Color Image Enhancement Using the Support Fuzzification in the Framework of the Logarithmic Model

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Summary

The image enhancement method presented here uses point operations.

The particularity of the approach is that

- the logarithmic representation of images, i.e. the image values are elements of another (Euclidean) space, not R (the real line) in the case of gray images;
- and the image is structured using fuzzy partitions.

Simple though powerful methods for image enhancement can be obtained using affine transforms, defined in the framework of the logarithmic model.

Summary (cont)

Better results can be obtained if the image can be divided in statistically uniform subimages i.e. defining a partition on the image support and allowing a different transform in each subimage of the partition.

Using classical partitions we are faced with a block effect at the border of the subimages. To avoid this drawback the classical partitions can be replaced by fuzzy partitions. Their elements will be the “fuzzy windows”.

In each of them an affine transform will be defined using the fuzzy mean and fuzzy variance computed for the pixels of the analyzed window.

The final image is obtained by summing up the images of every fuzzy window in a weight way. The weights used are membership degrees, which define the fuzzy partition.

The Vector Space of Gray Levels

- the space of gray levels is the set: $E = (-1,1)$

we shall use the following notations:

- the addition: $\langle + \rangle$
- the scalar multiplication: $\langle \times \rangle$
- the scalar product: $(\cdot \mid \cdot)_E$
- the norm: $\| \, . \, \|_E$

The Addition

The sum of two gray levels $v_1\langle + \rangle v_2$

$$\forall v_1, v_2 \in E, \qquad v_1\langle + \rangle v_2 = \frac{v_1 + v_2}{1 + v_1 v_2} \tag{1.1}$$

The neutral element: $\theta = 0$

The opposite of $v \in E$: $w = -v$

The Subtraction

$$\forall v_1, v_2 \in E, \qquad v_1\langle - \rangle v_2 = \frac{v_1 - v_2}{1 - v_1 v_2} \tag{1.2}$$

The Multiplication by a Scalar

The multiplication $\langle \times \rangle$ of a gray level v by a real scalar λ:

$\forall v \in E, \forall \lambda \in R,$

$$\lambda \langle \times \rangle v = \frac{(1+v)^{\lambda} - (1-v)^{\lambda}}{(1+v)^{\lambda} + (1-v)^{\lambda}} \qquad (1.3)$$

The addition $\langle + \rangle$ and the scalar multiplication $\langle \times \rangle$ induce on E a real vector space structure.

The Fundamental Isomorphism

The vector space of gray levels $(E, \langle + \rangle, \langle \times \rangle)$ is isomorphic to the space of real numbers $(R, +, \cdot)$ by the function:

$\varphi : E \to R :\ \forall v \in E,$

$$\varphi(v) = \frac{1}{2}\ln\left(\frac{1+v}{1-v}\right) \tag{1.4}$$

The isomorphism φ verifies:

$\forall v_1, v_2 \in E,$

$$\varphi(v_1 \langle + \rangle v_2) = \varphi(v_1) + \varphi(v_2) \tag{1.5}$$

$\forall \lambda \in R, \forall v \in E,$

$$\varphi(\lambda \langle \times \rangle v) = \lambda \cdot \varphi(v) \tag{1.6}$$

The Euclidean Space of Gray Levels.

The scalar product of two gray levels:

$(\cdot \mid \cdot)_E : E \times E \to R$, $\forall v_1, v_2 \in E$,

$$(v_1 \mid v_2)_E = \varphi(v_1) \cdot \varphi(v_2) \tag{1.7}$$

With the scalar product $(\cdot \mid \cdot)_E$ the gray level space becomes an Euclidean space.

The norm

$\| \cdot \|_E : E \to R^+$, $\forall v \in E$,

$$\| v \|_E = \sqrt{(v \mid v)_E} = | \varphi(v) | \tag{1.8}$$

For each image f heaving the support D, the mean $\mu_\varphi(f)$ and the variance $\sigma_\varphi^2(f)$ are defined:

$$\mu_\varphi(f) = \underset{(x,y)\in D}{\langle + \rangle} \left(\frac{1}{card(D)} \langle \times \rangle f(x,y) \right) \tag{1.9}$$

$$\sigma_\varphi^2(f) = \sum_{(x,y)\in D} \frac{\| f(x,y) \langle - \rangle \mu_\varphi(f) \|_E^2}{card(D)} \tag{1.10}$$

where $card(D)$ is the cardinality of D.

The Logarithmic Model for the Color Space

Consider the cube E^3 as the color space. Let be $q=(r,g,b)\in E^3$, a color having the components r (*red*), g (*green*) and b (*blue*).

The Addition

$\forall q_1, q_2 \in E^3,$

$$q_1\langle+\rangle q_2 = (r_1\langle+\rangle r_2, g_1\langle+\rangle g_2, b_1\langle+\rangle b_2) \tag{2.1}$$

The Subtraction

$\forall q_1, q_2 \in E^3,$

$$q_1\langle-\rangle q_2 = (r_1\langle-\rangle r_2, g_1\langle-\rangle g_2, b_1\langle-\rangle b_2) \tag{2.2}$$

The Scalar Multiplication

$\forall \lambda \in \mathrm{R}, \forall q \in E^3,$

$$\lambda\langle\times\rangle q = (\lambda\langle\times\rangle r, \lambda\langle\times\rangle g, \lambda\langle\times\rangle b) \tag{2.3}$$

The Euclidean Space of the Colors

The scalar product:

$$(\cdot\,|\,\cdot)_{E^3} : E^3 \times E^3 \to R,\ \ \forall\, q_1 = (r_1, g_1, b_1),\ q_2 = (r_2, g_2, b_2) \in E^3,$$

$$(q_1\,|\,q_2)_{E^3} = \varphi(r_1)\varphi(r_2) + \varphi(g_1)\varphi(g_2) + \varphi(b_1)(b_2) \qquad (2.4)$$

The norm:

$$\|\cdot\|_{E^3} : E^3 \to R^+,\ \forall q = (r, g, b) \in E^3,$$

$$\|\,q\,\|_{E^3} = \sqrt{\varphi^2(r) + \varphi^2(g) + \varphi^2(b)} \qquad (2.5)$$

The Fuzzification of the Image Support

A gray level image is described by its intensity function:

$$f : D \rightarrow E \tag{3.1}$$

where $D \subset R^2$ is the image support.

Without loss of generality, the rectangle

$$D = [x_0, x_1] \times [y_0, y_1] \tag{3.2}$$

can be considered as the image support.

The coordinates of a pixel within the support D will be noted (x, y).

Let there be

$$P = \{W_{ij} \mid (i, j) \in [0, m] \times [0, n]\} \tag{3.3}$$

a fuzzy partition of the support D.

