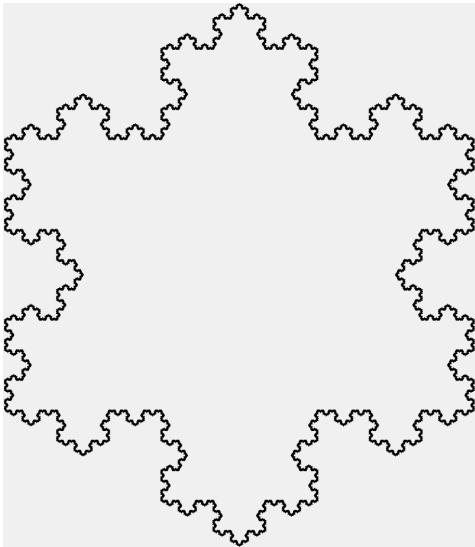
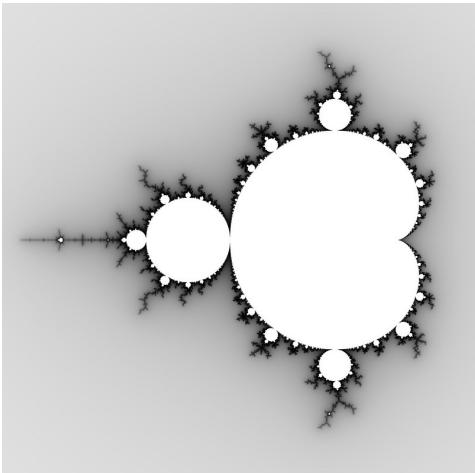
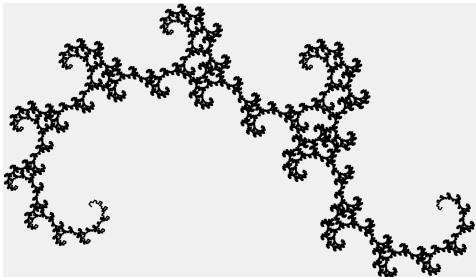
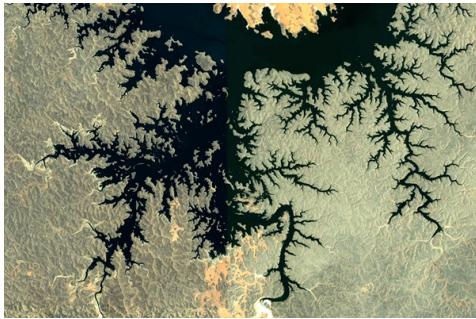




Introduction

Fractals are a type of mathematical pattern. If you zoom in on one part, the pattern looks similar to the original pattern (“self-similarity”). With math, we can imagine infinitely complex fractals. In nature, many patterns are fractal-like, from coastlines to romanesco cauliflower to clouds. Both mathematical and natural fractal structures can be beautiful.

