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Lappeenranta University of Technology
LUT Mathematics and Physics

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BM40A1200 Digital Imaging and Image Preprocessing

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Exercise 10 solutions: Imaging and geometry

1. Basic geometric transformations (1 point): The most common transformation marices are translation, rotation and scaling, which can be combined for more complex geometric transformations.

$$T = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

   (a) Rotation by $180° = \pi(rad)$:

$$R = \begin{bmatrix} \cos\pi & -\sin\pi & 0 \\ \sin\pi & \cos\pi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   (b) 1. Translation 0.5 units to the right and 0.5 down:

$$T = \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

   2. Scaling with factor 0.5 w.r.t. the y-axis:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   (c) 1. Rotation by $270° = \frac{3\pi}{2}(rad)$:

$$R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   2. Translation 0.5 units up:

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$$

Additional files: `s005.m`.

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2. Radial distortion (2 points): An example script transforming an image with radial distortion is provided as a separate file.

   If the purpose would be to correct a radially distorted image, it would be necessary to detect either points, lines or regions from the image, assume something (a model) about their locations w.r.t. each other or about their shape, estimate the parameters of the transformation by using an optimisation procedure, and carry out the transformation.

   Additional resources: `cart2pol, interp2, pol2cart`.

   Additional files: `s042.m`.

3. Basic camera model (1 point): The simplest solution to the task involves perspective projection of the given 3-D points onto the virtual plane which goes through the point (0, 0, 1) and is parallel to the x, y plane. A more profound example demonstrating the distortionless perspective camera with the given parameters is provided as a separate file.

   Additional files: `s043.m`.