Consider the polynomials,

$qx_i : [x_0, x_1] \to [0,1],$

$$qx_i(x) = C_m^i \frac{(x - x_0)^i (x_1 - x)^{m-i}}{(x_1 - x_0)^m} \tag{3.4}$$

$qy_j : [y_0, y_1] \to [0,1],$

$$qy_j(y) = C_n^j \frac{(y - y_0)^j (y_1 - y)^{n-j}}{(y_1 - y_0)^n} \tag{3.5}$$

$p_{ij} : D \to [0,1],$

$$p_{ij}(x, y) = qx_i(x) \cdot qy_j(y) \tag{3.6}$$

where $C_m^i = \dfrac{m!}{i!(m-i)!}$, $C_n^j = \dfrac{n!}{j!(n-j)!}$ and $(i, j) \in [0, m] \times [0, n]$.

The membership degrees of a point $(x,y) \in D$ to the fuzzy window W_{ij} are given by the functions:

$w_{ij} : D \to [0,1],$

$$w_{ij}(x,y) = \frac{\left(p_{ij}(x,y)\right)^{\gamma}}{\sum_{j=0}^{n} \sum_{i=0}^{m} \left(p_{ij}(x,y)\right)^{\gamma}} \qquad (3.7)$$

The membership degrees $w_{ij}(x,y)$ describe the position of the point (x,y) within the support D.

The parameter $\gamma \in (0,\infty)$ has the role of a tuning parameter offering a greater flexibility in building the fuzzy partition P. In other words, γ controls the fuzzification-defuzzification degree of the partition P.

