Power Question 12200

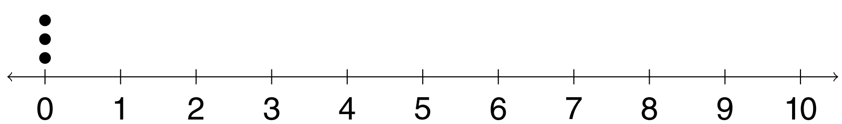
This Power Question deals with *chip firing*. *Chips* are placed at integer points on the number line (in Part One) or lattice points in the plane (in Part Two). The chips are indistinguishable, except when noted otherwise (for instance, in Problem 2), and more than one chip can be at the same location. Chips can be *fired* according to rules given in each part, causing one or more chips to move to new locations.

The following 10 questions are worth 10 points each.

Part One: Multitude Amplification

Chips lie at integer coordinates on a number line and they can be fired under the following rule: if there are c chips at point n, then a single one of these chips can be fired forward a distance of at most c. In other words, the chip moves to point $n' \in \mathbb{N}$ where $n < n' \leq n + c$.

As an example, consider the three chips at point 0 on the number line below. Any one of them can be fired forward up to 3 units. However, once one chip is fired forward, the remaining two chips at point 0 can be fired at most 2 units forward, not 3 units.



For an ordered pair of positive integers (c, d), suppose that c chips begin at point 0 on the number line. Then, let $m(c, d)$ be the minimum number of fires needed to move all of the chips to point d on the number line.

1. Compute the following:
   (a) $m(1, 10)$
   (b) $m(2, 2)$
   (c) $m(10, 2)$
   (d) $m(2, 5)$
   (e) $m(3, 5)$

2. Suppose we label the c chips $s_1, s_2, \ldots, s_c$ and fire chips under the condition that of all chips at a given point on the number line, only the chip of maximal index can be fired. For example, if point 3 on the number line contains chips $\{s_1, s_2, s_5\}$, then only s_5 can be fired. Prove that for any $1 \leq k \leq c$, chip s_k must be fired at least $\left\lceil \frac{d}{k} \right\rceil$ times to reach point d. (The notation $\lceil x \rceil$ denotes the least integer greater than or equal to x.)

3. For any ordered pair of positive integers (c, d), prove in general that $m(c, d) \geq \displaystyle\sum_{k=1}^{c} \left\lceil \frac{d}{k} \right\rceil$. That is, prove that this is true, disregarding the labeling condition imposed in Problem 2.

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Part Two: Hall of Mirrors

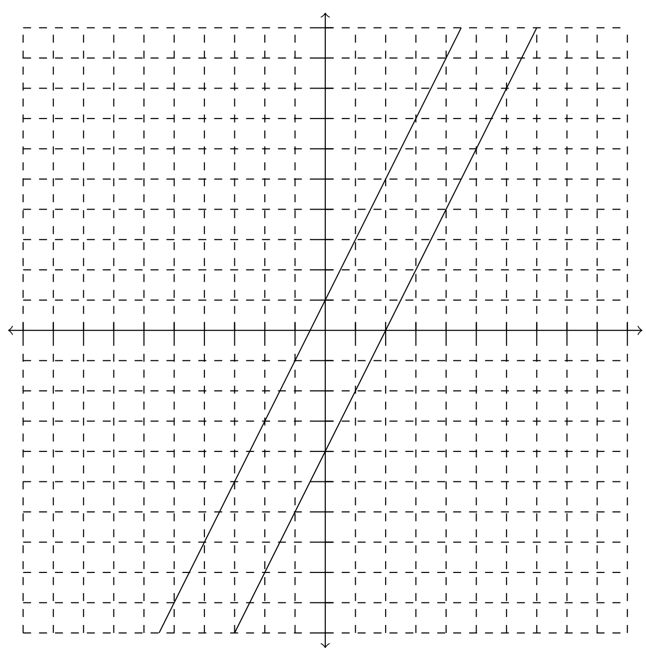
In the *Mirror Problem*, chips lie on lattice points on the xy-plane. Chips can only be fired in the following way: if $c \geq 2$ chips lie at (x, y), then two of those chips can be fired simultaneously, one moving to $(x+1, y)$ and the other to $(x, y+1)$. Given two relatively prime positive integers a and b, suppose there are two "mirrors", represented by the parallel lines $\mathcal{M}_1 : ay - bx = a^2$ and $\mathcal{M}_2 : ay - bx = -b^2$, forming an infinite hallway. Let $f(x, y) = ay - bx$. The mirrors interact with the chips in the following way:

(i) If a chip is fired to a point (x, y) where $f(x, y) > a^2$, then it instead moves to $(x+b, y-a)$.

