



Contest Problem Set 12310

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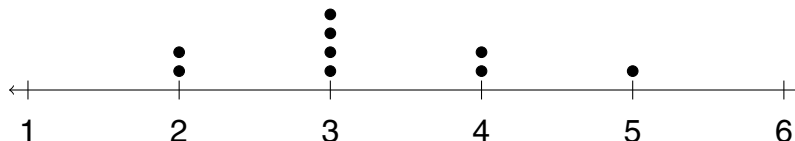
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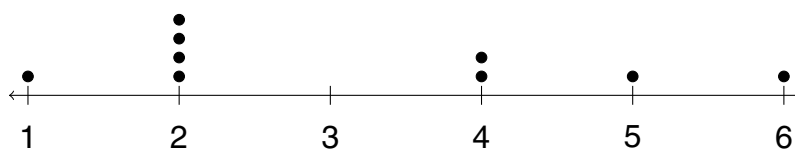
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1. What is the greatest number of right interior angles that a triangle can have?
2. Patrick has a twenty-dollar bill and 2 one-dollar bills. If he earns \$2 per week for his allowance, how many weeks will it take Patrick to have enough money to buy a new construction set worth \$30?
3. In water polo, a fair coin is flipped to determine which team goes first. If the coin lands on heads, then Lily's team gets the ball first. Otherwise, Natalie's team gets the ball first. What's the probability that Natalie's team gets the ball first? Express your answer as a common fraction.
4. In football, the "completion percentage" is the percent of the passes that are completed. Chase attempted 15 passes, and his completion percentage, when rounded to the nearest percent, is 73%. How many passes did Chase complete?
5. A triangle and a square have the same perimeter and share a side of length 6. The triangle also has a side of length 10. What is the length of the longest side of the triangle?
6. After a casual game of golf, Connor and Elise each made a dot plot shown below that represents the number of strokes done per hole. What is the mode of the list of numbers representing the number of strokes Connor or Elise did for a hole?



Strokes for a Hole (Connor)

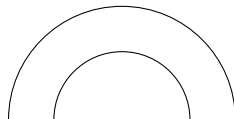


Strokes for a Hole (Elise)

7. Mark's goal is to plant 900 trees. He pilots a drone that can plant trees at a rate of 5 trees per minute. However, once he is halfway done, another drone that also plants trees at a rate of 5 trees per minute starts to work. How many minutes elapses from the time Mark starts planting trees to the time his goal is reached?
8. Rohan measures the three distances between one vertex of a rectangle and the other three vertices. If his shortest measurement is 5 inches and his longest measurement is 13 inches, how many inches are in Rohan's remaining measurement?

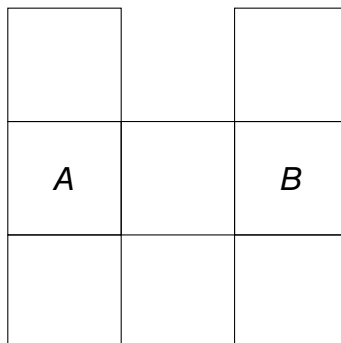
9. Amelia is working out a subtraction problem where she subtracts a three-digit number from 1624. Among the two numbers in the subtraction problem, no digits are repeated. Amelia always forgets to carry and borrow when she subtracts, but fortunately she does not need to borrow at any point when doing this subtraction problem and gets the correct answer. What answer does Amelia get?
10. A number is selected uniformly at random from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the probability that the number is odd or a multiple of 3? Express your answer as a common fraction.
11. Brian is getting stronger! His power level is currently at 300, and every hour, his power level doubles. What is the smallest whole number of hours it would take for Brian's power level to go over 8000?
12. Thomas has a robot that can use a powerful gear technique, where each use lasts 5 seconds, but after each use, the robot needs to cool down before activating again. The cool down time for the first use is 10 seconds, but each subsequent cool down time is 10 seconds longer than that of the previous cool down time. How many times could the robot activate the gear technique within 3 minutes, assuming the robot first uses the gear technique as the timer starts?
13. At a party, there are 8 people. Each pair of people shakes hands with each other once, except for one pair, who are best friends and refuse to shake hands with anyone else other than themselves. In total, how many handshakes take place?
14. Trunan now has 110 batteries available. An RC toy requires 4 batteries, which last for 30 days before they all need to be replaced, while an RC controller requires 2 batteries, which last for 40 days. If Trunan puts in all batteries at once for the RC toy and the RC controller on the first day, and he replaces batteries as soon as any batteries need to be replaced, how many days will the RC toy and the RC controller last before Trunan runs out of batteries?
15. Point A is at $(21, -12)$, point B is at $(40, 4)$, point C is at $(-78, -42)$, and point D is at $(66, -5)$. There is only one way to select two of the four points such that the slope of the line passing through the two points is negative. What is the sum of the coordinates of those two points? Note that you will be adding four whole numbers to get your answer.
16. Let x and y be real numbers such that $x^3 + y^3 = 12$ and $2x^3 + 3y^3 = 12$. What is the value of x^3 ?
17. What is the 6th smallest positive multiple of 6 that is divisible by neither 2^3 nor 3^3 ?
18. What is the probability that a randomly chosen divisor of 324 is both a multiple of 6 and a multiple of 9? Express your answer as a common fraction.

19. Collin is sketching an arch bridge below, where he draws an upward-facing semicircle with diameter 10 centimeters, then a straight horizontal line with length 2 centimeters, then another upward-facing semicircle with diameter 6 centimeters, and finally another horizontal line with length 2 centimeters going back to the starting point. To the nearest whole number, how many square centimeters is the area of Collin's arch bridge?



20. Audrey and her friends plan to visit Heaven's Park and Sakura Mountain, but they have to travel by vehicle, where each vehicle can have up to four passengers. The entrance fee for Heaven's Park is a fixed amount per vehicle, while the entrance fee for Sakura Mountain is a fixed amount per passenger. Audrey finds that the minimum cost if a group of 3 travels to both places is \$60 while the minimum cost if a group of 8 travels to both places is \$140. How many dollars does Audrey need if she, Melody, Lily, Jaedyn, and Madison all plan on travelling as a group to both places?
21. Jon has two 52-card decks where each deck has the positive whole numbers from 1 to 52, inclusive. One deck has all cards red while another deck has all cards blue. He is playing a modified version of Survival, where two of the 104 cards are marked as "harm cards." Jon knows that if A and B denote the values of the two harm cards, $AB = 90$, A is prime, and B is greater than 26. Based on this information, how many possible harm card configurations are there?
22. A positive number a is selected so that the equation $(3x + 2)(4x + 7) = (ax + 3)(x + 5)$ has exactly one solution. What is a ?
23. Right triangle ABC with right angle at B has area 9. The lengths of both AB and BC are whole numbers, and the altitude from B to \overline{AC} has length strictly greater than 2. What is AC^2 ?
24. Luke does a 2-minute street workout where he punches a speed bag. At first, he hits the bag at a rate of 1 punch per second, but every 10 seconds, he instantaneously speeds up by 1 punch per second until he plateaus at 4 punches per second. Starting at the the time when there are 60 seconds left in his workout, he instantaneously slows down by 1 punch per second every 30 seconds, until the workout ends. How many punches does he land in total during his training session?
25. Andy kept track of his sparring matches at a neo arena and found he won $53\frac{1}{3}\%$ of the matches. If he wins the next one, his win percentage will increase to $56\frac{1}{4}\%$. How many matches has Andy done so far?
26. In every calendar year, May k th and October $2k$ th fall on the same day of the week. Find the greatest possible value of k , where k is a positive whole number.

