Contest Problem Set 12220

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Number Sense 12220

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Name ______________________

Grade ______________________

School ______________________

1. $12 + 12 =$ ______.
2. The tens digit of 70476 is ______.
3. $293 + 128 =$ ______.
4. $4 + 7 \times 4 =$ ______.
5. $514 - 318 =$ ______.
6. $13 \times 20 =$ ______.
7. The remainder of $63 \div 5$ is ______.
8. $17 + 43 + 43 + 17 =$ ______.
9. $65 \div 13 =$ ______.
10. (estimate) $126 + 231 + 273 =$ ______.
11. $314 - 109 - 106 =$ ______.
12. $12^2 =$ ______.
13. The remainder of $134 \div 9$ is ______.
14. $11 \times 34 =$ ______.
15. $8 \times 5 \times 7 =$ ______.
16. *CCLVI* in Arabic numerals is ______.
17. $22 \times 18 =$ ______.
18. $26 + 39 + 65 =$ ______.
19. $18 \times 35 =$ ______.
20. (estimate) $102 \times 299 =$ ______.
21. $25 \times 28 =$ ______.
22. The GCD of 8 and 18 is ______.
23. If 1 gallon is equal to 4 quarts, then 12 quarts is equal to ______ gallons.
24. $12 \times 15 \div 20 =$ ______.
25. $22^2 =$ ______.
26. $527 \div 17 =$ ______.
27. The greater of $\frac{3}{5}$ and $\frac{5}{8}$ is ______ (fraction).
28. $18 + 24 + 30 + 36 + 42 =$ ______.
29. $\frac{1}{3} + \frac{2}{21} =$ ______ (fraction).
30. (estimate) $29 \times 30 \times 31 =$ ______.
31. $\frac{1}{2} =$ ______ %.
32. $16 \times 99 =$ ______.
33. The perimeter of a rectangle with length 8 and width 6 is ______.
34. 10% of 120 is ______.
35. $45^2 =$ ______.
36. The number of odd whole numbers between 8 and 28 is ______.
37. $34 \times 36 =$ ______.
38. The LCM of 8 and 18 is ______.
39. 180 minutes is ______ hours.
40. (estimate) $23456 \div 112 =$ ______.

41. The remainder of $221 \div 11$ is ______.

42. $8 \times 12 + 4 \times 16 =$ ______.

43. The perimeter of a regular octagon with a side length of 14 is ______.

44. $96 \times 16 =$ ______.

45. The eighth term in the arithmetic sequence $5, 10, 15, \ldots$ is ______.

46. $41^2 - 31^2 =$ ______.

47. $101 \times 21 =$ ______.

48. $3^5 =$ ______.

49. 63_9 in base 10 is ______.

50. (estimate) $539 \times 333 =$ ______.

51. $8\frac{1}{3}\% =$ ______ (fraction).

52. $103 \times 105 =$ ______.

53. The sum of the terms of the arithmetic sequence $4, 8, 12, \ldots, 40$ is ______.

54. 110001_2 in base 8 is ______$_8$.

55. $9^3 =$ ______.

56. The measure of an interior angle in an equilateral triangle is ______$^\circ$.

57. $\sqrt{2116} =$ ______.

58. The mode of the list $1, 2, 2, 3, 3, 3, 4$ is ______.

59. $125 \times 17 =$ ______.

60. (estimate) $142857 \times 14 =$ ______.

61. Two fair dice are rolled. The probability the sum of the numbers shown is 3 is ______ (fraction).

62. $5\frac{1}{2} \times 5\frac{1}{2} =$ ______ (mixed number).

63. $111 \times 207 =$ ______.

64. $0.\overline{6} =$ ______ (fraction).

65. If $v = 8$, then $v^2 + 8v + 16 =$ ______.

66. The area of a right triangle with a leg of length 6 and a hypotenuse of length 10 is ______.

67. $9 \times 99 \times 11 =$ ______.

68. $79^2 =$ ______.

69. The number 66 written in base 4 is ______$_4$.

70. (estimate) $17^4 =$ ______.

71. The sum of the prime divisors of 1001 is ______.

72. $1002 \times 1003 =$ ______.

73. The number of positive whole number divisors of 30 is ______.

74. If $2^x = \frac{1}{2}$, then $2^{3-x} =$ ______.

75. $\sqrt{18} \times \sqrt{8} =$ ______.

76. The sum of the lengths of the edges of a $4 \times 8 \times 11$ right rectangular prism is ______.

77. $48 \times 25 \times 18 =$ ______.

78. $\frac{1}{3}$ of 25 is $\frac{5}{6}$ of ______.

79. The sum of the terms of the infinite geometric sequence $\frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots$ is ______.

80. (estimate) $2.1^6 =$ ______.

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Name ______________________

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1.	A	B	C	D	E	11.	A	B	C	D	E	21.	A	B	C	D	E
2.	A	B	C	D	E	12.	A	B	C	D	E	22.	A	B	C	D	E
3.	A	B	C	D	E	13.	A	B	C	D	E	23.	A	B	C	D	E
4.	A	B	C	D	E	14.	A	B	C	D	E	24.	A	B	C	D	E
5.	A	B	C	D	E	15.	A	B	C	D	E	25.	A	B	C	D	E
6.	A	B	C	D	E	16.	A	B	C	D	E	26.	A	B	C	D	E
7.	A	B	C	D	E	17.	A	B	C	D	E	27.	A	B	C	D	E
8.	A	B	C	D	E	18.	A	B	C	D	E	28.	A	B	C	D	E
9.	A	B	C	D	E	19.	A	B	C	D	E	29.	A	B	C	D	E
10.	A	B	C	D	E	20.	A	B	C	D	E	30.	A	B	C	D	E

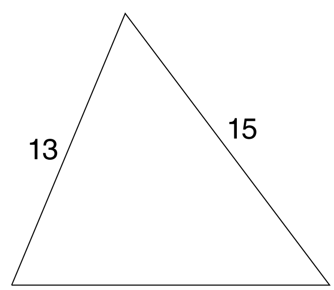
1. In the cafeteria, drinks cost \$1 each, and lunches cost \$2 each. Samantha purchases three drinks and two lunches for herself and a friend. What is the total cost, in dollars, of Samantha's purchase?