(ii) If a chip is fired to a point (x, y) where $f(x, y) \leq -b^2$, then it instead moves to $(x-b, y+a)$.

You can think of these chips as being "relocated" by the mirrors, entering one and exiting out the other.

As an example, if $(a, b) = (1, 2)$, mirrors will be the solid lines shown in the diagram below (tick marks are 1 unit). If there are two chips at $(2, 1)$ and the two chips are fired simultaneously, then one chip ends at $(2, 2)$ but the other chip ends at $(1, 2)$ (as $(3, 1)$ lies outside the boundary, the x-coordinate is decreased by 2 while the y-coordinate is increased by 1).



In other words, chips must remain strictly between the two mirrors or exactly on mirror $\mathcal{M}_1$. We also require that all chips begin within these boundaries.

Call a position *stable* if no chips can be fired. From some starting positions, it is possible to reach a stable position; that is, there exists a finite sequence of fires after which no more fires are possible. From other starting positions, chips can be "fired indefinitely": no finite sequence of fires yields a stable position. For example, with $a = b = 1$ and two chips starting at the origin, the chips can be fired indefinitely.

Our end goal is to prove the following claim: for any two relatively prime integers a and b, there exists an integer c such that, given c chips starting at the origin, no stable position can be reached.

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4. In each part, a and b are given, as well as the initial positions of some chips; these chips are fired until no more fires are possible. Find the positions of the chips after no more can be fired.
   (a) $(a, b) = (1, 2)$; 2 chips start at $(1, 2)$
   (b) $(a, b) = (1, 2)$; 3 chips start at $(0, 1)$
   (c) $(a, b) = (2, 3)$; 2 chips start at $(2, 0)$
   (d) $(a, b) = (2, 3)$; 2 chips start at $(0, 2)$ and 1 chip starts at $(3, 1)$

5. For the following pairs (a, b) and c chips starting at the origin, compute the set of positions of the chips once the chips can no longer be fired, or state (without proof) that they can be fired indefinitely. For example, $(a, b) = (2, 1)$ and $c = 2$ results in $\{(0, 1), (0, 2)\}$.
   (a) $(a, b) = (2, 3)$ and $c = 4$
   (b) $(a, b) = (4, 3)$ and $c = 8$
   (c) $(a, b) = (1, 2)$ and $c = 4$

6. Prove that $h(x, y) = ax + by$ remains invariant under mirror interactions. That is, if a chip is fired to a point (x, y) where it is relocated by the mirrors to position (x', y'), then $h(x, y) = h(x', y')$.

7. Prove that for any stable position and positive integer n, at most one chip lies on the line $ax+by = n$.

Now, suppose we start with c chips at the origin and eventually reach a stable position. For each integer n, let $v(n)$ be the number of chips that move to (or start at) a point on the line $h(x, y) = ax + by = n$ at some time during the process. Note that $v(n) = 0$ if it is not possible to have a chip on the line $ax+by = n$, and $v(0) = c$.

8. Prove that if $n \geq \max(a, b)$, then $v(n) = \left\lfloor \frac{v(n-a)}{2} \right\rfloor + \left\lfloor \frac{v(n-b)}{2} \right\rfloor$, where $\lfloor x \rfloor$ denotes the greatest integer function.

9. Use the result of Problem 8 to show that $v(ap + bq) > \dfrac{v(0)}{2^{p+q}} - 2$ for all ordered pairs (p, q) of nonnegative integers.

