

Algorithms for Dynamic Right-Sizing in Data Centers

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July 25, 2017

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Model and Problem Formulation

Model Description

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Notation:

- $x_1, \dots, x_T \in \{0, \dots, m\} \dots$ Numbers of active servers

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Goal: Minimize total costs

$$\text{minimize} \quad \sum_{t=1}^T \underbrace{\left(c_{op}(x_t, \lambda_t) + \beta \max\{0, x_t - x_{t-1}\} \right)}_{c(x_{t-1}, x_t, \lambda_t)}$$

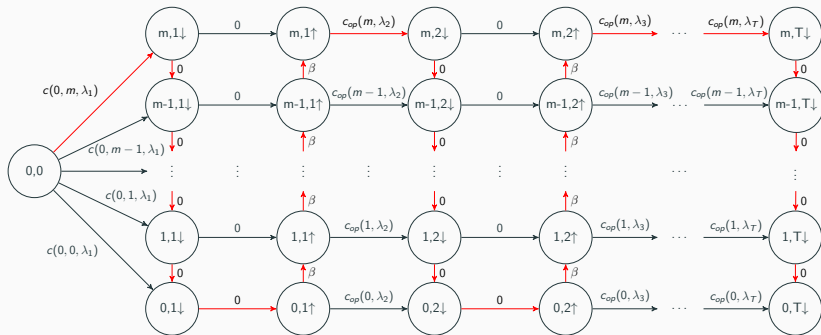
Optimal Offline Algorithm

Pseudo-Linear-Time Algorithm

Fundamental idea: reduce problem to shortest path problem

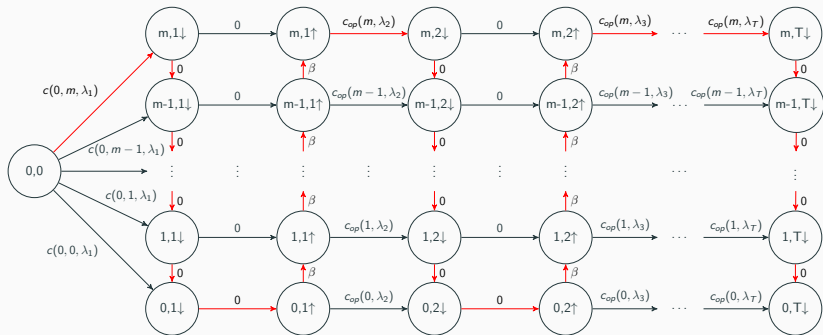
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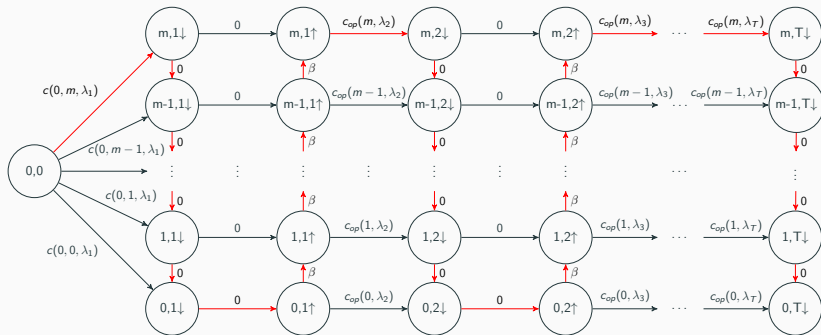
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... only $\log_2(m)$ bits required to encode m .

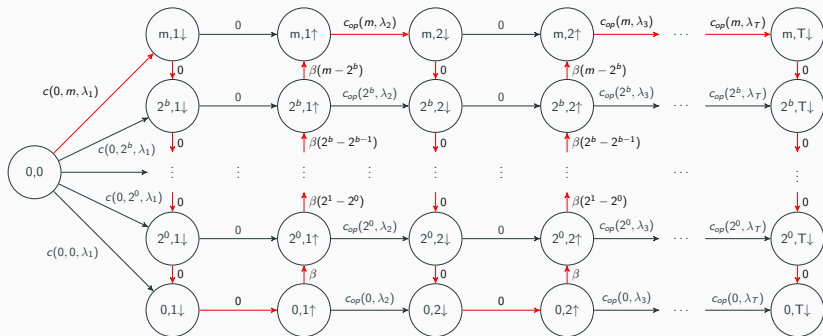
Offline Approximation Algorithm

2-Optimal Linear-Time Algorithm

Use logarithmic steps to reduce number of nodes

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where $b := \lfloor \log_2(m) \rfloor$

Approximative Scheduling

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Approach can be generalized to allow for arbitrary precisions
with time complexity $\Theta(T \log_{1+\varepsilon}(m)) = \Theta\left(T \frac{\log(m)}{\log(1+\varepsilon)}\right)$

Summary and Prospects

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$(1 + \epsilon)$ -Optimal Offline Algorithm with runtime
 $\Theta\left(T \frac{\log(m)}{\log(1+\epsilon)}\right)$

Prospects

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Open question: Is there a polynomial optimal algorithm or is it an **NP** problem?