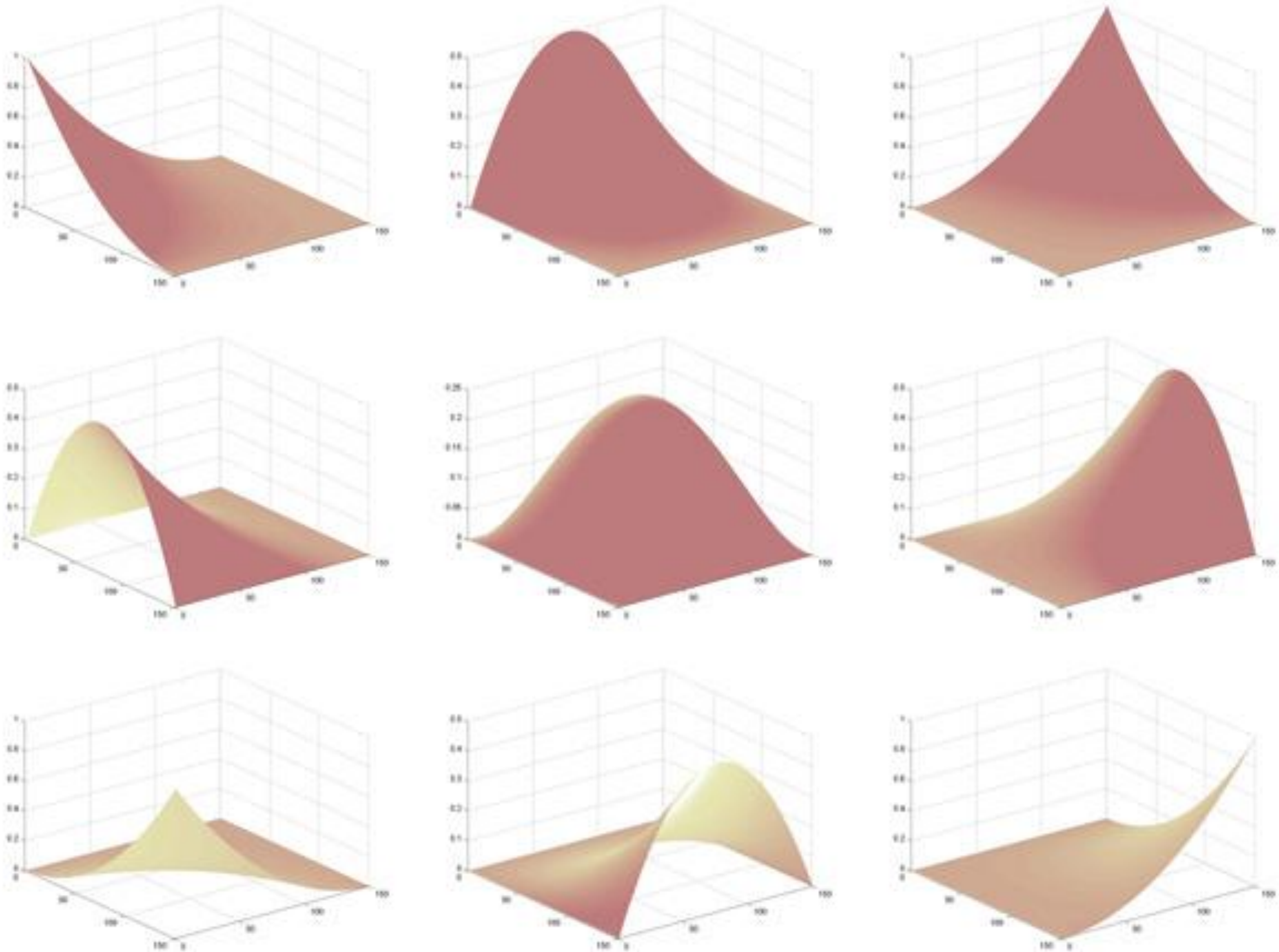

Fig.1 The graphics of membership degrees for $m=2$, $n=2$ and $\lambda=1$

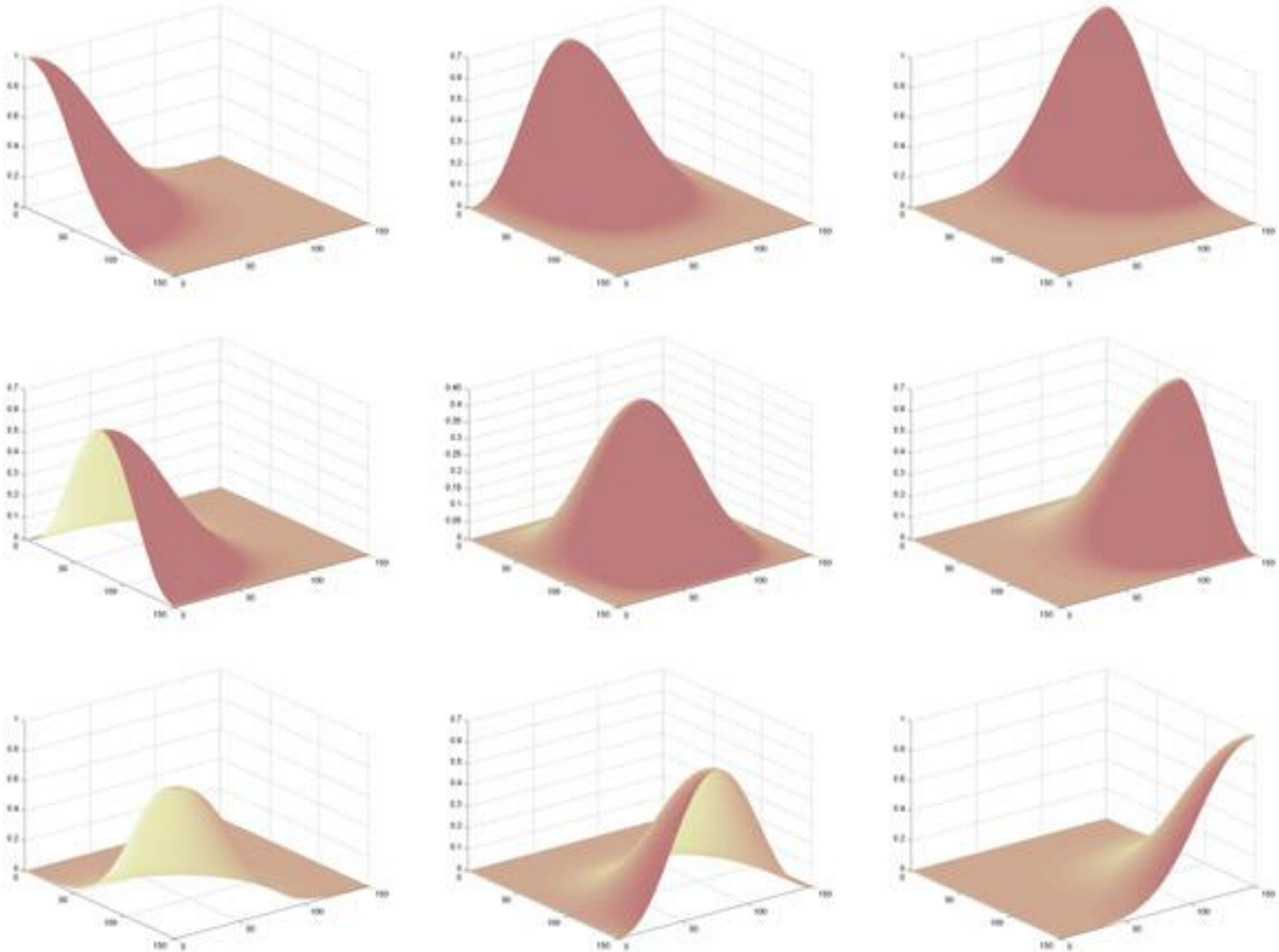

Fig.2 The graphics of membership degrees for $m=2$, $n=2$ and $\lambda=2$

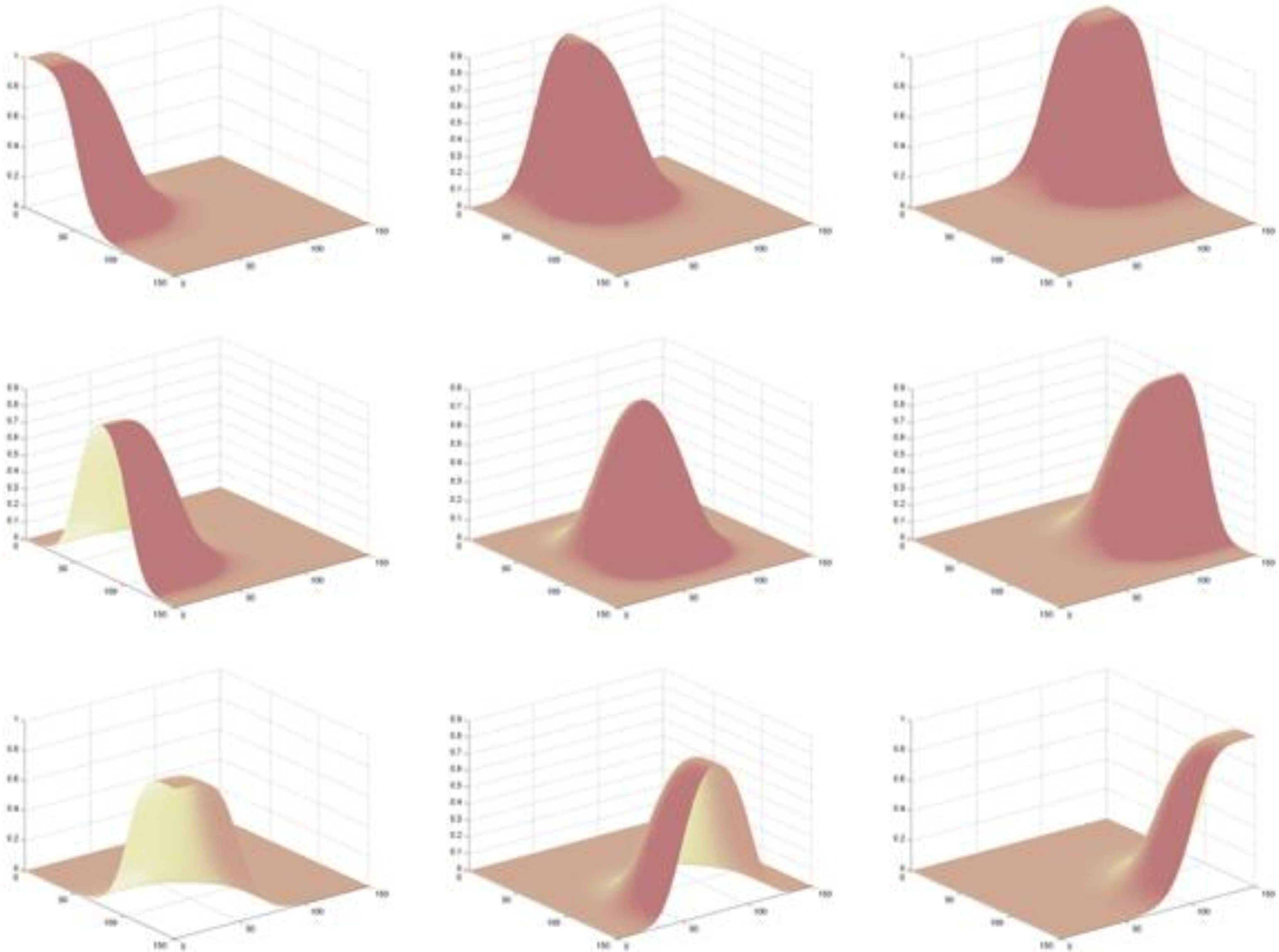

Fig.3 The graphics of membership degrees for $m=2$, $n=2$ and $\lambda=4$

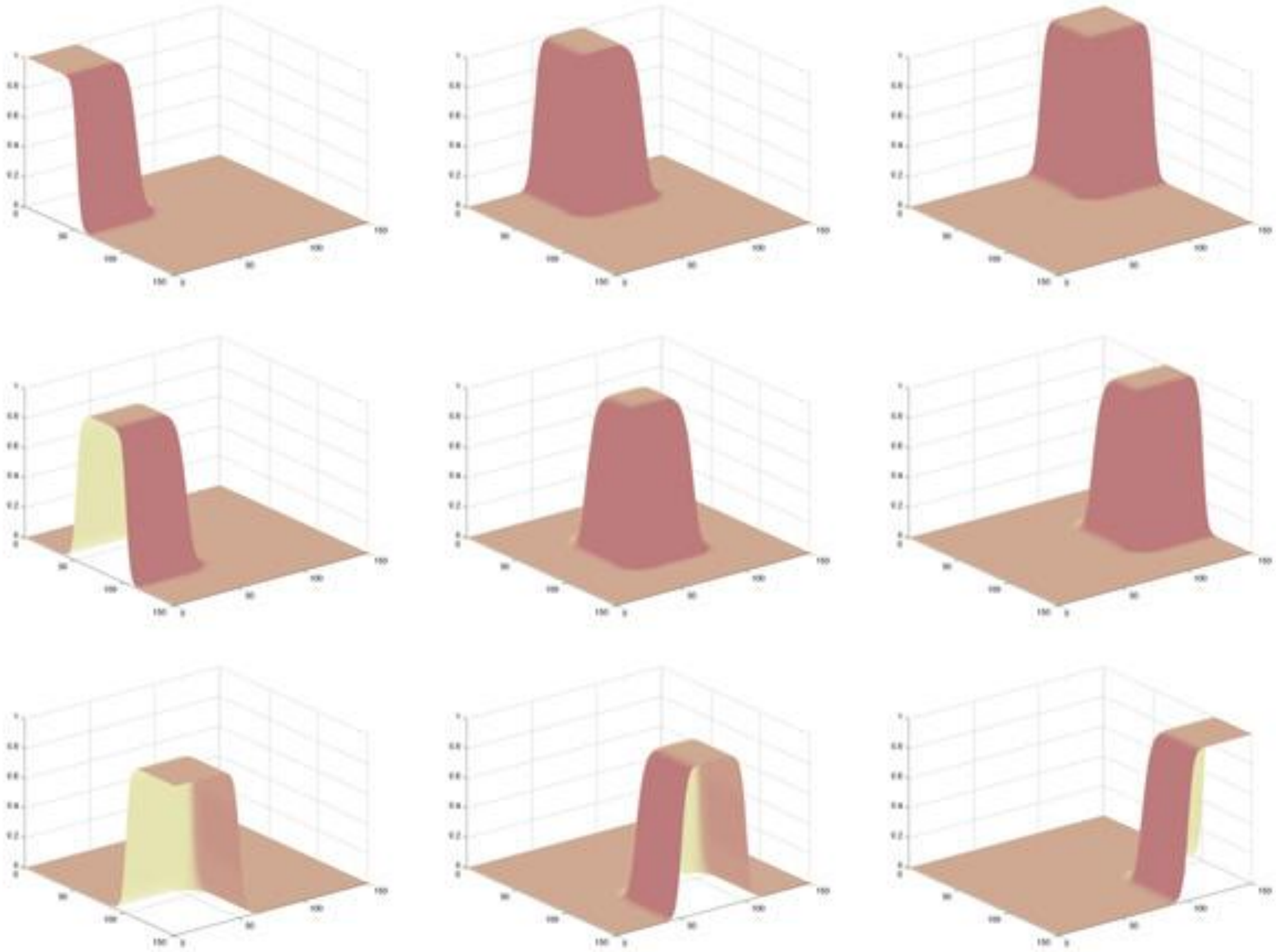

Fig.4 The graphics of membership degrees for $m=2$, $n=2$ and $\lambda=16$

For each window W_{ij} the fuzzy cardinality $card(W_{ij})$, the fuzzy mean $\mu_\varphi(f, W_{ij})$ and the fuzzy variance $\sigma^2_\varphi(f, W_{ij})$ of the image f are defined:

$$card(W_{ij}) = \sum_{(x,y)\in D} w_{ij}(x, y) \tag{3.8}$$

$$\mu_\varphi(f, W_{ij}) = \left\langle + \right\rangle_{(x,y)\in D} \left(\frac{w_{ij}(x, y)}{card(W_{ij})} \langle\times\rangle f(x, y) \right) \tag{3.9}$$

$$\sigma^2_\varphi(f, W_{ij}) = \sum_{(x,y)\in D} \frac{w_{ij}(x, y) \| f(x, y) \langle-\rangle \mu_\varphi(f, W_{ij}) \|^2_E}{card(W_{ij})} \tag{3.10}$$

The Enhancement Method for Gray Level Image

Let us consider these affine transforms on the images set $F(D,E)$, defined as following: $\psi : F(D,E) \to F(D,E)$, $\forall f \in F(D,E)$

$$\psi(f) = \lambda\langle\times\rangle(f\langle+\rangle\tau) \tag{4.1}$$

where $\lambda \in R$, $\lambda \neq 0$ and $\tau \in E$.

An image can be processed in two steps:

- a translation

$$f\langle+\rangle\tau \tag{4.2}$$

with a constant value τ, which leads to a change in the image brightness

- a scalar multiplication

$$\lambda\langle\times\rangle f \tag{4.3}$$

by the factor λ - leading to a change in the image contrast.

The determination of the two parameters (λ, τ) will be made, so that the new image will have the mean zero and the variance $\frac{1}{3}$.

$$\lambda = \frac{\sigma_u}{\sigma_\varphi(f)} \tag{4.4}$$

where $\sigma_u^2 = \frac{1}{3}$.

$$\tau = \langle - \rangle \mu_\varphi(f) \tag{4.5}$$

From statistical point of view, this means that the resulted image will be very close to an image with a uniform distribution of the gray levels.

