# Algorithms for Dynamic Right-Sizing in Data Centers

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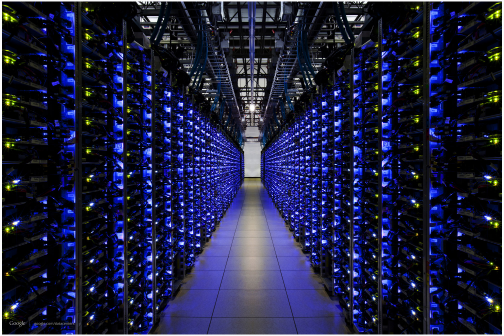
# Dynamic Right-Sizing

Dynamically adjust the number of active servers and efficiently distribute the varying workloads



# Model and Problem Formulation

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- All servers are shut down before and after the scheduling process.

# Problem Statement

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**Goal: Minimize total costs**

$$\text{minimize} \quad \sum_{t=1}^{T} \Big( \underbrace{c_{op}(x_t, \lambda_t) + \beta \max\{0, x_t - x_{t-1}\}}_{c(x_{t-1}, x_t, \lambda_t)} \Big)$$

# Optimal Offline Algorithm

# Pseudo-Linear-Time Algorithm

Fundamental idea: reduce problem to shortest path problem

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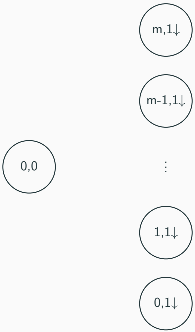

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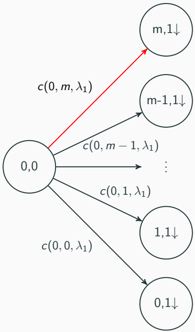

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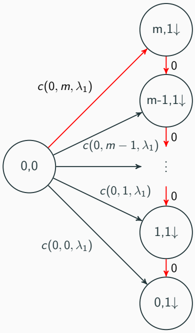

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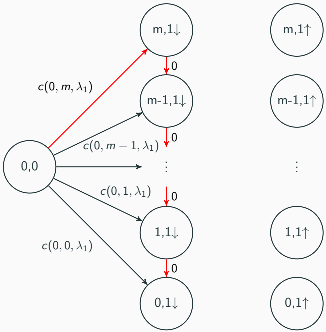

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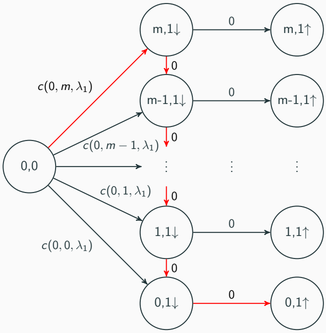

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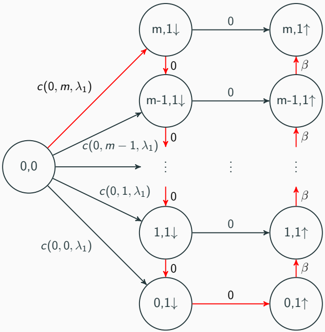

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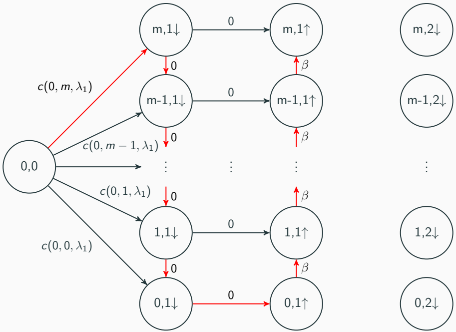

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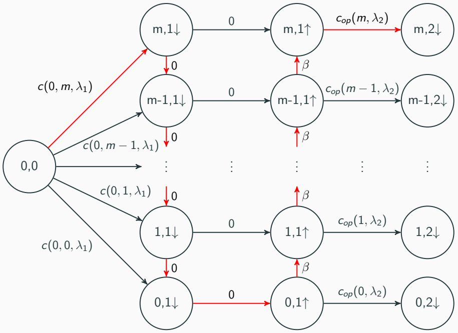

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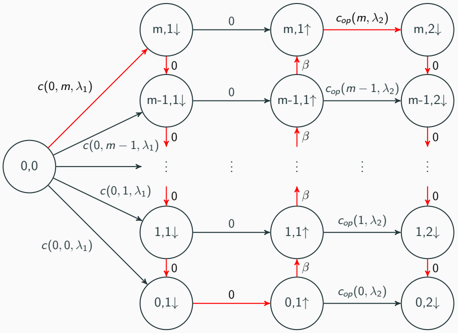

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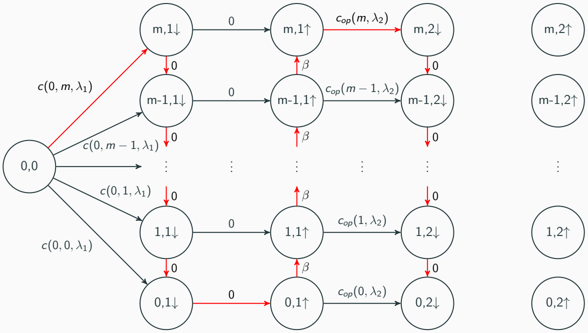