(A) \$5 (B) \$8 (C) \$6 (D) \$7 (E) \$4

2. What is the value of $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$?

(A) 120 (B) 144 (C) 64 (D) 100 (E) 125

3. Maxim bent a wire of length 42 inches into a triangle. One side of the triangle had length 13 inches. Another side of the triangle had length 15 inches. What was the length of the third side of Maxim's triangle, in inches?



(A) 10 (B) 14 (C) 12 (D) 13 (E) 11

4. Daniel is doing a trade with a merchant, where for every 5 gold blocks Daniel gives, the merchant will give 2 diamond blocks to Daniel. Daniel has 45 gold blocks. If he trades all of them to the merchant, how many diamond blocks will Daniel receive from the merchant?

(A) 12 (B) 15 (C) 10 (D) 18 (E) 9

5. Jeremiah spends \$2.50 to play five rounds of an arcade game. At the same price per arcade round, how much would seven rounds cost?

(A) \$2.50 (B) \$0.50 (C) \$2.10 (D) \$3.50 (E) \$4.20

6. April calculated the value of 36 divided by 3, and Melcka calculated the value of 39 divided by 13. What is the product of April's result and Melcka's result?

(A) 32 (B) 28 (C) 24 (D) 40 (E) 36

7. This past week, Ellen spent 15 minutes starting an essay on Monday. Each day after Monday, she spent 15 more minutes working on the essay than she did the day before. After she finished her essay on Thursday, how many total minutes had she spent doing the essay?

(A) 135 (B) 60 (C) 225 (D) 150 (E) 120

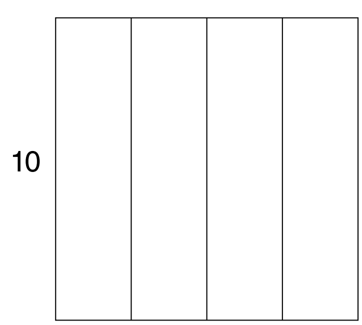
8. What is the hundreds digit of 56×99?

(A) 6 (B) 4 (C) 5 (D) 8 (E) 7

9. A whole number greater than 680 and less than 687 is a multiple of 4. What is the units digit of that number?

(A) 6 (B) 0 (C) 4 (D) 8 (E) 2

10. As shown below, a square of side length 10 is split into four rectangles by three line segments parallel to two sides of the square. What is the total length of all the segments in the figure?



(A) 70 (B) 60 (C) 80 (D) 100 (E) 110

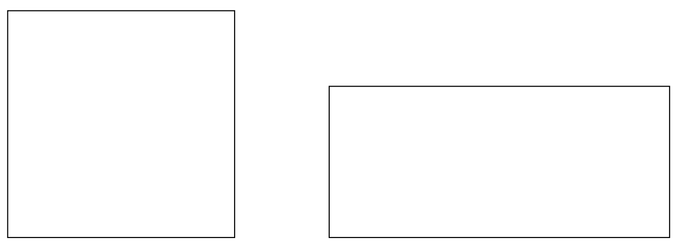
11. Juliet prepared a pancake and decided to cut some slices. She cut one piece that was 25% and two slices that were each 20% of the pancake. The final piece was the remainder of the pancake. What fraction of the pancake was the final piece?

(A) $\frac{11}{14}$ (B) $\frac{7}{20}$ (C) $\frac{11}{20}$ (D) $\frac{13}{20}$ (E) $\frac{3}{14}$

12. How many minutes pass from noon on December 6th until noon on December 14th?

(A) 11400 (B) 11640 (C) 11520 (D) 11340 (E) 11280

13. A rectangle with whole number side lengths has the same area as a square with a perimeter of 48. If the shorter side of the rectangle has a length that differs by 4 from the side length of the square, then what is the perimeter of the rectangle?



(A) 52 (B) 50 (C) 58 (D) 54 (E) 56

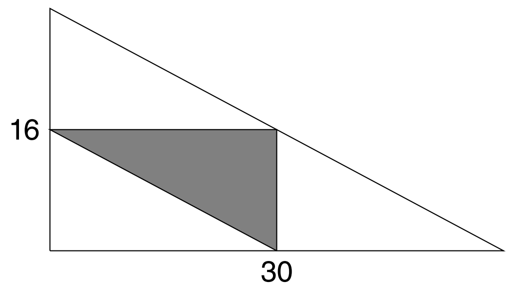
14. Justin writes all possible arrangements of four letters that contain the letters *M*, *E*, *T*, and *A* exactly once. For example, two of the possible arrangements are *META* and *TEAM*. How many arrangements that Justin wrote do not have the *M* and *T* next to each other?

(A) 2 (B) 12 (C) 6 (D) 4 (E) 8

15. Charlotte is managing a factory that produces boots. The factory produces boots 10 at a time and finishes all of them at the same time, and it takes 15 minutes to finish all 10 boots. However, every 30 minutes, one boot has to get discarded. How long, in minutes, will it take Charlotte to have 190 boots available?

(A) 285 (B) 225 (C) 200 (D) 150 (E) 300

16. The right triangle shown below has legs of length 16 and 30. The smaller gray triangle is formed by connecting the midpoints of the right triangle. What fraction of the total area is not gray?