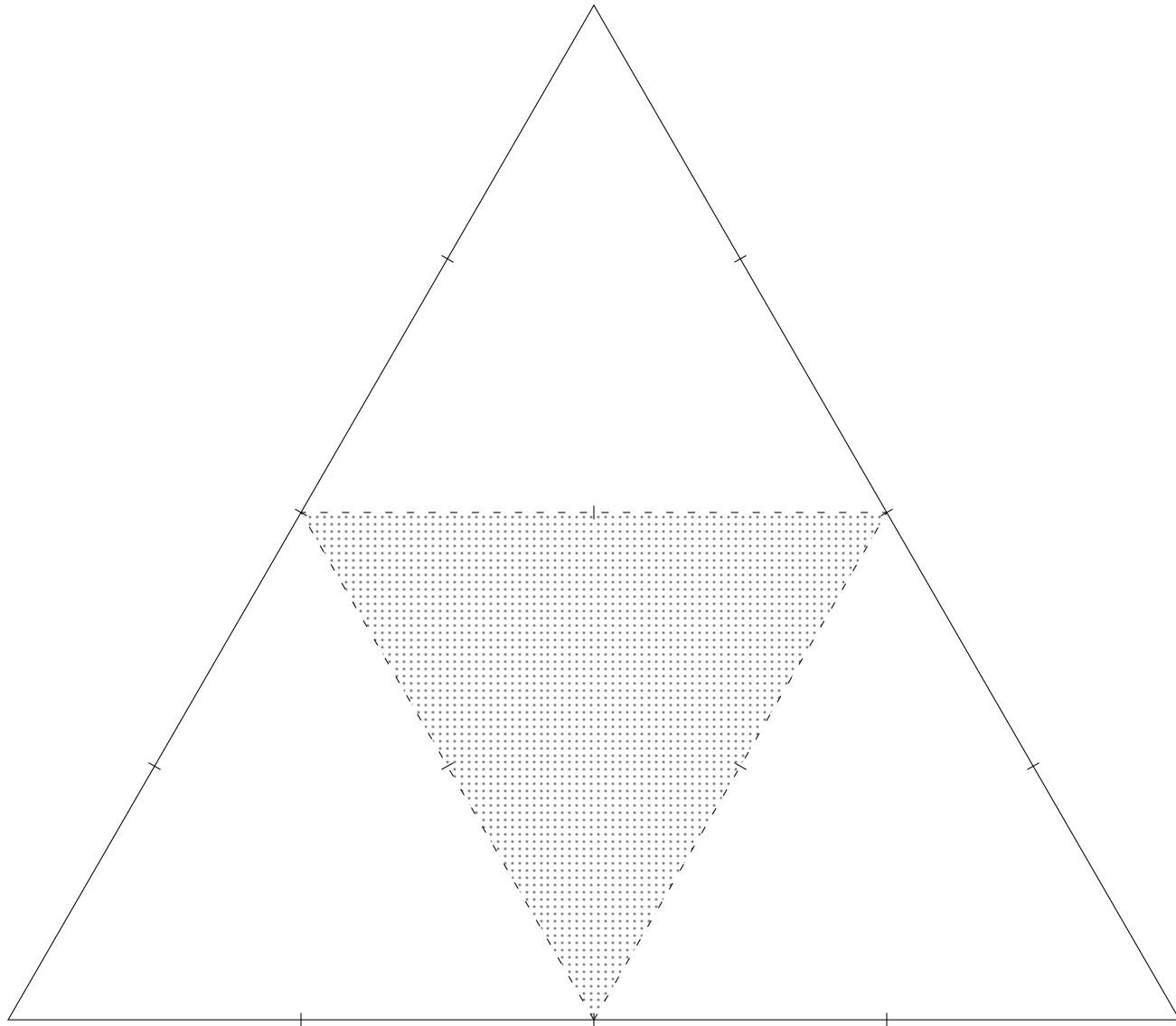


The following pages introduce you to some classic fractal patterns. There is drawing, geometry, and number patterns. As always, if you need help or want a hint, just raise your hand!

Enjoy! :)

**Sierpiński triangle**

Instructions: each round (or “iteration,” or “stage”), shade the middle of each \triangle to make it look like  and then keep repeating. (Hint: as in the template, mark the halfway point along each side, then connect the three points to make the upside-down triangle, and remember to shade it in.)



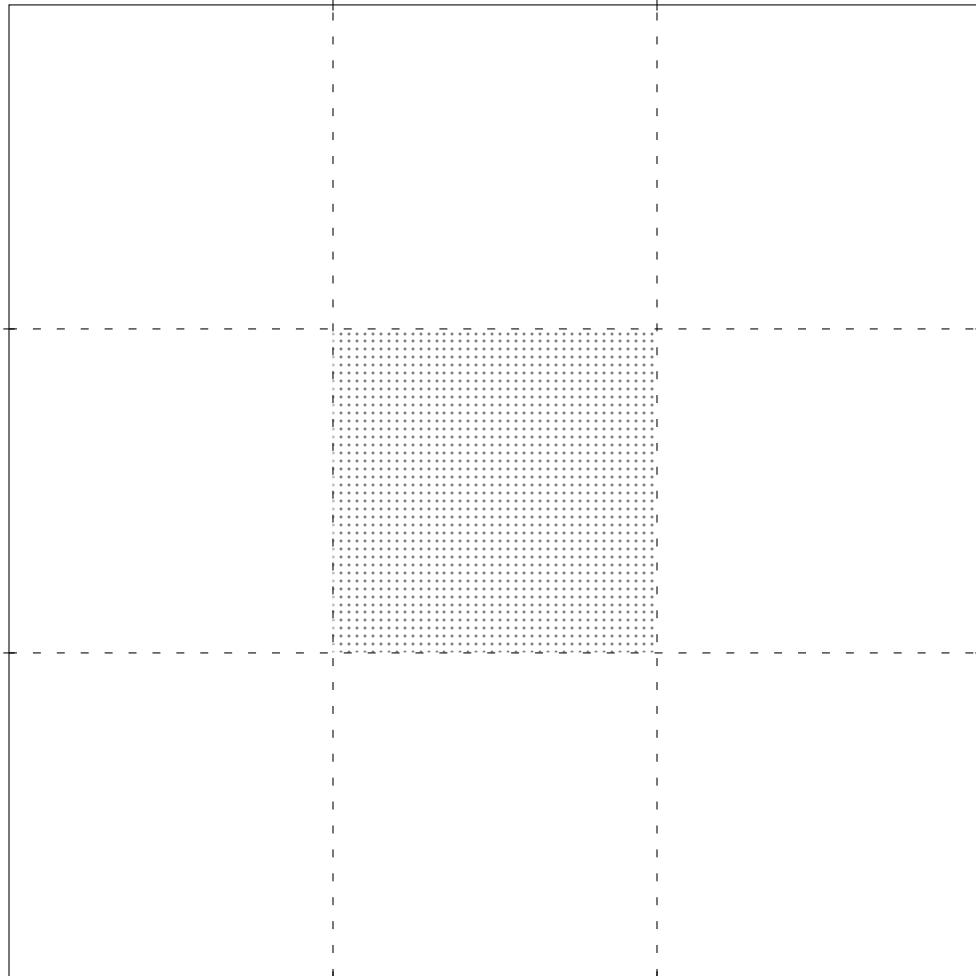
Questions (\star = bonus/challenge)