10. Finally, we will prove the original claim. Suppose $c = 2^{a+b}$, and $a > b$. (The same argument works for $b > a$, and $a = b = 1$ is a special case; you may ignore these.)
    (a) Prove that if a stable position were to be reached, then $v(n) > 2$ for all integers n with $(a - 1)(b - 1) \leq n < ab$.
    (b) Prove that there exists no finite sequence of fires that can produce a stable position.

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Select E only if you cannot determine a uniquely correct answer between A, B, C, and D.

1. A B C D E	11. A B C D E	21. A B C D E
2. A B C D E	12. A B C D E	22. A B C D E
3. A B C D E	13. A B C D E	23. A B C D E
4. A B C D E	14. A B C D E	24. A B C D E
5. A B C D E	15. A B C D E	25. A B C D E
6. A B C D E	16. A B C D E	26. A B C D E
7. A B C D E	17. A B C D E	27. A B C D E
8. A B C D E	18. A B C D E	28. A B C D E
9. A B C D E	19. A B C D E	29. A B C D E
10. A B C D E	20. A B C D E	30. A B C D E

1. Melody gives away pencils to classrooms, and each box of pencils has 12 pencils. In the first building, Melody gives away 12 boxes of pencils, and in the second building, Melody gives away 18 boxes of pencils. How many total pencils did Melody give away in both buildings?

   (A) 30 (B) 288 (C) 144 (D) 240 (E) Other

2. Compute the product $1\frac{1}{2} \cdot 2\frac{1}{3} \cdot 3\frac{1}{7}$.

   (A) 11 (B) 10 (C) $6\frac{41}{42}$ (D) 9 (E) Other

3. Jon is running a local fair where the ticket price is \$2 per child and \$5 per adult. If there are ten times as many adults who bought tickets for the fair as there are children, and the total revenue from tickets one day is \$780, how many adults bought tickets for the fair?

   (A) 140 (B) 39 (C) 150 (D) 144 (E) Other

4. At a store, an N95 mask costs \$5, while a KN95 mask costs \$2. Bella has \$16 to spend on masks for her family of four people, and she must buy one mask per person. What is the greatest number of N95 masks she can buy?

   (A) 2 (B) 4 (C) 3 (D) 0 (E) Other

5. A cube has side length 2. What is the positive numerical difference between the sum of the lengths of its edges and its surface area?

   (A) 12 (B) 0 (C) 6 (D) 18 (E) Other

6. Jeremiah's plant is 9 inches tall at noon on Thursday. For the next two weeks, the plant grows by 1 inch every 48 hours. Jeremiah plans on cutting the plant once the plant reaches 15 inches tall. On what day of the week should Jeremiah cut the plant?

   (A) Friday (B) Wednesday (C) Tuesday (D) Thursday (E) Other

7. The Dream Kingdom has 86 members in its parliament, and in order to pass a law, at least 75% of the parliament must vote “Yes”. In a recent vote, a food law did not pass, and would have passed if and only if at least 3 more members had voted “Yes”. How many members voted “Yes” in that vote?

(A) 64 (B) 63 (C) 66 (D) 62 (E) Other

8. What is the probability that a randomly chosen positive factor of 140 is not a multiple of 5?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{5}$ (E) Other

9. The zeroes of the function $f(x) = x^2 - 60x + 896$ are a and b, where a and b are integers and $a \leq b$. Compute $10a + b$.

(A) 366 (B) 294 (C) 348 (D) 312 (E) Other

10. Square $WXYZ$ has side length 12. Point P is inside the square such that the area of WPY is 18. What is the largest possible area of quadrilateral $WPYZ$?

(A) 54 (B) 68 (C) 90 (D) 72 (E) Other

11. When read from left to right, the 16th, 17th and 18th digits after the decimal point of Euler’s number e are 2, 3, and 5. What is the 16th digit after the decimal point of $3e$, when read from left to right?

(A) 6 (B) 3 (C) 0 (D) 9 (E) Other

12. Amee asks a select number of students and teachers if they like pineapple on pizza, and she records the results. Amee then selects someone she interviewed at random. The probability that she selects a teacher is $\frac{1}{3}$, and the probability that she selects someone who does not like pineapple on pizza is $\frac{5}{6}$, and the probability that she selects a teacher, given that she selects someone who does not like pineapple on pizza, is $\frac{9}{25}$. What is the probability that Amee selects someone who is a teacher or someone who does not like pineapple on pizza?

