Contest Problem Set 12320

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Please be sure to let us know if you find typographical or mathematical errors.

Number Sense 12320

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Name ______________________

Grade ______________________

School ______________________

1. $23 + 32 =$ ____________.
2. The tens digit of 1232 is ____________.
3. $218 + 286 =$ ____________.
4. $543 - 397 =$ ____________.
5. The remainder of $142 \div 5$ is ____________.
6. $11 + 29 + 42 + 18 =$ ____________.
7. $4 + 4 \times 9 =$ ____________.
8. The product of 6 and 13 is ____________.
9. $34 \times 20 =$ ____________.
10. (estimate) $111 + 333 + 777 =$ ____________.
11. $51 \times 11 =$ ____________.
12. $242 - 98 - 34 =$ ____________.
13. $13^2 =$ ____________.
14. $15 \times 24 =$ ____________.
15. The remainder of $181 \div 9$ is ____________.
16. $8 \times 9 \times 5 =$ ____________.
17. *LXVII* in Arabic numerals is ____________.
18. $999 + 1999 =$ ____________.
19. $21 \times 19 =$ ____________.
20. (estimate) $199 \times 202 =$ ____________.
21. $36 \times 25 =$ ____________.
22. If 12 inches is equal to 1 foot, then 48 inches is equal to ____________ feet.
23. $24^2 =$ ____________.
24. The GCD of 16 and 20 is ____________.
25. $13 + 16 + 19 + 22 + 25 =$ ____________.
26. The greater of $\frac{2}{3}$ and $\frac{3}{4}$ is ____________ (fraction).
27. $403 \div 13 =$ ____________.
28. $44 + 132 + 264 =$ ____________.
29. $7 \times 15 \div 35 =$ ____________.
30. (estimate) $29 \times 30 \times 31 =$ ____________.
31. The perimeter of a square with area 9 is ____________.
32. $99 \times 27 =$ ____________.
33. 10% of 150 is ____________.
34. 180 minutes is ____________ hours.
35. $35^2 =$ ____________.
36. The number of multiples of 5 between 21 and 51 is ____________.
37. $12 \times 6 \times 15 =$ ____________.
38. The LCM of 16 and 20 is ____________.
39. $27 \times 23 =$ ____________.
40. (estimate) $189591 \div 94 =$ ____________.

41. $31^2 - 21^2 =$ ________.

42. The greatest prime divisor of 42 is ________.

43. $4 \times 11 + 22 \times 8 =$ ________.

44. The remainder of $982 \div 11$ is ________.

45. $2^8 =$ ________.

46. The tenth term of the arithmetic sequence $7, 13, 19, \ldots$ is ________.

47. $101 \times 23 =$ ________.

48. The area of a right triangle with legs of length 35 and 12 is ________.

49. $23 \times 83 =$ ________.

50. (estimate) The area of a circle with radius 10 is ________.

51. $104 \times 103 =$ ________.

52. The measure of the third exterior angle of a triangle with two exterior angles measuring 100° and 140° is ________ $^\circ$.

53. $8\frac{1}{3}\%$ of 72 is ________.

54. 32_4 in base 2 is ________ $_2$.

55. If $x + 2 = 9$, then $3x - 1 =$ ________.

56. $98 \times 94 =$ ________.

57. The sum of the terms of the arithmetic sequence $1, 2, 3, \ldots, 19$ is ________.

58. $88 \times 125 =$ ________.

59. The length of the hypotenuse of a right triangle with legs of length 9 and 12 is ________.

60. (estimate) $142857 \times 14 =$ ________.

61. $41^2 =$ ________.

62. Two standard dice are rolled. The probability the sum of the numbers shown is 11 is ________ (fraction).

63. $4\frac{1}{3} \times 4\frac{2}{3} =$ ________ (mixed number).

64. If $y = 8$, then $y^2 + 6y + 9 =$ ________.

65. $0.\overline{18} =$ ________ (fraction).

66. The area of a rhombus with diagonals of length 24 and 18 is ________.

67. $11 \times 9 \times 7 \times 14 =$ ________.

68. The number 56 in base 6 is ________ $_6$.

69. $314 \times 111 =$ ________.

70. (estimate) The length of the diagonal of a square with side length 50 is ________.

71. $512 \times 999 =$ ________.

72. The surface area of a cube with volume 729 is ________.

73. $16^{1.75} =$ ________.

74. The number of positive whole number divisors of 30 is ________.

75. $\sqrt{17689} =$ ________.

76. The arithmetic mean of four 16's and three 9's is ________.

77. $\sqrt{18} \times \sqrt{8} =$ ________.

78. The sum of the terms of the infinite geometric sequence $2, 1, \frac{1}{2}, \ldots$ is ________.

79. $60 \times 75 \times 45 =$ ________.

80. (estimate) $2.3^6 =$ ________.

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1.	A	B	C	D	E	11.	A	B	C	D	E	21.	A	B	C	D	E
2.	A	B	C	D	E	12.	A	B	C	D	E	22.	A	B	C	D	E
3.	A	B	C	D	E	13.	A	B	C	D	E	23.	A	B	C	D	E
4.	A	B	C	D	E	14.	A	B	C	D	E	24.	A	B	C	D	E
5.	A	B	C	D	E	15.	A	B	C	D	E	25.	A	B	C	D	E
6.	A	B	C	D	E	16.	A	B	C	D	E	26.	A	B	C	D	E
7.	A	B	C	D	E	17.	A	B	C	D	E	27.	A	B	C	D	E
8.	A	B	C	D	E	18.	A	B	C	D	E	28.	A	B	C	D	E
9.	A	B	C	D	E	19.	A	B	C	D	E	29.	A	B	C	D	E
10.	A	B	C	D	E	20.	A	B	C	D	E	30.	A	B	C	D	E

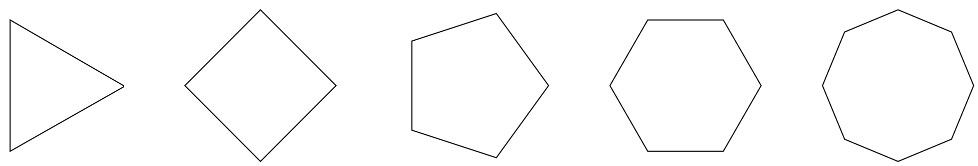
1. Which of the following sums is even?