- F1. After one iteration, when it looks like , what fraction of the original triangle is shaded?
- F2. After two iterations, what fraction of the original triangle is shaded?
- F3. If you keep going forever (infinite iterations), will the triangle ever get completely shaded???
- F4. \star What fraction is shaded after three iterations? Four iterations? N iterations?

- M1. After one iteration, how many unshaded triangles \triangle are there?
- M2. After two iterations, how many unshaded triangles \triangle are there?
- M3. \star After N iterations, how many unshaded triangles are there?
- M4. \star If there are N unshaded triangles now, then how many will there be after the next iteration?

**Sierpiński carpet**

Instructions: each iteration, shade the middle of every unshaded square to make it look like and then keep repeating and repeating and repeating.



Questions (\star = bonus/challenge)

- F1. After one iteration, what fraction of the original square is shaded?
- F2. After two iterations, what fraction of the original square is shaded?
- F3. If you keep going forever (infinite iterations), will it ever become totally shaded???
- F4. \star What fraction is shaded after three iterations? Four iterations? N iterations?

- M1. Before the first iteration, there's one unshaded square ; after the first iteration, how many unshaded squares are there?
- M2. After the second iteration, how many unshaded squares are there?
- M3. \star After the third iteration, how many unshaded squares are there?
- M4. \star After N iterations, how many unshaded squares are there?

- P1. \star Imagine the shaded parts are holes cut out of a carpet. If the original big square has perimeter 36, then what's the perimeter of the square that's cut out in the first iteration?
- P2. \star In the second iteration, what's the perimeter of one of the eight squares you cut out?
- P3. \star In iteration N , what's the perimeter of one of the squares you cut out?



Koch curve (note: “curve” in math does not mean curvy, just that you can draw it without lifting your pen)

Instructions: each iteration, replace every straight line segment _____ with \triangle where all four line segments are the same length; and then keep repeating. Some examples: $/$ becomes \swarrow and \backslash becomes \searrow

..... |

..... |

Questions ($*$ = bonus/challenge)

- L1. If the original _____ has length 9, then how long is the \triangle after one iteration?
- L2. How long is the curve after two iterations?
- L3. Does the curve keep getting longer and longer after each iteration? Does it get infinitely long after infinite iterations??
- L4. $*$ If at some point the curve is L units long, then how long will it be after the next iteration?
- L5. $*$ After N iterations, how many times longer is the curve than the original line?