(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

17. Charlie averaged a score of 95 on three quizzes. On two of the quizzes, he had the same score. On the third quiz, he scored a 91. What was the range of Charlie's scores on the three quizzes?

(A) 5 (B) 6 (C) 3 (D) 4 (E) 7

18. Daisy calculated the value of $\frac{1}{0.1} + \frac{11}{0.01} + \frac{10}{0.001}$ and then summed the digits of her result. What was Daisy's sum?

(A) 4 (B) 5 (C) 7 (D) 6 (E) 3

19. How many positive whole numbers evenly divide 72 but do not evenly divide 80?

(A) 6 (B) 4 (C) 8 (D) 3 (E) 2

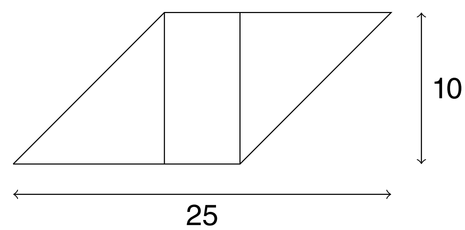
20. If $a \heartsuit b = a^2 - 2ab + b^2$, then what is the value of $(66 \heartsuit 12) - (36 \heartsuit 12)$?

(A) 2400 (B) 2310 (C) 2280 (D) 2340 (E) 2370

21. Christine has at least 3 each of jump stickers, hammer stickers, and flower stickers. Stickers of the same type are considered the same, but stickers of different types are considered different. She wants to select 5 stickers (without regard to order) such that she selects at least 1 jump sticker, 1 hammer sticker, and 1 flower sticker. How many different ways can Christine select 5 stickers? For example, one such possibility is 2 jump stickers, 2 hammer stickers, and 1 flower sticker.

(A) 5 (B) 9 (C) 6 (D) 7 (E) 8

22. A rectangular piece of paper measures 10 inches on the shorter side and 25 inches on the longer side. Opposite corners of the piece of paper are folded until they meet at opposite edges, and the folded over portions are then cut off and removed. To the nearest whole number, in inches, what is the perimeter of the resulting piece of paper?



(A) 64 (B) 58 (C) 62 (D) 56 (E) 60

23. Mabel and Ann each chose a positive whole number. Twice Mabel's number plus Ann's number summed to 110. Twice Ann's number plus Mabel's number summed to 115. What is the product of Mabel's number and Ann's number?

(A) 1200 (B) 1050 (C) 1400 (D) 1575 (E) 1800

24. Ashley is observing some bugs in one area of the laboratory. In the area, every bug is either a stag beetle or a single-horned beetle, and every bug is either shiny or not shiny. There are 69 bugs in the area. There are 15 more single-horned beetles than there are stag bettles, and there are 55 more non-shiny bugs than there are shiny bugs. Ashley observes that there are 25 non-shiny stag beetles in the area. How many shiny single-horned beetles are in the area?

(A) 5 (B) 37 (C) 27 (D) 2 (E) 42

25. Emma's team took part in a soccer league along with three other teams. During the season, every team played every other team in the league twice, and none of the games ended in a tie. The standings of teams in the league are calculated using a point system where a win is worth 2 points and a loss is worth 0 points. At the end of the season, Emma's team got first place while the other three teams got 4 points each. How many points did Emma's team get?

(A) 8 (B) 12 (C) 10 (D) 6 (E) 14

26. From least to greatest, what is the order of 5^{21}, 2^{42}, and 11^{14}?

(A) $2^{42}<5^{21}<11^{14}$ (B) $2^{42}<11^{14}<5^{21}$ (C) $11^{14}<2^{42}<5^{21}$ (D) $5^{21}<2^{42}<11^{14}$ (E) $5^{21}<11^{14}<2^{42}$

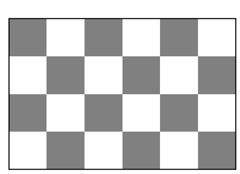
27. How many positive even whole numbers divide 10!?

(A) 128 (B) 270 (C) 64 (D) 240 (E) 120

28. Farmer John has a field with some grass, and the grass grows at a constant rate. He wants to feed some of his cows on this field. All of his cows consume the same amount of grass per day. If Farmer John places 180 cows in the field, the field will be bare in twelve weeks. If Farmer John places 200 cows in the field, the field will be bare in ten weeks. If Farmer John only needs to feed cows in the field for four weeks, then what is the greatest number of cows he can place in the field?

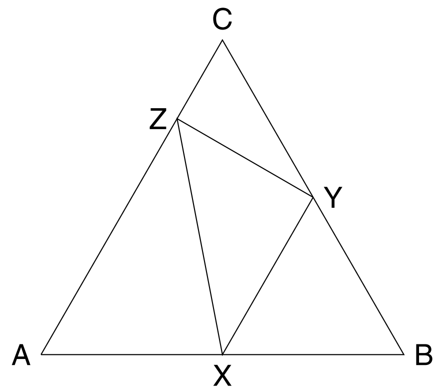
(A) 380 (B) 305 (C) 300 (D) 375 (E) 400

29. A 6×4 checkerboard is shown below. A number of rectangles exist on this board, with boundaries of each rectangle that are boundaries of the small squares or boundaries of the board. A single small square is one such rectangle, and the entire board is another such rectangle. One of these rectangles is randomly chosen. The probability that it is a square can be expressed as a common fraction. What is the sum of the numerator and denominator of that fraction?



(A) 87 (B) 6 (C) 253 (D) 26 (E) 49

30. In equilateral triangle ABC, shown below, points X and Y are the midpoints of sides AB and BC, respectively. Additionally, point Z lies on side AC such that the length of AZ is three times the length of CZ. What is the ratio of the area of triangle XYZ to the area of triangle ABC?



