



Contest Problem Set 12210

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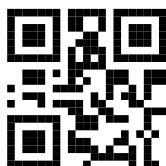
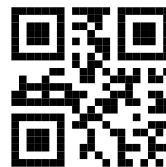
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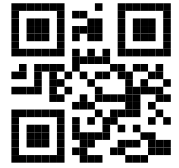
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1. A box of chocolates has 4 rows containing 6 chocolates each. Anshul takes them all out, eats three of them, and splits the remaining chocolates into 3 equal groups. How many chocolates does each group have?
2. John draws a rectangle with side lengths 7 and 6. What is the numerical difference between the area and perimeter of John's rectangle?
3. Maggie reads two comic books on Monday. Starting on Tuesday, she reads twice as many comic books as she does the previous day. How many comic books does she read from Monday to Friday of the same week?
4. Drake plans on eating $\frac{1}{3}$ of a pizza, but the pizza he ordered is divided into 6 equal slices! How many slices should Drake NOT eat?
5. Elvis rolls a fair six-sided die. What is the probability that the die shows a number that is not prime? Express your answer as a common fraction.
6. A rectangular computer screen has a width of 15 inches and a length of 22 inches. A larger rectangular computer screen has a width of 19 inches and has 90% more area. What is the length of the larger screen in inches?
7. MacKenzie, Jacob, and Ameer each have a positive whole number of trading cards. MacKenzie and Jacob together have a total of 17 trading cards. Ameer has twice as many trading cards as MacKenzie. Ameer and Jacob have a total of 22 trading cards. How many trading cards do all three of them have together?
8. Kaycee is running a lemonade stand, where she sells one lemonade per person. Each lemonade costs \$1 to make, and Kaycee sells lemonade for \$1.20. Kaycee estimates that 500 people will buy lemonade one hot day, but the actual number of people who bought the lemonade was 25% more than Kaycee's estimate. If Kaycee makes exactly as many lemonade as she sells, how much profit, in dollars, does Kaycee make that day?
9. Barbara is doing a 10K race, where she has to run 10 km, and she must finish the race in 1 hour. If Barbara runs the first 8 km at a rate of 5 meters per second, what is the minimum speed, in meters per second, that Barbara needs to run the remainder of the race and finish on time?
10. What is the positive difference between the minimum and maximum area of a rectangle with whole number side lengths and perimeter 44?
11. It is 2:24 AM right now. Isaac's flight arrived yesterday at 11:20 PM. How many minutes have passed since?

12. Antonio wrote down the number of touchdown passes from the top 10 football players in the area. His results are in the list $[7, 2, 9, 8, 1, 55, 3, 4, 5, 6]$. Antonio realizes that William has the highest number of touchdown passes in the area. The mean of the ten numbers in Antonio's list is a , and the mean of the numbers after removing the number of touchdown passes that William did is b . What is $a - b$?
13. At an intramural competition, Team Power, Team Wisdom, and Team Courage each have 6 people. Brandon is a leader who plans on selecting a number of people at random to do the master trial. If he wants to guarantee that at least one person from each team participates, how many people must Brandon select at random?
14. Cole's dormitory is 6 kilometers from the stadium. Cole takes the bus until he is halfway to the stadium, and for half the remaining distance, Cole walked towards the stadium before taking a water break. After his water break, how far, in *meters*, does Cole need to still walk in order to reach the stadium?
15. One week from Monday to Friday inclusive, Sam does some number of pushups in the martial arts dojo. On Monday and Wednesday, Sam does 8 pushups. On Tuesday, Sam does 12 pushups. On Thursday, Sam does a whole number of pushups, and on Friday, Sam also does a whole number of pushups. After Friday, Sam calculated the median number of pushups he did over the past five days. What is the sum of all possible values of Sam's result?
16. The point $(25, -25)$ is reflected across the line $y = x$ and then rotated 30° counterclockwise about the origin. What is the smallest positive number of degrees counterclockwise does the point then need to be rotated about the origin to end up back where it started?
17. Chris bought 200 candy bars that were \$0.99 each and a car that is worth \$12000. The total cost for the items included the price of the candy bars and car as well as a 7% sales tax. Chris plans on paying with only pennies, which are worth one cent each. How many pennies does Chris need to pay?
18. What is the number of different arrangements of the letters of the word *VIVIDRIA*?
19. The parabola $f(x) = x^2 - x - 6$ intersects the x -axis and y -axis at three different points. What is the area of the triangle formed by these points?
20. An octagon has two opposite vertices colored white and blue, and all other vertices are colored gray. A move consists of swapping the colors of the white vertex and a vertex adjacent to it by an edge. At most how many moves are required in order to swap the positions of the white and blue vertices?
21. When $\frac{2}{63}$ is written in decimal form, what is the 100th digit to the right of the decimal point?