(A) $\frac{7}{10}$ (B) $\frac{8}{15}$ (C) $\frac{13}{15}$ (D) $\frac{17}{30}$ (E) Other

13. A fully-pumped wall ball is a sphere with diameter 12 inches. Jake kicks away a deflated wall ball, which only holds 62.5% of the volume of a fully-pumped wall ball. What is the volume of that deflated wall ball (in cubic inches)?

(A) 288π (B) 20π (C) 180π (D) 108π (E) Other

14. Compute $12^{10} + 5 \cdot 12^8 \cdot 16^2 + 10 \cdot 12^6 \cdot 16^4 + 10 \cdot 12^4 \cdot 16^6 + 5 \cdot 12^2 \cdot 16^8 + 16^{10}$.

(A) 120000000000 (B) 800000000000 (C) 10240000000000 (D) 400000000000 (E) Other

15. Dawsen plans on flipping a coin 19 times. The probability that exactly 3 heads are flipped is given by $\frac{n}{2^{19}}$, where n is a positive integer. Compute n.

(A) 1141 (B) 5814 (C) 3360 (D) 82782 (E) Other

16. On a remote island, there are 50 people, each of whom is either a truth-teller (and only says true statements) or a liar (and only says false statements). Each person on the island makes one statement describing each other person on the island, saying whether that person is a truth-teller or liar, for a total of 49 statements per person. Compute the maximum possible number of these statements that can call the other person a liar.

(A) 4901 (B) 1250 (C) 5000 (D) 2500 (E) Other

17. A hexagon has five right angles and one angle measuring 270°. Its side lengths, in some order, are 6, 7, 8, 9, 13, and 17. What is the greatest possible area of the hexagon?

(A) 173 (B) 158 (C) 167 (D) 179 (E) Other

18. Say a positive integer is *deciduous* if, each time consecutive digits in the number do not increase from left to right, the digits always decrease by at least 5. For example, 49 and 92 are both deciduous, but 66 and 85 are not. Compute the number of two-digit deciduous positive integers.

(A) 45 (B) 69 (C) 51 (D) 55 (E) Other

19. Quadrilateral $ABCD$ is a square with side length 50. Triangles ABE, BCF, CDG, and DAH are all congruent isosceles triangles such that $AE = EB = 65$, and for each of the four isosceles triangles, each plane it resides on is perpendicular to the plane of $ABCD$ and E, F, G, H are all on the same side of $ABCD$. Additionally, mark points I, J, K, L such that EI, FJ, GK, HL are the altitudes of the respective isosceles triangles. Compute the volume of solid $EFGHIJKL$.

(A) 15000 (B) 75000 (C) $15000\sqrt{2}$ (D) $37500\sqrt{2}$ (E) Other

20. A square table can fit up to four chairs around it, one for each edge. Meri adjoins 70 of these tables together to form one rectangular table. The maximum number of chairs that can fit around this large table is N, one for each edge of a small table comprising the sides of the large table. Compute the sum of all possible values of N.

(A) 576 (B) 254 (C) 288 (D) 280 (E) Other

21. A rectangular prism has edge lengths a, b, and c. Three new rectangular prisms are formed, each by doubling a different edge length of the original prism. The new prisms have surface areas of 18, 20, and 22. Compute the surface area of the original prism.

(A) 12 (B) $\frac{121}{25}$ (C) $\frac{34}{5}$ (D) $\frac{66}{5}$ (E) Other

22. Compute the sum of all positive integers $n \leq 10$ for which $10^{2^n} - 1$ is divisible by at least one of $10^n - 1$ and $10^n + 1$.

(A) 7 (B) 4 (C) 12 (D) 55 (E) Other

23. For a matrix $M = \begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}$, the determinant of M^{128} equals 2^n, where n is a positive integer. Compute the value of n.