27. Jessica and Rebecca are playing a mini game where Jessica stands on square A and Rebecca stands on square B in the diagram below. On each turn, Jessica and Rebecca jump to a square chosen at random that shares a side with their respective squares at the same time, then return to their previous squares. What is the probability that after two turns, there will be at least one turn where Jessica and Rebecca jumped to the same square? Express your answer as a common fraction.



28. Quadrilateral $ABCD$ has side lengths $AB = 4$, $BC = 6$, $CD = 9$, and $DA = 9$, and diagonal length $AC = 6$. Diagonals AC and BD intersect at point E . What is $\frac{BD}{BE}$? Express your answer as a common fraction.
29. Kendra wants to arrange and divide the letters in her name to create 3 strings, each of which must contain at least one letter. In how many ways can she do so, assuming that the order in which she creates the strings is irrelevant, and she uses each letter exactly once?
30. Brandon is making a staircase out of blocks with at least one column where column 1 has 1 block, column 2 has 2 blocks, and so on. In general, column n should have n blocks. If Brandon has a collection of some number of blocks, he can use up all his blocks to make a staircase if there were either 1 more block or 8 fewer blocks. What is the sum of all possible number of blocks in Brandon's collection?



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Problems 1 & 2

1. Reagan is going on a road trip. She first travels 100 miles per day due north for seven days. Then she travels 30 miles per day due south for three days. At the end of the tenth day, how many miles due north is Reagan from her starting point?

1.

2. At the end of a soccer game, Ava's team and Grace's team line up in a line to shake hands. Ava and Grace are standing 10 meters apart and facing each other. Both Ava and Grace have 14 teammates from their respective teams standing behind them such that the teammates of each team are standing 2 meters apart. How many meters is the furthest distance from one player on Ava's team to one player on Grace's team?

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Problems 3 & 4

3. Jimmy plans on buying every food item from another store. The store has 10 rows where each row has 80 food items. Jimmy notes that 1% of all the food items are canned beans and that the total cost of all the canned beans is \$9.20. What is the average price of the canned beans in the store, in dollars? Express your answer as a decimal to the nearest hundredth.

3.

4. Let A be the intersection point of the graphs of $y = 2x^3 + 3x^2 - 4x$ and $x = -1.5$. Let B be the point $(0, 0)$. What is the area of a right triangle whose legs are parallel to the coordinate axes and whose hypotenuse is AB ? Express your answer as a common fraction.

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Problems 5 & 6

5. On a mathleague.org high school test, the Sprint portion is worth 120 points and the Target portion is worth 80 points. The lowest percentage score that one can earn on the Sprint test and still get an overall score of 70 percent is $n\%$. What is n ?

5.

6. Emma is putting together a jigsaw puzzle where all the jigsaw pieces are equilateral triangles with side length 2 centimeters. Once all of the jigsaw pieces are correctly placed, the completed jigsaw puzzle is a regular hexagon with side length 12 centimeters. Emma creates a starting pile consisting of every “edge piece,” which are jigsaw pieces that have at least one side that is not bordered by another jigsaw piece once the entire jigsaw puzzle is complete, and additional 60 “non-edge” pieces in the jigsaw puzzle. What fraction of all the pieces in the jigsaw puzzle are not in Emma’s starting pile? Express your answer as a common fraction.

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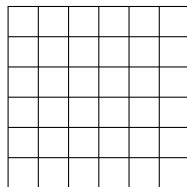
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Problems 7 & 8

7. The 6×6 rectangle below can be split into four rectangles that each have dimensions 1×5 as well as another rectangle with whole number side lengths. What is the perimeter of the fifth rectangle?



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8. An amusement park opens at 9 AM and closes at 5 PM the same day. One day, Farmer John showcases his animals in two different sessions that each last one hour. The first session starts at 11 AM while the second session starts at 2 PM. What is the probability that some part of his sessions happens somewhere in between two randomly selected times that the park is open? Express your answer as a common fraction.

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School or Team

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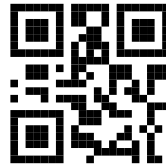
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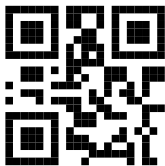
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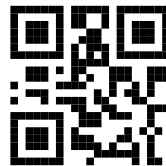
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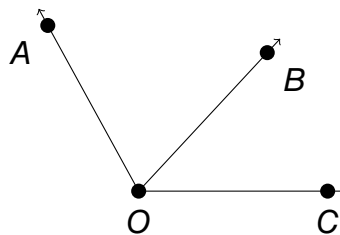
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1. A lecture hall has 10 rows where each row has 10 seats, and each person who attends a lecture sits in a seat such that each seat has at most one student. Ellen keeps track of the number of students who attend lecture each day in the below table. In how many lectures do there remain an odd number of empty seats at that time?

Lecture Number	1st	2nd	3rd	4th	5th
Students Attending	96	79	68	53	41

2. Ethan pays for a ride with the transportation service Speedy, which comes at a base fee of \$15, plus \$1.50 for each mile traveled. He would have had to pay three times the amount if he had traveled four times as far as he did. How many miles did Ethan travel?
3. Thomas turned in his final exam at 2:30 PM on June 10. He learned from his TA that the probability that the final is graded within 24 hours after the exam is turned in is $\frac{1}{2}$, the probability that the final is graded from 24 to 48 hours after the exam is turned in is $\frac{1}{3}$, and the probability that the final takes longer than 48 hours to be graded is $\frac{1}{6}$. What is the probability that Thomas's final is graded by 2:30 PM on June 12? Express your answer as a common fraction.
4. As shown in the below diagram, $\angle AOC = 118\frac{3}{4}^\circ$ and $\angle BOC = 47\frac{1}{8}^\circ$. How many degrees are in the measure of the angle supplementary to $\angle AOB$? Express your answer as a mixed number.