22. Jayden encounters moving walkways in an airport while trying to find his gate. If he walks on an operating walkway, it only takes him 25 seconds. If he walks along a non-operating walkway, it takes him 100 seconds to get through. How many seconds does it take Jayden to ride down an operating walkway while just standing on it? Round your answer to the nearest whole number.
23. Zach got a new TV where the screen has a diagonal of length 60 inches. The dimensions of the new TV's screen are $\frac{3}{5}$ and $\frac{4}{5}$ of the diagonal length. The area of the new TV's screen is the same as the area of Zach's old TV screen, which is a square. Then the side length of the screen of Zach's old TV has a length of $a\sqrt{b}$ inches, where b is not divisible by the square of a prime. What is $a + b$?
24. One highway has 900 mile markers in a single-file line on the road such that the distance between two consecutive mile markers on the road is 1 mile. Daniel wants to install the lowest number of call boxes such that each mile marker is at most one mile from a call box. What is the lowest number of call boxes that Daniel needs to install?
25. Compute the product of all distinct negative values of x such that $(x^2 + 7x + 10)(x^2 + 2x - 8) + (x^2 + 3x - 10) = 0$.
26. Anthony has a circular piece of paper that is 30 centimeters in diameter. He draws two lines and then cuts the paper using scissors on the lines so that he has four pieces in total. The average perimeter of the pieces is 45 centimeters. To the nearest whole number, what is the total length of the two cuts, in centimeters?
27. The Greek alphabet has 24 uppercase letters. Elise tries to come up with a name for her college club by writing down all sequences of 3 letters that contain exactly two different Greek uppercase letters. How many different sequences did Elise write?
28. Percy is reading a book with 899 pages on Thanksgiving (that is, the page number of the last page is 899). The prologue is 100 pages long, so he starts reading from page 101. Each page of the book has the page number at the bottom of the page. Whenever he gets to a page where the page number is a multiple of 37, he writes down the sum of its digits on a piece of paper. After he finishes reading the book, what is the sum of all the numbers Percy wrote down?
29. Carson is designing a spaceship capsule, which is in the shape of a frustum. He designs the capsule by taking a right circular cone with height 24 and base diameter 20, then making a cut parallel to the base such that he can take out a right circular cone with height 12. The remaining piece is Carson's frustum, and the surface area of the frustum can be written as $k\pi$ for some positive whole number k . What is the value of k ?
30. A function f is defined such that $f(x) = \frac{f(x+1)+f(x-1)}{2}$ for all positive whole numbers $x > 1$. If $f(1) = 1$ and $f(3)$ is a positive whole number less than 6, what is the sum of all possible values of $f(15)$?



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Problems 1 & 2

1. What is the sum of all positive even whole numbers that are less than 32?

1.

2. The side lengths of a rectangle are positive whole numbers. The area of the rectangle is numerically equal to the perimeter of a square with area 12.25. What is the sum of all possible values of the perimeter of the rectangle?

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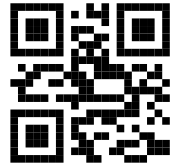
Problems 3 & 4

3. In college, Evan went to bed at exactly 11 : 55 PM and woke up at exactly 8 : 01 AM the next day. The minimum recommended number of hours to sleep for young adults is 7 hours. How many more minutes did Evan sleep compared to the minimum recommended number of sleep hours for young adults?

3.

4. Becca and Madison are playing a game during their lunch break. Becca flips four fair coins and wins if the number of heads is greater than the number of tails. What is the probability that Becca wins the game? Express your answer as a common fraction.

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Problems 5 & 6

5. Nicole forms a four-digit number that uses each of the digits 2, 4, 7, and 8 exactly once. Out of all possible numbers Nicole can form, what fraction of them are multiples of four? Express your answer as a common fraction.

5.

6. Claudia has five flower beds that each have a *distinct* positive whole number of flowers. One flower bed has 19 flowers, and the rest each have at least 29 and at most 36 flowers. Among all five flower beds, the median number of flowers is 30, and the range is 17 flowers. What is the smallest possible average number of flowers among the five flower beds?

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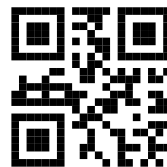
Problems 7 & 8

7. A line that is perpendicular to the line $y = 2x + 3$ is drawn. The lines intersect at a point with a y -coordinate of 3. What is the sum of the coordinates of the y -intercept of the line that was drawn?

7.

8. Let x be the value when $\frac{n}{60}$ is rounded to the nearest hundredth. How many positive whole numbers $n \leq 2021$ are there such that $x < \frac{n}{60}$? For example, if $n = 40$, then $\frac{n}{60} = 0.\overline{6}$, so $x = 0.67$, which is greater than $\frac{n}{60}$.

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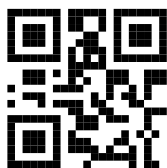
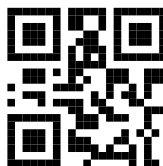
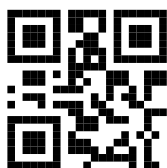
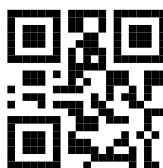
School or Team

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1. Sienna is planning on sowing a rectangular quilt from patches. All of her patches are squares with the same side length, and she has just enough patches for the quilt to be exactly 10 patches long by 4 patches wide. Of the patches she has, 10 of them are red, and the rest are blue. How many blue patches does Sienna have?
2. Alex plans on working 5 total hours from Monday to Friday for mathleague.org. He worked for 0.25 hours on Monday, 2.65 hours on Tuesday, 0.4 hours on Wednesday, and 0.5 hours on Thursday. How long in hours should Alex work on Friday? Express your answer as a decimal to the nearest tenth.
3. The product of two positive fractions is $\frac{1}{25}$. One of the fractions is 36 times the other fraction. What is the positive difference between the two fractions? Express your answer as a common fraction.
4. The *half-life* of an object is the amount of time it takes for that object's mass to decrease by half its size. Collin finds a rock that has a half-life of 1 year. To the nearest whole number, how many years will it take for the rock to reach 0.8% of its original mass?
5. Suppose x and y are positive whole numbers such that $\frac{x}{5} + \frac{y}{7} = \frac{74}{35}$ and $\frac{5}{x} + \frac{7}{y} = \frac{74}{35}$. What is the value of $x + y$?

6. A list of four positive whole numbers is created. For each pair of numbers in the list, the larger of the two numbers is written down. If the six numbers that are written down are 1, 2, 2, 5, 5, and 5, then what is the sum of the four numbers in the original list?
7. A vehicle shipping container is shaped like a rectangular prism with dimensions 2 meters by 2 meters by 10 meters. Phil transports safari pod vehicles in boxes that are cubes with side length 2 meters. However, Phil discovers a shrink function in each safari pod vehicle, and to store a shrunk vehicle, he only needs a box with 12.5% of the dimensions of the original box. How many more shrunk safari pod vehicles can Phil store in the shipping container compared to when they are their original size?
8. Andrea, Belinda, Chelina, Diana, and Evelyn are standing in a line for a group photo. Andrea is not next to Diana and Chelina is not next to Evelyn. How many different ways can the five people be standing in a line?
9. The decibel system is a measure of sound. An $X + 10$ decibel sound is 10 times as loud as an X decibel sound. For any positive whole number n , there is a fixed number k such that a $X + n$ decibel sound is always k times as loud as an X decibel sound. Phoenix, a lawyer, usually speaks at a volume of 62 decibels, but in court, he speaks with a volume of 87 decibels. How many times as loud is he in court compared to his normal voice? Round your answer to the nearest integer.
10. Hilda and Marianne are playing a cooperative game of Ones, which is played with colored cards with numbers. In the beginning, a green 5 is in the middle. Hilda has a green 2, a green 8, a yellow 2, a yellow 8, and a blue 6; Marianne has a green 3, a yellow 7, a blue 2, and a blue 8. Starting with Hilda, they take turns placing down cards in the middle. A turn consists of placing a card on top of the card in the middle that has either the same color or the same number; if this isn't possible, the game ends. At the end of the game, both Hilda and Marianne have no cards left. Compute the sum of the numbers on all possible cards that can be the last card placed down.

