Macroeconomics Qualifying Exam

May 25, 2022

You have 120 minutes to answer the following questions. Do not consult any source, animate or inanimate.

Read the questions carefully. Feel free to make any extra assumptions that you deem appropriate to get results.

1. Suppose there is a technology that combines private and government capital to produce units of the single, perishable consumption good. The economy is divided into equal-sized period with $t = 1, 2, 3, ...$ there is a measure one of identical infinitely lived consumers. Formally, let $Y_t = A(K_{t-1} + K_{t-1}^g)$ where Y denotes aggregate quantity of the good produced, K is aggregate private capital and K^g is aggregate government capital. Let $A > 0$ denote productivity. Assume that K_0 and K_0^g are given. Both private capital and government capital depreciate at rate δ. There exists a sequence of lump-sum taxes, denoted by $\{\tau_t\}_{t=1}^{\infty}$ that is fixed.

   (a) Write down the government budget constraint in which lump-sum taxes are the only source of spending. If the government needs to borrow or lend, let B_t denote this quantity. Assume that $B_0 = 0$

   (b) Write down the consumer's problem where $U(C_t)$ stands for the consumer's preferences. Derive the first-order conditions for the consumer's problem.

   (c) Write down the planner's problem for this economy. Write down the first-order conditions. Is there a unique solution to the planner's problem?

   (d) Can K_{t-1}^g be either "too big" or "too small"? How would you interpret this statement.

2. Consider an economy with aggregate production $Y_t = F(K_t, N_t)$ and household utility $\sum_{t=0}^{\infty} \beta^t u(c_t)$. Households inelastically supply 1 unit of labor with stochastic efficiency $y_t \in Y$, which follows a Markov process with transitions $\pi_y(y_{t+1}|y_t)$. The wage per unit of efficiency is w_t, and capital, which depreciates at rate δ, receives rental rate r_t.

   (a) Suppose that households have access to a complete set of state-contingent contracts for saving. Write down the household's problem in sequence form.

   (b) Derive the household's Euler equation.

   (c) Now suppose that households can only save in non-state-contingent capital. Write down the functional equation for the household's problem.

   (d) Derive the household's Euler equation from (c).

   (e) Define a stationary recursive equilibrium for the economy in (c).

   (f) Under standard assumptions for u and F, what is the impact of a mean-preserving spread in labor income risk π_y on equilibrium prices w_t and r_t and output Y_t?

   (g) Now suppose that the government unexpectedly introduces a permanent tax on capital with proportional rate τ and rebates all the revenue to households through lump-sum transfers $\mathcal{T}$. Define the appropriate equilibrium concept and outline how one would go about solving for the equilibrium computationally.

3. There is a continuum of workers with unit mass. Each worker has preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t$$

where $0 < \beta < 1$ and c_t is consumption at time t. A worker (regardless of his/her employment status) dies at the end of a period with probability s and will be replaced by a new worker. A new worker enters the economy unemployed. An unemployed worker receives unemployment benefit $b > 0$ at the beginning of the period, and then receives a wage offer w that is drawn from a probability distribution $F(w)$, where $0 \leq w \leq \bar{w}$. Assume $\bar{w} > b$. If she/he accepts the offer and survives, she/he will become employed next period. When employed, the worker receives w units of consumption goods at the beginning of the period, then he/she dies with probability s. If he/she survives, he/she will continue to be employed at the same wage next period (i.e., an employed worker will be employed forever until he/she dies).

(a) Write down the equations for the worker's problem.

(b) Derive the expression for the reservation wage, w^*.

(c) Is w^* strictly higher than b? Explain.

(d) How does an increase in s change w^*? Explain.