Answers 12320

Sprint Round

1. D
2. B
3. B
4. E
5. D
6. A
7. D
8. A
9. B
10. A
11. B
12. C
13. A
14. A
15. B
16. D
17. E
18. D
19. A
20. D
21. D
22. B
23. A
24. A
25. A
26. C
27. B
28. E
29. E
30. D

Target Round

1. 51
2. 80
3. 8
4. 7
5. 622
6. 64
7. 1500
8. 40

Team Round

1. 100
2. 4
3. 18
4. 615
5. 104
6. 81
7. 41
8. 12
9. 5
10. 3248

Number Sense

1. 55
2. 3
3. 504
4. 146
5. 2
6. 100
7. 40
8. 78
9. 680
10. $[1160, 1282]$
11. 561
12. 110
13. 169
14. 360
15. 1
16. 360
17. 67
18. 2998
19. 399
20. $[38189, 42207]$
21. 900
22. 4
23. 576
24. 4
25. 95
26. $\frac{3}{4}$
27. 31
28. 440
29. 3
30. $[25622, 28318]$
31. 12
32. 2673
33. 15
34. 3
35. 1225
36. 6
37. 1080
38. 80
39. 621
40. $[1917, 2117]$
41. 520
42. 7
43. 220
44. 3
45. 256
46. 61
47. 2323
48. 210
49. 1909
50. $[299, 329]$
51. 10712
52. 120
53. 6
54. 1110
55. 20
56. 9212
57. 190
58. 11000
59. 15
60. $[1899999, 2099997]$
61. 1681
62. $\frac{1}{18}$
63. $20\frac{2}{9}$
64. 121
65. $\frac{2}{11}$
66. 216
67. 9702
68. 132
69. 34854
70. $[68, 74]$
71. 511488
72. 486
73. 128
74. 8
75. 133
76. 13
77. 12
78. 4
79. 202500
80. $[141, 155]$

Solutions 12320

Sprint Round Solutions

1. One way to solve is by computing each sum and then finding the one with an even number sum. A quicker way to solve is by observing that an even number plus an odd number equals an odd number, but an odd number plus an odd number equals an even number. Either way, we get the sum $\boxed{9+9}$.

2. The sum of the first four terms is 30. The sum of the last three terms is 110. Therefore, the sum totals to $30 + 110 = \boxed{140}$.

3. We can count the sides of each shape, which are often called polygons. The triangle has 3 sides, the square has 4 sides, the pentagon has 5 sides, the hexagon has 6 sides, and the octagon has 8 sides. Altogether, the total number of sides is $3 + 4 + 5 + 6 + 8 = \boxed{26}$.

4. After five minutes, the time would be 3:00 PM. At that time, there will still be 20 minutes left, so the competition would end at $\boxed{\text{3:20 PM}}$.

5. The cost of two energy canteens is $700 \times 2 = 1400$ yen, and the cost of five hyper cartridges is $3000 \times 5 = 15000$. This means that the amount of yen that Barbara would need to pay is $1400 + 15000 = \boxed{16400}$.

6. The total amount Maxim got so far is $4 \times 100 + 50 + 2 \times 20 + 5 = 495$ dollars. This means that the number of \$1 bills that Maxim still needs is $500 - 495 = \boxed{5}$.

7. Time is equal to distance divided by rate, or speed. Therefore, the number if hours it would take Cat to fly through the serpent road is $10000 \div 625 = \boxed{16}$.

8. The perimeter of the triangle is $10 + 24 + 26 = 60$, which is also the perimeter of the square. This makes the side length of the square equal to $60 \div 4 = 15$, so the area of the square is $15 \cdot 15 = \boxed{225}$.

9. November 30th is day $365 - 31 = 334$ of the year 2022, so October 31st is day $334 - 30 = \boxed{304}$.

10. Kaycee has 5 gallons, which is $5 \times 4 = 20$ quarts, or $20 \times 4 = 80$ cups. If she sells 80 cups for \$2 per cup, the total amount of dollars she will receive is $80 \times 2 = \boxed{160}$.