1. In the following geometric sequence, what is the product of a and b ? 4, a , b , 36
2. What is the fifth triangular number?
3. The measures of the angles of a triangle are in the ratio 2 : 3 : 4. What is the degree measure of the smallest angle in the triangle?
4. Simplify: $-1 + 2 - 3 + 4 - 5 + 6 + \cdots + 2012$.
5. An isosceles right triangle has legs of length 5. What would the area of the circle circumscribing this triangle be? Express your answer as a common fraction in terms of π .
6. Melinda places 11 coins into a stack, where each coin is equally likely to face up or down. What is the expected number of pairs of touching faces that show the same side (heads or tails)?
7. A fair 6-sided die is rolled three times. Find the probability that the values of the rolls are strictly increasing. Express your answer as a common fraction.
8. Niah is volunteering at a summer camp that has 40 students. The students can choose to rest or either play dodgeball or knockout (but not both). There are 2 more students who choose dodgeball than knockout. If 16 students choose to play knockout, how many students choose to rest?
9. What is the area of the quadrilateral with vertices at $(3, 5)$, $(8, 3)$, $(6, -2)$, and $(1, 0)$?
10. What is the base-16 number $4A_{16}$ in base 10? (The base 16 digits in order are 0, 1, 2, 3, ..., 9, A, B, C, D, E, F.)
11. What is the greatest common divisor of 198 and 165?
12. If x , y , and z are positive integers such that $xyz = 120$, what is the least possible value of $x + y + z$?
13. A train travels 50 miles per hour for the first 40 miles, then slows down to 40 miles per hour for the next 50 miles. What is the average speed of the train in miles per hour? Round your answer to the nearest whole number.
14. In $\triangle ABC$, $AB = 5$, $BC = 6$, and $\angle B = 120^\circ$. Find AC^2 .
15. A circle fits inside a 4×6 rectangle. Find the largest possible area of the circle in terms of π .
16. When a whole number is divided by 23, the beginning of the decimal expansion of the result is $0.478 \dots$. What is the value of this whole number?

17. A fair six-sided die is rolled 6 times. What is the expected value of the sum of the 6 rolls?
18. How many 2-digit integers are relatively prime to 98?
19. Let x be a positive integer. Given that $x = 13y$ and $x - y = 564$, what is x ?
20. Evaluate: $3 + 6 + 9 + \cdots + 33$.
21. Euler's theorem states that any convex three-dimensional polyhedron satisfies the equation $V - E + F = 2$, where V , E , and F are the numbers of vertices, edges, and faces of the polyhedron, respectively. An dodecahedron has 20 vertices and 12 faces. How many edges does it have?
22. Find the 11th term in the arithmetic sequence: $9, -3, -15, \dots$
23. Two positive integers have a GCD of 2 and LCM of 240. When one number is divided by the other, the result is less than 1. What is the greatest possible value of the result? Express your answer as a common fraction.
24. If a random integer between 1 and 100, inclusive, is chosen, what is the probability that it is composite? Express your answer as a common fraction.
25. For what value of k is there no solution to the equations $4x + 5y = 20$ and $-16x + ky = 60$?
26. Three of the faces of a rectangular prism have areas 20, 24, and 30. What is half of its volume?
27. Three mechanics can fix two cars in five days. Assuming all mechanics work at the same rate, how many cars can five mechanics fix in six days?
28. Find the area of a triangle with side lengths 10, 12, and 14. Express your answer in simplest radical form.
29. Elaine chooses 10 distinct positive integers from $\{1, 2, \dots, 19\}$. Surprisingly, she does not choose any two numbers that differ by exactly one. How many possible values are there for the sum of the 10 numbers chosen?
30. If $a \# b = a^b - b^a$, what is $5 \# 7$?
31. How many positive integer factors does $2^3 4^5 6^7$ have?
32. What is the radius of a circle whose area is numerically equivalent to twice its circumference?
33. Michael has a collection of 15 Russian coins, which are either 2 kopeyka coins or 5 kopeyka coins. The coins are worth in total 48 kopeykas. What percent of the coins are 2 kopeyka coins?
34. What is the greatest common divisor of 978 and 1020?
35. Suppose x and y^2 are inversely proportional. If y is multiplied by $\frac{1}{2}$, by what value is x multiplied by?