(A) $5+8$ (B) $6+7$ (C) $2+3$ (D) $9+9$ (E) $1+4$

2. What is the value of $1+4+9+16+25+36+49$?

(A) 110 (B) 140 (C) 150 (D) 130 (E) 120

3. A triangle, a square, a pentagon, a hexagon, and an octagon are shown below. How many sides do the shapes have altogether?



(A) 29 (B) 26 (C) 27 (D) 30 (E) 33

4. A tower-stacking competition is 25 minutes long and began at 2:55 PM. At what time did the competition end?

(A) 3:30 AM (B) 2:30 PM (C) 3:20 AM (D) 3:30 PM (E) 3:20 PM

5. At a junk shop, each energy canteen costs 700 yen, and each hyper cartridge costs 3000 yen. Barbara plans to buy two energy canteens and five hyper cartridges. How many yen would Barbara need to pay?

(A) 15140 (B) 2900 (C) 14150 (D) 16400 (E) 15500

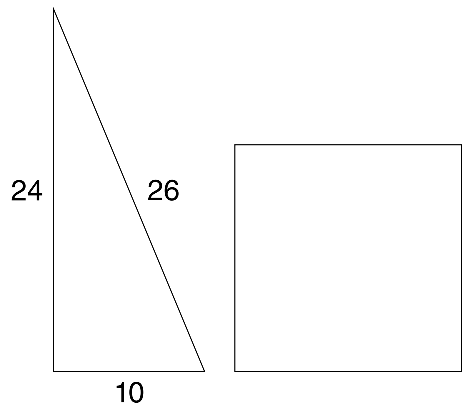
6. Maxim wants to exchange one \$500 bill for the same amount using only bills less than \$500. So far, he has gotten back four \$100 bills, one \$50 bill, two \$20 bills, and one \$5 bill. How many \$1 bills does Maxim still need to get?

(A) 5 (B) 20 (C) 10 (D) 15 (E) 25

7. The length of the serpent road is 10000 miles. If Cat flies at 625 miles per hour, then how many hours will it take Cat to fly through the entire path of the serpent road?

(A) 14 (B) 12 (C) 8 (D) 16 (E) 10

8. A triangle has side lengths of 10, 24, and 26. A square has the same perimeter as the triangle. What is the area of the square?



(A) 225 (B) 200 (C) 120 (D) 100 (E) 60

9. The year 2022 has 365 days. The months of October and December each have 31 days, and the month of November has 30 days. If December 31st is day 365 of the year 2022, then what day of the year is October 31, 2022?

(A) 306 (B) 304 (C) 305 (D) 303 (E) 302

10. Kaycee is selling lemonade. She has five gallons of lemonade and is selling the lemonade at a price of $2 per cup. If there are four cups in one quart and four quarts in one gallon, then how many dollars will Kaycee receive for selling all of her lemonade?

(A) 160 (B) 80 (C) 100 (D) 128 (E) 64

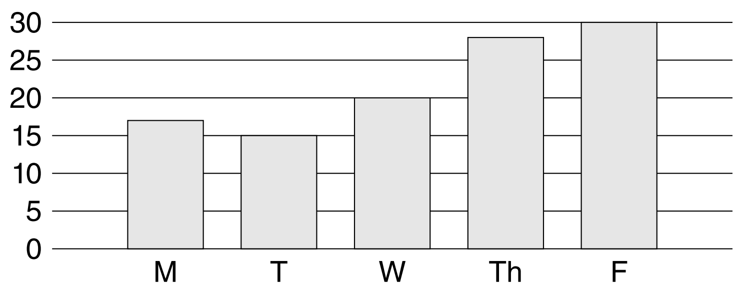
11. What is the remainder when $11 \times 13 \times 17 \times 19$ is divided by 5?

(A) 2 (B) 4 (C) 3 (D) 1 (E) 0

12. Daniel, Spencer, and Kai are lining up to swing at a home-run derby. If Daniel does not want to swing at the home-run derby at any time after Spencer, how many ways can the three line up to swing at the home-run derby?

(A) 4 (B) 1 (C) 3 (D) 2 (E) 6

13. Sarah sold tickets for the school band concert on the weekend. The number of tickets sold each day is displayed in the bar graph below. Which of the following is closest to the average number of tickets sold per day?



(A) 22 (B) 24 (C) 20 (D) 18 (E) 26

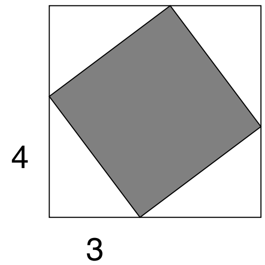
14. A prime number is a whole number that is both greater than 1 and is only evenly disible by 1 and itself. For example, since 5 is only divisible by 1 and 5, the number 5 is a prime number. Of the answer choices, which number is prime?

(A) 101 (B) 121 (C) 91 (D) 111 (E) 81

15. Aaron, Manuel, and Phil are going on a three day road trip on a car that has a driver seat, a side seat, and a back seat. The three agreed to sit in different seats on each of the three days. Aaron sits in the back seat on the first day, and Manuel sits in the side seat on the second day. Using the letter *A* to represent Aaron, the letter *M* to represent Manuel, and the letter *P* to represent Phil, in what order, from the first day to the third day, did the three sit in the driver seat?

(A) P, M, A (B) M, A, P (C) A, P, M (D) P, A, M (E) M, P, A

16. A square of side length 7 is divided into a smaller shaded quadrilateral as well as four right triangles each with legs of length 3 and 4 as shown in the below diagram. What is the area of the shaded quadrilateral?



(A) 30 (B) 16 (C) 20 (D) 25 (E) 24

17. Which of the following fractions is greater than $\frac{3}{5}$ and less than $\frac{9}{14}$?

(A) $\frac{22}{37}$ (B) $\frac{2}{3}$ (C) $\frac{23}{35}$ (D) $\frac{4}{7}$ (E) $\frac{22}{35}$

18. Alex gets paid for helping with writing and editing problems, where he is paid per minute at a rate of $15 per hour. On Monday, Alex helped for two hours. On Wednesday, Alex helped for 80 minutes. On Friday, Alex helped from 3:50 PM to 5:10 PM. Alex records the three activities in his timesheet that week. How many dollars would Alex earn from that timesheet?