11. The remainder when a number is divided by 5 is determined by the units digit of the number. The units digit of the product $11 \times 13 \times 17 \times 19$ is the units digit of $1 \times 3 \times 7 \times 9$, which is the units digit of 3×3, or 9, since $1 \times 3 = 3$ and $7 \times 9 = 63$ both have a units digit of 3. The remainder when a number with a units digit of 9 is divided by 5 is $\boxed{4}$.

12. Daniel must go before Spencer, so Daniel can not go last. If Daniel is second-to-last, then Spencer must be last and so Kai must be first. If Daniel is first, then there are 2 ways for Spencer and Kai to be in a line as no matter what, Daniel will swing before Spencer. Therefore, the total number of ways for the three to line up is $2 + 1 = \boxed{3}$.

13. We know that Tuesday's total is 15, Wednesday's total is 20, and Friday's total is 30. Additionally, we know that Monday's total is greater than 15 but less than 20, and Thursday's total is greater than 25 but less than 30. This means that the average must be greater than $\frac{15+15+20+25+30}{5} = 21$ and less than $\frac{20+15+20+30+30}{5} = 23$. The closest answer choice given this range is $\boxed{22}$.

14. Notice that both 81 and 111 have digit sums that are divisible by 3, so they too are divisible by 3, meaning that they are not prime. In addition, $121 = 11^2$, so it is also not prime. Finally, we can check divisibility by small prime numbers to find that $91 = 7 \cdot 13$, so it is not a prime number. Therefore, the only prime number among the answer choices is $\boxed{101}$.

15. On Day 1, the seats available for Manuel are the driver seat and side seat, but since Manuel sits in the side seat on Day 2, Manuel would sit in the driver seat. On Day 2, the seats available for Aaron are the driver seat and back seat, but since Aaron sits in the back seat on Day 1, Aaron would sit in the driver seat. This means that on Day 3, Phil would sit in the driver seat. In the process of solving, we can organize our information in a table like the one below.

	Day 1	Day 2	Day 3
Driver	M	A	P
Side	P	M	A
Back	A	P	M

So in summary, the order for the driver seat is $\boxed{M, A, P}$.

16. One way to solve this is by subtracting the area of the four right triangles from the area of the square of side length 7. The area of the large square is $7 \times 7 = 49$, while the area of each of the four triangles is $\frac{3 \times 4}{2} = 6$. Another way to solve this is by recognizing that the shaded quadrilateral is a square, which has a side length of 5 by the Pythagorean Theorem. Either way, the area of the shaded quadrilateral is $\boxed{25}$.

17. When $\frac{4}{7}$ is compared to $\frac{3}{5}$, $4 \cdot 5 < 3 \cdot 7$, so $\frac{3}{5}$ is greater. Similarly, for $\frac{22}{37}$, $22 \cdot 5 < 3 \cdot 37$, so $\frac{3}{5}$ is greater. When $\frac{2}{3}$ is compared to $\frac{9}{14}$, $2 \cdot 14 > 9 \cdot 3$, so $\frac{9}{14}$ is smaller. Similarly, for $\frac{23}{35}$, $23 \cdot 14 > 9 \cdot 35$, so $\frac{23}{35}$ is greater. However, for $\frac{22}{35}$, $3 \cdot 35 < 22 \cdot 5$, so $\frac{22}{35}$ is greater than $\frac{3}{5}$, and $9 \cdot 35 > 22 \cdot 14$, so $\frac{22}{35}$ is less than $\frac{9}{14}$, so the only fraction that meets both conditions is $\boxed{\dfrac{22}{35}}$.