(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{7}{24}$ (D) $\frac{1}{2}$ (E) $\frac{\sqrt{3}}{6}$

Target Round
12220

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Name ______________________

Grade ______________________

School ______________________

Problems 1 & 2

1. Paige summed all of the whole numbers greater than 5 and less than 25 that are not even and also do not have a units digit of five. What is the value of Paige's sum?

1.

2. A four-digit number is odd, a multiple of 5, and a multiple of 11. The hundreds digit of the number is a 3, and the thousands digit is the same value as the tens digit. What is the four-digit number?

2.

Target Round 12220

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	Grade ______
	School ______

Problems 3 & 4

3. Marian's birthday is March 15th. In the year 2022, the month of January has 31 days, and the month of February has 28 days. How many days in 2022 occur before Marian's birthday (not including the exact day itself)?

3.

4. Silver studs are worth 10 points and gold studs are worth 100 points. Chase has 5 more silver studs than gold studs, and the total value of Chase's silver and gold studs is 1700 points. What is the total number of silver and gold studs that Chase has?

4.

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12220

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Grade

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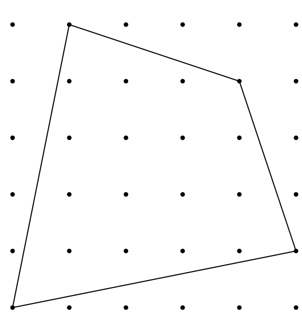
Problems 5 & 6

5. Jeff does roll call by counting all the positive whole numbers from 1 to 30 inclusive, but any time a number is a multiple of 7 or has a digit that is equal to 7, Jeff says “buzz” instead. How many times does Jeff say “buzz” in the count?

5.

6. In the figure below, dots are in a square grid so that each dot has a horizontal and vertical distance of 1 unit from neighboring dots, and the area of the entire grid is $5 \times 5 = 25$. What is the area of the quadrilateral shown?

6.



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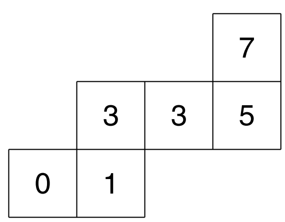
Grade ______________________

School ______________________

Problems 7 & 8

7. Becca cuts out the figure below and folds it into a cube, with the squares shown becoming faces of the cube. Becca writes down the number on the side opposite the face numbered 0, and she writes down the number on the face opposite the face numbered 1. What is the product of the two numbers Becca wrote down?

7.



8. Lindsey has three fair dice. Each die is a cube that has either red or blue faces. One cube has exactly 1 face that is red, one cube has exactly 3 faces that are red, and one cube has exactly 5 faces that are red. Lindsey rolls all three dice. What is the probability that exactly one die shows a red face? Express your answer as a common fraction.

8.

Team Round 12220

School or Team

Name

Name

Name

Name

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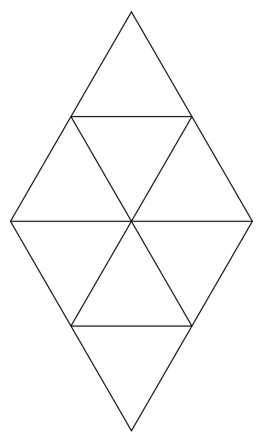
1.	2.	3.	4.	5.
6.	7.	8.	9.	10.

1. Patrick started watching a TV episode about a yellow sponge. The episode started at 1:46 PM and finished at 2:00 PM. How many minutes long was the episode?

2. At an amusement park, tickets cost $60 for children who are under 13 years old and $75 for everyone else. Andrew, Reagan, and Brandon are planning on attending the theme park. Andrew is 14 years old, Reagan is 13 years old, and Brandon is 11 years old. How many dollars do the three of them have to pay for all of them to attend the amusement park?

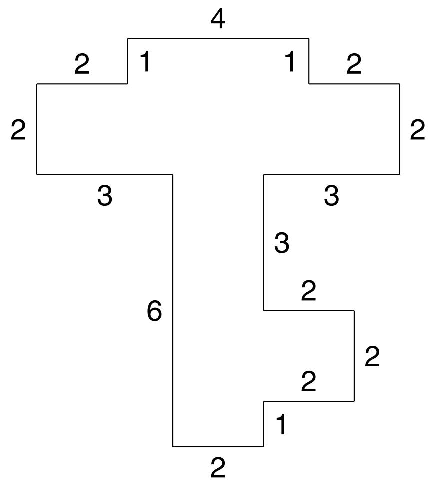
3. The figure below shows a rhombus partitioned into smaller identical equilateral triangles. How many equilateral triangles, of any size, exist in the figure?



4. In the stock market, the Sigma stock falls by 15 points every month, while the X stock increases by 12 points every month. This month, the Sigma stock and X stock each have 120 points. Three months ago, what was the sum of the point values of the Sigma stock and the X stock?

5. When Nariyaki measured his height on his 8th birthday, he was 51 inches tall. When Nariyaki measured his height on his 12th birthday, he was 62 inches tall. On average, how many inches per year did Nariyaki grow between these two measurements? Express your answer as a mixed number.

6. In the figure below, all angles are right angles, and side lengths are as shown. What is the total area enclosed by the figure?

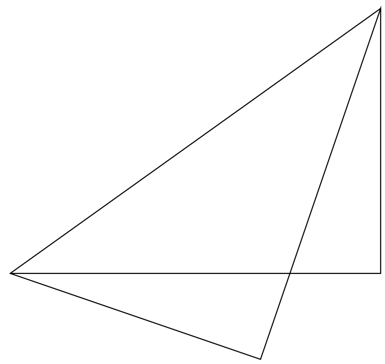


7. Three bells on a clocktower ring at precise times. The north bell rings every four hours. The east bell rings every six hours. The south bell rings every eight hours. If all three bells ring at the same time at 12 : 00 PM on a Monday, then how many times over the next 42 hours will exactly two of the bells ring at the same time?