5. Tania is giving a presentation in which she speaks at 100 words per minute. Each slide of her presentation consists of anywhere between 40 and 125 words. If her presentation is between 10 and 12 minutes long, what is the positive difference between the minimum and maximum number of slides that she has?

6. Lucas has 15 acorns, 10 metal fragments, and 20 smooth stones available to make capsules. The below table lists the number of resources needed to make each capsule. What is the greatest number of capsules that Lucas could make?

Capsule	Acorns Needed	Metal Fragments Needed	Smooth Stones Needed
Standard	1	1	1
Light	1	1	0
Heavy	1	0	2

7. Garrett found a way to fix his leak and drain the water such that the water exits the bathtub at a constant rate. At 6:00 PM, the bathtub has 33 gallons of water, and at 6:03 PM on the same day, the bathtub has 16 gallons and 8 cups of water. How many *seconds* past 6:03 PM will it take for the bathtub to be completely empty? Note that 1 gallon equals 16 cups.

8. Chloe encounters a plane with four engines, but all but one engine has two broken parts. Chloe fixes one part every day, but for every day after the first, there is a $\frac{1}{2}$ probability that a friend joins Chloe and fixes one part every day. What is the probability that all the broken parts are fixed within four days? Express your answer as a common fraction.

9. Four equilateral triangles in a plane, each with side length 4, all share one common vertex. The eight unshared vertices form an equiangular octagon. The closest distance between any two unshared vertices can be expressed as $m\sqrt{n} - p\sqrt{q}$, where m , n , p , and q are positive whole numbers, and n and q are not divisible by the square of any prime. What is $m + n + p + q$?

10. Professor Christopher devises a method to encrypt a secret positive whole number n . He creates the sequence a_0, a_1, a_2, \dots such that $a_0 = n$ and a_{k+1} is the result when dividing a_k by 2 then rounding down to the nearest whole number for $k > 0$. His encrypted number would then be the sum of all non-zero numbers in the sequence. For example, if his secret number is 5, then the encrypted number is $5 + 2 + 1 = 8$. How many positive whole numbers have encrypted numbers that are less than or equal to 50?

1. Bella's bubble soccer team has 9 players, including Drake. She has to pick 5 players to be on the field, but she wants one of the players picked to be Drake. How many ways can Bella pick the rest of the players?
2. How many seconds are in a third of a day?
3. Matt is in a hall of mirrors that has 100 mirrors, each in a shape of a parallelogram. He observes that 56 mirrors are not rectangles, 42 mirrors are not rhombuses, and 14 mirrors are squares. How many mirrors are rectangles but not squares?
4. Point A has coordinates (24, 18) and the midpoint of AB is (3, -9). What is the product of the coordinates of B?
5. Daniel has a collection of trading cards about legendary dragons that are either fire, electric, or ice. He has 8 fewer fire cards compared to electric cards and ice cards combined, 12 fewer electric cards compared to fire cards and ice cards combined, and 4 fewer ice cards compared to fire cards and electric cards combined. If Daniel stores half of the total number of cards in the black case, how many cards are in the black case?
6. Evaluate: $38 + 16 \cdot 7 - 29$.
7. Calculate: $19 \cdot 301 \cdot 1001$.
8. The bullseye of one target is a circular area with area 30 square inches. The below table lists the distances from two arrows of each player to the center of that bullseye. How many arrows landed in the bullseye?

Player	1st Arrow Distance	2nd Arrow Distance
Andrea	2 inches	4 inches
Belinda	3 inches	3 inches
Chelina	3.5 inches	2.5 inches
Diana	1.5 inches	4 inches
Evelyn	3 inches	2 inches

9. Find the GCD of 8099 and 7800.
10. Rebecca's fan is a sector of a circle with radius 7.2 centimeters and a central angle of 140° . How many square centimeters is the area of Rebecca's fan when rounded to the nearest whole number?
11. An unfair coin lands heads $\frac{6}{7}$ of the time. If the coin is flipped 3 times, what is the probability that at most two of the flips land heads? Express your answer as a common fraction.

12. How many times does $\frac{18}{35}$ go into $\frac{1}{14}$? Express your answer as a common fraction.
13. Andy divides a rectangular field with area 240 square feet into smaller plots that are squares that each have area 16 square feet. The tomato plots do not share an edge with the edge of the entire rectangular field, and the total area of the tomato plots is 48 square feet. What is the perimeter of Andy's rectangular field?
14. Find the largest prime divisor of 7198.
15. Express $\frac{8}{21} + \frac{7}{18}$ as a common fraction.
16. Becca needs to obtain the books shown in the below table. Fortunately, a friend gave her used copies of the two most expensive books on the list, so Becca only needs to buy the rest of the books. If Becca has \$3.28 left after buying the books, how many dollars does she have before buying the books? Express your answer as a decimal to the nearest hundredth.

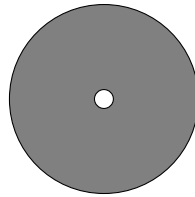
Book	Cost
Blazing World	\$12.99
Fahrenheit 451	\$15.99
Frankenstein	\$8.99
Secret Life of Bees	\$11.99
The Odyssey	\$10.99

17. The area of a 12-sided regular polygon inscribed in a circle with radius r is given by $A = 3r^2$. If a circle has a circumference of 28π , what is the area of a 12-sided regular polygon inscribed in it?
18. How many digits are in one of the numbers 24750 and 128053 but not both?
19. A circle is divided into a number of pieces by 6 straight lines, where each pair of lines is either parallel or perpendicular. What is the maximum number of pieces the circle is divided into?
20. Eugene and Marian are working on a jigsaw puzzle, where the puzzle once completed has 20 rows with each row having 25 pieces. So far, Marian completed the space ranger part, which used up 20% of the total pieces. Then Eugene completes the tricky green slime monster part, which uses 37.5% of the pieces not used for the space ranger part. How many remaining pieces are left?
21. If $f(x) = 303x + 33$, then find $f(33)$.
22. Below are five whole numbers. What is the sum of all the numbers that are not divisible by 7?
- 109
 - 110
 - 111
 - 112
 - 113
23. A basketball game is 48 minutes long and has four quarters with each quarter lasting the same time. At the end of the third quarter, the Tunes have 73 points while the Monsters have 81 points. The Tunes average 3 points per minute in the fourth quarter, while the Monsters average 2 points per minute in the fourth quarter. At the end of the fourth quarter, what is the score of the team with the most points?