36. In Japan, there are coins worth 1 yen, coins worth 5 yen, and coins worth 10 yen. How many different combinations involving any number (or none) of these three coin types are worth 25 yen? Two combinations are considered identical if they consist of the same number of coins for all three coin types.
37. If $a + b = 4$ and $a^2 + b^2 = 10$, what is $a^4 + b^4$?
38. If the radius of a sphere is quadrupled, then its surface area is multiplied by what?
39. If $2x^2 - 25x + 9 = 0$, what is the product of the values for x ? Express your answer as a common fraction.
40. For all real x , $f(3x) = x^2 - 3x + 1$. Find $f(12)$.
41. How many 3-digit integers have their tens digit greater than their hundreds digit and their units digit greater than their tens digit?
42. Evaluate: 111111^2 .
43. What is the greatest integer less than $\sqrt[3]{-129}$?
44. Simplify: $\frac{286^5}{143^5}$.
45. What is the largest prime divisor of 15015?
46. Lucas is buying boxes of chicken nuggets, where each box either has 5 nuggets or 8 nuggets. What is the maximum number of nuggets that Lucas couldn't get in exact amount from buying some number of boxes that can be either size?
47. What is the positive difference between the sum of the first 40 positive integers and the sum of the first 40 positive odd integers?
48. What is the surface area of a regular octahedron with a side length of 2? Express your answer in simplest radical form.
49. The side length of a regular hexagon is 6. What is the positive difference between the areas of the circumscribed and inscribed circles of the hexagon? Express your answer in terms of π .
50. Riley is picking 3 people from a soccer team of 11 people to have a role of defender. If the order Riley assigns defenders does not matter, how many ways can Riley assign three defenders?
51. Simplify: 72×11 .
52. What is the value of $-20 + 15 \cdot 17 + 5$?
53. What is the sum of all integers that are one more than a one-digit prime?
54. What is the greatest common factor of 125 and 80?

55. Let x , y , and z satisfy the equation $x^2 + 9y = 9z$. If $x = 6$ and $y = 5$, what is z ?
56. Find 6^3 .
57. If $\frac{x}{y} = 25$ and $x = 125$, then what is y ?
58. The area of a 12-sided regular polygon inscribed in a circle with radius r is given by $A = 3r^2$. If a circle has a circumference of 24π , what is the area of a 12-sided regular polygon inscribed in it?
59. Simplify: $100,000 - 34,933$.
60. How many of the following numbers are rational: 0.6666666 , $\sqrt{2/9}$, $\sqrt{-9}$, $\sqrt{7.84}$?
61. The side lengths of a rectangle are both integers. If the perimeter of the rectangle is 18, what is the smallest possible value for the area of the rectangle?
62. If 1% of 10% of a number is 66, then what is half of the number?
63. If Eugene drove at 84 miles per hour for 25 minutes, how many miles did he travel?
64. Evaluate: $13^3 - 8^3$.
65. What is the greatest 3-digit number that is exactly divisible by 15, 25, and 30?
66. The ratio between two consecutive even integers is $\frac{11}{12}$. Find the product of the integers.
67. What is the positive difference between the largest 3-digit number and the smallest 3-digit number?
68. If the lengths of two sides of a right triangle are 8 and 10 units, what is the largest possible length of the third side? Express your answer in simplest radical form.
69. 70% of a number is 490. What is 110% of that number?
70. Express in scientific notation: $\frac{3 \times 10^5}{8 \times 10^8}$.
71. What is the least perfect square greater than 4000?
72. Three fair six-sided dice are rolled. What is the probability that the sum of those three numbers is odd? Express your answer as a common fraction.
73. Find the sum of the tens and millions digits of $101 \times 202 \times 303$.
74. $\sqrt{2000}$ lies between two consecutive integers. Find the product of these integers.
75. Find the range of the set $\{99, 33, 8, 78, 13, 38, 28, 48, 9, 30\}$.
76. What is 20% of 60% of 500?

77. Simplify: $41 \cdot 72$.

78. Compute: $\frac{5.3}{1.5} + \frac{3.7}{1.5}$.

79. What is the product of the greatest common divisor and the least common multiple of 16 and 30?

80. How many times does $\frac{1}{49}$ go into $\frac{2}{7}$?

Sprint Round

- | | | |
|------------------|---------------------|----------|
| 1. 7 | 11. 184 | 21. 7 |
| 2. 16 | 12. 5 | 22. 33 |
| 3. 62 | 13. 13 | 23. 27 |
| 4. 4 | 14. 1500 | 24. 300 |
| 5. $\frac{1}{2}$ | 15. 50 | 25. 15 |
| 6. 33 | 16. $150(^{\circ})$ | 26. 43 |
| 7. 27 | 17. 1305186 | 27. 1656 |
| 8. (\$) 125 | 18. 3360 | 28. 297 |
| 9. 1 | 19. 15 | 29. 320 |
| 10. 100 | 20. 28 | 30. 75 |

Target Round

1. 240
2. 48
3. 66
4. $\frac{5}{16}$
5. $\frac{5}{12}$
6. 29
7. 3
8. 674

Team Round

1. 30
2. 1.2
3. $\frac{7}{6}$
4. 7
5. 12
6. 9
7. 2555
8. 48
9. 316
10. 16

Countdown

- | | | | |
|----------------------|---------------------|-----------------|---------------------------|
| 1. 144 | 21. 30 | 41. 84 | 61. 8 |
| 2. 15 | 22. -111 | 42. 12345654321 | 62. 33,000 |
| 3. $40(^{\circ})$ | 23. $\frac{8}{15}$ | 43. -6 | 63. 35 |
| 4. 1006 | 24. $\frac{37}{50}$ | 44. 32 | 64. 1685 |
| 5. $\frac{25\pi}{2}$ | 25. -20 | 45. 13 | 65. 900 |
| 6. 5 | 26. 60 | 46. 27 | 66. 528 |
| 7. $\frac{5}{54}$ | 27. 4 | 47. 780 | 67. 899 |
| 8. 6 | 28. $24\sqrt{6}$ | 48. $8\sqrt{3}$ | 68. $2\sqrt{41}$ |
| 9. 29 | 29. 1 | 49. 9π | 69. 770 |
| 10. 74 | 30. 61,318 | 50. 165 | 70. 3.75×10^{-4} |
| 11. 33 | 31. 168 | 51. 792 | 71. 4096 |
| 12. 15 | 32. 4 | 52. 240 | 72. $\frac{1}{2}$ |
| 13. 44 | 33. $60(\%)$ | 53. 21 | 73. 6 |
| 14. 91 | 34. 6 | 54. 5 | 74. 1980 |
| 15. 4π | 35. 4 | 55. 9 | 75. 91 |
| 16. 11 | 36. 12 | 56. 216 | 76. 60 |
| 17. 21 | 37. 82 | 57. 5 | 77. 2952 |
| 18. 39 | 38. 16 | 58. 432 | 78. 6 |
| 19. 611 | 39. $\frac{9}{2}$ | 59. 65,067 | 79. 480 |
| 20. 198 | 40. 5 | 60. 2 | 80. 14 |