(A) 105 (B) 90 (C) 42 (D) 70 (E) 84

19. How many whole numbers greater than 1 and less than 10 evenly divide 210?

(A) 5 (B) 8 (C) 7 (D) 6 (E) 9

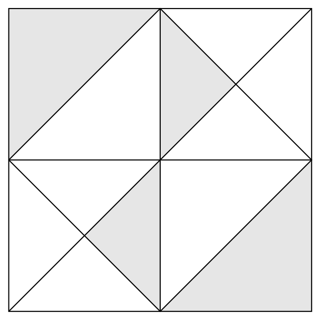
20. Rebecca has 7 cards, Ellen has 8 cards, and Calvin has 9 cards. On each turn, one of the three discards a card. What is the minimum number of turns needed to guarantee that each player discards at least one card?

(A) 15 (B) 14 (C) 16 (D) 18 (E) 17

21. Anthony's class has 30 students, where 60% of the students play golf. Michael's class has 20 students, where 40% of the students play golf. One day, Anthony and Michael teach together with all 50 students from the combined two classes in the classroom. What percent of the students do *not* play golf?

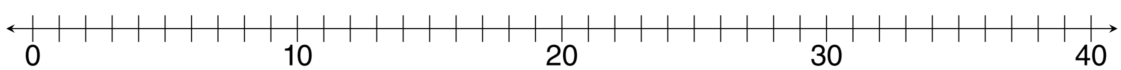
(A) 51% (B) 49% (C) 52% (D) 48% (E) 50%

22. The figure below shows a square, with one diagonal of the square drawn. A second square is formed by connecting the midpoints of the sides of the original square, with both diagonals of that smaller square drawn as well. If the area of the original square is 64, then what is the total area of the shaded regions?



(A) 12 (B) 24 (C) 28 (D) 16 (E) 20

23. Wentinn fired a chip to a number on the number line. The distance between the chip and 21 was twice the distance between the chip and 12. What is the sum of all possible numbers that the chip could be at?

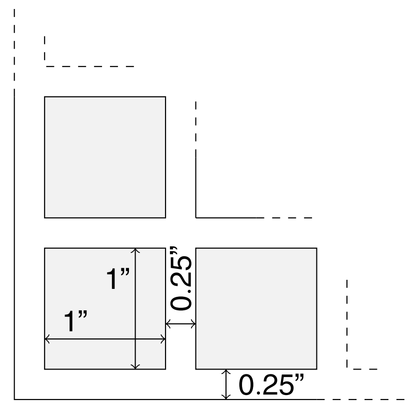


(A) 18 (B) 20 (C) 22 (D) 24 (E) 16

24. In a list of four whole numbers, the unique mode is 8, the mean, or average, is 10, and the range is 5. What is the greatest number in the list?

(A) 13 (B) 14 (C) 11 (D) 15 (E) 12

25. Jada is doing a sketch of a city. In her sketch, she draws squares with side length 1 inch to represent buildings, and the squares have a border of 0.25 inches around each square, including around the outside of the outer edge of squares. One corner of her sketch is shown below. If the completed sketch has 7 rows, with each row having 7 squares that are all equally spaced apart, then how many square inches is the total area of Jada's sketch?



(A) 81 (B) 49 (C) 63 (D) 64 (E) 56

26. In the 3×3 grid of squares below, each of the whole numbers from 1 through 9 is placed into one of the nine squares of the grid so that the sum of the numbers in any row or column of the grid is odd. Five of the squares already have numbers. In how many different ways can the remaining four numbers be placed into the remaining four squares?

	4	
3	1	5
	2	

(A) 4 (B) 2 (C) 8 (D) 6 (E) 0

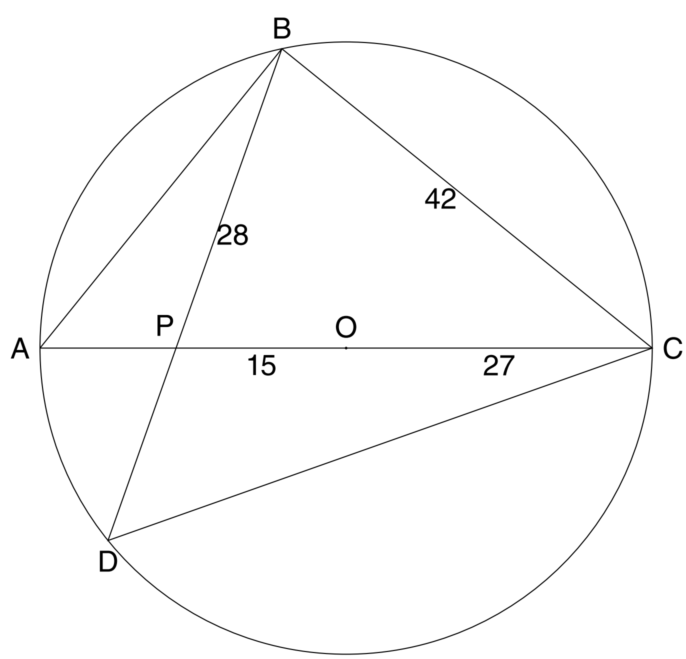
27. Once every four years in the Dream Kingdom, a single lead knight is chosen to serve for the next four years. If each chosen lead knight must complete their full four years as lead knight, and there is only one lead knight at a time, what is the greatest number of different lead knights that can serve at some time between April 27, 1992 and April 27, 2022?

(A) 8 (B) 9 (C) 6 (D) 10 (E) 7

28. Brendan has a collection of creatures that are either grass type, fire type, or water type. The ratio of water creatures to fire creatures is $4 : 5$. The ratio of water creatures to grass creatures is $3 : 5$. After Brendan got an additional two dozen water creatures to his collection, the ratio of water creatures to fire creatures became $8 : 5$, and the ratio of water creatures to grass creatures became $6 : 5$. How many creatures in Brendan's collection are not water type?

(A) 48 (B) 54 (C) 88 (D) 64 (E) 70

29. Points A, B, C, and D lie on a circle with center O. Chord $\overline{BD}$ and diameter $\overline{AC}$ intersect at point P. The length of segment $\overline{OP}$ is 15, the length of segment $\overline{OC}$ is 27, the length of segment $\overline{BC}$ is 42, and the length of segment $\overline{PB}$ is 28. What is the square of the length of segment $\overline{CD}$?