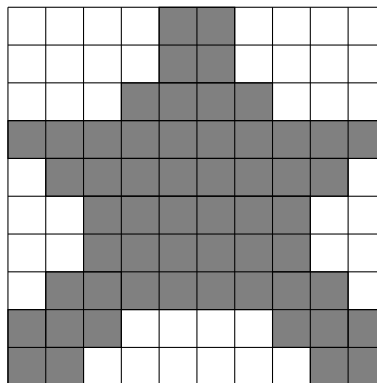
24. Find the distance between the point $(0,0)$ and the line $5x + 12y = 60$. Express your answer as a decimal to the nearest tenth.
25. Maia kept track of how many pictures she took at national parks this year as well as the state of the national park in the below table. Last year, Maia took 20 pictures in Yosemite, but this year, she took 25% more pictures at Yosemite compared to last year. What percent of the national park pictures that Maia took this year were in California?

National Park	State	Pictures Taken
Crater Lake	Oregon	15
Grand Canyon	Arizona	18
Sequoia	California	12
Yosemite	California	
Yellowstone	Idaho, Montana, Wyoming	30

26. What is the sum of the three smallest integers n that satisfy the inequality $|n - 7| < 12$?
27. Find the 101st number in the arithmetic sequence: 275, 371, 467, 563, ...
28. Andrew needs four wheels for his gravity vehicle. The diagram below is a cross-section of one cylindrical wheel that is parallel to one of the faces, where the shaded sections are part of the wheel. The hole is a cylinder with radius 0.5 inch that runs through the entire larger cylinder, which has radius 5 inches. The height of the cylinder is 1 inch. How many cubic inches is the total volume he needs? Express your answer in terms of π .



29. If $x = 15$, what is the value of $2x^2 + (-2x)^2$?
30. A large square is divided into 100 unit squares, as shown below. What percent of the area of the large square is shaded?



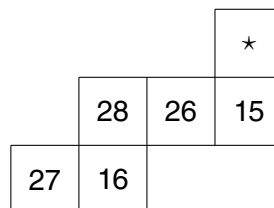
31. What is the second largest possible integer value of $30 - x^2$?

32. Cate is selling corsages for \$12 and boutonnieres for \$8. If John has \$76 and buys two corsages, what is the greatest amount of boutonnieres that John can buy with the remaining cash?
33. Find the shortest distance between the graphs of the equations $(x + 4)^2 + (y - 2)^2 = 64$ and $(x - 8)^2 + (y + 3)^2 = 4$.
34. What is the surface area, in square inches, of a rectangular prism with side lengths 21 in, 19 in, and 10 in?
35. How many ways are there to draw a rectangle from a 9×2 grid of squares if the sides of the rectangle are on the lines of the grid?
36. Olivia needs 2 boxes of spinach and 1 onion to make 4 servings of spinach salad. She currently has 36 onions and 25 boxes of spinach, but a sailor who loves spinach gave her 45 boxes of spinach. After getting the extra boxes of spinach, what is the greatest number of servings of spinach salad that Olivia can make?
37. How many diagonals are there in a regular pentagon?
38. What is the least common multiple of the first five even positive integers?
39. Evaluate: $193 + 359$.
40. At a carnival, blue tickets are worth 2 points while gold tickets are worth 5 points. In one activity, the reward for scoring less than 5 points is a gold ticket, while scoring otherwise would reward a blue ticket. Jake earned 5 points in his first time, 4 points in his second time, and 6 points in his third time. Ashley earned 4 points in her first time, 4 points in her second time, and 5 points in her third time. How many points did the two get after all their games?
41. What is the degree measure of the supplement of an angle whose complement has a measure of 86.555 degrees? Express your answer as a decimal without trailing zeroes.
42. Evan went to sleep at 10:20 PM and woke up at 6:30 AM the next day. He spent 20% of the time sleeping in deep sleep. How many minutes did Evan spend in deep sleep?
43. Simplify: $\frac{18}{\frac{35}{\frac{7}{2}}}$. Express your answer as a common fraction
44. How many distinct positive integers n are there such that $n > 6$ and the remainder when 153 is divided by n is 6?
45. A garden has 4 white sage plants, 1 elderberry plant, and 8 coyote brush plants. Chloe selects one plant at random. What is the probability that the plant is a coyote brush plant? Express your answer as a common fraction.
46. Two fair six-sided dice are thrown. What is the probability that at least one of them lands on a 2 or a 5? Express your answer as a common fraction.
47. Solve for a if $5a + 4b = 535$ and $-8a + 2b = -520$.

48. What is the smallest integer value of y such that the equation $x^2 = 58 - 2y$ has no real solutions?
49. Calculate: $16^3 - 15^3$.
50. Dillon is watching one pair of twins doing three laps for an exercise drill in PE. He recorded the results in the below table. How many seconds is the average time of the twin with the faster average time?

Lap	Time for Twin 1	Time for Twin 2
1	5 minutes 3 seconds	5 minutes 10 seconds
2	4 minutes 45 seconds	4 minutes 55 seconds
3	4 minutes 57 seconds	4 minutes 48 seconds

51. In $\triangle ABC$, $\angle A = 28\frac{9}{10}^\circ$ and $\angle B = 59\frac{1}{5}^\circ$. What is the degree measure of the exterior angle to $\angle C$? Express your answer as a mixed number
52. What is the value of $939 + 949 + 959 + 961 + 971 + 981$?
53. Jacob draws the lines with equations $y = 3x + 8$ and $y = -3x + 8$ and $y = 3x + 2$, and Desi draws the lines with equations $y = 2x + 3$ and $y = 3x - 8$ and $y = 2x + 5$. How many intersection points are there between one of Jacob's lines and one of Desi's lines?
54. A cube has a surface area of 13.5 square inches. What is the number of cubic inches in the volume of the cube? Express your answer as a common fraction.
55. The ratio between two consecutive positive multiples of 5 is $\frac{11}{12}$. Find the sum of the integers.
56. When the figure shown below is folded into a cube such that the squares shown become faces of the cube, what is the sum of all the numbers on all the faces that share an edge with the face marked with a star?



57. Ann is putting up flyers in 5 different non-overlapping square city boroughs that each have side length 15 kilometers for an resilience festival. The city has an area of 3750 square kilometers and a population of 25000, and she expects that the population density, with the units of people per square kilometer, will be the same within each borough as the entire city. Based on the given information, how many people would Ann expect to live in the square city borough with the petitions?
58. What is the least integer greater than $5\sqrt{13}$?
59. Find the number such that the sum of its double, triple, and quadruple is 711.