18. We can calculate the number of minutes that Alex worked that week. We know that Alex worked for $2 \cdot 60 = 120$ minutes on Monday, and we already know that Alex worked for 80 minutes on Wednesday. On Friday, Alex helped for $60+20 = 80$ minutes. The total time that Alex worked that week is $120+80+80 = 280$ minutes. This is $\frac{280}{60} = \frac{14}{3}$ hours, so the number of dollars Alex would earn is $\frac{14}{3} \cdot 15 = \boxed{70}$.

19. The number 210 can be expressed as $21 \times 10 = 2 \times 3 \times 5 \times 7$. Thus, it is divisible by 2, 3, 5, $2 \times 3 = 6$, and 7, for a total of $\boxed{5}$ whole numbers between 1 and 10, exclusive.

20. The three of them have a total of $7+8+9 = 24$ cards. After 17 turns, it is possible for Ellen and Calvin to discard all their cards and for Rebecca to not discard a single card. However, on the next turn, Rebecca would have to discard a card. Therefore, the minimum number of turns needed to guarantee that each player discards a card is $\boxed{18}$.

21. First, we observe that 60% of 30 is $0.6 \cdot 30 = 18$, and 40% of 20 is $0.4 \cdot 20 = 8$. This means that 18 students of Anthony's class play golf, while 8 students of Michael's class play golf. In the combined class, a total of $18+8 = 26$ students play golf. Since the problem asks for the percent of students who do not play golf, we need to find that $50 - 26 = 24$ students do not play golf, resulting in a percentage of $\frac{24}{50} = \boxed{48\%}$.

22. The two large triangular regions are each $\frac{1}{8}$ of the original square, and the two small triangular regions are each $\frac{1}{16}$ of the original square. Altogether, the shaded regions total $\frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{3}{8}$ of the square, so they have an area of $\frac{3}{8} \cdot 64 = \boxed{24}$.

23. There are two possibilities. The first possibility is that the number is between 12 and 21. Then the distance from 12 to 21, which is $21 - 12 = 9$, is split into two segments by the chip, with one segment twice as long as the other. Therefore the distance from the chip to 12 must be $9 \div 3 = 3$, which means the chip's number is $12 + 3 = 15$. The second possibility is that the number is less than 12. The distance of this number from 21 is always 9 more than the distance from 12, so for the distance from 21 to be twice the distance from 12, the distance from 12 must be 9, and the chip's number must be $12 - 9 = 3$. The sum of the two possible values for the chip's number is $3 + 15 = \boxed{18}$.

24. If the average of the 4 numbers is 10, then the sum of the 4 numbers is $4 \cdot 10 = 40$. If the unique mode is 8, then at least 2 of the numbers must be 8, so the other two numbers must sum to $40-8-8 = 24$. If the range is 5, then either the remaining two numbers must differ by 5, with the lesser being less than or equal to 8, or the lesser is greater than 8 and the greater is $8+5 = 13$. The first case is impossible, as two whole numbers cannot have an odd difference and an even sum. In the second case, the greater number is 13, and the lesser is $24 - 13 = 11$. The range of the four numbers is $13 - 8 = 5$. Therefore, the greatest of the four numbers is $\boxed{13}$.

25. Since there is a border on all four edges of the the sketch, there are $7 + 1 = 8$ borders of 0.25 inches added to the width, for a total horizontal width of $7 + 8 \cdot 0.25 = 9$ inches. The height has the same dimension of 9 inches, so the total area, in square inches, is $9 \cdot 9 = \boxed{81}$.

26. The remaining four numbers are 6, 7, 8, and 9. We need to examine the parity of the numbers, where we look at whether a number is even or odd. Since we want the sum of the rows to be odd, we cannot have the two remaining numbers in the top row both have the same parity, and we cannot have the two remaining numbers in the bottom row both have the same parity. Similarly, we cannot have the two remaining numbers in the left row have different parity, and we cannot have the two remaining numbers in the right column have different parity.

    In summary, we must have one column of the remaining numbers all be odd numbers while the other column of the remaining numbers all be even numbers. Since there are two ways to rearrange the remaining numbers in each column and two ways to decide where the even and odd numbers go, the total number of ways to place the remaining four numbers is $2 \times 2 \times 2 = \boxed{8}$.

