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Lappeenranta University of Technology
LUT Machine Vision and Pattern Recognition 2015-09-29

BM40A0700 Pattern Recognition

Lasse Lensu

Exercise 4: Bayesian classification and normal distribution

1. Classification with single-variable normal distribution (1 point): Consider two normally distributed probability distributions

$$p(x|\omega_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_i}{\sigma}\right)^2\right] \tag{1}$$

   with equal standard deviation (STD) $\sigma = 1$ and a priori probabilities $P(\omega_1) = P(\omega_2)$.

   Determine a classifier with a minimum classification error as follows: using the above formula for the normal distribution, derive the equation for the discriminant function

$$g_i(x) = P(\omega_i|x) \tag{2}$$

   Calculate then the natural logarithm of the discriminant function

$$g'_i(x) = \ln P(\omega_i|x). \tag{3}$$

   Use the result to determine the decision boundary for the classifier. That is, determine x where $g'_1(x) = g'_2(x)$. Verify the result using values $\mu_1 = 3$ and $\mu_2 = 6$.

2. Effect of unequal a priori (1 point): Consider two normally distributed probability distributions

$$p(x|\omega_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_i}{\sigma}\right)^2\right] \tag{4}$$

   with **unequal** a priori probabilities $P(\omega_1) \neq P(\omega_2)$ and with **equal** STD $\sigma = 1$. Determine a minimum-error classifier, derive the natural logarithm form of the discriminant function and use that to determine the decision boundary for the classifier in the case $P(\omega_1) = 2/3$, $P(\omega_2) = 1/3$, $\mu_1 = 3$ and $\mu_2 = 6$.

   Calculate the decision boundary by hand, and verify by plotting the distributions in Matlab.

3. Classification in two dimensions (2 points): Consider the following three-class classification problem. Each class is normally distributed with equal covariance matrix and a priori probabilities

$$\Sigma = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \qquad P(\omega_1) = P(\omega_2) = P(\omega_3), \qquad \mu_1 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad \mu_3 = \begin{pmatrix} 11 \\ -2 \end{pmatrix} \tag{5}$$

   Calculate the decision boundaries and draw a figure that shows the class centers and the decision boundaries. You can use the equations for linear discriminant functions to determine the decision boundaries.

   Additional files: `plotclass.m`.

PO BOX 20, FI-53851 LAPPEENRANTA, FINLAND
E-mail: lasse.lensu@lut.fi