60. Lani measured how much storage four programs take up and recorded the results in the below table, where 1 terabyte (TB) equals 1024 gigabytes (GB). How many gigabytes is the median storage?

Program	Storage
rock.exe	3000 GB
roll.exe	0.25 TB
blues.exe	512 GB
forte.exe	1 TB

61. Evaluate: 6.2×0.22 . Express your answer as a decimal without trailing zeroes.
62. What is the slope of the line $140x + 142y = 144$? Express your answer as a common fraction.
63. There are 61 points on a circle. How many ways are there to draw a triangle with vertices chosen from the 61 points?
64. Ellie has 5760 crystal beads and 4608 pieces of ribbon string. She plans on putting the supplies into as many boxes as she could such that the crystal beads are divided evenly among the boxes and the ribbon strings are divided evenly among the boxes. She then plans to place as many boxes as she could into a big-rig that could carry at most 1000 boxes. How many boxes would not be in the big-rig?
65. Given that $a^b = 729$ and a and b are both integers, what is the smallest possible value of a ?
66. Find n if $2^{10} + 2^{10} + 2^{10} + 2^{10} = 4^n$.
67. Joeli observes some laths, which are rectangular pieces of wood, while surveying houses. She records the length and width of each lath in the below table. How many square centimeters is the range of the areas of the laths with the three biggest perimeters?

Lath	Length (centimeters)	Width (centimeters)
A	29	37
B	17	28
C	6	85
D	23	56
E	62	83

68. This morning, Melcka answered 27 customer care emails. Then April answered some emails. Afterwards, Roselle answered twice as many emails as April did. Finally, Yuna answered 32 emails so that the total number of emails answered today was 110. How many emails were answered by Roselle?
69. It takes three mathleague.org staff members 40 minutes to produce a sonnet. One staff member assembles the words, one edits for style and grammar, and one performs it in front of a focus group of would-be suitors. How many sonnets can fourteen mathleague.org staff members produce in a summer's day? Remember, like all days, a summer's day contains 24 hours, but it just happens to be more lovely and more temperate.
70. 20% of what number is 50% of 286?
71. What is the volume of a cube with side length 19?

72. How many two-digit prime numbers have no prime digits?

73. At a robot colosseum, each attendee roots for either the buzz-saw hammer robot or the wild cutter robot. Kyle keeps track of how many attendees rooted for each robot over six rounds in the below table. On which round number is the probability that a random attendee of that round rooted for the buzz-saw hammer robot the greatest?

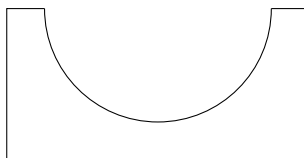
Round Number	Buzz-Saw Hammer	Wild Cutter
1	1769236	1856354
2	1856295	1856385
3	1856425	1254853
4	1859476	1649265
5	1967262	1967251
6	1866251	1856254

74. Compute: $\frac{24}{3} + 4 \cdot 7$.

75. In how many ways can the letters in the word NATIONAL be arranged?

76. What is the maximum number of acute angles that can be formed from four lines that intersect in a single point?

77. Andersen is designing one of the walls of a seat for a roller coaster. On the blueprint, he made sketch of that wall, where a semicircle of diameter 6 centimeters is taken out of the top of the rectangle with length 8 centimeters and height 4 centimeters, as shown in the below diagram. The scale indicates that 1 centimeter on the blueprint equals 0.5 meters in real life. To the nearest tenth of a square meter, what is the area of the real life wall?



78. How many proper subsets does the set $\{1, 2, 4, 5, 6, 7\}$ have?

79. The average of three numbers is 60. If the largest number is 68, then what is the average of the two smallest numbers?

80. Nathan and Grace are running along the same path starting from the same point. Nathan runs at 10 km/h and Grace runs at 15 km/h. If Nathan has a 500 meter head start, how many minutes must Grace run before catching up to Nathan?

Sprint Round

- | | | |
|-------------------|-------------------|---------------------|
| 1. 1 | 11. 5 | 21. 8 |
| 2. 4 | 12. 6 | 22. 12 |
| 3. $\frac{1}{2}$ | 13. 16 | 23. 45 |
| 4. 11 | 14. 600 | 24. 330 |
| 5. 10 | 15. 105 | 25. 15 |
| 6. 2 | 16. 24 | 26. 15 |
| 7. 135 | 17. 42 | 27. $\frac{17}{81}$ |
| 8. 12 | 18. $\frac{2}{5}$ | 28. $\frac{13}{4}$ |
| 9. 1121 | 19. 25 | 29. 1200 |
| 10. $\frac{2}{3}$ | 20. 110 | 30. 67 |

Target Round

1. 610
2. 66
3. 1.15
4. $\frac{9}{2}$
5. 50
6. $\frac{5}{9}$
7. 16
8. $\frac{13}{16}$

Team Round

1. 3
2. 20
3. $\frac{5}{6}$
4. $108\frac{3}{8}$
5. 22
6. 15
7. 180
8. $\frac{3}{4}$
9. 12
10. 27

Countdown

- | | | | |
|-----------------------|--------------|-----------------------------|----------------------|
| 1. 70 | 21. 10032 | 41. 176.555 | 61. 1.364 |
| 2. 28800 | 22. 443 | 42. 98 | 62. $-\frac{70}{71}$ |
| 3. 30 | 23. 109 | 43. $\frac{36}{245}$ | 63. 35990 |
| 4. 648 | 24. 4.6 | 44. 4 | 64. 152 |
| 5. 12 | 25. 37(%) | 45. $\frac{8}{13}$ | 65. -81 |
| 6. 121 | 26. -9 | 46. $\frac{5}{9}$ | 66. 6 |
| 7. 5724719 | 27. 9875 | 47. 75 | 67. 4906 |
| 8. 7 | 28. 99 π | 48. 30 | 68. 34 |
| 9. 13 | 29. 1350 | 49. 721 | 69. 168 |
| 10. 63 | 30. 56(%) | 50. 295 | 70. 715 |
| 11. $\frac{127}{343}$ | 31. 29 | 51. $88\frac{1}{10}(\circ)$ | 71. 6859 |
| 12. $\frac{245}{9}$ | 32. 6 | 52. 5760 | 72. 6 |
| 13. 64 | 33. 3 | 53. 7 | 73. 3 |
| 14. 61 | 34. 1598 | 54. $\frac{27}{8}$ | 74. 36 |
| 15. $\frac{97}{126}$ | 35. 135 | 55. 115 | 75. 10080 |
| 16. (\$)35.25 | 36. 35 | 56. 96 | 76. 12 |
| 17. 588 | 37. 5 | 57. 7500 | 77. 4 |
| 18. 5 | 38. 120 | 58. 19 | 78. 63 |
| 19. 16 | 39. 552 | 59. 79 | 79. 56 |
| 20. 250 | 40. 21 | 60. 768 | 80. 6 |