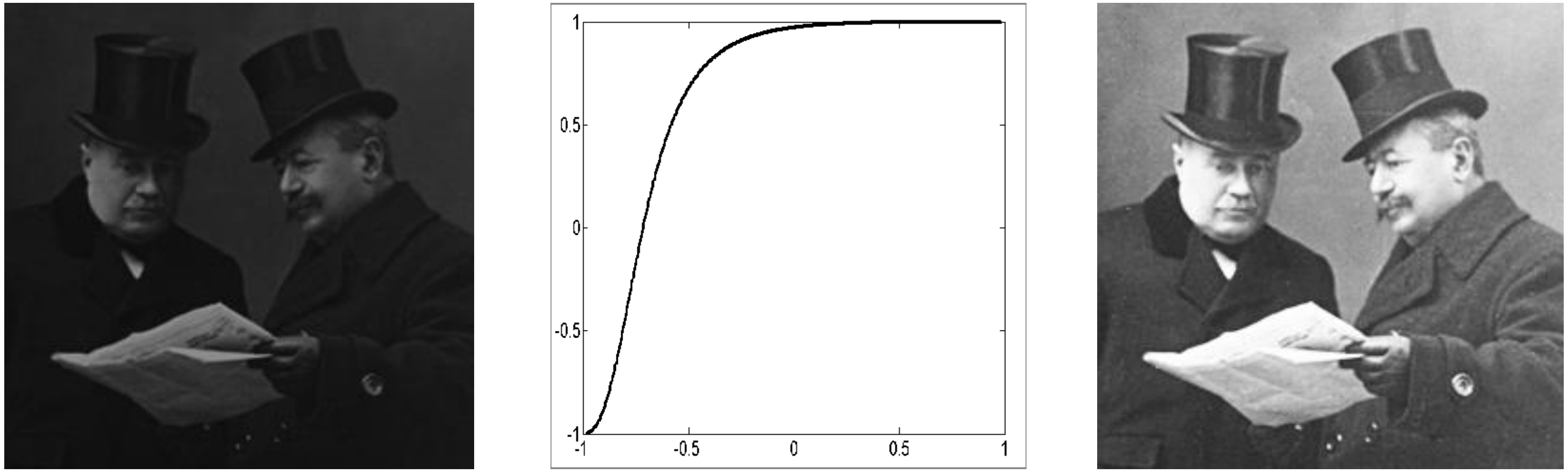

Fig. 5. The original image "news", the affine transform and the enhanced image.

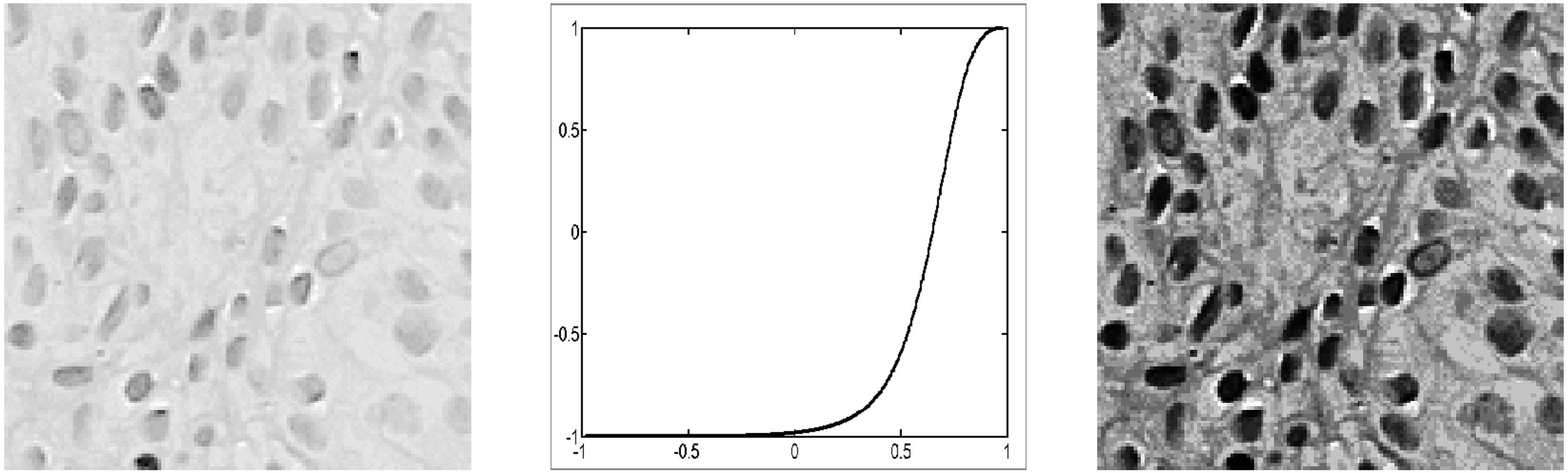

Fig. 6. The original image "cells", the affine transform and the enhanced image

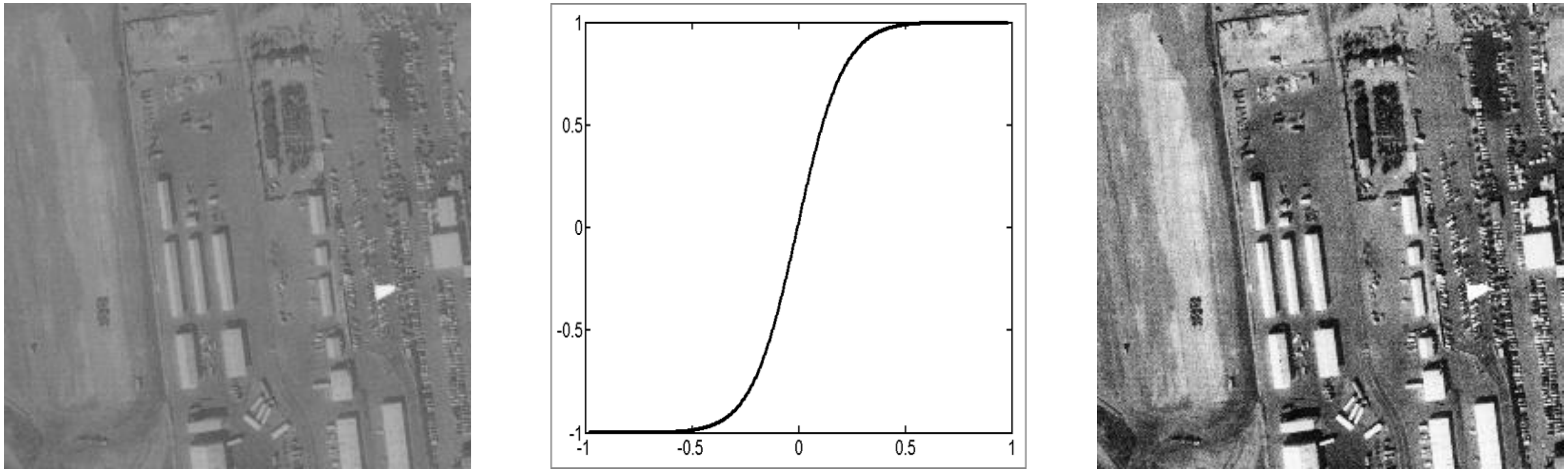

Fig. 7 The original image “lax”, the affine transform and the enhanced image.