8. Manuel and Eric live three miles away from each other. Eric leaves his house walking towards Manuel's house at a speed of 3 miles per hour. Manuel leaves his house for Eric's house at the same time, but he is riding his skateboard at a speed that is three times as fast as Eric's walking speed. When Manuel and Eric meet, how many miles more has Manuel travelled than Eric walked? Express your answer as a mixed number.

9. Tomi prepared a number of chemical solutions that were a mixture of hydrochloric acid and distilled water. Each solution measured 25 milliliters. One particular solution used $\frac{1}{4}$ of the total hydrochloric acid she used for all solutions and $\frac{1}{6}$ of the total distilled water she used for all solutions. How many solutions did Tomi prepare?

10. A rectangular piece of paper measuring 70 by 98 is folded along its diagonal. After folding, what is the area of the region where the paper has double its original thickness?



Answers 12220

Sprint Round

1. D
2. D
3. B
4. D
5. D
6. E
7. D
8. C
9. C
10. A
11. B
12. C
13. A
14. B
15. E
16. E
17. B
18. A
19. C
20. D
21. C
22. B
23. C
24. A
25. B
26. B
27. D
28. A
29. D
30. B

Target Round

1. 120
2. 4345
3. 73
4. 35
5. 6
6. 16
7. 21
8. $\frac{31}{72}$

Team Round

1. 14
2. ($)210
3. 10
4. 249
5. $2\frac{3}{4}$
6. 36
7. 6
8. $1\frac{1}{2}$
9. 5
10. 2590

Number Sense

1. 24
2. 7
3. 421
4. 32
5. 196
6. 260
7. 3
8. 120
9. 5
10. $[599, 661]$
11. 99
12. 144
13. 8
14. 374
15. 280
16. 256
17. 396
18. 130
19. 630
20. $[28974, 32022]$
21. 700
22. 2
23. 3
24. 9
25. 484
26. 31
27. $\frac{5}{8}$
28. 150
29. $\frac{3}{7}$
30. $[25622, 28318]$
31. 50
32. 1584
33. 28
34. 12
35. 2025
36. 10
37. 1224
38. 72
39. 3
40. $[199, 219]$
41. 1
42. 160
43. 112
44. 1536
45. 40
46. 720
47. 2121
48. 243
49. 57
50. $[170513, 188461]$
51. $\frac{1}{12}$
52. 10815
53. 220
54. 61
55. 729
56. 60
57. 46
58. 3
59. 2125
60. $[1899999, 2099997]$
61. $\frac{1}{18}$
62. $30\frac{1}{4}$
63. 22977
64. $\frac{2}{3}$
65. 144
66. 24
67. 9801
68. 6241
69. 1002
70. $[79345, 87697]$
71. 31
72. 1005006
73. 8
74. 16
75. 12
76. 92
77. 21600
78. 10
79. 1
80. $[82, 90]$

Solutions 12220

Sprint Round Solutions

1. The total cost of 3 drinks is $3 \cdot \$1 = \$3$, and the total cost of 2 lunches is $2 \cdot \$2 = \$4$, so the total cost of Samantha's purchase is $\$3 + \$4 = \boxed{\$7}$.

2. This can be rewritten as $(1+19)+(3+17)+(5+15)+(7+13)+(9+11)$, which is $20 \times 5 = \boxed{100}$.

3. The sum of the lengths of the three sides of the triangle must be 42 inches. The two given sides have a total length of $13 + 15 = 28$ inches, so the length of the third side, in inches, is $42 - 28 = \boxed{14}$.

4. Daniel gives the merchant $45 \div 5 = 9$ groups of 5 gold blocks, so Daniel will receive 9 groups of 2 diamond blocks, for a total of $9 \cdot 2 = \boxed{18}$.

5. One round of an arcade game costs $\$2.50 \div 5 = \$0.50$, so 7 rounds of the arcade game would cost $7 \cdot \$0.50 = \boxed{\$3.50}$.

6. The two values are $36 \div 3 = 12$ and $39 \div 13 = 3$, and $12 \cdot 3 = \boxed{36}$.

7. Ellen spent 15 minutes on Monday, 30 minutes on Tuesday, 45 minutes on Wednesday, and 60 minutes on Thursday. Altogether, the number of minutes Ellen spent doing the essay was $15 + 30 + 45 + 60 = \boxed{150}$.

8. This is $56 \times 100 - 56 \times 1$, which is $5600 - 56$. Since 56 is less than 100, the hundreds digit of the result will be $6 - 1 = \boxed{5}$.

9. The number 600 is a multiple of 4, so the number between 80 and 87 (exclusive) that is 600 less than the original number must also be a multiple of 4. The only multiple of 4 in that range is 84, which has a units digit of $\boxed{4}$.

10. The perimeter of the square is $4 \times 10 = 40$. Each segment creates 1 side of length 10. There are 3 such segments, so the total length of all the segments exceeds the perimeter of the square by $3 \times 1 \times 10 = 30$. Thus, the total length of all the segments is $40 + 30 = \boxed{70}$.

11. As a fraction, 25% equals $\frac{1}{4}$, and 20% equals $\frac{1}{5}$. The pieces that were cut totaled $\frac{1}{4} + \frac{1}{5} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} + \frac{4}{20} = \frac{13}{20}$ of the pancake, so as a fraction of the pancake, the final piece was $1 - \frac{13}{20} = \boxed{\frac{7}{20}}$.