Sprint Round Solutions

1. There are $4 \cdot 6 = 24$ chocolates to start, and after Anshul eats three of them, there are $24 - 3 = 21$ remaining, so each group has $\frac{21}{3} = \boxed{7}$ chocolates.
2. The perimeter of John's rectangle is $7 + 6 + 7 + 6 = 26$, and the area is $7 \cdot 6 = 42$. The numerical difference between the area and perimeter is $42 - 26 = \boxed{16}$.
3. On Tuesday, she reads $2 \cdot 2 = 4$ comic books. On Wednesday, she reads $2 \cdot 4 = 8$ comic books. On Thursday, she reads $2 \cdot 8 = 16$ comic books. Finally, on Friday, the number of comic books Maggie reads is $2 \cdot 16 = 32$. So the total number of books Maggie read from Monday to Friday is $2 + 4 + 8 + 16 + 32 = \boxed{62}$.
4. If Drake eats $\frac{1}{3}$ of a pizza, he does not eat $1 - \frac{1}{3} = \frac{2}{3}$ of the pizza. If there are 6 slices, the number of slices he does not eat is $\frac{2}{3} \cdot 6 = \boxed{4}$.
5. The prime numbers are 2, 3, or 5, so the numbers that are not prime are 1, 4, or 6. The probability of rolling one of these numbers is $\frac{3}{6} = \boxed{\frac{1}{2}}$.
6. Comparing the larger screen to the smaller screen, the width is $\frac{19}{15}$ as large, and the area is $\frac{100+90}{100} = \frac{19}{10}$ as large, so the length should be $\frac{19/10}{19/15} = \frac{3}{2}$ as large. So the length of the larger screen is $\frac{3}{2} \cdot 22 = \boxed{33}$.
7. Since Amee has twice as many trading cards as MacKenzie, Amee and Jacob will have a total of 17 trading cards plus the number of trading cards that MacKenzie has. Therefore, MacKenzie has $22 - 17 = 5$ trading cards. This means that Amee has $5 \cdot 2 = 10$ trading cards and Jacob has $17 - 5 = 12$ trading cards, so the three of them have in total $5 + 10 + 12 = \boxed{27}$ trading cards.
8. The total number of people who bought Kaycee's lemonade that day is $500 \cdot 1.25 = 625$ people. Each person buys one lemonade, so 625 lemonades are sold. We could calculate the total amount of money the people paid (also known as the total revenue), but a quicker route is to note that for each lemonade, Kaycee has a net earning of $1.2 - 1 = 0.2$ dollars per lemonade. Therefore, Kaycee earns $625 \cdot 0.2 = \boxed{125}$ dollars that day.
9. Barbara ran 8 km, or 8000 meters, at a rate of 5 meters per second, so it took Barbara 1600 seconds to run that far. Since 1 hour equals 3600 seconds, Barbara has 2000 seconds to run the remaining 2000 meters, so Barbara needs to run at $\boxed{1}$ meter per second.

10. The least possible area occurs when the length of two of the sides is maximized (21), and the other two sides have minimum length 1. The resulting area is $21 \cdot 1 = 21$. The area of the rectangle is maximized by making each side of the rectangle equal to each other, that is, by making it a square with side length $\frac{44}{4} = 11$. The perimeter of this square is $11 \cdot 4 = 44$. The positive difference between the minimum and maximum areas is therefore $121 - 21 = \boxed{100}$.
11. There are 3 hours, or 180 minutes, in between 11:20 PM yesterday and 2:20 AM today. Adding 4 minutes to that to get to 2:24 AM today, $\boxed{184}$ minutes have passed since Isaac's flight arrived.
12. The sum of the ten numbers is 100, so the mean of the ten numbers is $\frac{100}{10} = 10$. However, when removing the number of touchdown passes that William did, the sum of the nine numbers is $100 - 55 = 45$, and the mean of the nine numbers is $\frac{45}{9} = 5$. Then $a - b = \boxed{5}$.
13. Brandon needs to select at least 3 people in order for one to represent each team, but when selecting at random, the 3 people could be all on the same team. Brandon can select at most $6 \cdot 2 = 12$ people from exactly two teams, so to guarantee that at least one person from each team participates, the minimum number of people required is $12 + 1 = \boxed{13}$.
14. After the bus, Cole is currently halfway there, so Riley is $6 \cdot \frac{1}{2} = 3$ kilometers from the stadium. After the water break, Cole is $3 \cdot \frac{1}{2} = 1.5$ kilometers from the stadium. Since 1.5 kilometers is equal to 1500 meters, the distance that Cole still needs to walk, in meters, is $\boxed{1500}$.
15. If we consider a list of the pushups done from Monday to Wednesday, the list is 8, 8, 12. Then we consider the remaining days' pushup count and the possible cases. For the case where Sam does less than 8 pushups on one day, the median would be 8 regardless of the number of pushups done on the other day. For the case where Sam does greater than 8 pushups but less than 12 pushups for one day and greater than 8 pushups for the other day, the median would be 9, 10, or 11. For the case where Sam does greater than 12 pushups for both days, the median would be 12. The sum of all possible values of Sam's result is $8 + 9 + 10 + 11 + 12 = \boxed{50}$.
16. If the point $(25, -25)$ is reflected across the line $y = x$, then it would land at $(-25, 25)$, which is also the same point if $(25, -25)$ is rotated 180° counterclockwise about the origin. Thus, after the first rotation that happens after the reflection, the point is in the same place as if the original point is rotated 210° about the origin. Since a circle has 360° , the minimum number of degrees that the point needs to be rotated about the origin counterclockwise in order to get back to the starting place is $\boxed{150^\circ}$.
17. The total cost before the tax was $0.99 \cdot 200 + 12000 = 12198$ dollars, and including the 7% sales tax means that the total cost with tax is $12198 \cdot 1.07 = 13051.86$ dollars. Therefore, Chris would need $\boxed{1305186}$ pennies to pay for the items.
18. There are 8 letters with 2 copies of V and 3 copies of I, so the number of arrangements is $\frac{8!}{2!3!} = \boxed{3360}$.