Sprint Round Solutions

1. We can easily draw a triangle that has one right angle. However, if a triangle has two or more right angles, then the sum of the angles would be greater than 180° . Therefore, a triangle cannot have two or more right interior angles, so the greatest number of right interior angles must be 1.
2. Patrick currently has \$22. We could skip count by twos to get the answer, or we can write an equation where x is the number of weeks elapsed, which yields $22 + 2x = 30$. Either way, we find that it will take 4 weeks.
3. A fair coin has two sides that come up with equal probability – heads or tails. Of the two possibilities, only tails results in Natalie going first. Therefore, the probability that Natalie goes first is $\frac{1}{2}$.
4. We can reasonably expect the answer to be less than 12 as $16 \cdot \frac{3}{4} = 12$. In fact, $\frac{11}{15}$ equals 0.733 to three decimal places, so Chase completed 11 passes.
5. The perimeter of the square is $4 \cdot 6 = 24$. Thus, the perimeter of the triangle is also 24, and the length of the third side is $24 - 6 - 10 = 8$. Thus, the sides of the triangle are 6, 8, and 10, and the longest length is 10.
6. Recall that the mode is the data value that appears the most often. When combining the data from the two dot plots, we can observe that the mode would be 2.
7. For the first 450 trees, one drone would take 90 minutes. For the next 450 trees, two drones would be working. The combined rate is 10 trees per minute, and so it would take 45 minutes. Therefore, the total amount of time elapsed in minutes is $90 + 45 =$ 135.
8. The longest measurement will be the diagonal of the rectangle, and the other two measurements will be the side lengths of the rectangle. By the Pythagorean theorem, the sum of the squares of the two side lengths is equal to the square of the diagonal, so $5^2 + a^2 = 13^2$ where a is the remaining measurement. Solving gives $a =$ 12 inches.
9. If no borrowing occurs, then in the three-digit number the hundreds digit must be less than 6, the tens digit must be less than 2, and the ones digit must be less than 4. Since 1 is already used, the tens digit must be 0. Then the ones digit must be 3 and the hundreds digit must be 5. The answer to the subtraction problem is $1624 - 503 =$ 1121.

10. The elements in the set that are odd or multiples of 3 are 1, 3, 5, 7, 9, and 6, for a probability of $\frac{2}{3}$.
11. One way is to make a list where we keep track of the number of times Brian's power level doubles until the power level is greater than 8000. Alternatively, we can write an equation where x is the number of hours, and the equation is $300 \cdot 2^x > 8000$. Either way, we find that the minimum positive whole number x must be $\boxed{5}$.
12. We could consider the time during the gear technique as well as the cool down time right before that as one block of time. The first block is 5 seconds, but the next block is 15 seconds, then 25 seconds, and so on. Thus, we want to find the point where the sum $5 + 15 + 25 + \dots$ grows larger than 180. By adding our way up, we find that $5 + 15 + \dots + 55 = 180$ exactly. This means that immediately after the robot uses the gear technique for the sixth time, the three minutes has elapsed. In total, the robot can use the gear technique for $\boxed{6}$ times in 3 minutes.
13. There are $\binom{8-2}{2} = 15$ handshakes among the 6 people who are not in the last pair, and 1 handshake among the best friends pair, giving $\boxed{16}$ handshakes in total.
14. Every $\text{lcm}(40, 30) = 120$ days, the RC toy uses 16 batteries while the RC controller uses 6 batteries, so 22 batteries in total are used every cycle. This means 110 batteries are used up after five cycles of 120 days, which is $\boxed{600}$ days.
15. First we need to determine what the two points are. We could calculate the slope, but a faster way is to do a rough sketch of the points on the graph and notice that the two points are B and D. The sum of the coordinates of points B and D is $40 + 4 + 66 - 5 = \boxed{105}$.
16. By subtracting the first equation from the second, we obtain $x^3 + 2y^3 = 0$. Thus, $y^3 = -12$ since $x^3 + y^3 = 12$; therefore, $x^3 = \boxed{24}$.
17. A multiple of 6 must contain at least one factor of both 2 and 3. Thus, the powers of 2 and 3 in the prime factorization must be either 1 or 2. We can count manually to get that the 6th smallest whole number with this property is $7 \cdot 6 = \boxed{42}$, as 24 is divisible by 2^3 .
18. Being a multiple of 6 and 9 is the same as being a multiple of 18. $324 = 2^2 \cdot 3^4$ is small enough that we can count the factors which are multiples of 18 by hand: of the $(2 + 1)(4 + 1) = 15$ factors of 324, the factors 18, 36, 54, 108, 162, and 324 are divisible by 18. There are 6 such factors, giving a probability of $\frac{6}{15} = \frac{2}{5}$. (An alternative way of finding the factors that are multiples of 18 is to factor $18 = 2 \cdot 3^2$, and observe that there are $(1 + 1)(2 + 1) = 6$ multiples of 18 that go into 324.)
19. The arch bridge is essentially a large semicircle with radius 5 cm with a smaller semicircle with radius 3 cm taken out. Its area is then $\frac{1}{2}\pi(5^2 - 3^2) = 8\pi$. By using either 3.14 or $\frac{22}{7}$ for π as an approximation, we find that the area of the arch bridge is approximately $\boxed{25}$ square centimeters.