(A) 1296 (B) 1152 (C) 2304 (D) 3072 (E) 2592

30. Alecsis and Bettina each chose a rational number. The sum of Alecsis's number and the reciprocal of Bettina's number is 2. The sum of Bettina's number and the reciprocal of Alecsis's number is $\frac{9}{4}$. The least possible value of the product of Alecsis's number and Bettina's number is a common fraction. What is the sum of the numerator and denominator of that fraction?

(A) 5 (B) 2 (C) 4 (D) 3 (E) 6

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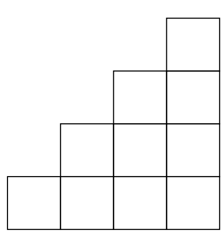
Problems 1 & 2

1. Becca, Nicole, and Sophie are making friendship bracelets. Becca made 15 friendship bracelets, which is nine fewer than the number of bracelets Nicole made. Additionally, Sophie made half as many bracelets as Nicole. How many friendship bracelets did Becca, Nicole, and Sophie make altogether?

1.

2. The figure below is made up of ten identical squares. Each square has a side length of 5. What is the perimeter of the figure?

2.



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Problems 3 & 4

3. Bryce leads 20 scouts in a camping trip. Three scouts did not swim or hike in the trip. A total of 13 scouts swam, and a total of 12 scouts hiked. How many scouts both swam and hiked?

3.

4. Lois is playing a taiko drum mini game at an arcade. The normal stages earn her 6 coins, while the hard stages earn her 10 coins. So far, Lois has played 3 normal stages and 2 hard stages. What is the fewest number of additional stages that Lois needs to play so that she would have exactly 100 coins to buy a prize?

4.

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School ______________________

Problems 5 & 6

5. The product of the tens and units digit of a three-digit whole number is 4. The product of the tens and hundreds digits of the same three-digit whole number is 12. What is the greatest possible value of the three-digit whole number?

5.

6. The mean, or average, of a list of five different positive whole numbers is also a whole number. If four of the numbers are 2, 10, 21, and 23, then what is the sum of all possible values of the median, or middle number, of the list?

6.

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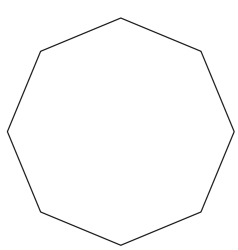
Problems 7 & 8

7. Lily is swimming laps in a swimming pool of length 50 meters, where one lap is the length of the swimming pool. This week, Lily swam 2 laps on Monday. Each day after Monday, she swam two more laps compared to the previous day. From Monday through Friday of this week, how many meters did Lily swim altogether?

7.

8. A regular octagon is shown below. How many different triangles have vertices that are also vertices of the octagon, and share at least one side with the octagon?



8.

Team Round 12320

School or Team

Name

Name

Name

Name

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1.	2.	3.	4.	5.
6.	7.	8.	9.	10.

1. What is the sum of the odd whole numbers that are greater than 0 and less than 20?

2. For Amanda's eighteenth birthday party, she went to see a western play with five of her friends. Some of her friends are younger than 18 years old, and some of her friends are 18 years old. The below table lists the prices of the tickets as well as the age range for those who can buy those tickets. The total cost of the six tickets for Amanda and her friends was $42. How many of those tickets were adult tickets?

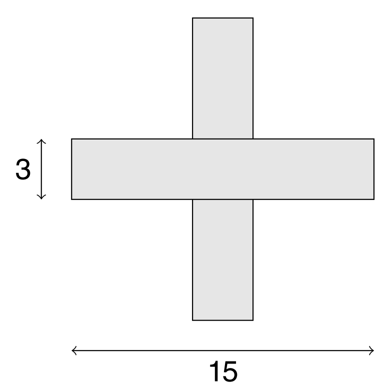
Ticket	**Age Range**	**Price**
Youth	0 to 17	$5
Adult	18 and up	$8

3. An art teacher is passing out colored pencils from a bag. The teacher gives half of the pencils to Bella. The teacher then gives three pencils to Kay. After that, the teacher gives half of the remaining pencils to Kendall. After this, the art teacher has three colored pencils left in the bag. How many colored pencils were originally in the bag?

4. Ian went to a sleepover that started at 10:15 PM and ended at 8:30 AM the next morning. How many minutes long was the sleepover?

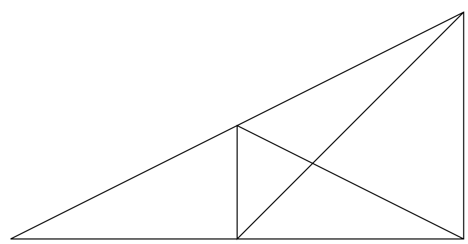
5. Ethan can buy 3 red balloons for 7 banana coins or 7 red balloons for 15 banana coins. He wants to buy at least 48 red balloons for a monkey. What is the fewest number of banana coins that Ethan will need in order to do so?

6. Two rectangular strips of paper, each measuring 3 inches by 15 inches, are placed on a table as shown. One strip is placed perfectly vertical, and the other strip is perfectly horizontal. What is the total number of square inches in the part of the table covered by the strips of paper?



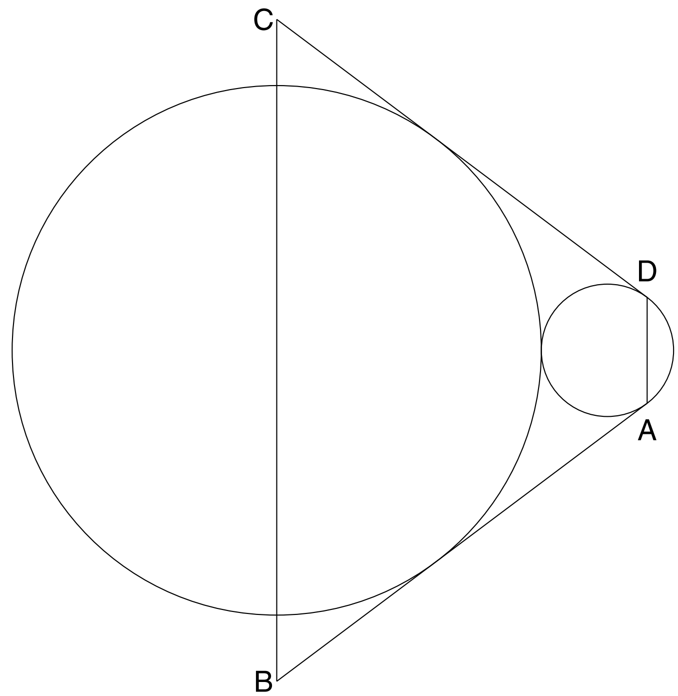
7. Darin is trying to divide some number of players into teams that each have the same number of players. If Darin had one more player, then the players could be divided into seven teams. If Darin had one fewer player, then the players could be divided into eight teams. What is the fewest possible number of players?