27. The number of years between these two times is $2022 - 1992 = 30$, so we wanted to find the greatest number of knights within that time frame. We see that $30 \div 4$ is 7 with a remainder of 2, so the period must cover 7 full terms. The remaining two years could be split between two different knights. Therefore, the greatest number of knights chosen is $7 + 2 = \boxed{9}$.

28. Let the number of water type creatures be $12b$. Then, the number of fire creatures is $15b$, and the number of grass creatures is $20b$. After getting an additional 12 water creatures, the ratio of water creatures to fire creatures will be $\frac{12b+24}{15b}$, which is $\frac{8}{5}$. Setting these quantities equal and simplifying, we get $b = 2$. Then, the number of creatures that are not water type, or the number of creatures that are grass or fire type, is $15 \times 2 + 20 \times 2 = \boxed{70}$.

29. Because $\overline{OC}$ is a radius, the length of $\overline{AC}$ is $2 \cdot 27 = 54$. Because $\overline{AC}$ is a diameter, triangle ABC must be a right triangle, so by the Pythagorean Theorem, the length of $\overline{AB}$ is $\sqrt{54^2 - 42^2} = 24\sqrt{2}$. Angles CAB and CDB both subtend arc CB and are therefore congruent, and angles DBA and DCA both subtend arc DA and are therefore congruent. Thus, triangle DPC is similar to triangle APB. Then, $\frac{CP}{BP} = \frac{CD}{BA}$, or $\frac{42}{28} = \frac{CD}{24\sqrt{2}}$, so the length of $\overline{CD}$ is $36\sqrt{2}$, and the square of this length is $\boxed{2592}$.

30. Let Alecsis's number be a and Bettina's number be b. Then, $a + \frac{1}{b} = 2$, and $b + \frac{1}{a} = \frac{9}{4}$. Multiplying these equations gives $ab + 1 + 1 + \frac{1}{ab} = \frac{9}{2}$. Letting $ab = k$, this is $k + 2 + \frac{1}{k} = \frac{9}{2}$. We can multiply both sides by $2k$ to get the quadratic $2k^2 - 5k + 2 = 0$. This quadratic factors as $(2k-1)(k-2)$, so k is either $\frac{1}{2}$ or 2. The smaller of the two is $\frac{1}{2}$, so our answer is $1 + 2 = \boxed{3}$.

Target Round Solutions

1. Since Becca made nine fewer bracelets compared to Nicole, we find that Nicole made $15 + 9 = 24$ bracelets. Since Sophie made half as many bracelets as Nicole, we find that Sophie made $24 \div 2 = 12$ bracelets. Altogether, the total number of bracelets the three made is $15 + 24 + 12 = \boxed{51}$.

2. We can find the perimeter by counting the number of sides of squares around the figure. We count a total of 16 of these sides, which each have length 5, and so the perimeter would be $16 \times 5 = \boxed{80}$.

3. There are $20 - 3 = 17$ scouts that swam, hiked, or did both. However, the total number of scouts that swam or hiked is $13 + 12 = 25$. This counts twice the number of scouts that swam and hiked, so the number of scouts that did both is $25 - 17 = \boxed{8}$.

4. Lois earned $6 \times 3 = 18$ coins from the normal stages and $10 \times 2 = 20$ coins from the hard stages, so she so far has $18 + 20 = 38$ coins. This means that she needs $100 - 38 = 62$ coins. The key to minimizing the number of stages played is by playing more hard stages, but since hard stages earn her 10 coins, she would need to play enough normal stages such that the amount of coins earned from the normal stages she played is a multiple of 10. This can be obtained by just playing two more normal stages, which would result in $62 - 12 = 50$ coins left that could be earned from 5 hard stages. Altogether, the fewest number of stages that Lois needs to play is $2 + 5 = \boxed{7}$.