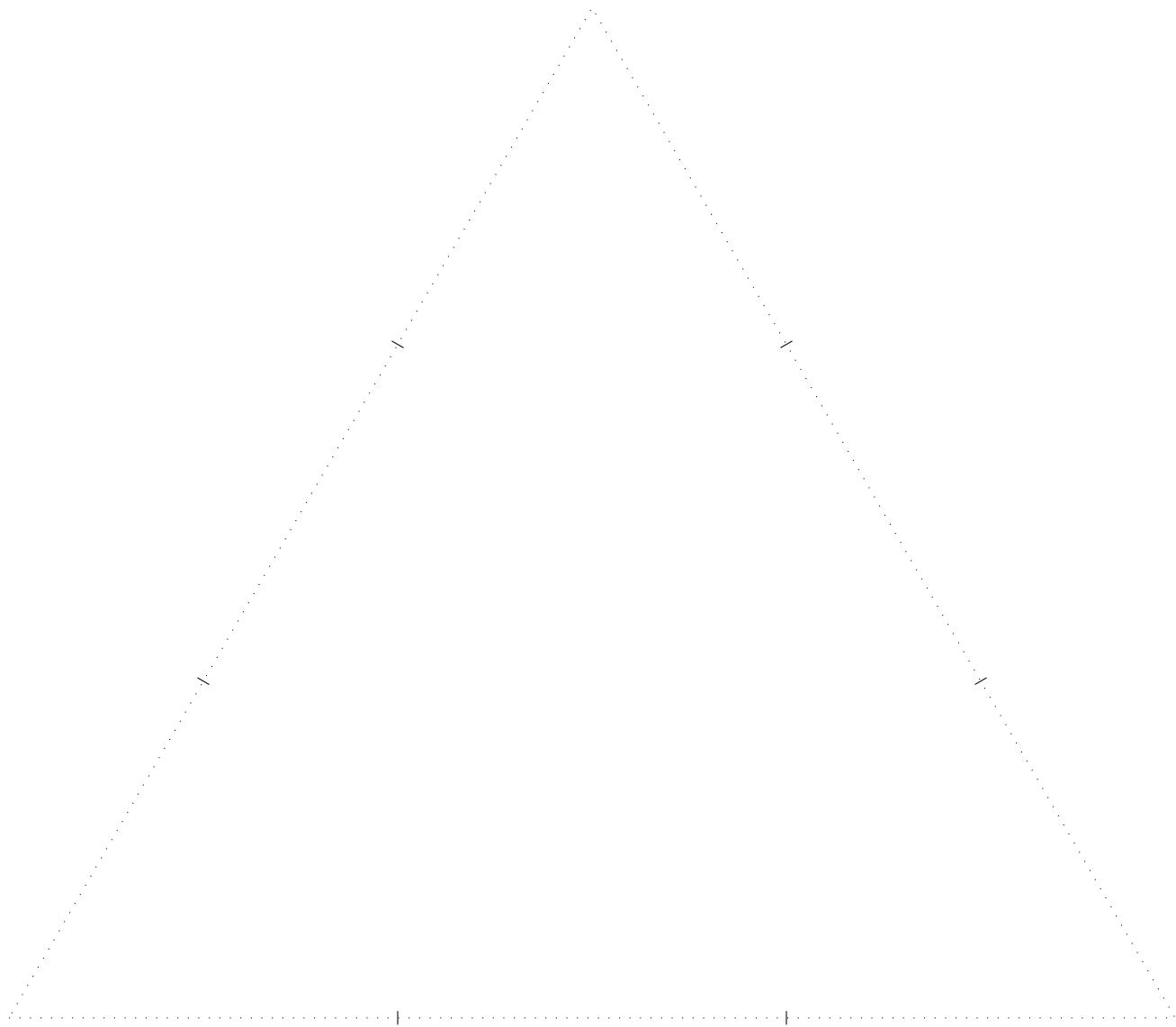
Fun fact: if you go forever (infinite iterations), then the Koch curve is not one-dimensional (like a line), but also not two-dimensional (like a square); it has dimension $\frac{\log(4)}{\log(3)} = 1.26$! Now you can name a 1.26-dimensional shape.

(The reason is complicated, but if you imagine making everything on this paper twice as wide and twice as tall, then each one-dimensional line $_$ increase to two times the length $--$ and each square \square increases to four times the area \boxplus while each Koch curve gets approximately 2.4 times bigger.)

**Koch snowflake**

Instructions (paper): same as for the Koch curve, just make sure all the “bumps” go on the outside. For example, in the first iteration, replace _____ with $\sqrt{}$ and replace \diagup with \swarrow and replace \diagdown with \searrow

Instructions (board): same as above, but don’t erase, just use a new color for each iteration. Start with light colors, then use darker colors.



**Extra templates for Sierpiński triangle and carpet**