12. This is a period of $14-6=8$ days. There are 24 hours in 1 day, so this is $8 \times 24 = 192$ hours. There are 60 minutes in 1 hour, so the number of minutes is $192 \times 60 = \boxed{11520}$.

13. The side length of the square is $48 \div 4 = 12$, so the area of the square is $12 \cdot 12 = 144$. The shorter side of the rectangle must have a length less than 12, so it has length $12-4=8$. Then the other side of the rectangle has length $144 \div 8 = 18$. The perimeter of the rectangle is $2 \cdot (8+18) = \boxed{52}$.

14. If the arrangement starts with the letter *M* or the letter *T*, then the other letter, *M* or *T*, must be in the third or fourth position, yielding *META*, *MATE*, *MAET*, *MEAT*, *TEMA*, *TAME*, *TAEM*, and *TEAM*. If the arrangement starts with the letter *E* or *A*, then the letters *M* and *T* must be in the second and fourth positions, yielding *ETAM*, *EMAT*, *ATEM*, and *AMET*. Altogether, the number of arrangements with the *M* and *T* not next to each other is $\boxed{12}$.

15. Every 15 minutes, Charlotte has 10 boots available, so every 30 minutes, there are 20 boots available. But also every 30 minutes, one boot gets discarded, so the total change in 30 minutes is gaining $20-1=19$ boots. Thus, the number of minutes required to have a total of 190 boots is $\frac{190}{19} \cdot 30 = \boxed{300}$.

16. The area of the entire right triangle is $\frac{1}{2} \cdot 30 \cdot 16 = 240$. The grey triangle formed by connecting the midpoints is also a right triangle, with legs that are half the length of the legs of the larger triangle, so its area is $\frac{1}{2} \cdot 15 \cdot 8 = 60$. The fraction of the total area of the right triangle that is not gray is $\frac{240-60}{240} = \boxed{\frac{3}{4}}$.

17. The sum of Charlie's scores on the 3 quizzes is 3 times his average score, or $3 \cdot 95 = 285$. Because the third quiz had a score of 91, the other two quizzes had scores summing to $285-91=194$. As the scores on these two quizzes were equal, they were each $\frac{194}{2} = 97$, so the range of Charlie's scores is $97-91=\boxed{6}$.

18. Multiplying the first fraction by 10, the second by 100, and the third by 1000 yields $\frac{10}{1} + \frac{1100}{1} + \frac{10000}{1}$. This is 11110, and the sum of the digits of this number is $\boxed{4}$.

19. One way is to manually count and list factors, but another way is to observe the prime factorization of 72 and 80. The prime factorization of 72 is $2^3 \cdot 3^2$, and the prime factorization of 80 is $2^4 \cdot 5$. Since 2^3 evenly divides 80, any number that divides 72 but does not divide 80 must be a multiple of 3. These numbers are 3, 6, 9, 12, 18, 24, 36, and 72, for a total of $\boxed{8}$.

20. The given expression can be rewritten as $a^2 - 2ab + b^2 = (a-b)^2$, so $66 \heartsuit 12 = (66-12)^2$, and $36 \heartsuit 12 = (36-12)^2$. Finally, $(66-12)^2 - (36-12)^2$ is $54^2 - 24^2$. By difference of squares, this is $(54+24)(54-24) = 78 \cdot 30$, or $\boxed{2340}$.

21. We know that there is at least 1 jump sticker, at least 1 hammer sticker, and at least 1 flower sticker used, meaning that Christine can then select any two stickers to finish off the selection. If the two stickers are the same type, then Christine has 3 ways. If the two stickers are different type, then Christine also has 3 ways. The total number of possible sticker selections is $3+3=\boxed{6}$.

22. After the pieces are removed, the paper is a parallelogram. The remaining edges of the original piece of paper are 2 segments of length 15 inches, for a combined length of $2 \cdot 15 = 30$ inches. The additional 2 edges are hypotenuses of isosceles right triangles with legs of length 10. Each has a length of $10\sqrt{2}$, and their combined length is $20\sqrt{2}$. The total perimeter is $30 + 20\sqrt{2}$ inches, and since $\sqrt{2} \approx 1.41$, the whole number nearest to $30 + 20\sqrt{2}$ is $\boxed{58}$.

23. Let Mabel's number be M and Ann's number be A. Then $2M + A = 110$ and $M + 2A = 115$. Subtract the first equation from the second equation to get $A - M = 5$. Add this to the second equation to get $3A = 120$, so $A = 40$. Then $M = 35$, and $A \cdot M = \boxed{1400}$.

24. Of the 69 bugs, 27 are stag beetles and 42 are single-horned beetles. Additionally, 62 are non-shiny and 7 are shiny. Since 25 bugs are non-shiny stag beetles, $27 - 25 = 2$ bugs are shiny stag beetles. Since 7 bugs are shiny, the number of shiny single-horned beetles must be $7 - 2 = \boxed{5}$.

25. Each team plays each of the other three teams twice, so each team plays $3 \cdot 2 = 6$ games. For each of the four teams to play 6 games, there must be a total of $\frac{1}{2} \cdot 4 \cdot 6 = 12$ games, since each game always involves two teams. In each of the 12 games, exactly 2 total points are always awarded. Therefore, the total number of points in the league standings must equal $12 \cdot 2 = 24$. The three known point totals sum to $4 \cdot 3 = 12$, so the number of points earned by Emma's team is $24 - 12 = \boxed{12}$.

26. Each exponent is a multiple of 7, so raising each number to the $\frac{1}{7}$ power yields 5^3, 2^6, and 11^2. These can now be compared. Since $5^3 = 125$, $2^6 = 64$, and $11^2 = 121$, the numbers can be ordered as $\boxed{2^{42} < 11^{14} < 5^{21}}$.

