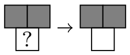
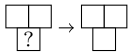
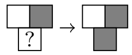
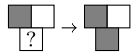
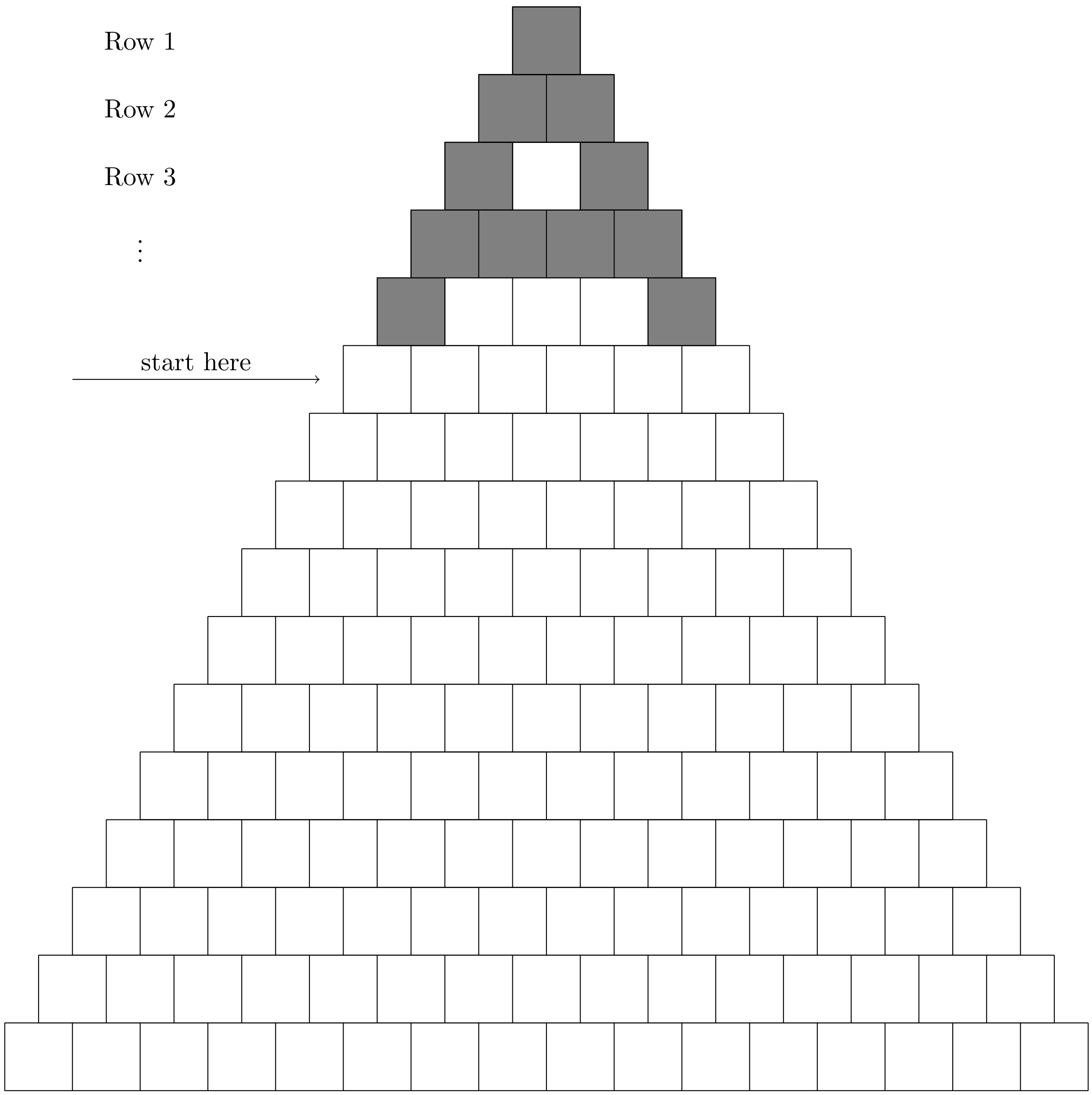
Pascal's triangle: black and white

Instructions: work from top to bottom. Anything outside the "triangle" is considered white. If both squares above are "black" (shaded in), then leave it white. If both squares above a square are white, then leave it white, too. If one square above is black and the other white, then shade it in gray/black. The first five rows have been completed to help you get started.

