Question 1

Given vectors v $= \begin{bmatrix} 5 \\ -1 \end{bmatrix}$, $b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

all written in the standard basis, what is v in the basis defined by b_1 and b_2?
You are given that b_1 and b_2 are orthogonal to each other.

Answer

$$\frac{v \cdot b_1}{|b_1|^2} = \frac{4}{2} = 2$$

$$\frac{v \cdot b_2}{|b_1|^2} = \frac{6}{2} = 3$$

$$\boxed{v_b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}}$$

Question 2

Given vectors v $= \begin{bmatrix} 10 \\ -5 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $b_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

all written in the standard basis, what is v in the basis defined by b_1 and b_2?
You are given that b_1 and b_2 are orthogonal to each other.

Answer

$$\frac{v \cdot b_1}{|b_1|^2} = \frac{10}{25} = \frac{2}{5}$$

$$\frac{v \cdot b_2}{|b_1|^2} = \frac{55}{25} = \frac{11}{5}$$

$$\boxed{v_b = \begin{bmatrix} 2/5 \\ 11/5 \end{bmatrix}}$$