5. If the product of the tens and units digits is 4, then the tens and units digits are either 4 and 1 in some order, or 2 and 2. If the units digit is 4, then the tens digit is 1, and the hundreds digit would be $12 \div 1 = 12$, which is impossible. If the units digit is 1, then the tens digit is 4, and the hundreds digit would be $12 \div 4 = 3$, so the number would be 341. If the units digit is 2, then the tens digit is 2, and the hundreds digit is $12 \div 2 = 6$, so the number would be 622. Of the two possibilities, the greater is $\boxed{622}$.

6. The sum of four numbers in the list is $2 + 10 + 21 + 23 = 56$. Additionally, since the mean is a whole number, the sum of all five numbers must be a multiple of 5. There are three different situations to consider. If the fifth number is less than 10, then the median is 10. This occurs when the fifth number is 4 or 9, and the sum is 55 or 60, respectively. If the fifth number is greater than 23, then the median is 21. This occurs when the fifth number is 24 or a multiple of 5 more than 24. However, if the fifth number is between 10 and 21, then the median is the fifth number. The only numbers between 10 and 21 that can be added to 56 to produce a multiple of 5 are 14 and 19. The sum of all possible values of the median is $10 + 14 + 19 + 21 = \boxed{64}$.

7. Lily swam 2 laps on Monday, 4 laps on Tuesday, 6 laps on Wednesday, 8 laps on Thursday, and 10 laps on Friday. This is a total of $2 + 4 + 6 + 8 + 10 = 30$ laps. Since each lap is 50 meters, this distance is $30 \times 50 = \boxed{1500}$ meters.

8. As no triangle can share all three sides with the octagon, there are two remaining possibilities to consider.
   - The first is that the triangle shares exactly 2 sides with the octagon. These two sides of the octagon must be adjacent, so there are 8 such triangles.
   - The second possibility is that the triangle shares exactly 1 side with the octagon. Once one of the 8 sides of the octagon is selected for the triangle, the third vertex must be one of the $6 - 2 = 4$ vertices that is not adjacent to the selected side, so there are $8 \times 4 = 32$ such triangles.

   Altogether, the total number of possible triangles is $8 + 32 = \boxed{40}$.

Team Round Solutions

1. We are essentially adding the odd numbers from 1 to 19 inclusive, so we are computing $1+3+5+7+9+11+13+15+17+19$. Now we could manually add the numbers up, or we could make five pairs of numbers with a sum of 20. Then, we can observe that the sum, which is notably a perfect square, is $\boxed{100}$.

2. If every ticket purchased was a youth ticket, the total cost would be $6 \times 5 = 30$ dollars. However, for each ticket that is an adult ticket instead of a youth ticket, the total cost is increased by $8-5=3$ dollars. Since the dollar difference is $42-30=12$, the number of adult tickets is $12 \div 3 = \boxed{4}$.

3. Before the art teacher gave pencils to Kendall, the bag had $3 \times 2 = 6$ pencils. Before the art teacher gave pencils to Kay, the bag had $3+6=9$ pencils. Before the art teacher gave pencils to Bella, the bag had $9 \times 2 = 18$ pencils. This means the number of pencils was $\boxed{18}$.

4. We can work our way up from the starting time to the finish time. After 45 minutes from the starting time, it would be 11 PM. After an additional hour, it would be midnight. After an additional eight hours, it would be 8 AM. After an additional 30 minutes, the sleepover would end at 8:30 AM. Since there are 60 minutes in an hour, the total number of minutes elapsed was $45+(1+8)\times 60+30 = \boxed{615}$.

5. The unit price is lower when buying 7 red balloons for 15 banana coins compared to buying 3 red balloons for 15 banana coins. Now, note that $48 \div 7$ is 6 with a remainder of 6. The cost of buying the set of 7 red balloons six times is $15 \times 6 = 90$ banana coins. Now, Ethan can buy the set of 3 red balloons twice for $7 \times 2 = 14$ banana coins, or he can buy the set of 7 red balloons once for 15 banana coins. The cheaper option is buying the set of 3 red balloons twice, and so the fewest number of banana coins Ethan will need is $90+14 = \boxed{104}$.