19. The parabola intersects the y -axis once, at the y -intercept $(0, -6)$. We can factor the quadratic into $f(x) = (x + 2)(x - 3)$ to find the roots $x = 3, -2$, so the points of intersection with the x -axis are $(3, 0)$ and $(-2, 0)$. The triangle thus has vertices $(0, -6)$, $(3, 0)$, and $(-2, 0)$. The base lying along the x -axis is 5, and the height relative to the base is 6, so the area of the triangle is $\frac{1}{2} \cdot 5 \cdot 6 = \boxed{15}$.
20. We move the blue vertex by swapping it with the white vertex, and this must be done four times in the same direction. It takes 3 moves to get the white vertex next to the blue vertex, and it takes 1 move to swap. Then it takes 6 moves to go around the octagon to return to the same side of the blue vertex as before, and it takes 1 move to swap again. This step is repeated twice more, and finally it takes 3 moves for the white vertex to move to where the blue vertex originally started. This process takes $3 + 1 + (6 + 1) \cdot 3 + 3 = \boxed{28}$ moves in all.
21. To solve this, we realize that $\frac{2}{63} = \frac{1}{7} - \frac{1}{9}$. Then the decimal form of $\frac{2}{63}$ is just the decimal form of $\frac{1}{7}$, except all digits are decreased by 1, and this works since all of the digits in the decimal form of $\frac{1}{7}$ are nonzero. The period of $\frac{1}{7}$ is 6, so the 100th digit of $\frac{1}{7}$ is the 4th digit of $\frac{1}{7}$, which is 8. Then the 100th digit of $\frac{2}{63}$ is one less than this, which is $\boxed{7}$.
22. Let j be Jayden's speed and w be the speed of the moving walkway. The formula where distance equals rate times time will be useful, and keep in mind that when calculating the speed of Jayden walking along a moving walkway, the speed is the sum of Jayden's and the walkway's speeds. By setting d to be the length of the walkway, $d = 24(w + j)$ when Jayden walks on an operating walkway. In addition, $d = 100j$ when he walks on a non-operating walkway. Since the lengths are the same, $25w + 25j = 100j$, and so $w = 3j$. Therefore, the time it takes for Jayden to ride down the walkway is $100j \div 3j \approx \boxed{33}$ seconds.
23. The dimensions of the new TV's screen are $60 \cdot \frac{3}{5} = 36$ inches and $60 \cdot \frac{4}{5} = 48$ inches, so the area of the screen is $36 \cdot 48 = 12^2 \cdot 3 \cdot 4$ square inches. Since the area of a square is the square of its side length, the square has side length $\sqrt{12^2 \cdot 3 \cdot 4} = 24\sqrt{3}$ inches, so $a + b = \boxed{27}$.
24. Notice that 900 is a multiple of 3, so we can consider groups of three consecutive mile markers. We can put a call box in the center mile marker of each group, and the other mile markers in the group are indeed within one mile of the call box. We cannot do any better than this, as each call box can only reach up to three mile markers. Therefore, the lowest number of call boxes that Daniel needs to install is $900 \div 3 = \boxed{300}$.
25. First, note that $x^2 + 7x + 10 = (x + 5)(x + 2)$ and $x^2 + 2x - 8 = (x - 2)(x + 4)$ and $x^2 + 3x - 10 = (x + 5)(x - 2)$. Then the left side factors as $(x + 5)(x + 2)(x - 2)(x + 4) + (x + 5)(x - 2)$, and taking out a common factor of $(x + 5)(x - 2)$, we have $(x + 5)(x - 2)[(x + 2)(x + 4) + 1] = (x + 5)(x - 2)(x^2 + 6x + 9)$. Notice that $x^2 + 6x + 9$ is a perfect square trinomial that equals $(x + 3)^2$, and so the left hand side also equals $(x + 5)(x - 2)(x + 3)^2$. Since that equals zero, by the Zero Product Property, x can be -5, -3, or 2, and the product of all distinct negative values of x is $-5 \cdot -3 = \boxed{15}$.
26. Let P be the total length of the two cuts in centimeters. The total perimeter of the pieces is $45 \cdot 4 = 180$ centimeters. If we put the pieces into a circle again, then the total consists of the circumference, which has length 30π centimeters, and twice the length of the two cuts since each length contributes its length to the edges of bordering pieces. This gives us the equation $30\pi + 2P = 180$, and solving results in $P = 90 - 15\pi$, and by using 3.14 (or $\frac{22}{7}$) as an approximation for π , we find that $P \approx \boxed{43}$.

