Algorithms for Dynamic Right-Sizing in Data Centers

Kevin Kappelmann

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Technical University of Munich

Motivation and Problem Statement

Data Centers' Costs

Fast-growing demand for data collection, processing, and storage

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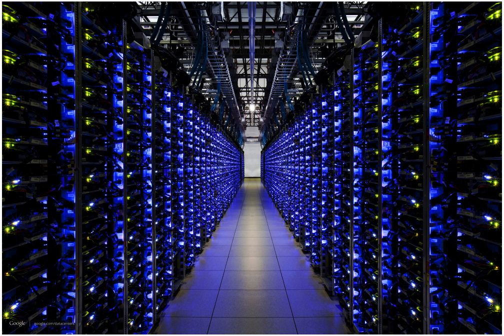

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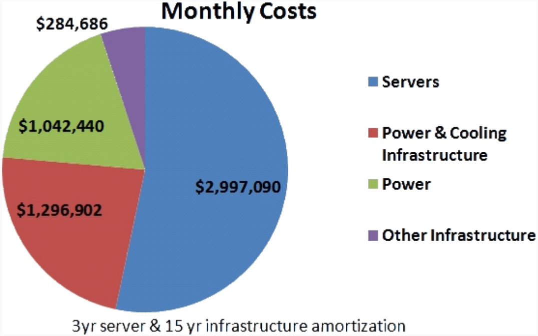

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- $\lambda_1, \dots, \lambda_T \in [0, m]$ ... Arrival rates
- $x_1, \dots, x_T \in \{0, \dots, m\}$ ... Numbers of active servers

Problem Statement

Operating costs for one time step

$$c_{op}(x_t, \lambda_t) \coloneqq \begin{cases} \infty, & \text{if } \lambda_t > x_t \quad \text{//too few servers} \\ x_t f(\lambda_t / x_t), & \text{if } x_t \neq 0 \wedge \lambda_t \leq x_t \quad \text{//even distribution} \\ 0, & \text{if } x_t = \lambda_t = 0 \quad \text{//no active servers} \end{cases}$$

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Goal: Minimize total costs

$$\text{minimize} \quad \sum_{t=1}^{T} \Big(\underbrace{c_{op}(x_t, \lambda_t) + \beta \max\{0, x_t - x_{t-1}\}}_{c(x_{t-1}, x_t, \lambda_t)} \Big)$$

Optimal Offline Algorithm

Pseudo-Linear-Time Algorithm

Fundamental idea: reduce problem to shortest path problem

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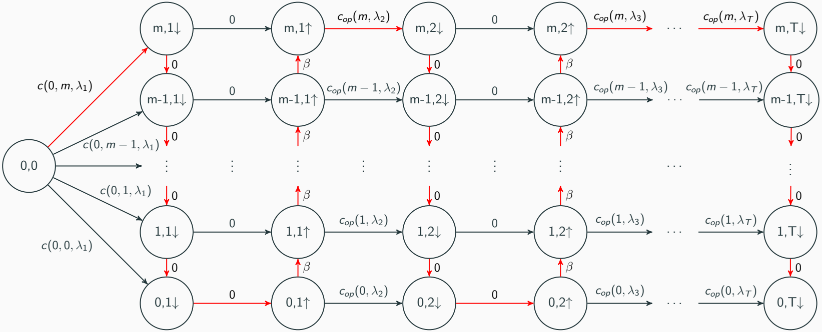

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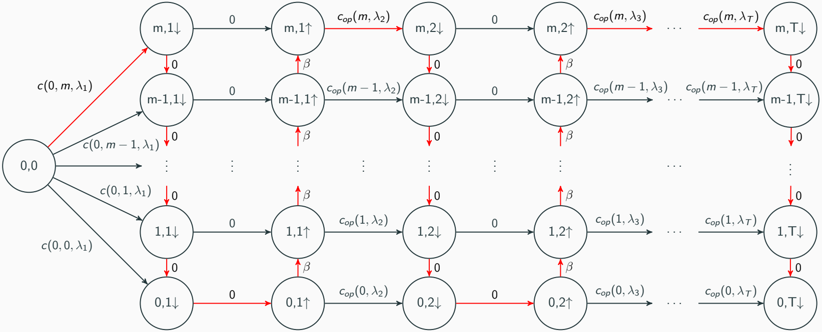

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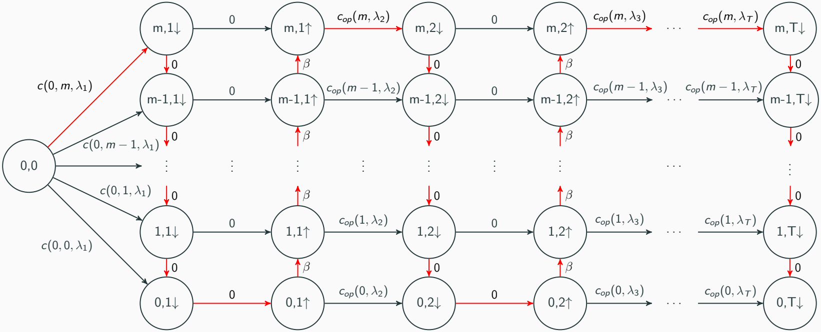

Time complexity: $\Theta(Tm)$

...only $\log_2(m)$ bits required to encode m.