Visually:  and  and  and 

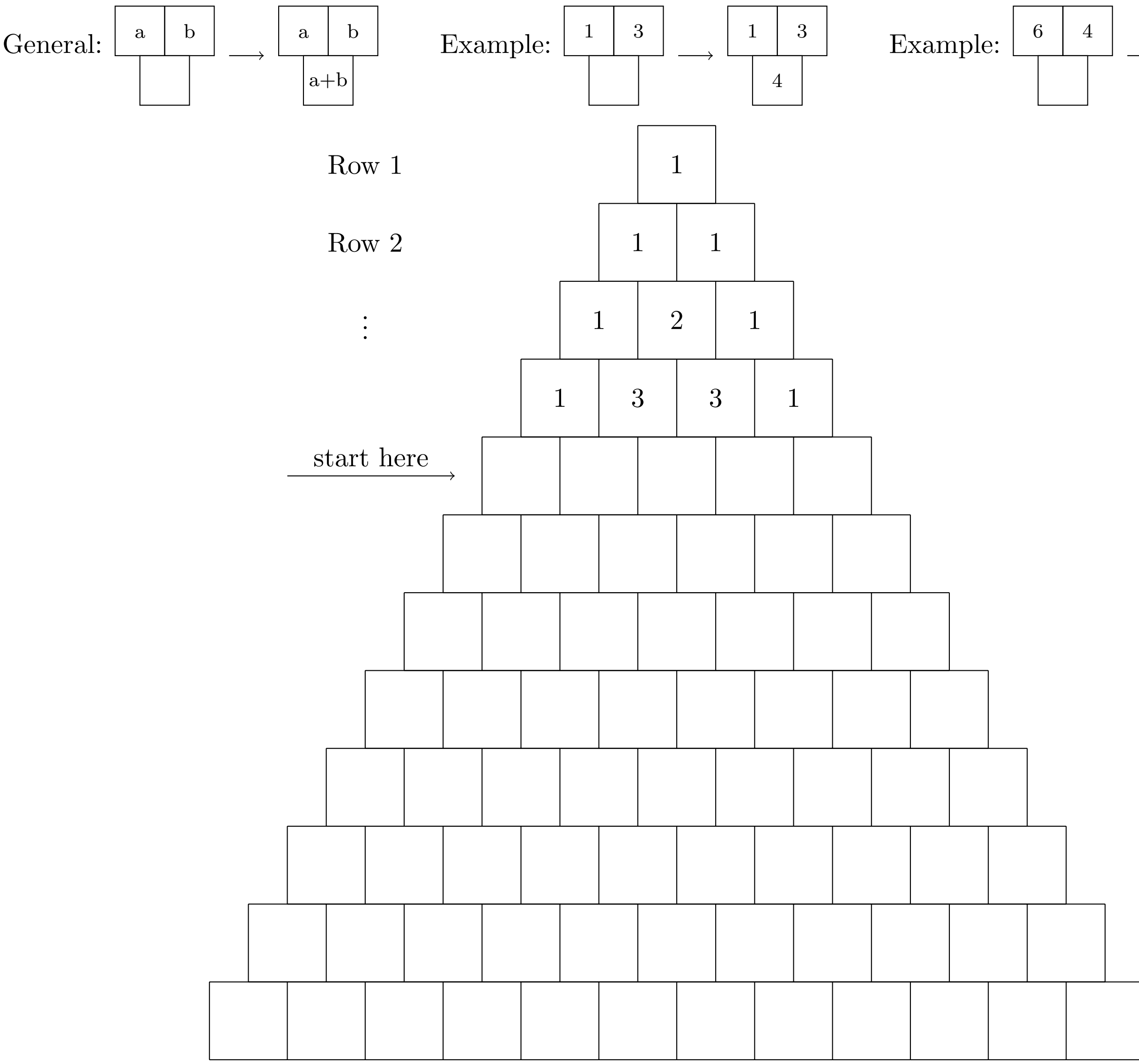


Questions:

Q1. Does this remind you of one of our fractals from last time? Which one?
Q2. What do you see in the row numbers that are powers of two? ($2^1 = 2$, $2^2 = 2 \times 2 = 4$, $2^3 = 2 \times 2 \times 2$, etc.)
Q3. Look at the pattern in Rows 1–8. Do you see a copy of that pattern anywhere else?
Q4. Do you see any symmetry? What kind?

Pascal's triangle: sums

Instructions: work from top to bottom. For each square in the current row, write the sum of the two numbers above it. Anything outside the "triangle" is considered zero, so the first and last square in each row should always be 1. The first four rows have been completed to help you get started.



Questions (continued next page):

- Q1. What type of symmetry does the triangle have?
- Q2. Can you find the sequence $1, 3, 6, 10, 15, \ldots$? Can you find it in two places? These are known as the "triangle numbers"; for example, the fourth triangle number is 10 because there are 10 squares in Rows 1–4 of the triangle above.
- Q3. If you sum consecutive pairs of triangle numbers, you get $1 + 3 = 4$, $3 + 6 = 9$, $6 + 10 = 16$, $10 + 15 = 25$, $\ldots$; do you recognize these special numbers? (Hint: what shape might you get by putting two triangles together?)
- Q4. Start at the first 1 in Row 2, and go diagonally down-right two spaces, then go diagonally down-left one space; altogether, your path is 1, 2, 3, then 6. Hey, $1 + 2 + 3 = 6$! Try the same pattern (down-right,

down-right, down-left) from a different 1 along the left side of the triangle; does the last number again equal the sum of the first three? What if you do more down-rights before you finally do a down-left? What if you start along the right side of the triangle and do a lot of down-lefts before a final down-right?