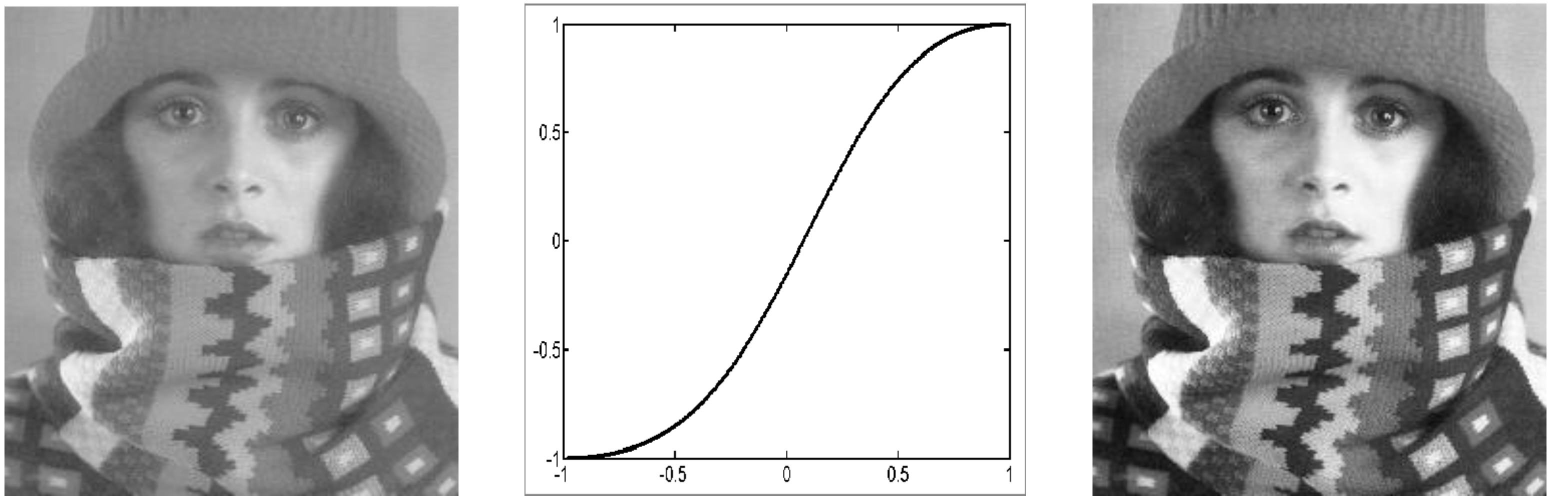

Fig. 8 The original image “miss”, the affine transform and the enhanced image.

The fuzzy window W_{ij} will supply a couple of parameters (λ, τ), which reflects the gray level statistics:

$$\lambda_{ij} = \frac{\sigma_u}{\sigma_\varphi(f, W_{ij})} \tag{4.6}$$

where $\sigma_u^2 = 1/3$.

$$\tau_{ij} = \langle - \rangle \mu_\varphi(f, W_{ij}) \tag{4.7}$$

The function for the fuzzy window W_{ij}:

$$\psi_{ij}(f) = \frac{\sigma_u}{\sigma_\varphi(f, W_{ij})} \langle\times\rangle \left(f \langle-\rangle \mu_\varphi(f, W_{ij}) \right) \tag{4.8}$$

The transform ψ_{enh} is built as a sum of the affine transforms ψ_{ij} weighted with the degrees of membership w_{ij}:

$$\psi_{enh}(f) = \sum_{j=0}^{n} \sum_{i=0}^{m} w_{ij} \langle\times\rangle \psi_{ij}(f) \tag{4.9}$$

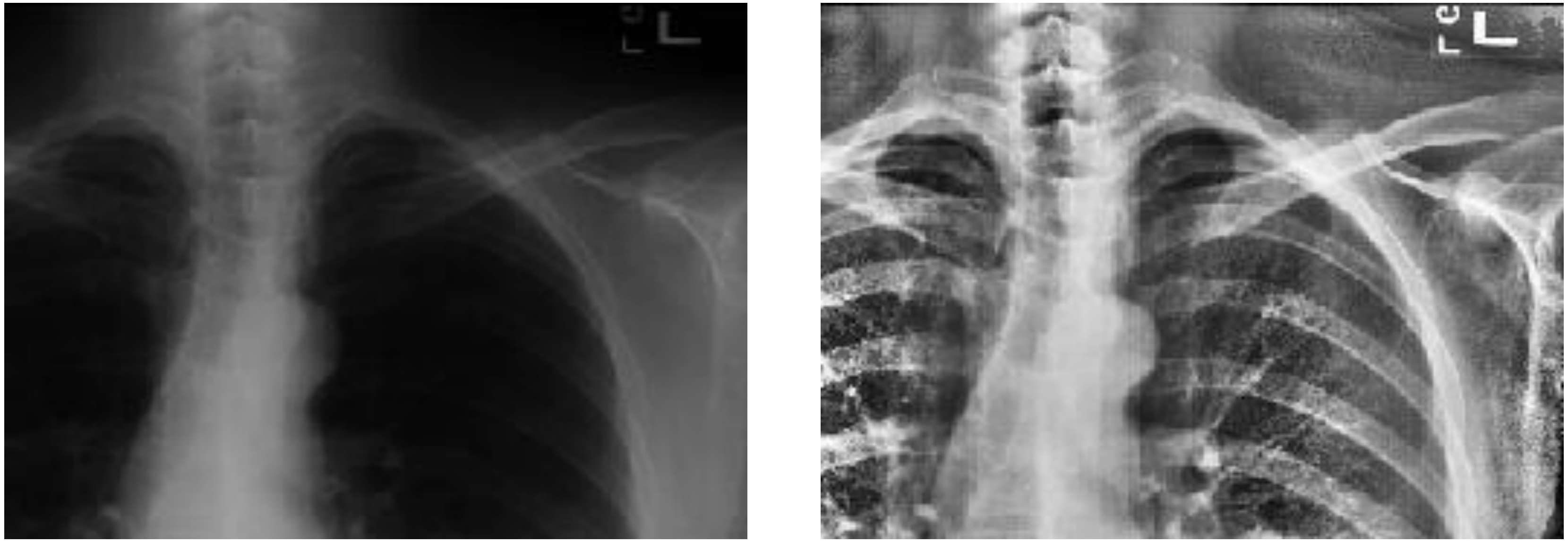

Fig.9 The original image “med” and the enhancement with fuzzy partition.

