

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Answer 2-1		Answer 2-2		Answer 2-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

Relay Round
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(2-1) Let $T = 34$. Let $f(x)$ be a polynomial of the smallest possible positive degree with integer coefficients and a leading coefficient of 1 such that one of its roots is $x = (T+1) + \sqrt{T+2}$. Let K be the absolute value of the sum of the coefficients of this polynomial. Find the remainder when $K + 36$ is divided by 100.

(2-2) Let $T = TNYWR$. Let N be the number of positive factors of $T + 1$. Let K be the area of a rectangle with sides of length $T + 1$ and N. Find the remainder when $K + 70$ is divided by 100.

(2-3) Let $T = TNYWR$. A $T \times 2T$ rectangle is inscribed in a semicircle with one side on the diameter of the semicircle. The smallest possible area of the semicircle is $K\pi$. Find the remainder when $K + 19$ is divided by 100.

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Relay Round 12300

Relay 3

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Answer 3-1		Answer 3-2		Answer 3-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

Relay Round 12300

(3-1) Let $T = 64$. Let K be the $(T+1)$st digit after the decimal point in the decimal expansion of $\frac{1}{17}$. Find the remainder when $K + 92$ is divided by 100.

(3-2) Let $T = TNYWR$. Then $\sin(30T^\circ) + \cos(30T^\circ) = \frac{a+\sqrt{b}}{c}$, where a, b, and c are integers (with b nonnegative), and $|c|$ is as small as possible. Let $K = a + b + c$. Find the remainder when $K + 9$ is divided by 100.

(3-3) Let $T = TNYWR$. Let $V = 2T + 1$. The integers $\{1, 2, \ldots, V^2\}$ are placed in the cells of a $V \times V$ checkerboard, one integer per cell. Each row, column, and diagonal is then added up, and the sums are themselves added up. Let M be the greatest possible value of the result. Let K be the remainder when M is divided by V. Find the remainder when $K + 23$ is divided by 100.

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Relay Round 12300

Relay 4

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Answer 4-1		Answer 4-2		Answer 4-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

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(4-1) Let $T = 20$. Let K be the greatest composite number less than $4(T+2)^2$. Find the remainder when $K+69$ is divided by 100.

(4-2) Let $T = TNYWR$. The graph of the equation $x^2 - 4T^2 = 2(x+2T)(y-5T)$ consists of two lines in the coordinate plane. Let K be the y-coordinate of the point where these two lines intersect. Find the remainder when $K+31$ is divided by 100.

(4-3) Let $T = TNYWR$. Let U be the sum of all $T+1$-digit palindromes, and let K be the remainder when U is divided by 100. Find the remainder when $K+58$ is divided by 100.

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Relay 5

Name ______________________

Name ______________________

Name ______________________

School ______________________

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Answer 5-1		Answer 5-2		Answer 5-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

Relay Round
12300

(5-1) Let $T = 38$. Let K be the coefficient of the x^2 term in the expansion of $(1 + 0x)(1 + x)(1 + 2x)(1 + 3x) \cdots (1 + Tx)$. Find the remainder when $K + 65$ is divided by 100.

(5-2) Let $T = TNYWR$. Let $f(x) = |200 - 2x|$, and let $K = f^{T+1}(T)$ (in other words, start with T and apply the function $T + 1$ times). Find the remainder when $K + 23$ is divided by 100.

(5-3) Let $T = TNYWR$. A car averages 60 miles per hour on a trip from Kansas City to Omaha and N miles per hour on the return trip, with an average speed for the entire round trip of $T + 1$ miles per hour. Let $K = a + b$, where a and b are relatively prime positive integers such that $N = \frac{a}{b}$. Find the remainder when $K + 63$ is divided by 100.