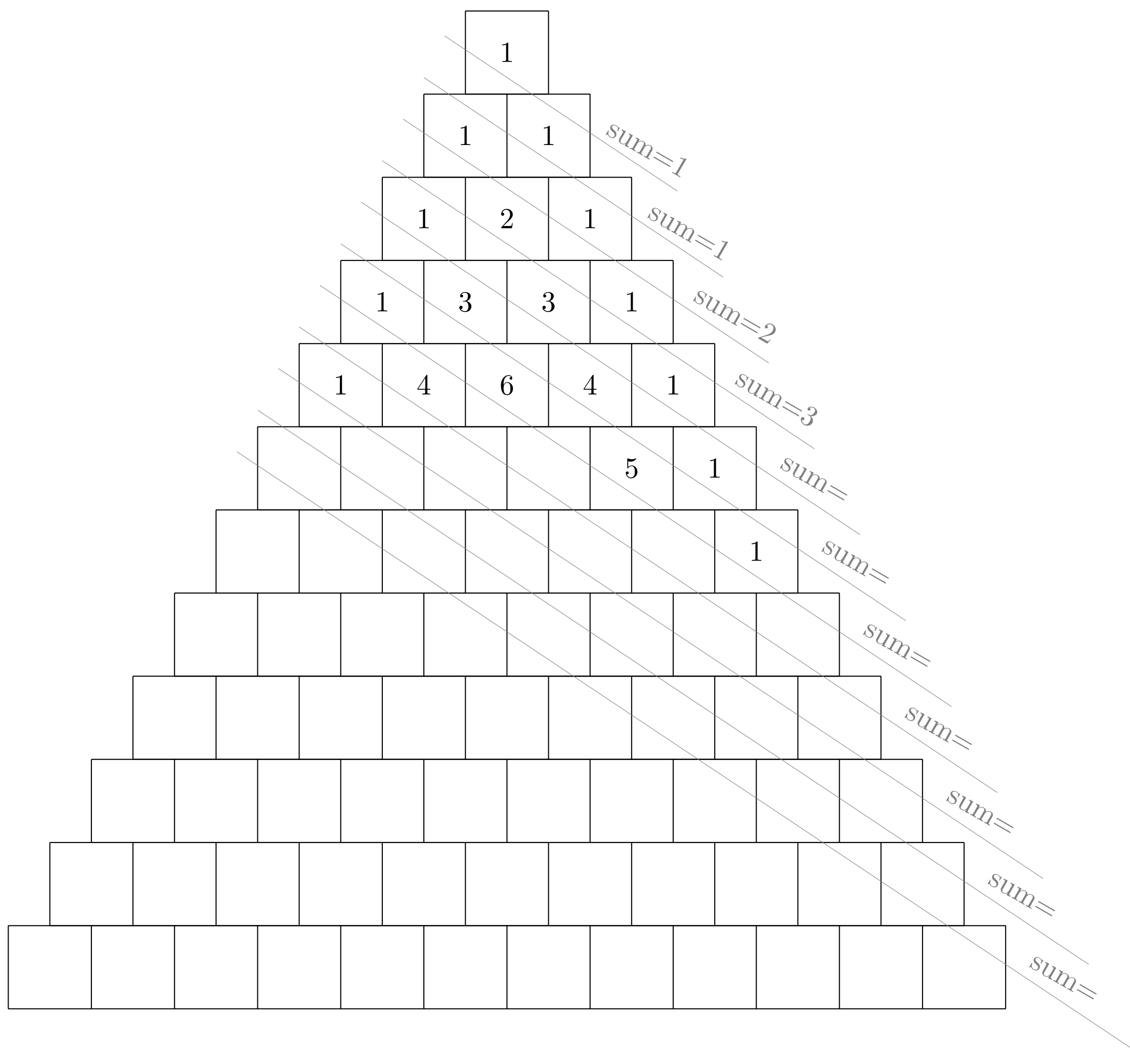
Q5. Consider the first five rows. Imagine each row contains the digits of a single number: the first row is 1, then 11, then 121, then 1,331, then 14,641. To get from 1 to 11, you multiply by 11. What do you multiply 11 by to get 121? What do you multiply 121 by to get 1,331? What do you multiply 1,331 by to get 14,641?

Q6. What would it look like if you shade in all squares with odd numbers (but not squares with even numbers)?