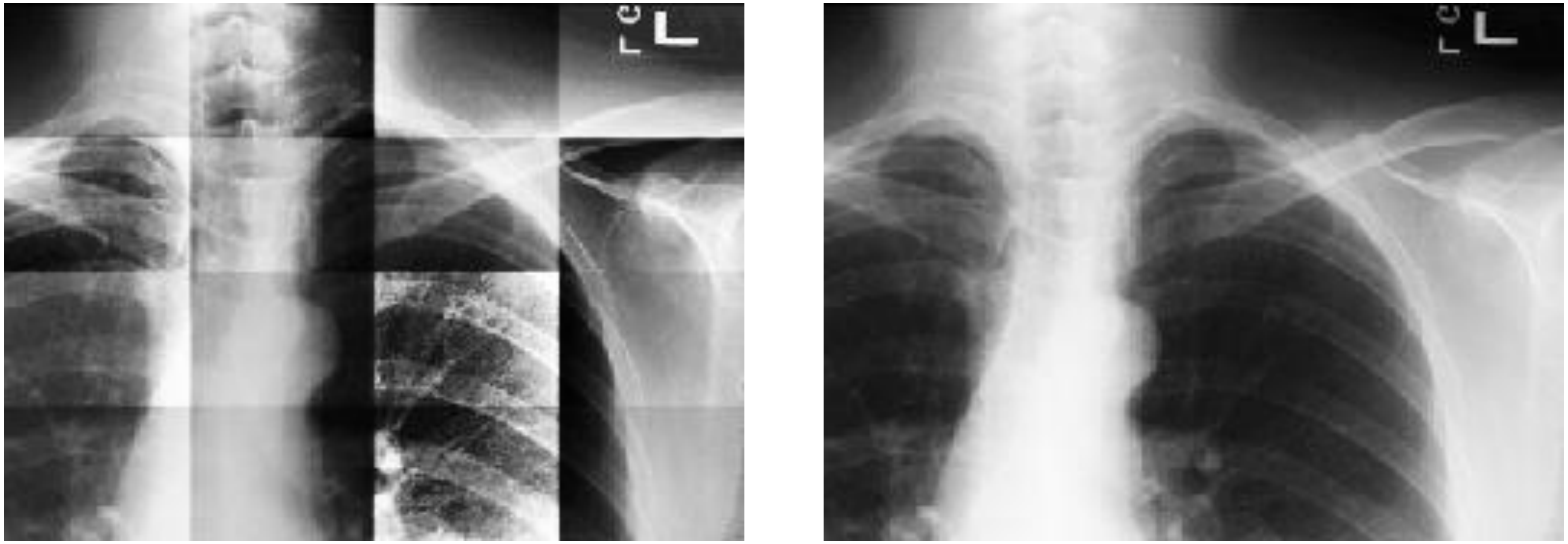

Fig.10 The enhancement with classical partition and the enhancement without partition

Fig.11 The original image “street” and the enhancement with fuzzy partition.

The Enhancement Method for Color Images

A color image is defined by three scalar functions *red*, *green* and *blue:*

$$r : D \to E \quad (5.1)$$

$$g : D \to E \quad (5.2)$$

$$b : D \to E \quad (5.3)$$

The image luminosity: $l : D \to E$,

$$l = \frac{1}{3} \langle\times\rangle \left(r \langle+\rangle g \langle+\rangle b \right) \quad (5.4)$$

The enhanced image r_{enh}, g_{enh}, b_{enh}:

$$r_{enh} = \sum_{j=0}^{n} \sum_{i=0}^{m} \frac{w_{ij}\sigma_u}{\sigma_\varphi(l, W_{ij})} \langle\times\rangle \left(r \langle-\rangle \mu_\varphi(l, W_{ij}) \right) \tag{5.5}$$

$$g_{enh} = \sum_{j=0}^{n} \sum_{i=0}^{m} \frac{w_{ij}\sigma_u}{\sigma_\varphi(l, W_{ij})} \langle\times\rangle \left(g \langle-\rangle \mu_\varphi(l, W_{ij}) \right) \tag{5.6}$$

$$b_{enh} = \sum_{j=0}^{n} \sum_{i=0}^{m} \frac{w_{ij}\sigma_u}{\sigma_\varphi(l, W_{ij})} \langle\times\rangle \left(b \langle-\rangle \mu_\varphi(l, W_{ij}) \right) \tag{5.7}$$

Fig. 12 The original image “aerial1” and the enhanced with fuzzy partition.

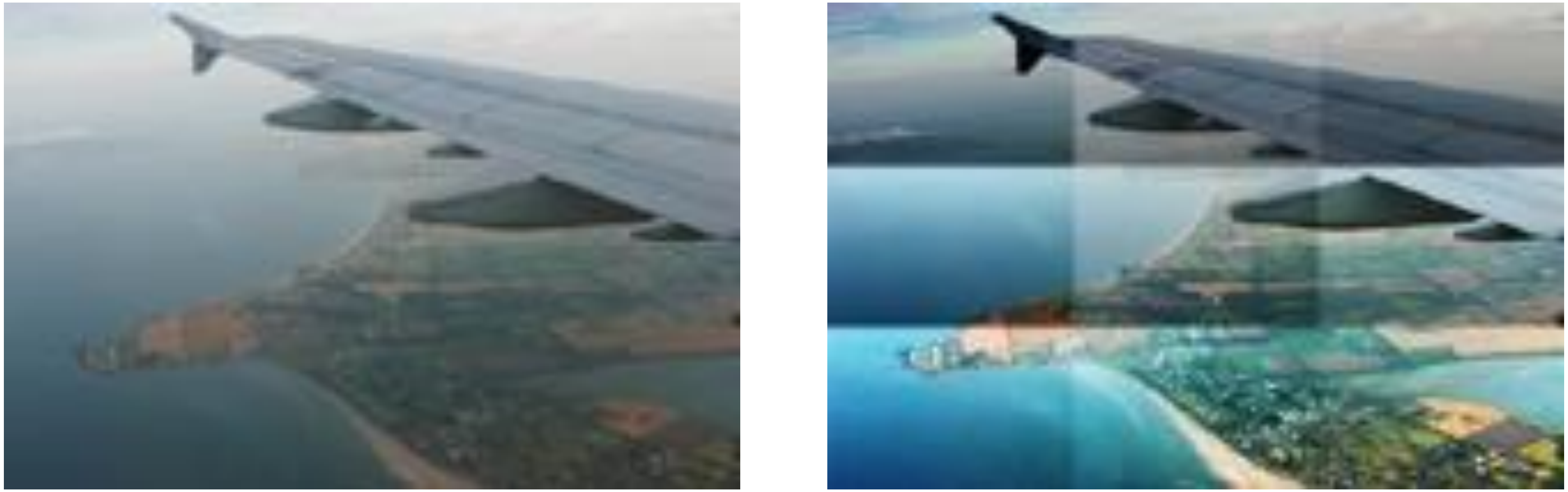

Fig. 13 Enhanced without partition and the enhanced with classical partition.