8. Using only segments in the figure below as sides of the triangle, how many different triangles can be formed?



9. A shipping clerk got a new scale and four new boxes of differing weights that are each less than 10 pounds, and the clerk has to weigh the boxes in pairs in order to figure out their weights. The shipping clerk weighed every possible pair of boxes, and the scale showed weights, in pounds, of 11, 12, 13, 14, 15, and 16. Using these results, the clerk was able to determine the weight of each of the five boxes, which each weigh a whole number of pounds. How many pounds does the lightest box weigh?

10. The figure below shows a small circle of radius 10 tangent to a larger circle of radius 40. Segment $\overline{AB}$ is tangent to the smaller circle at A as well as the larger circle. Segment $\overline{CD}$ has the same length as $\overline{AB}$ and is tangent to the smaller circle at D as well as the larger circle. Segment $\overline{BC}$ passes through the center of the larger circle. What is the area of quadrilateral $ABCD$?



Answers 12320

Sprint Round

1. D
2. B
3. B
4. E
5. D
6. A
7. D
8. A
9. B
10. A
11. B
12. C
13. A
14. A
15. B
16. D
17. E
18. D
19. A
20. D
21. D
22. B
23. A
24. A
25. A
26. C
27. B
28. E
29. E
30. D

Target Round

1. 51
2. 80
3. 8
4. 7
5. 622
6. 64
7. 1500
8. 40

Team Round

1. 100
2. 4
3. 18
4. 615
5. 104
6. 81
7. 41
8. 12
9. 5
10. 3248

Number Sense

1. 55
2. 3
3. 504
4. 146
5. 2
6. 100
7. 40
8. 78
9. 680
10. $[1160, 1282]$
11. 561
12. 110
13. 169
14. 360
15. 1
16. 360
17. 67
18. 2998
19. 399
20. $[38189, 42207]$
21. 900
22. 4
23. 576
24. 4
25. 95
26. $\frac{3}{4}$
27. 31
28. 440
29. 3
30. $[25622, 28318]$
31. 12
32. 2673
33. 15
34. 3
35. 1225
36. 6
37. 1080
38. 80
39. 621
40. $[1917, 2117]$
41. 520
42. 7
43. 220
44. 3
45. 256
46. 61
47. 2323
48. 210
49. 1909
50. $[299, 329]$
51. 10712
52. 120
53. 6
54. 1110
55. 20
56. 9212
57. 190
58. 11000
59. 15
60. $[1899999, 2099997]$
61. 1681
62. $\frac{1}{18}$
63. $20\frac{2}{9}$
64. 121
65. $\frac{2}{11}$
66. 216
67. 9702
68. 132
69. 34854
70. $[68, 74]$
71. 511488
72. 486
73. 128
74. 8
75. 133
76. 13
77. 12
78. 4
79. 202500
80. $[141, 155]$

Solutions
12320

Sprint Round Solutions

1. One way to solve is by computing each sum and then finding the one with an even number sum. A quicker way to solve is by observing that an even number plus an odd number equals an odd number, but an odd number plus an odd number equals an even number. Either way, we get the sum $\boxed{9+9}$.

2. The sum of the first four terms is 30. The sum of the last three terms is 110. Therefore, the sum totals to $30 + 110 = \boxed{140}$.

3. We can count the sides of each shape, which are often called polygons. The triangle has 3 sides, the square has 4 sides, the pentagon has 5 sides, the hexagon has 6 sides, and the octagon has 8 sides. Altogether, the total number of sides is $3+4+5+6+8 = \boxed{26}$.

4. After five minutes, the time would be 3:00 PM. At that time, there will still be 20 minutes left, so the competition would end at $\boxed{\text{3:20 PM}}$.

5. The cost of two energy canteens is $700 \times 2 = 1400$ yen, and the cost of five hyper cartridges is $3000 \times 5 = 15000$. This means that the amount of yen that Barbara would need to pay is $1400 + 15000 = \boxed{16400}$.

6. The total amount Maxim got so far is $4 \times 100 + 50 + 2 \times 20 + 5 = 495$ dollars. This means that the number of \$1 bills that Maxim still needs is $500 - 495 = \boxed{5}$.

7. Time is equal to distance divided by rate, or speed. Therefore, the number if hours it would take Cat to fly through the serpent road is $10000 \div 625 = \boxed{16}$.

8. The perimeter of the triangle is $10 + 24 + 26 = 60$, which is also the perimeter of the square. This makes the side length of the square equal to $60 \div 4 = 15$, so the area of the square is $15 \cdot 15 = \boxed{225}$.

9. November 30th is day $365 - 31 = 334$ of the year 2022, so October 31st is day $334 - 30 = \boxed{304}$.

10. Kaycee has 5 gallons, which is $5 \times 4 = 20$ quarts, or $20 \times 4 = 80$ cups. If she sells 80 cups for \$2 per cup, the total amount of dollars she will receive is $80 \times 2 = \boxed{160}$.

11. The remainder when a number is divided by 5 is determined by the units digit of the number. The units digit of the product $11 \times 13 \times 17 \times 19$ is the units digit of $1 \times 3 \times 7 \times 9$, which is the units digit of 3×3, or 9, since $1 \times 3 = 3$ and $7 \times 9 = 63$ both have a units digit of 3. The remainder when a number with a units digit of 9 is divided by 5 is $\boxed{4}$.