27. The prime factorization of 10! is $2^8 \cdot 3^4 \cdot 5^2 \cdot 7$. When an even divisor is prime factored, it will have at least 1 power of 2. Then there are 8 possibilities for the exponent of 2 in such a prime factorization, $4 + 1 = 5$ possibilities for the exponent of 3 (as 0 is a possible exponent), $2 + 1 = 3$ possibilities for the exponent of 5, and $1 + 1 = 2$ possibilities for the exponent of 7. Therefore, the number of even divisors is $8 \cdot 5 \cdot 3 \cdot 2 = \boxed{240}$.

28. To support 180 cows for 12 weeks, the field must supply $12 \cdot 180 = 2160$ cow-weeks of grass. To support 200 cows for 10 weeks, the field must supply $10 \cdot 200 = 2000$ cow-weeks of grass. The additional $2160 - 2000 = 160$ cow-weeks of grass means the grass in the field grows at a rate of $\frac{160}{12-10} = 80$ cow-weeks per week. At the start of the 10 week period, the field has $2000 - 10 \cdot 80 = 1200$ cow-weeks of grass, so at the end of a four week period, the field would be able to provide a total of $1200 + 4 \cdot 80 = 1520$ cow-weeks of grass, which is enough to supply a number of cows equal to $\frac{1520}{4} = \boxed{380}$.

29. Including the boundaries of the board, there are $6 + 1 = 7$ possible vertical boundaries and $4 + 1 = 5$ possible horizontal boundaries. A rectangle can be formed by choosing any 2 of the 7 vertical boundaries and any 2 of the 5 horizontal boundaries, for a total of $\binom{7}{2} \cdot \binom{5}{2} = 210$ possible rectangles. There are $4 \cdot 6 = 24$ squares that are 1×1. For a 2×2 square, there are 3 horizontal possibilities for the lower left corner, and 5 vertical possibilities for the same lower left corner, for a total of $3 \cdot 5 = 15$ squares that are 2×2. For 3×3, the number is $2 \cdot 4 = 8$, and for 4×4, the number is $1 \cdot 3 = 3$. The total number of squares is $24 + 15 + 8 + 3 = 50$. The probability that a randomly chosen rectangle is a square is $\frac{50}{210} = \frac{5}{21}$, and $5 + 21 = \boxed{26}$.

30. Since X and Y are the midpoints of sides AB and BC, respectively, the length of XY is half that of AC. Moreover, the length of a perpendicular line segment from any point on AC to XY is half that of the altitude from B to AC because XY and AC are parallel. In other words, the base and altitude lengths of triangle XYZ are both half those of triangle ABC. Since the area of a triangle scales with the base and altitude lengths, the ratio of the area of triangle XYZ to that of triangle ABC is $\frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{4}}$.

Target Round Solutions

1. The sum of the odd whole numbers from 6 through 24, inclusive, is $7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 135$. But 15 has a units digit of 5 and must be removed, so the final value of the sum is $135 - 15 = \boxed{120}$.

2. For a four-digit number to be divisible by 11, the sum of the tens and thousands digits must differ from the sum of the units and hundreds digits by a value that is a multiple of 11. Since the number is also a multiple of 5, the units digit is 0 or 5, and it cannot be 0 since the number must be odd, so the units digit is 5. The sum of the hundreds and units digits is $3 + 5 = 8$. Therefore, the sum of the tens and thousands digits must be 8, as $8 - 8 = 0$ and any larger sums are not possible for two digits. Therefore, the tens and thousands digit must both be $8 \div 2 = 4$, and the number is $\boxed{4345}$.

3. In the first two months of the year, $31 + 28 = 59$ days occur. In March, $15 - 1 = 14$ days occur before Marian's birthday. The number of days in 2022 that occur before Marian's birthday is $59 + 14 = \boxed{73}$.

4. Without the five extra silver studs, Chase's coins are worth $1700 - 5 \cdot 10 = 1650$ points. The remaining studs can be grouped into pairs of 1 silver stud and 1 gold stud, with each pair worth $100 + 10 = 110$ points. There are $1650 \div 110 = 15$ such pairs, which are comprised of $2 \cdot 15 = 30$ studs. Including the five extra silver studs, the total number of studs is $30 + 5 = \boxed{35}$.

5. The numbers less than 30 that are multiples of 7 are 7, 14, 21, and 28, for a total of 4. The numbers less than 30 that have a digit that is 7 are 7, 17, and 27, for a total of 3. However, the number 7 appears in both lists, so the total number of numbers that appear in at least one of the lists is $4 + 3 - 1 = \boxed{6}$.

6. The entire grid has an area of 25. The area of the quadrilateral can be calculated by removing the regions of the grid that are outside the quadrilateral. The area of the triangle in the upper left is $\frac{1}{2} \cdot 5 \cdot 1 = \frac{5}{2}$. The area of the triangle in the lower right is also $\frac{1}{2} \cdot 5 \cdot 1 = \frac{5}{2}$. The upper region can be partitioned into three regions. In the upper left, there is a triangle with base 3 and height 1, and an area of $\frac{1}{2} \cdot 3 \cdot 1 = \frac{3}{2}$. In the upper right, there is a square that has area $1 \cdot 1 = 1$. Finally, below the rectangle there is a triangle with base 1 and height 3, and an area of $\frac{1}{2} \cdot 1 \cdot 3 = \frac{3}{2}$. The area of the quadrilateral is $25 - \frac{5}{2} - \frac{5}{2} - \frac{3}{2} - 1 - \frac{3}{2} = \boxed{16}$.