27. There are $\binom{24}{2} = 276$ ways to select two letters from the Greek alphabet to form a sequence of 3 letters. For each letter position, we can have the letter be one of the two chosen letters, so there are $2^3 = 8$ ways to make the sequence. However, 2 of the sequences have all of the same letters, so $8 - 2 = 6$ of these sequences have exactly 2 letters. In total, Elise wrote $276 \cdot 6 = \boxed{1656}$ sequences for her college club.
28. Let $N = abc$ be a three-digit positive integer less than 900 that is a multiple of 37. The sum of the digits of N is $a + b + c$. Then $K = 999 - N$ is also a three-digit positive integer less than 900 that is a multiple of 37, since 37 divides 999. The hundreds digit of K will be $9 - a$, the tens digit of K will be $9 - b$, and the units digit of K will be $9 - c$. Therefore, the sum of the digits of K is $(9 - a) + (9 - b) + (9 - c) = 27 - (a + b + c)$. This means that the sum of the digits of N and the sum of the digits of $999 - N$ add up to 27. Since there are $\frac{888-111}{37} + 1 = 22$ three-digit multiples of 37 less than 900, the sum of all the numbers that Percy wrote down is $\frac{22}{2} \cdot 27 = \boxed{297}$.
29. First, let's determine the base radius of the smaller cone. Since both cones are right circular, their heights will coincide so we will have two similar right triangles. Since the ratio of the height to radius is $24 \div \frac{20}{2} = \frac{12}{5}$, the base radius of the smaller cone has length 5. The surface of the bigger piece is composed of two circles, one with radius 5 and the other with radius 10, and the lower part of the lateral surface from the original cone. The lateral surface of the original cone had an area of $\pi \cdot 10 \cdot \sqrt{10^2 + 24^2} = 260\pi$. The lateral surface of the smaller cone has an area of $\pi \cdot 5 \cdot \sqrt{5^2 + 12^2} = 65\pi$. The area of the circular bases of the larger piece is $\pi \cdot 5^2 + \pi \cdot 10^2 = 125\pi$. Therefore, the total surface area of the bigger piece is $125\pi + 260\pi - 65\pi = 320\pi$ so $k = \boxed{320}$.
30. We can rearrange the expression to get $2f(x) = f(x+1) + f(x-1)$, or $f(x) - f(x-1) = f(x+1) - f(x)$. Then $f(x) - f(x-1)$ is the same of all positive whole numbers $x > 1$, so the sequence $f(1), f(2), f(3), f(4), \dots$ is an arithmetic sequence with a common difference of $\frac{f(3)-f(1)}{2}$. Therefore, $f(15) = f(1) + 14 \cdot \frac{f(3)-f(1)}{2} = 7f(3) - 6f(1) = 7f(3) - 6$. However, since $f(3)$ is a positive whole number less than 6, it must be equal to 1, 2, 3, 4, or 5. Therefore, the sum of all possible values of $f(15)$ is $1 + 8 + 15 + 22 + 29 = \boxed{75}$.

Target Round Solutions

1. We want to find the sum $2 + 4 + 6 + \cdots + 26 + 28 + 30$. We can group together the following sums: $2 + 28$, $4 + 26$, $6 + 24$, $8 + 22$, $10 + 20$, $12 + 18$, and $14 + 16$. Notice that all of these and so on since all these sums are equal to 30 and there are a total of 7 groups. This gives us a total sum of $30 \cdot 7 = 210$. However, we haven't included 30 yet, which changes our sum to $210 + 30 = \boxed{240}$.
2. Since the square has an area of 12.25, each of its sides have length $\sqrt{12.25} = 3.5$. Therefore, the perimeter of the square and the area of the rectangle are $4 \cdot 3.5 = 14$. Since the rectangle has positive whole number side lengths, its dimensions are one of 1×14 or 2×7 . The perimeters of these rectangles are $2 \cdot 1 + 2 \cdot 14 = 30$ or $2 \cdot 2 + 2 \cdot 7 = 18$, respectively. Therefore, the sum of all possible values of the perimeter of the rectangle is $30 + 18 = \boxed{48}$.
3. We can calculate the number of minutes of Evan's sleep by adding up time between set intervals. There are 5 minutes between 11 : 55 PM and midnight. There are $8 \cdot 60 = 480$ minutes between midnight and 8 AM. Finally, there is 1 minute between 8 : 00 AM and 8 : 01 AM. Thus, Evan slept for $5 + 480 + 1 = 486$ minutes. The minimum recommended number of sleep minutes is $7 \cdot 60 = 420$ minutes, so Evan slept for $486 - 420 = \boxed{66}$ more minutes compared to the minimum recommended number of sleep hours.
4. In order for Becca to win, she must flip at least 3 heads. There are a total of $2^4 = 16$ possibilities. One way to determine the winning probability is by casework: there are 4 ways for the flips to result in exactly 3 heads and only 1 way for the flips to result in all four heads. Another way is to note that there are three scenarios: head count is greater than tail count, head count is less than tail count, and head count is equal to tail count. By symmetry, the number of possibilities for the first scenario is equal to the number of possibilities for the second scenario. The third scenario happens if there are exactly 2 heads, and there are $\binom{4}{2} = 6$ ways for that to happen. Therefore, the number of possibilities for the first scenario is $\frac{16-6}{2} = 5$, so the probability that Becca wins is $\boxed{\frac{5}{16}}$.
5. In order for Nicole's number to be a multiple of 4, we need the last two digits to form a number that is a multiple of 4. Then the last two digits must be 24, 28, 48, 72, or 84. In each case, there are 2 ways to arrange the thousands and hundreds digits. Thus, there are $2 \cdot 5 = 10$ possible values of Nicole's number if it is a multiple of 4. In total, there are $4 \cdot 3 \cdot 2 \cdot 1 = 24$ possibilities for Nicole's number. Out of all possible numbers Nicole can form, the fraction of them that are multiples of four is $\frac{10}{24} = \boxed{\frac{5}{12}}$.
6. We're given that the smallest number of flowers in a flower bed is 19, and the range is 17, so the largest number of flowers in a flower bed is $19 + 17 = 36$. The third largest number of flowers on a flower bed is 30. To make the average as small as possible, we need to make the other number of flowers as small as possible, and we can achieve this by choosing the smallest remaining distinct numbers of 29 and 31. Therefore, the average number of flowers among the five flower beds is $(19 + 29 + 30 + 31 + 36) \div 5 = \boxed{29}$.
7. The lines intersect at a point $(a, 3)$ satisfying $3 = 2a + 3$. Solving, we get $a = 0$, so $(0, 3)$ is the y-intercept of the line that was drawn, and $0 + 3 = \boxed{3}$.

8. We want to figure out how many numbers $\frac{n}{60}$ round down when rounding to the nearest hundredth. Notice that if x rounds down when rounding to the nearest hundredth, then for any integer k , $x + \frac{k}{100}$ also rounds down when rounding to the nearest hundredth. With this in mind, since $\frac{3}{60} = 0.05$, we can just check the cases where $1 \leq n \leq 3$. From some trial and error, $\frac{n}{60}$ rounds down only if $n = 2$, so $\frac{n}{60}$ rounds down if n is 2 more than a multiple of 3 (in other words, $n \equiv 2 \pmod{3}$). Since $3 \cdot 673 + 2 = 2021$, there are $673 + 1 = \boxed{674}$ values of n such that $n \equiv 2 \pmod{3}$, which is our desired condition.