12. Daniel must go before Spencer, so Daniel can not go last. If Daniel is second-to-last, then Spencer must be last and so Kai must be first. If Daniel is first, then there are 2 ways for Spencer and Kai to be in a line as no matter what, Daniel will swing before Spencer. Therefore, the total number of ways for the three to line up is $2 + 1 = \boxed{3}$.

13. We know that Tuesday's total is 15, Wednesday's total is 20, and Friday's total is 30. Additionally, we know that Monday's total is greater than 15 but less than 20, and Thursday's total is greater than 25 but less than 30. This means that the average must be greater than $\frac{15+15+20+25+30}{5} = 21$ and less than $\frac{20+15+20+30+30}{5} = 23$. The closest answer choice given this range is $\boxed{22}$.

14. Notice that both 81 and 111 have digit sums that are divisible by 3, so they too are divisible by 3, meaning that they are not prime. In addition, $121 = 11^2$, so it is also not prime. Finally, we can check divisibility by small prime numbers to find that $91 = 7 \cdot 13$, so it is not a prime number. Therefore, the only prime number among the answer choices is $\boxed{101}$.

15. On Day 1, the seats available for Manuel are the driver seat and side seat, but since Manuel sits in the side seat on Day 2, Manuel would sit in the driver seat. On Day 2, the seats available for Aaron are the driver seat and back seat, but since Aaron sits in the back seat on Day 1, Aaron would sit in the driver seat. This means that on Day 3, Phil would sit in the driver seat. In the process of solving, we can organize our information in a table like the one below.

	Day 1	Day 2	Day 3
Driver	M	A	P
Side	P	M	A
Back	A	P	M

So in summary, the order for the driver seat is $\boxed{M, A, P}$.

16. One way to solve this is by subtracting the area of the four right triangles from the area of the square of side length 7. The area of the large square is $7 \times 7 = 49$, while the area of each of the four triangles is $\frac{3 \times 4}{2} = 6$. Another way to solve this is by recognizing that the shaded quadrilateral is a square, which has a side length of 5 by the Pythagorean Theorem. Either way, the area of the shaded quadrilateral is $\boxed{25}$.

17. When $\frac{4}{7}$ is compared to $\frac{3}{5}$, $4 \cdot 5 < 3 \cdot 7$, so $\frac{3}{5}$ is greater. Similarly, for $\frac{22}{37}$, $22 \cdot 5 < 3 \cdot 37$, so $\frac{3}{5}$ is greater. When $\frac{2}{3}$ is compared to $\frac{9}{14}$, $2 \cdot 14 > 9 \cdot 3$, so $\frac{9}{14}$ is smaller. Similarly, for $\frac{23}{35}$, $23 \cdot 14 > 9 \cdot 35$, so $\frac{23}{35}$ is greater. However, for $\frac{22}{35}$, $3 \cdot 35 < 22 \cdot 5$, so $\frac{22}{35}$ is greater than $\frac{3}{5}$, and $9 \cdot 35 > 22 \cdot 14$, so $\frac{22}{35}$ is less than $\frac{9}{14}$, so the only fraction that meets both conditions is $\boxed{\frac{22}{35}}$.

18. We can calculate the number of minutes that Alex worked that week. We know that Alex worked for $2 \cdot 60 = 120$ minutes on Monday, and we already know that Alex worked for 80 minutes on Wednesday. On Friday, Alex helped for $60+20 = 80$ minutes. The total time that Alex worked that week is $120+80+80 = 280$ minutes. This is $\frac{280}{60} = \frac{14}{3}$ hours, so the number of dollars Alex would earn is $\frac{14}{3} \cdot 15 = \boxed{70}$.

19. The number 210 can be expressed as $21 \times 10 = 2 \times 3 \times 5 \times 7$. Thus, it is divisible by 2, 3, 5, $2 \times 3 = 6$, and 7, for a total of $\boxed{5}$ whole numbers between 1 and 10, exclusive.

20. The three of them have a total of $7+8+9 = 24$ cards. After 17 turns, it is possible for Ellen and Calvin to discard all their cards and for Rebecca to not discard a single card. However, on the next turn, Rebecca would have to discard a card. Therefore, the minimum number of turns needed to guarantee that each player discards a card is $\boxed{18}$.

21. First, we observe that 60% of 30 is $0.6 \cdot 30 = 18$, and 40% of 20 is $0.4 \cdot 20 = 8$. This means that 18 students of Anthony's class play golf, while 8 students of Michael's class play golf. In the combined class, a total of $18+8 = 26$ students play golf. Since the problem asks for the percent of students who do not play golf, we need to find that $50 - 26 = 24$ students do not play golf, resulting in a percentage of $\frac{24}{50} = \boxed{48\%}$.

22. The two large triangular regions are each $\frac{1}{8}$ of the original square, and the two small triangular regions are each $\frac{1}{16}$ of the original square. Altogether, the shaded regions total $\frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{3}{8}$ of the square, so they have an area of $\frac{3}{8} \cdot 64 = \boxed{24}$.

23. There are two possibilities. The first possibility is that the number is between 12 and 21. Then the distance from 12 to 21, which is $21 - 12 = 9$, is split into two segments by the chip, with one segment twice as long as the other. Therefore the distance from the chip to 12 must be $9 \div 3 = 3$, which means the chip's number is $12 + 3 = 15$. The second possibility is that the number is less than 12. The distance of this number from 21 is always 9 more than the distance from 12, so for the distance from 21 to be twice the distance from 12, the distance from 12 must be 9, and the chip's number must be $12 - 9 = 3$. The sum of the two possible values for the chip's number is $3 + 15 = \boxed{18}$.

24. If the average of the 4 numbers is 10, then the sum of the 4 numbers is $4 \cdot 10 = 40$. If the unique mode is 8, then at least 2 of the numbers must be 8, so the other two numbers must sum to $40-8-8 = 24$. If the range is 5, then either the remaining two numbers must differ by 5, with the lesser being less than or equal to 8, or the lesser is greater than 8 and the greater is $8+5 = 13$. The first case is impossible, as two whole numbers cannot have an odd difference and an even sum. In the second case, the greater number is 13, and the lesser is $24 - 13 = 11$. The range of the four numbers is $13 - 8 = 5$. Therefore, the greatest of the four numbers is $\boxed{13}$.