Offline Approximation Algorithm

2-Optimal Linear-Time Algorithm

Use logarithmic steps to reduce number of nodes

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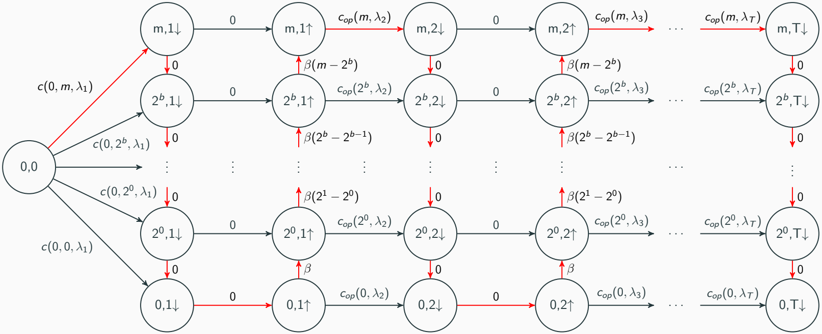

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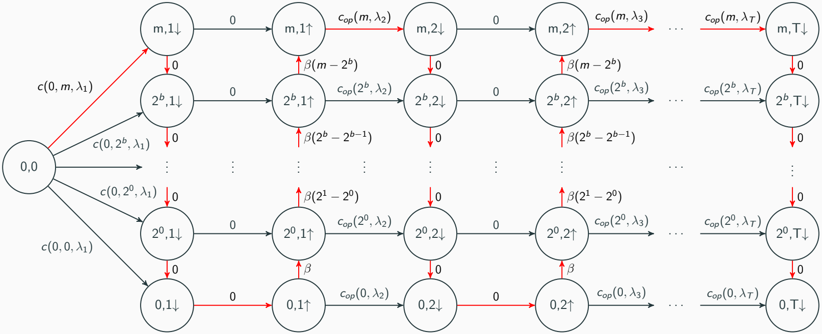

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Claim: Shortest Path in graph corresponds to 2-optimal schedule.

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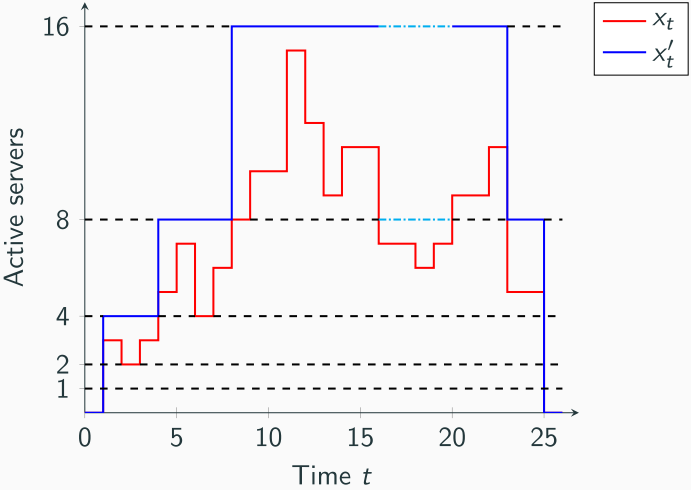

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Approach can be generalized to allow for arbitrary precisions with time complexity $\Theta\big(T \log_{1+\varepsilon}(m)\big)$

Summary and Prospects

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We reduced the scheduling problem to the shortest path problem of acyclic graphs.

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Optimal Offline Algorithm with runtime $\Theta(Tm)$

$(1+\varepsilon)$-Optimal Offline Algorithm with runtime
$$\Theta(T \log_{1+\varepsilon}(m)) = \Theta\left(T \, \frac{\log(m)}{\log(1+\varepsilon)}\right)$$

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Open question: Is there a polynomial optimal algorithm or is it an **NP** problem?

Thanks for your attention!

Any questions?

Image Sources I

- Data center: `datacentervoice.com/wp-content/uploads/2015/12/data-center.jpg`
- Data center costs: `perspectives.mvdirona.com/2008/11/cost-of-power-in-large-scale-data-centers/`