(A) 7 (B) 64 (C) 128 (D) 1 (E) Other

24. Consider a regular octagon. Some of the sides are painted red, but all of the sides painted red are consecutive. If rotations of the octagon are considered different, then how many different colorings have at least 2 red sides?

(A) 45 (B) 48 (C) 42 (D) 56 (E) Other

25. On the complex plane, if the complex number z is rotated by 30° counterclockwise about the origin, the result is equal to $z^{\frac{3}{2}}$. Assuming $|z| = 1$, what is the sum of the real and imaginary parts of z? (If $x = a + bi$ where a and b are real, then $|x| = \sqrt{a^2 + b^2}$.)

(A) $\frac{1+\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $\frac{\sqrt{3}-1}{2}$ (E) Other

26. Lucas plans on getting pet creatures for an exhibition, and he wants to be able to evenly put the creatures in teams of 4. He has a total of 15 different creatures to choose from. In how many ways (including not getting any creatures at all) can Lucas choose creatures such that the number of creatures chosen is divisible by 4?

(A) 8256 (B) 8512 (C) 8192 (D) 34048 (E) Other

27. Let s_k be the combined surface area of k^3 unit cubes, and let S_k be the surface area of the large cube formed when all of those k^3 unit cubes are glued together. Given that $\sum_{k=1}^{n}(s_k - S_k) = 2100$, compute n.

(A) 7 (B) 5 (C) 9 (D) 8 (E) Other

28. For a positive integer n, the value $\prod_{k=1}^{100}(n+k)$ equals the product of all integers from $n+1$ to $n+100$ inclusive. Given that $n \leq 100$ and this product ends in exactly 25 zeroes when written in base-10, compute the sum of the possible values of n.

(A) 4775 (B) 4750 (C) 4800 (D) 4700 (E) Other

29. How many ways are there to divide a group of 8 different marbles into one or more unlabeled groups so that each group has an equal number of marbles?

(A) 96 (B) 115 (C) 76 (D) 86 (E) Other

30. Call a nondecreasing sequence of positive integers (a_n) *budding* if all of its elements are less than or equal to 5 and *developed* if the median of its elements is greater than or equal to 5. Let s_1, s_2, s_3, s_4, s_5 be a nondecreasing sequence of five positive integers, with $s_5 \leq 10$. Compute the number of such sequences that are either budding or developed but not both.

(A) 1382 (B) 1397 (C) 1442 (D) 1412 (E) Other

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Problems 1 & 2

1. Nico received a jar of 80 jellybeans as a gift. The colors of the jellybeans are red, green, and blue. Upon eating 7 red jellybeans, 9 blue jellybeans, and 13 green jellybeans from the jar, Nico observed that there are an equal number of remaining jellybeans of each color. How many green jellybeans were originally in the jar before Nico ate any?

1.

2. Suppose 50% of 50% of 80% of A is 30% of 40% of 60% of B. What percent of $\sqrt{B}$ is $\sqrt{A}$?

2.

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Problems 3 & 4

3. On a multiple choice practice problem with 5 answer choices and only 1 right answer choice, Daniel guesses an answer choice at random. If Daniel gets the question wrong, he selects a different answer choice at random, continuing until he gets it right, and never guessing the same answer choice more than once. Compute the expected number of guesses Daniel must make in order to get the question right.

3.

4. A circle is inscribed in an isosceles triangle with side lengths 10, 10, and 12. To the nearest whole percent, what percent of the area inside the triangle lies outside the circle?

4.

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Problems 5 & 6

5. A quadratic polynomial $P(x)$ with leading coefficient 1 satisfies $P(0) = 12$ and $P(1) = 8$. Determine the value of n such that $P(3n) = 9P(n)$. Express your answer as a common fraction.

5.

6. Andrew's frog can hop 91 spaces forward or backward or 117 spaces forward or backward on a number line. If his frog starts at -2 on the number line, what is the sum of the twenty smallest positive numbers that Andrew's frog can eventually reach?

6.

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Problems 7 & 8

7. Hardworking Hudson works 40 hours per week (Monday-Friday). Each weekday, he works an integer number of hours between 7 and 10, inclusive. He also never works fewer hours than he did the day before. In how many different ways can he distribute his work hours between the five weekdays?