7. When folded, the face opposite the face numbered 0 has the number 3, and the face opposite the face numbered 1 has the number 7. The product of 3 and 7 is $\boxed{21}$.

8. As a cube has 6 faces, the probabilities that each die shows a red face are $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{5}{6}$. The probabilities that each die does not show a red face is $\frac{5}{6}$, $\frac{1}{2}$, and $\frac{1}{6}$. The probability that only the first die shows a red face is $\frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{72}$. The probability that only the second die shows a red face is $\frac{5}{6} \cdot \frac{1}{2} \cdot \frac{1}{6} = \frac{5}{72}$. The probability that only the third die shows a red face is $\frac{5}{6} \cdot \frac{1}{2} \cdot \frac{5}{6} = \frac{25}{72}$. The probability that exactly one die shows a red face is $\frac{1}{72} + \frac{5}{72} + \frac{25}{72} = \boxed{\frac{31}{72}}$.

Team Round Solutions

1. There are 60 minutes in an hour. The time 1:46 PM is 46 minutes after 1 PM, while 2:00 PM is 60 minutes after 1 PM. Therefore, the length of the episode, in minutes, was $60 - 46 = \boxed{14}$.

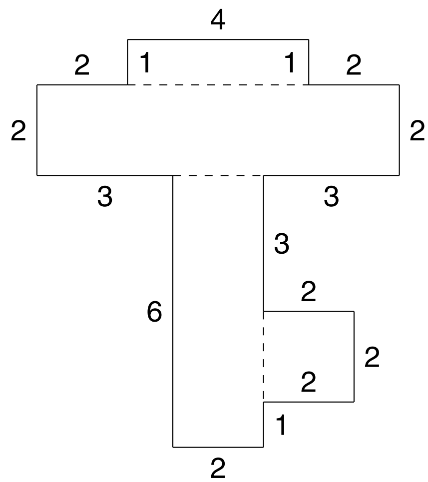
2. Among the three people, two of them (Andrew and Reagan) have to pay \$75 each, and one of them (Brandon) only has to pay \$60. The total amount that the three have to pay is $\$75+\$75+\$60 = \boxed{\$210}$.

3. There are a total of 8 small equilateral triangles. However, we can put four small equilateral triangles together to form one large equilateral triangle, and there are 2 such triangles – one facing up and one facing down. Altogether, the total number of equilateral triangles is $8 + 2 = \boxed{10}$.

4. The Sigma stock falls by 15 points every month, so three months ago, the Sigma stock had $120 + 3 \cdot 15 = 165$ points. The X stock increases by 12 points each month, so three months ago, the X stock had $120 - 3 \cdot 12 = 84$ points. The sum of the point values of the two stocks three months ago is $165 + 84 = \boxed{249}$.

5. The total height Nariyaki gained was $62 - 51 = 11$ inches. The time between the measurements was $12 - 8 = 4$ years. Therefore, the average increase in inches per year for Nariyaki is $\frac{11}{4} = \boxed{2\frac{3}{4}}$.

6. There are many ways to partition the figure into rectangles. One way is shown below. This creates a 4×1 rectangle, an 8×2 rectangle, a 6×2 rectangle, and a square of side length 2. The total area is $4 \cdot 1 + 8 \cdot 2 + 6 \cdot 2 + 2 \cdot 2 = \boxed{36}$.



7. We check each case where a pair of bells ring at the same time. The north and east bells ring at the same time every 12 hours, the east and south bells ring at the same time every 24 hours, the south and north bells ring at the same time every 8 hours, and all three bells ring at the same time every 24 hours. So exactly two bells ring at the same time when the number of hours that pass is a multiple of 8 or 12 but not a multiple of 24. The multiples of 8 under 42 that are not multiples of 24 are 8, 16, 32, and 40, and the multiples of 12 under 42 that are not multiples of 24 are 12 and 36, so the number of times exactly two bells ring at the same time over the next 42 hours is $\boxed{6}$.

8. The absolute speed each travels at does not matter. Since they have both traveled for the same amount of time, Manuel has traveled 3 times as far as Eric. Therefore, Manuel has traveled $\frac{3}{3+1} = \frac{3}{4}$ of the total distance, while Eric has traveled $\frac{1}{3+1} = \frac{1}{4}$ of the total distance. The difference in between the two distances each travels is $\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$ of 3 miles, or $\boxed{1\frac{1}{2}}$ miles.

9. Let w be the total number of milliliters of water that Tomi used in preparing the solutions, a be the total number of mlliliters of acid that Tomi used, and n be the number of solutions she prepared. From the total volume, $w + a = 25n$. Additionally, from the known solution, $\frac{a}{4} + \frac{w}{6} = 25$. Substituting the second equation into the first, $w + a = \left(\frac{a}{4} + \frac{w}{6}\right) \cdot n$. Simplifying, $24w + 24a = 6an + 4wn$, and $24w - 4wn = 6an - 24a$. Factoring, $4w \cdot (6 - n) = 6a \cdot (n - 4)$. Both $4w$ and $6a$ are positive, and n must also be positive. The only whole number value of n for which both sides of the equation produce a positive value is $\boxed{5}$.

10. Let the distance from an unfolded corner to the the point where the overlapped paper begins be x. This is the shorter leg of a right triangle where the other leg has length 70, and the hypotenuse has length $98-x$. By the Pythagorean Theorem, $x^2+70^2=(98-x)^2$, or $x^2+4900=x^2-196x+9604$. Solving this equation, $x=24$, so the hypotenuse of this triangle has length $98-24=74$. The region where the paper has double its original thickness is half of the rectangle minus a 24-70-74 right triangle. This is also be viewed as an obtuse triangle with a base of 74 and a height of 70, which has area $\frac{1}{2}\cdot 74\cdot 70=\boxed{2590}$.