6. Each strip has an area of $3 \cdot 15 = 45$ square inches. The total area of two of those strips is $2 \cdot 45 = 90$ square inches, but we need to account for the overlap. That overlap is counted twice and is a square of side length 3 inches, which has area $3 \times 3 = 9$ square inches. So, the total area, in square inches, covered by the two strips is $90 - 9 = \boxed{81}$.

7. The number must be one more than a multiple of 8 and one fewer than a multiple of 7. We can then approach this by skip counting. The first few whole numbers that are one more than a multiple of 8 are 1, 9, 17, 25, 33, 41, 49, and 57. The first few whole numbers that are one fewer than a multiple of 7 are 6, 13, 20, 27, 34, 41, 48, and 55. The first number to appear in both lists is $\boxed{41}$.

8. The figure is divided into 5 regions. Using exactly one region per triangle, there are 5 triangles. Using exactly two regions per triangle, there are 4 triangles. Using exactly three regions per triangle, there are 2 triangles. There are no triangles that use exactly four regions, and only 1 triangle that uses all five regions. Altogether, the number of triangles in the figure is $5 + 4 + 2 + 0 + 1 = \boxed{12}$.

9. We can make use of the boxes having different weights that are less than 10 pounds. Looking at the pair weighing 16 pounds, we conclude that there is a box weighing 9 pounds and a box weighing 7 pounds, since the other option would be having two boxes weighing 8 pounds, which is not allowed. Looking at the pair weighing 15 pounds, we can conclude that these two boxes must be 6 pounds and 9 pounds. If there was an 8 pound box, then we can form a pair with boxes weighing 8 pounds and 9 pounds to give 17 pounds, which is not a weight that is listed. Thus, our weights so far are 6, 7, and 9 pounds, giving pair weights of 13, 15, and 16 pounds. Finally, if we add a box of with weight 5 pounds, then we have additional pair weights of $5 + 6 = 11$, $5 + 7 = 12$, and $5 + 9 = 14$ pounds. Thus, the number of pounds that the lightest box weighs is $\boxed{5}$.

10. As shown below, let point E be the point on the larger circle where segment $\overline{AB}$ is tangent, let O and P be the centers of the circles, let Q be the point on $\overline{AD}$ so that points P, O, and Q are collinear, and let F be on $\overline{EP}$ so that $\overline{OF}$ is parallel to $\overline{AE}$. Then, quadrilateral $AOFE$ is a rectangle, and the length of $\overline{FP}$ is $40-10=30$. The length of $\overline{OP}$ is $40+10=50$. By the Pythagorean Theorem, the length of $\overline{OF}$ is $\sqrt{50^2-30^2}=40$. Note from symmetry that $\angle OPB=\angle OPC=90°$, and so angles OPF and FPB are complementary. Then, from some angle chasing, we find that $\triangle EPB$ and $\triangle QAO$ are both similar to $\triangle FOP$. As such, $\frac{OF}{OP}=\frac{PE}{PB}$, or $\frac{40}{50}=\frac{40}{PB}$, and $PB=50$. Additionally, $\frac{OF}{OP}=\frac{AQ}{AO}$, or $\frac{40}{50}=\frac{AQ}{10}$, and $AQ=8$. By the Pythagorean Theorem, $QO=6$. Note that quadrilateral $AQPB$ is a trapezoid with bases of length 8 and 50 and an altitude of $6+50=56$, so its area is $\frac{1}{2}\cdot 56\cdot(8+50)=1624$. Quadrilateral $DQPC$ is congruent to trapezoid $AQPB$, so the area of quadrilateral $ABCD$ is $2\cdot 1624=\boxed{3248}$.