7.

8. Let w, x, y, and z be distinct complex numbers such that $|w| = |x| = |y| = |z|$, w and y both have real part a, and x and z both have imaginary part b. Let $v = a + bi$. Given that $|v - w| = 28$, $|v - x| = 14$, and $|v - y| = 18$, compute $|v - z|$.

8.

Team Round 12200

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1.	2.	3.	4.	5.
6.	7.	8.	9.	10.

1. A right triangle has one leg of length x, an area of x, and a hypotenuse of length $2\sqrt{10}$. Compute x.

2. Let $f(x) = \sqrt{x^2 - 4}$ and $g(x) = \sqrt{x^2 + 4}$. What is the value of $f\left(100 + \frac{1}{100}\right) + g\left(100 - \frac{1}{100}\right)$?

3. Addison plays three games, and the probability that he wins in each of them is $\frac{1}{2}$. Addison gets two tokens if he wins the first game, three tokens if he wins the second game, and four tokens if he wins the third game, and nothing for any games he loses. What is the expected value of tokens that Addison gets after playing the three games? Express your answer as a common fraction.

4. Candice rotates her pencil by 90 degrees clockwise about one end, and finds that the total distance traveled by the midpoint was 24 centimeters. To the nearest centimeter, how many centimeters long is Candice's pencil?

5. Dante is saving for a new game console, which costs \$349.99. On Sunday, November 28, Dante sets aside \$100 from his savings to be spent on the new console. Every weekday (excluding Saturday and Sunday), Dante works at a pizza shop from 4 PM to 6 PM and earns \$15 per hour. After 6 PM on that day, Dante collects the money he earns and sets aside half of his daily earnings to be spent on the new game console. December N (where N is a whole number) is the first day where Dante will have enough allocated money to buy the new game console. Determine the value of N.

6. Keneya's figurine of a stop motion video is 7 centimeters high initially. The difference in elevation between the figurine and a ceiling that stays at the same place in the entire video halves every second, and Keneya's figurine is 22 centimeters high after 4 seconds. Compute the height of the ceiling in centimeters.

7. Juliet baked 100 cookies and wants to share them evenly among $p \geq 1$ people, not including herself, so that she has no more than 2 cookies left over. Compute the sum of all possible values of p.

8. An arithmetic sequence has first term a and common difference 2, where a is a positive integer. The first ten terms in the sequence have product P_1. Another arithmetic sequence has first term $a + 1$ and common difference 2, and its first ten terms have product P_2. If $P_1 P_2$ ends in exactly ten zeros when written in base 10, let S the sum of all possible values of a less than 5^7. Compute the smallest integer greater than $\frac{S}{5^6}$.

9. Define the *fractional part* of a real number r as $\{r\} = r - \lfloor r \rfloor$, where $\lfloor r \rfloor$ is the greatest integer less than or equal to r. For example, $\{5.5\} = 0.5$. How many real values of $r \geq 1$ satisfy the equation $\{r\} + \left\{\frac{1}{r}\right\} = \frac{6}{5}$?

10. Triangle ABC has $AB = 13$, $BC = 14$, $CA = 15$. The circle with diameter $\overline{CA}$ passes through point P_1 on $\overline{AB}$ and point P_2 on $\overline{BC}$. The circle with diameter $\overline{P_1P_2}$ passes through point P_3 on $\overline{BP_1}$ and point P_4 on $\overline{BP_2}$. We continue in this fashion, constructing circles with diameter $\overline{P_{2i-1}P_{2i}}$ passing through point P_{2i+1} on $\overline{BP_{2i-1}}$ and point P_{2i+2} on $\overline{BP_{2i}}$. Let K_i be the area of $\triangle BP_{2i+1}P_{2i+2}$. Compute $\displaystyle\sum_{i=0}^{\infty}(-1)^i K_i$.

Express your answer as a common fraction.

Relay Round 12200

Relay 1

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Answer 1-1		Answer 1-2		Answer 1-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

Relay Round 12200

(1-1) Let $T = 50$. Let K be the number of ways to form a T-member committee from a group of $T + 3$ people. Find the remainder when $K + 91$ is divided by 100.