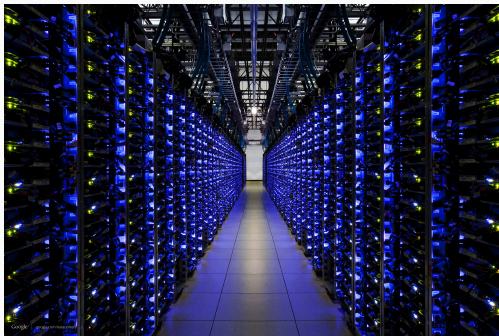
Thanks for your attention!

Any questions?

Fast-growing demand for data collection,
processing, and storage

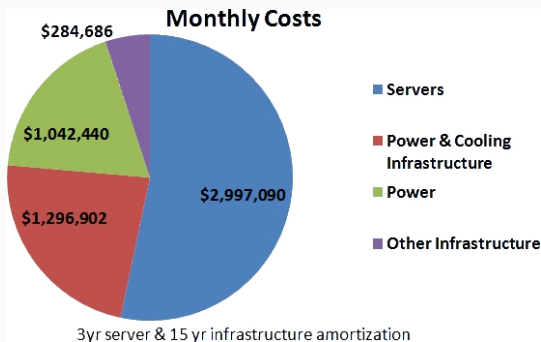
Data Centers' Costs

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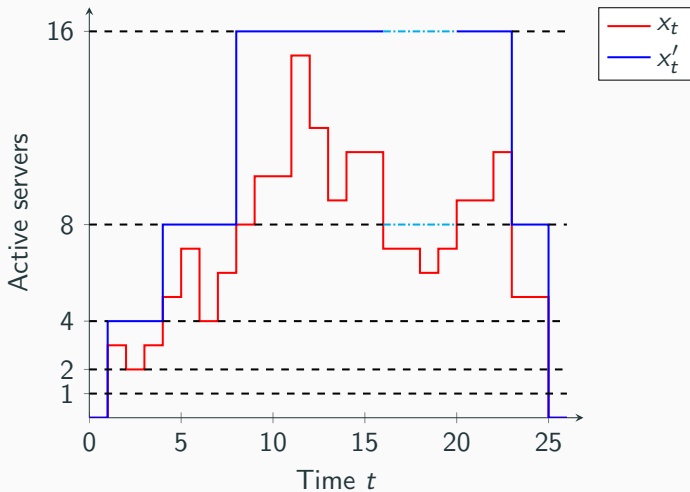


Proof Idea

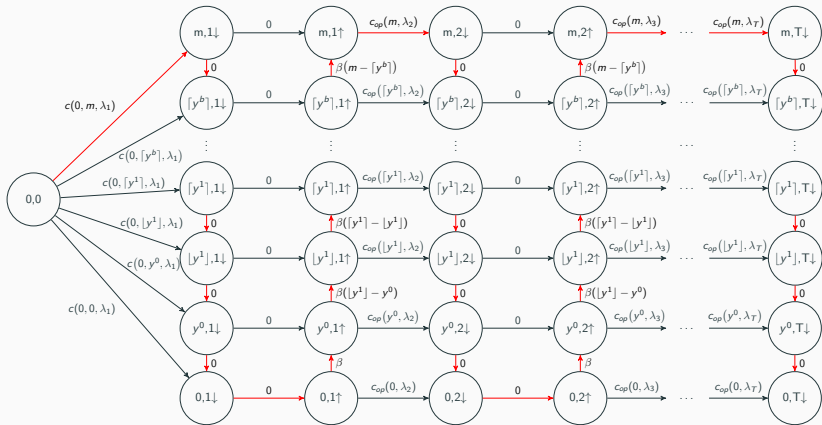
Take a schedule and transform periods between two powers of 2

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$(1 + \varepsilon)$ -Optimal Offline Algorithm



where $y := 1 + \varepsilon$

Image Sources I

- Data center: datacentervoice.com/wp-content/uploads/2015/12/data-center.jpg
- Data center costs: perspectives.mvdirona.com/2008/11/cost-of-power-in-large-scale-data-centers/