Fig. 14. The original image “island” and the enhanced with fuzzy partition.

Fig. 15. The original image “aerial2” and the enhanced with fuzzy partition.

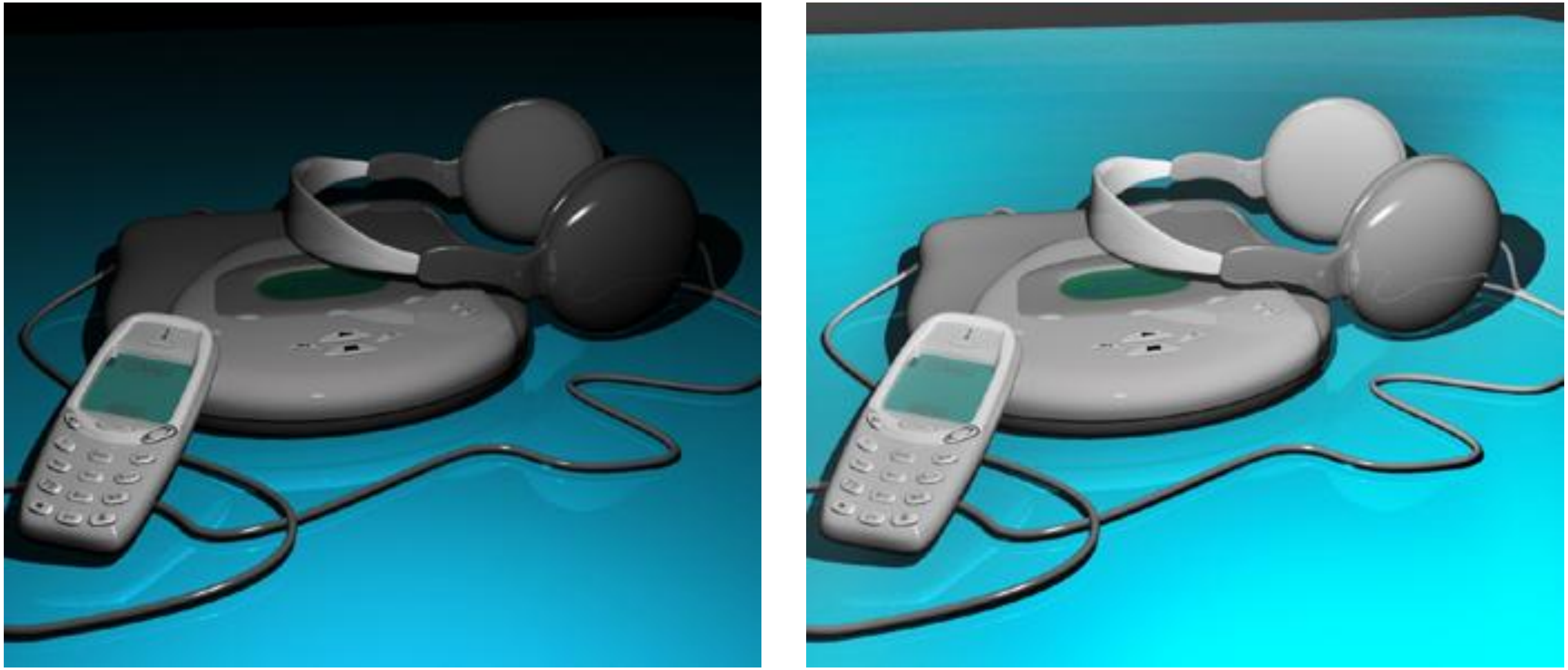

Fig. 16 The original image “player” and the enhanced with fuzzy partition.

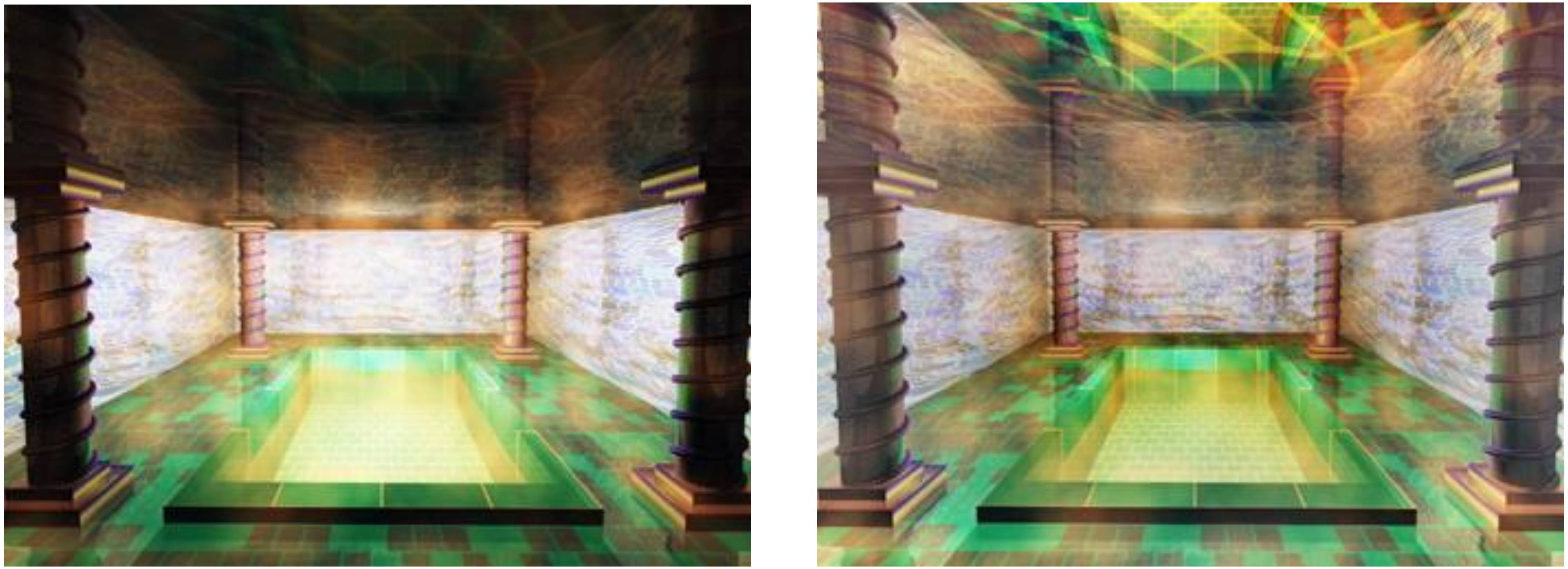

Fig. 17 The original image “Egyptian bath” and the enhanced with fuzzy partition.

Fig. 18 The original image “puerta”, the enhanced with fuzzy partition and enhanced without partition.

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