25. Since there is a border on all four edges of the the sketch, there are $7 + 1 = 8$ borders of 0.25 inches added to the width, for a total horizontal width of $7 + 8 \cdot 0.25 = 9$ inches. The height has the same dimension of 9 inches, so the total area, in square inches, is $9 \cdot 9 = \boxed{81}$.

26. The remaining four numbers are 6, 7, 8, and 9. We need to examine the parity of the numbers, where we look at whether a number is even or odd. Since we want the sum of the rows to be odd, we cannot have the two remaining numbers in the top row both have the same parity, and we cannot have the two remaining numbers in the bottom row both have the same parity. Similarly, we cannot have the two remaining numbers in the left row have different parity, and we cannot have the two remaining numbers in the right column have different parity.

    In summary, we must have one column of the remaining numbers all be odd numbers while the other column of the remaining numbers all be even numbers. Since there are two ways to rearrange the remaining numbers in each column and two ways to decide where the even and odd numbers go, the total number of ways to place the remaining four numbers is $2 \times 2 \times 2 = \boxed{8}$.

27. The number of years between these two times is $2022 - 1992 = 30$, so we wanted to find the greatest number of knights within that time frame. We see that $30 \div 4$ is 7 with a remainder of 2, so the period must cover 7 full terms. The remaining two years could be split between two different knights. Therefore, the greatest number of knights chosen is $7 + 2 = \boxed{9}$.

28. Let the number of water type creatures be $12b$. Then, the number of fire creatures is $15b$, and the number of grass creatures is $20b$. After getting an additional 12 water creatures, the ratio of water creatures to fire creatures will be $\frac{12b+24}{15b}$, which is $\frac{8}{5}$. Setting these quantities equal and simplifying, we get $b = 2$. Then, the number of creatures that are not water type, or the number of creatures that are grass or fire type, is $15 \times 2 + 20 \times 2 = \boxed{70}$.

29. Because $\overline{OC}$ is a radius, the length of $\overline{AC}$ is $2 \cdot 27 = 54$. Because $\overline{AC}$ is a diameter, triangle ABC must be a right triangle, so by the Pythagorean Theorem, the length of $\overline{AB}$ is $\sqrt{54^2 - 42^2} = 24\sqrt{2}$. Angles CAB and CDB both subtend arc CB and are therefore congruent, and angles DBA and DCA both subtend arc DA and are therefore congruent. Thus, triangle DPC is similar to triangle APB. Then, $\frac{CP}{BP} = \frac{CD}{BA}$, or $\frac{42}{28} = \frac{CD}{24\sqrt{2}}$, so the length of $\overline{CD}$ is $36\sqrt{2}$, and the square of this length is $\boxed{2592}$.

30. Let Alecsis's number be a and Bettina's number be b. Then, $a + \frac{1}{b} = 2$, and $b + \frac{1}{a} = \frac{9}{4}$. Multiplying these equations gives $ab + 1 + 1 + \frac{1}{ab} = \frac{9}{2}$. Letting $ab = k$, this is $k + 2 + \frac{1}{k} = \frac{9}{2}$. We can multiply both sides by $2k$ to get the quadratic $2k^2 - 5k + 2 = 0$. This quadratic factors as $(2k - 1)(k - 2)$, so k is either $\frac{1}{2}$ or 2. The smaller of the two is $\frac{1}{2}$, so our answer is $1 + 2 = \boxed{3}$.

Target Round Solutions

1. Since Becca made nine fewer bracelets compared to Nicole, we find that Nicole made $15 + 9 = 24$ bracelets. Since Sophie made half as many bracelets as Nicole, we find that Sophie made $24 \div 2 = 12$ bracelets. Altogether, the total number of bracelets the three made is $15 + 24 + 12 = \boxed{51}$.

2. We can find the perimeter by counting the number of sides of squares around the figure. We count a total of 16 of these sides, which each have length 5, and so the perimeter would be $16 \times 5 = \boxed{80}$.

3. There are $20 - 3 = 17$ scouts that swam, hiked, or did both. However, the total number of scouts that swam or hiked is $13 + 12 = 25$. This counts twice the number of scouts that swam and hiked, so the number of scouts that did both is $25 - 17 = \boxed{8}$.

4. Lois earned $6 \times 3 = 18$ coins from the normal stages and $10 \times 2 = 20$ coins from the hard stages, so she so far has $18 + 20 = 38$ coins. This means that she needs $100 - 38 = 62$ coins. The key to minimizing the number of stages played is by playing more hard stages, but since hard stages earn her 10 coins, she would need to play enough normal stages such that the amount of coins earned from the normal stages she played is a multiple of 10. This can be obtained by just playing two more normal stages, which would result in $62 - 12 = 50$ coins left that could be earned from 5 hard stages. Altogether, the fewest number of stages that Lois needs to play is $2 + 5 = \boxed{7}$.

5. If the product of the tens and units digits is 4, then the tens and units digits are either 4 and 1 in some order, or 2 and 2. If the units digit is 4, then the tens digit is 1, and the hundreds digit would be $12 \div 1 = 12$, which is impossible. If the units digit is 1, then the tens digit is 4, and the hundreds digit would be $12 \div 4 = 3$, so the number would be 341. If the units digit is 2, then the tens digit is 2, and the hundreds digit is $12 \div 2 = 6$, so the number would be 622. Of the two possibilities, the greater is $\boxed{622}$.

6. The sum of four numbers in the list is $2 + 10 + 21 + 23 = 56$. Additionally, since the mean is a whole number, the sum of all five numbers must be a multiple of 5. There are three different situations to consider. If the fifth number is less than 10, then the median is 10. This occurs when the fifth number is 4 or 9, and the sum is 55 or 60, respectively. If the fifth number is greater than 23, then the median is 21. This occurs when the fifth number is 24 or a multiple of 5 more than 24. However, if the fifth number is between 10 and 21, then the median is the fifth number. The only numbers between 10 and 21 that can be added to 56 to produce a multiple of 5 are 14 and 19. The sum of all possible values of the median is $10 + 14 + 19 + 21 = \boxed{64}$.

7. Lily swam 2 laps on Monday, 4 laps on Tuesday, 6 laps on Wednesday, 8 laps on Thursday, and 10 laps on Friday. This is a total of $2 + 4 + 6 + 8 + 10 = 30$ laps. Since each lap is 50 meters, this distance is $30 \times 50 = \boxed{1500}$ meters.