20. Let a be the entrance fee per passenger for Sakura Mountain and let b be the entrance fee per vehicle for Heaven's Park. A group of three needs one vehicle, so we get the equation $3a + b = 60$. A group of eight needs two vehicles, so we get the equation $8a + 2b = 140$. Then we have a system of equations that we can solve. One possible approach is elimination where subtracting $2(3a + b) = 2 \cdot 60$ from $8a + 2b = 140$ results in $2a = 20$, resulting in $a = 10$ then $b = 30$. Now if Audrey, Melody, Lily, Jaedyn, and Madison all travel as a group, they need two vehicles, so the cost would be $5 \cdot 10 + 2 \cdot 30 = \110 .
21. We begin solving this problem by determining the prime factors of 90. The prime factors of 90 are 2, 3, and 5. Then we find that $2 \cdot 45 = 90$ and $3 \cdot 30 = 90$ but $5 \cdot 18 = 90$. This means that there are only 2 ways to pick the numbers. However, there are no restrictions on color, so there are $2 \cdot 2^2 = 8$ possible harm card configurations.
22. There are two possibilities: the equation that results from expanding and simplifying either results in a linear equation or a quadratic with a double root. In the first case, the coefficient of x^2 must be 0; this can be attained when $3 \cdot 4 = a \cdot 1$, or when $a = 12$. In the second case, simplifying results in $(12 - a)x^2 + (26 - 5a)x - 1 = 0$. The discriminant should be zero, so $(26 - 5a)^2 + 4(12 - a) = 0$. However, that quadratic does not have any real solutions. Therefore, the only way there can be only one solution to the equation is when $a = 12$.
23. Let the altitude from B to \overline{AC} intersect \overline{AC} at D . We have $AC \cdot BD = 18$, with $BD > 2$, so $AC < 9$. Then since ABC is a right triangle, $AB \cdot BC = 18$. By the Pythagorean Theorem, $AB^2 + BC^2 = AC^2 < 9^2$. Therefore, AB and BC must be 3 and 6 since $2^2 + 9^2 = 85$ and $1^2 + 18^2 = 325$ are greater than 81, meaning that the only possible value of AC^2 is $3^2 + 6^2 = 45$.
24. In the first 30 seconds, Luke hits the bag $10 + 20 + 30 = 60$ times. Then in the next 30 seconds, Luke hits $40 + 40 + 40 = 120$ times. Finally, in the last minute, Luke hits $30(3 + 2) = 150$ times. Hence, his total is $60 + 120 + 150 = 330$ punches.
25. First, note that $53\frac{1}{3}\% = \frac{8}{15}$ and that $56\frac{1}{4}\% = \frac{9}{16}$. We could set up an equation, but a quicker way is observing how the difference between the numerators and the difference between the denominators are both 1. This means that Andy has done 15 matches.
26. From May k th to May 31st, there are $31 - k$ days. Between May 31st and September 30th, there are $30 + 31 + 31 + 30 = 122$ days. Finally, there are $2k$ more days until October 2kth. Thus, the total number of days that pass is $(31 - k) + 122 + 2k = 153 + k$, which should be a multiple of 7. This implies that k leaves a remainder of 1 when divided by 7. Additionally, since there are 31 days in October, $2k$ is at most 31, so $k \leq 15$. The greatest whole number up to 15 that leaves a remainder of 1 when divided by 7 is 15, which is the greatest possible value of k .
27. The only time where where Jessica and Rebecca can choose the same square is when Jessica and Rebecca chose the square right between square A and square B, and they each have a $\frac{1}{3}$ probability of choosing that square. This means that the probability that Jessica and Rebecca choose the same square at a turn is $\frac{1}{9}$. Since the probability that the two turns result in them picking different squares is $(\frac{8}{9})^2 = \frac{64}{81}$, the probability that there is at least one turn where the same square is chosen is $1 - \frac{64}{81} = \frac{17}{81}$.

28. By SSS similarity, triangles ABC and ACD are similar, so $\angle BAC = \angle CAD$. Therefore, AE bisects $\angle BAD$, and by the Angle Bisector Theorem, $\frac{DE}{BE} = \frac{DA}{BA} = \frac{9}{4}$. Then $\frac{BD}{BE} = \frac{DE}{BE} + \frac{BE}{BE} = \frac{13}{4}$.

29. There are $\binom{6}{3} = 20$ ways to choose the starting letters of the strings. Then, there are $3! = 6$ ways to order the remaining three letters, and there are $\binom{5}{2} = 10$ ways to split the letters using dividers into three strings to attach to the starting letters. Therefore, the number of sets of strings is $20 \cdot 6 \cdot 10 = 1200$.

30. Let B be the number of blocks in Brandon's collection. Then $B+1 = \frac{m(m+1)}{2}$ and $B-8 = \frac{n(n+1)}{2}$ for some positive whole numbers $m > n$. Taking the difference of these equations gives $9 = \frac{m(m+1)}{2} - \frac{n(n+1)}{2}$, which could be simplified to $18 = m^2 - n^2 + m - n$. Note that by difference of squares, $m^2 - n^2 = (m+n)(m-n)$, so the right hand side factors as $18 = (m+n+1)(m-n)$. Since $m+n+1$ and $m-n$ have to be positive whole numbers, and $m+n+1 > m-n$, we consider the factors of 18 as cases.

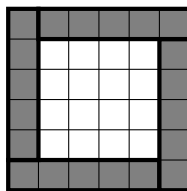
- If $m+n+1 = 18$ and $m-n = 1$, then $2m+1 = 19$, so $m = 9$ and $n = 8$. This corresponds to $\frac{9 \cdot 10}{2} - 1 = 44$ blocks.
- If $m+n+1 = 9$ and $m-n = 2$, then $2m+1 = 11$, so $m = 5$ and $n = 3$. This corresponds to $\frac{5 \cdot 6}{2} - 1 = 14$ blocks.
- If $m+n+1 = 6$ and $m-n = 3$, then $2m+1 = 9$, so $m = 4$ and $n = 1$. This corresponds to $\frac{4 \cdot 5}{2} - 1 = 9$ blocks.

Thus, the sum of the possible values of B is $44 + 14 + 9 = 67$.

Target Round Solutions

1. We can skip count or multiply to find out Reagan's position. Reagan travels $100 \cdot 7 = 700$ miles due north for the first seven days and $30 \cdot 3 = 90$ miles due south for the next three days. Thus, Reagan is $700 - 90 = \boxed{610}$ miles due north from her starting point.
2. We want the distance from Ava's teammate that is furthest from Ava to Grace's teammate that is furthest from Grace. When 15 teammates are standing 2 meters apart, there are 14 spaces of 2 meters and so the distance from the person in front to the person in the back is $14 \cdot 2 = 28$ meters. Since Ava and Grace are standing 10 meters apart, the distance would be $28 \cdot 2 + 10 = \boxed{66}$ meters.
3. The total number of food items in the store is $80 \cdot 10 = 800$. Since 1% of the food items in the store are canned beans, the store has a total of 8 canned beans. Thus, the average price of the canned beans (in dollars) is $\frac{\$9.2}{8} = \$\boxed{1.15}$.
4. First, find point A by substituting $x = -1.5$ into the equation $y = 2x^3 + 3x^2 - 4x$. This gives $y = 6$, so $A = (-1.5, 6)$. This indicates that the triangle has base 1.5 and height 6, so its area is $\boxed{\frac{9}{2}}$.
5. Assume a perfect Target score. 70 percent of 200 is 140, so 60 points can be missed in total. If all of these points are on Sprint, the resulting score is $60/120$, which is $\boxed{50}$ percent. (Another way is to recognize that Sprint is worth 60% and Target is worth 40%.)
6. A regular hexagon with side length 12 centimeters is made up of six equilateral triangles with side length 12 centimeters. Since the jigsaw pieces are equilateral triangles with side length 2 centimeters, it takes $1 + 3 = 4$ jigsaw pieces to make an equilateral triangle with side length 4 centimeters, $1 + 3 + 5 = 9$ jigsaw pieces to make an equilateral triangle with side length 6 centimeters, and $1 + 3 + 5 + 7 = 16$ jigsaw pieces to make an equilateral triangle with side length 8 centimeters. Following this pattern, it takes 36 jigsaw pieces to make an equilateral triangle with side length 12 centimeters. The completed puzzle is composed of 6 equilateral triangles of side length 12 centimeters, which equates to $36 \cdot 6 = 216$ jigsaw pieces. The edge pieces form the perimeter of the completed puzzle, and since each side of the hexagon is 12 centimeters long, the total number of edge pieces is $\frac{12}{2} \cdot 6 = 36$ pieces. Thus, Emma's starting pile has $36 + 60 = 96$ pieces, which means $216 - 96 = 120$ jigsaw pieces are not in Emma's starting pile, so $\frac{120}{216} = \boxed{\frac{5}{9}}$ of all the pieces in the jigsaw puzzle are not in Emma's starting pile.