(1-2) Let $T = TNYWR$. Let K be the product of the roots of the polynomial $x^2 - Tx + 6$. Find the remainder when $K + 33$ is divided by 100.

(1-3) Let $T = TNYWR$. Let J be the remainder when T is divided by 11. Let $K = 7^J + 6^J$. Find the remainder when $K + 62$ is divided by 100.

Relay Round 12200

Relay 2

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Answer 2-1		Answer 2-2		Answer 2-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

Relay Round 12200

(2-1) Let $T = 46$. Let K be the smallest triangular number greater than T. Find the remainder when $K+98$ is divided by 100.

(2-2) Let $T = TNYWR$. A line ℓ_1 with slope $\frac{1}{3}$ and a line ℓ_2 with slope $\frac{1}{5}$ intersect at the point $(17, 76)$ in the coordinate plane. Let P and Q be the points where the line $y = T$ intersects the lines ℓ_1 and ℓ_2, respectively. Let K be the distance PQ. Find the remainder when $K + 77$ is divided by 100.

(2-3) Let $T = TNYWR$. Let K be the remainder when $(T+1)^{T+1}$ is divided by 11. Find the remainder when $K + 30$ is divided by 100.

Relay Round 12200

Relay 3

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Answer 3-1		Answer 3-2		Answer 3-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

Relay Round 12200

(3-1) Let $T = 52$. Let D be the distance between the x- and y-intercepts of the function $y = x^2 - 2\sqrt{T}x + T$. Let $K = \frac{D^2}{2}$. Find the remainder when $K + 7$ is divided by 100.

(3-2) Let $T = TNYWR$. Let K be the minimum number of people in a room needed to guarantee that at least $T + 1$ people have the same birthday, assuming that no one is born on February 29th. Find the remainder when $K + 45$ is divided by 100.

(3-3) Let $T = TNYWR$. Let K be the smallest positive integer greater than $T - 20$ that is both a triangular number and a square number. Find the remainder when $K + 93$ is divided by 100.

Relay Round 12200

Relay 4

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Answer 4-1		Answer 4-2		Answer 4-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

Relay Round 12200

(4-1) Let $T = 1$. Consider a prism that has rectangular lateral faces and $(T+3)$-sided polygons for bases. Let K be the total number of diagonals of all faces of such a prism. Find the remainder when $K+40$ is divided by 100.

(4-2) Let $T = TNYWR$. Let a and b be natural numbers such that $\frac{T+1}{T+2} < \frac{a}{b} < \frac{T+2}{T+3}$. Let K be the smallest possible value of $a+b$. Find the remainder when $K+76$ is divided by 100.

(4-3) Let $T = TNYWR$. Let K be the number of integer triples (a, b, c), in degrees, where $0 \leq a, b, c \leq T$ and $\sin a + \sin b + 1 - \sin^2 c = \cos a + \cos b + \cos^2 c$. Find the remainder when $K+32$ is divided by 100.

Relay Round 12200

Relay 5

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Answer 5-1		Answer 5-2		Answer 5-3	
0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Be sure to fill in your answer to each question by fully darkening the appropriate number bubbles in the area provided. You may also write the digits in the boxes above the number bubbles, but in the event of a discrepancy what is bubbled in will count as your official answer.

Relay Round
12200

(5-1) Let $T = 10$. Let K be the number of digits in the base-21 representation of 3^T. (Note that $\log_{21} 7 \approx 0.64$.) Find the remainder when $K + 78$ is divided by 100.

(5-2) Let $T = TNYWR$. Let $\triangle ABC$ have side lengths $AB = T + 2$, $AC = T + 3$, and $BC = T + 4$. Let R be the circumradius and r be the inradius of this triangle. Let $K = 6Rr$. Find the remainder when $K + 39$ is divided by 100.

(5-3) Let $T = TNYWR$. Let $f(x) = |200 - 2x|$ and let $K = f^{T+1}(T)$ (in other words, start with T and apply the function $T + 1$ times). Find the remainder when $K + 64$ is divided by 100.