Team Round Solutions

1. The rectangular quilt essentially has 10 rows of patches, which each row having 4 patches. Thus, Sienna's quilt has $10 \cdot 4 = 40$ patches. Of the 40 patches, 10 of them are red while the rest are blue, so Sienna has $40 - 10 = \boxed{30}$ blue patches.
2. By adding up the hours, we find that Alex worked $0.25 + 2.65 + 0.4 + 0.5 = 3.8$ hours over four days. This means that on Friday, Alex should work for $5 - 3.8 = \boxed{1.2}$ hours.
3. Let x be the larger fraction and y be the smaller fraction. We have that $xy = \frac{1}{25}$ and $x = 36y$. This means $36y^2 = xy = \frac{1}{25}$ so $y^2 = \frac{1}{900}$, or $y = \pm \frac{1}{30}$, but y is positive, so we know $y = \frac{1}{30}$. Since $x = 36y$, we know that $x - y = 35y$, so the positive difference between x and y is $35 \cdot \frac{1}{30} = \boxed{\frac{7}{6}}$.
4. We can model the fraction of mass remaining as a function of time. In particular, if t years have passed, the mass of the object is $\frac{1}{2^t}$ of the original mass. Since 0.8% is equal to $\frac{1}{125}$, we wish to find t such that $2^t = 125$. By trying whole number values of t , we find that $2^6 = 64$ and $2^7 = 128$, and since $2^{6.5} = 64\sqrt{2} \approx 90.510$, to the nearest whole number, it takes $\boxed{7}$ years for the rock to reach 0.8% of its original mass.
5. After noting that $\frac{x}{5} + \frac{y}{7} = \frac{5}{x} + \frac{7}{y}$, we realize that replacing $x = 7$ and $y = 5$ leads to the same fractions. Then, we see that $\frac{5}{7} + \frac{7}{5} = \frac{74}{35}$, so the value of $x + y$ is $7 + 5 = \boxed{12}$.

6. Since 1, 2, and 5 appear in the second list, they must have been in the original list. Moreover, 1 must have been greater than or equal to one other number in this list, so the list must contain two 1's. The original list is 1, 1, 2, 5, and the sum of these numbers is $1 + 1 + 2 + 5 = \boxed{9}$.
7. Because 12.5% is equal to $\frac{1}{8}$, Phil only needs a box that is a cube with side length $\frac{1}{4}$ meter. Notice that when dividing both 2 and 10 by $\frac{1}{4}$ or 2, the result is a whole number, so we can safely divide the volume of the shipping container by the volume of the box (since there is no "unused space"). The volume of the shipping container is $2 \cdot 2 \cdot 10 = 40$ cubic meters. The volume of the box for a regular-sized safari pod is $2^3 = 8$ cubic meters, and the volume of the box for a shrunken safari pod is $\frac{1}{4}^3 = \frac{1}{64}$ cubic meters. Thus, the container can store $40 \div 8 = 5$ boxes of regular-sized safari pods, and the container can store $40 \div \frac{1}{64} = 2560$ boxes of shrunken safari pods. Phil can then store $2560 - 5 = \boxed{2555}$ more shrunken safari pods than regular-sized safari pods.
8. We can use complementary counting to find the number of possible lineups where Andrea is next to Diana or Chelina is standing next to Evelyn. Suppose Andrea is next to Diana. We can treat the two as one "unit" and the remaining three people as three units. The four units can be arranged in $4! = 24$ ways, and since Andrea and Diana can stand next to each other in 2 ways, there are a total of $24 \cdot 2 = 48$ possibilities. By symmetry, the number of lineups where Chelina is next to Evelyn is also 48. However, we overcounted the lineups where both Andrea is next to Diana and Chelina is next to Evelyn. In this case, we treat Andrea and Diana as a unit, Chelina and Evelyn as a unit, and Belinda as a unit. The three units can be arranged in $3! = 6$ ways. However, Andrea can stand next to Diana in 2 ways and Chelina can stand next to Evelyn in 2 ways, so the number of possible lineups where Andrea is next to Diana and Chelina is next to Evelyn is $6 \cdot 2 \cdot 2 = 24$. Thus, the number of lineups where Andrea is next to Diana or Chelina is next to Evelyn is $48 + 48 - 24 = 72$. Without restrictions, there are a total of $5! = 120$ ways the five people can line up, so the desired lineups where Andrea is not next to Diana and Chelina is not next to Evelyn is $120 - 72 = \boxed{48}$.
9. A 72 decibel sound is 10 times as loud as a 62 decibel sound, and an 82 decibel sound is 10 times as loud as a 72 decibel sound. Now, suppose an 87 decibel sound is k times as loud as an 82 decibel sound. Then a 92 decibel sound is also k times as loud as an 87 decibel sound. Putting these two statements together, a 92 decibel sound is k^2 times as loud as an 82 decibel sound. Since a 92 decibel sound is 10 times as loud as an 82 decibel sound, we know that $k^2 = 10$, so $k = \sqrt{10}$. Therefore, Phoenix's 87 decibel voice in court is $100\sqrt{10} \approx 316.22$ times as loud as his normal 62 decibel voice. The closest integer to the answer is $\boxed{316}$.
10. Note that Hilda will be the last player to place down a card. From there on, a helpful strategy is to draw a table for the cards, with rows representing the colors and the columns representing the numbers, and marking cells to ascertain the cards as Hilda's, Marianne's, or not in the game. This makes it much easier to experiment with possible directions the game can go in; starting from the green 5, one can alternate between the two players by traveling along the rows and the columns. Neither of Hilda's green cards can be the last card because both of them are needed to place down Marianne's green 3. Trying out possible games, Hilda's other cards can all possibly be the last card. These cards are a yellow 2, a yellow 8, and a blue 6, and the sum of the numbers on these cards is $2 + 8 + 6 = \boxed{16}$.