7. We could try experimentation but another way to approach this problem is by area. The area of the 6×6 rectangle is 36, and the area of each 1×5 rectangle is 5. This means that the area of the fifth rectangle is $36 - 4 \cdot 5 = 16$. Since the lengths of the fifth rectangle are whole numbers, we can consider pairs of factors that multiply to 16. Furthermore, by observing that the rectangle must actually fit in the 6×6 rectangle, we can conclude that the remaining rectangle must be a 4×4 rectangle, which has a perimeter of $\boxed{16}$, which indeed works as shown in the setup below.



8. There are a lot of cases if we approach directly, so a good approach is by complementary counting. The two sessions split the times where there isn't a session into three parts. Now observe that the first session ends at 12 PM and the second session ends at 3 PM. This means that each of the parts are 2 hours long each, and the total time the amusement park is open is 8 hours. A part of a session does not happen between the times if and only if both times are in the same part without a session, so the probability that there is no part of the session between the times is $3 \cdot \left(\frac{1}{4}\right)^2 = \frac{3}{16}$. With the complement found, we find that our desired probability and answer is $1 - \frac{3}{16} = \boxed{\frac{13}{16}}$.

Team Round Solutions

1. There are a total of $10 \cdot 10 = 100$ seats. We could do subtraction and count the number of even and odd numbers, but a faster way is to use parity. In particular, an even number minus an even number is an even number, and an even number minus an odd number is an odd number, so we just want to count the number of days where there are an odd number of students attending. Based on the table, we find that there are $\boxed{3}$ lectures with an odd number of empty seats.
2. Let m be the distance Ethan traveled. The cost for traveling $4m$ miles is $15 + 4m \cdot 1.50 = 15 + 6m$, while the cost for m miles is $15 + 1.50m$. Then $15 + 6m = 45 + 4.5m$, or $1.5m = 30$. Solving for m yields $m = \boxed{20}$.
3. If the final is graded by 2:30 PM on June 12, then the final must be graded within 48 hours. All the probabilities listed in the problem statement are mutually exclusive, so the probability that Thomas's final is graded by then is $\frac{1}{2} + \frac{1}{3} = \boxed{\frac{5}{6}}$.
4. By the Angle Addition Postulate, $\angle AOC = \angle AOB + \angle BOC$, so $\angle DBE = 118\frac{3}{4}^\circ - 47\frac{1}{8}^\circ = 71\frac{5}{8}^\circ$. Since supplementary angles add up to 180° , the measure of the angle supplementary to $71\frac{5}{8}^\circ$ is $180^\circ - 71\frac{5}{8}^\circ = \boxed{108\frac{3}{8}^\circ}$.
5. The length of the presentation multiplied by the number of words per minute, divided by the number of words per slide, equals the number of slides. The minimum number of slides is then $\frac{10 \cdot 100}{125} = 8$, and the maximum is $\frac{12 \cdot 100}{40} = 30$. Thus, the difference is $\boxed{22}$.

6. Each capsule requires an acorn, so the upper bound is at most 15. To show that 15 capsules are attainable, we can observe that Lucas can make 10 light capsules with 10 acorns and 10 metal fragments, then 5 heavy capsules with 5 acorns and 10 smooth stones. This means that Lucas can make up to $\boxed{15}$ capsules.
7. First we observe that 16 gallons and 8 cups of water equals 16.5 gallons of water. Then we find that water flows out at a rate of 5.5 gallons per minute. Now $16.5 = 5.5 \cdot 3$ meaning the remaining amount of water will drain in 3 minutes, and so $3 \cdot 60 = \boxed{180}$ seconds would have elapsed.
8. A total of $3 \cdot 2 = 6$ parts need to be fixed. The only cases where Chloe needs to take longer than four days is when she either works all by herself or only one friend joins for the fourth day. Both of these scenarios happen when no one joined for the second and third day, which has a $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ chance of happening. Thus, the probability that the plane gets fixed within four days is $1 - \frac{1}{4} = \boxed{\frac{3}{4}}$.
9. The shortest distance between any two unshared vertices is the shorter of the side lengths of the octagon. To find its length, we can draw a 45-45-90 triangle with that side as the hypotenuse. Then the distance between two opposite longer sides of the octagon, which is just two equilateral triangle heights or $4\sqrt{3}$, is two leg lengths plus a longer length of the octagon; if we let a be the length of a leg, then we get the equation $4\sqrt{3} = 2a + 2$, and solving gives $a = 2\sqrt{3} - 2$. Then the length of the hypotenuse is $\sqrt{2}(2\sqrt{3} - 2) = 2\sqrt{6} - 2\sqrt{2}$, so $m + n + p + q = 2 + 6 + 2 + 2 = \boxed{12}$.
10. For convenience, let $f(n)$ be the encrypted number. Now, $f(n) = \lfloor n \rfloor + \lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{4} \rfloor + \lfloor \frac{n}{8} \rfloor + \dots$. Based on the effect as n increases, we can conclude that $f(n)$ is a strictly increasing function over the positive whole numbers. This tells us that we can find the greatest possible n such that $f(n) \leq 50$ and all possible numbers whole numbers would just be the positive whole numbers less than or equal to n . At this point, we approach by estimation and trial-by-error, though a good first guess can be 25 as $f(n)$ is similar to a geometric series with common difference $\frac{1}{2}$. As it turns out, $f(27) = 27 + 13 + 6 + 3 + 1 = 50$, and the number of positive whole numbers satisfying the property is $\boxed{27}$.