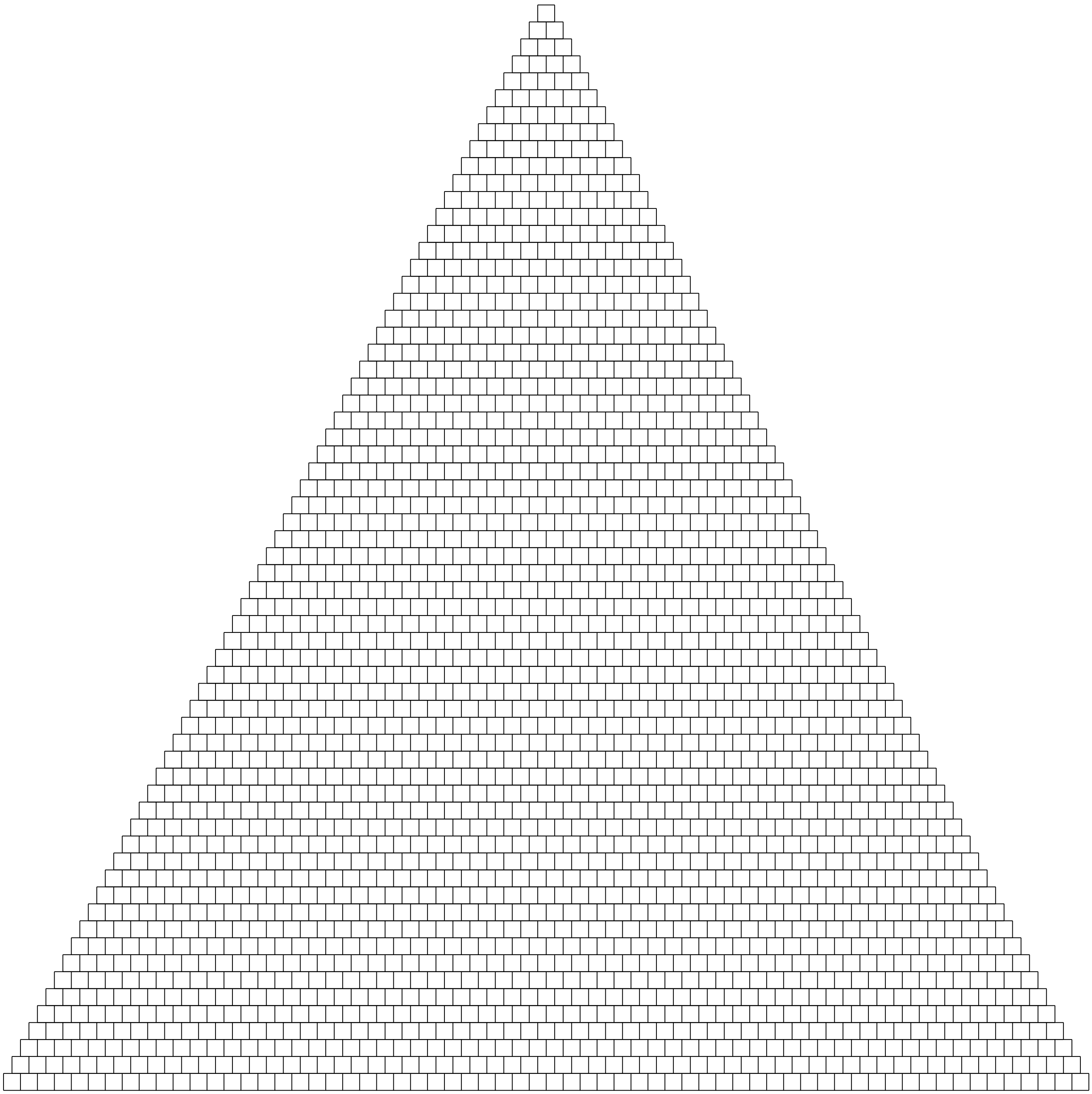
Q7. In the triangle below, sum up all the numbers inside each diagonal path. For example, above the top diagonal line is just a 1; between the top diagonal line and the diagonal line below it is just another 1; then $1 + 1 = 2$, then $2 + 1 = 3$, then $1 + 3 + 1 = 5$, etc. Keep adding like that; write the sums just to the right of the triangle, like has been done for the first few already. Do you recognize this sequence? Surprise: it's the Fibonacci numbers! $(1, 1, 2, 3, 5, 8, 13, \ldots$; to get the next number in the sequence, add the previous two togther: $1 + 1 = 2$, $1 + 2 = 3$, $2 + 3 = 5$, $3 + 5 = 8$, etc.)

Q8. What other patterns can you find? (Yes, there are more!)

This one helps you see the Fibonacci pattern hidden in Pascal's triangle.



Extra template: small grid (for shading/coloring)



Extra template: large grid (for numbers)