8. As no triangle can share all three sides with the octagon, there are two remaining possibilities to consider.
   - The first is that the triangle shares exactly 2 sides with the octagon. These two sides of the octagon must be adjacent, so there are 8 such triangles.
   - The second possibility is that the triangle shares exactly 1 side with the octagon. Once one of the 8 sides of the octagon is selected for the triangle, the third vertex must be one of the $6 - 2 = 4$ vertices that is not adjacent to the selected side, so there are $8 \times 4 = 32$ such triangles.

   Altogether, the total number of possible triangles is $8 + 32 = \boxed{40}$.

Team Round Solutions

1. We are essentially adding the odd numbers from 1 to 19 inclusive, so we are computing $1+3+5+7+9+11+13+15+17+19$. Now we could manually add the numbers up, or we could make five pairs of numbers with a sum of 20. Then, we can observe that the sum, which is notably a perfect square, is $\boxed{100}$.

2. If every ticket purchased was a youth ticket, the total cost would be $6 \times 5 = 30$ dollars. However, for each ticket that is an adult ticket instead of a youth ticket, the total cost is increased by $8-5 = 3$ dollars. Since the dollar difference is $42 - 30 = 12$, the number of adult tickets is $12 \div 3 = \boxed{4}$.

3. Before the art teacher gave pencils to Kendall, the bag had $3 \times 2 = 6$ pencils. Before the art teacher gave pencils to Kay, the bag had $3 + 6 = 9$ pencils. Before the art teacher gave pencils to Bella, the bag had $9 \times 2 = 18$ pencils. This means the number of pencils was $\boxed{18}$.

4. We can work our way up from the starting time to the finish time. After 45 minutes from the starting time, it would be 11 PM. After an additional hour, it would be midnight. After an additional eight hours, it would be 8 AM. After an additional 30 minutes, the sleepover would end at 8:30 AM. Since there are 60 minutes in an hour, the total number of minutes elapsed was $45 + (1 + 8) \times 60 + 30 = \boxed{615}$.

5. The unit price is lower when buying 7 red balloons for 15 banana coins compared to buying 3 red balloons for 15 banana coins. Now, note that $48 \div 7$ is 6 with a remainder of 6. The cost of buying the set of 7 red balloons six times is $15 \times 6 = 90$ banana coins. Now, Ethan can buy the set of 3 red balloons twice for $7 \times 2 = 14$ banana coins, or he can buy the set of 7 red balloons once for 15 banana coins. The cheaper option is buying the set of 3 red balloons twice, and so the fewest number of banana coins Ethan will need is $90 + 14 = \boxed{104}$.

6. Each strip has an area of $3 \cdot 15 = 45$ square inches. The total area of two of those strips is $2 \cdot 45 = 90$ square inches, but we need to account for the overlap. That overlap is counted twice and is a square of side length 3 inches, which has area $3 \times 3 = 9$ square inches. So, the total area, in square inches, covered by the two strips is $90 - 9 = \boxed{81}$.

7. The number must be one more than a multiple of 8 and one fewer than a multiple of 7. We can then approach this by skip counting. The first few whole numbers that are one more than a multiple of 8 are 1, 9, 17, 25, 33, 41, 49, and 57. The first few whole numbers that are one fewer than a multiple of 7 are 6, 13, 20, 27, 34, 41, 48, and 55. The first number to appear in both lists is $\boxed{41}$.

8. The figure is divided into 5 regions. Using exactly one region per triangle, there are 5 triangles. Using exactly two regions per triangle, there are 4 triangles. Using exactly three regions per triangle, there are 2 triangles. There are no triangles that use exactly four regions, and only 1 triangle that uses all five regions. Altogether, the number of triangles in the figure is $5 + 4 + 2 + 0 + 1 = \boxed{12}$.

9. We can make use of the boxes having different weights that are less than 10 pounds. Looking at the pair weighing 16 pounds, we conclude that there is a box weighing 9 pounds and a box weighing 7 pounds, since the other option would be having two boxes weighing 8 pounds, which is not allowed. Looking at the pair weighing 15 pounds, we can conclude that these two boxes must be 6 pounds and 9 pounds. If there was an 8 pound box, then we can form a pair with boxes weighing 8 pounds and 9 pounds to give 17 pounds, which is not a weight that is listed. Thus, our weights so far are 6, 7, and 9 pounds, giving pair weights of 13, 15, and 16 pounds. Finally, if we add a box of with weight 5 pounds, then we have additional pair weights of $5 + 6 = 11$, $5 + 7 = 12$, and $5 + 9 = 14$ pounds. Thus, the number of pounds that the lightest box weighs is $\boxed{5}$.

10. As shown below, let point E be the point on the larger circle where segment $\overline{AB}$ is tangent, let O and P be the centers of the circles, let Q be the point on $\overline{AD}$ so that points P, O, and Q are collinear, and let F be on $\overline{EP}$ so that $\overline{OF}$ is parallel to $\overline{AE}$. Then, quadrilateral $AOFE$ is a rectangle, and the length of $\overline{FP}$ is $40 - 10 = 30$. The length of $\overline{OP}$ is $40 + 10 = 50$. By the Pythagorean Theorem, the length of $\overline{OF}$ is $\sqrt{50^2 - 30^2} = 40$. Note from symmetry that $\angle OPB = \angle OPC = 90^\circ$, and so angles OPF and FPB are complementary. Then, from some angle chasing, we find that $\triangle EPB$ and $\triangle QAO$ are both similar to $\triangle FOP$. As such, $\frac{OF}{OP} = \frac{PE}{PB}$, or $\frac{40}{50} = \frac{40}{PB}$, and $PB = 50$. Additionally, $\frac{OF}{OP} = \frac{AQ}{AO}$, or $\frac{40}{50} = \frac{AQ}{10}$, and $AQ = 8$. By the Pythagorean Theorem, $QO = 6$. Note that quadrilateral $AQPB$ is a trapezoid with bases of length 8 and 50 and an altitude of $6 + 50 = 56$, so its area is $\frac{1}{2} \cdot 56 \cdot (8 + 50) = 1624$. Quadrilateral $DQPC$ is congruent to trapezoid $AQPB$, so the area of quadrilateral $ABCD$ is $2 \cdot 1624 = \boxed{3248}$.