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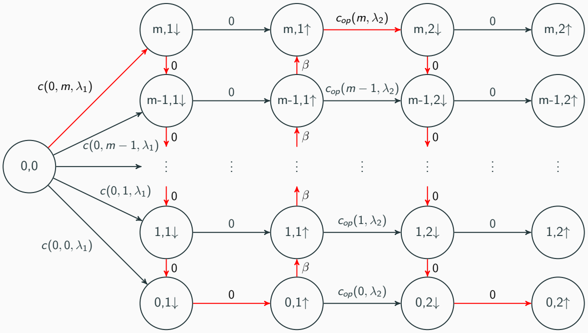

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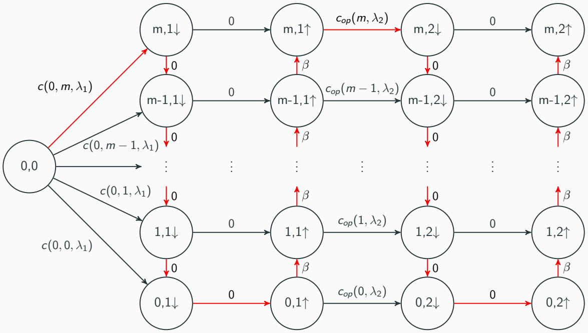

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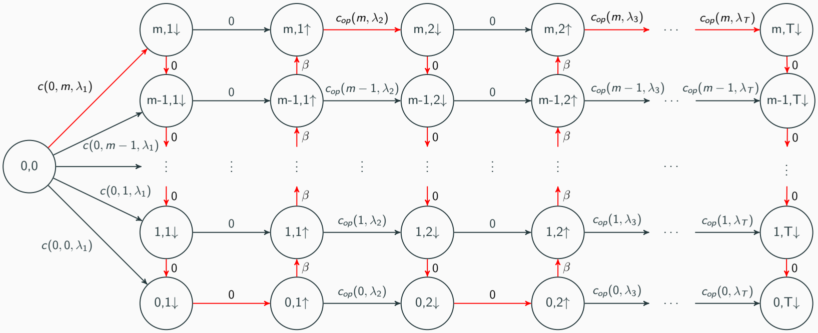

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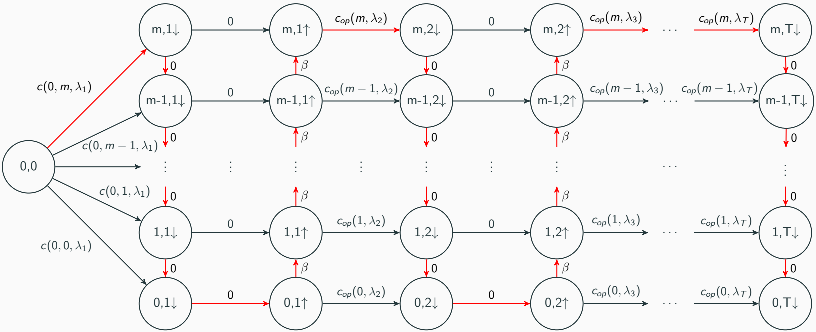


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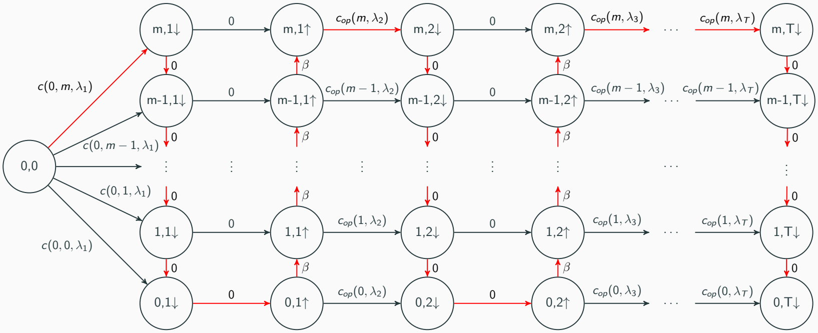


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. . . only $\log_2(m)$ bits required to encode $m$.

# Offline Approximation Algorithm

# 2-Optimal Linear-Time Algorithm

Use logarithmic steps to reduce number of nodes

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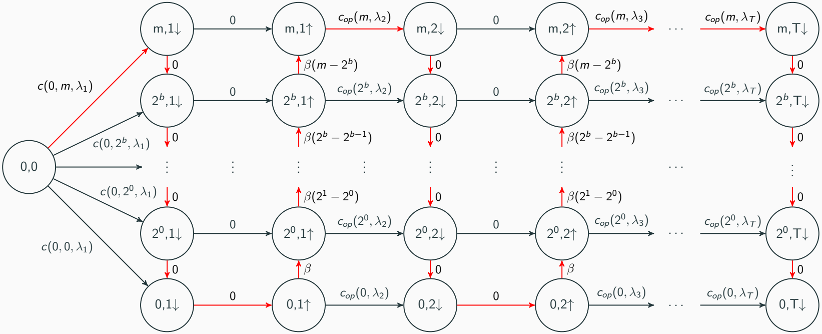


where $b := \lfloor \log_2(m) \rfloor$

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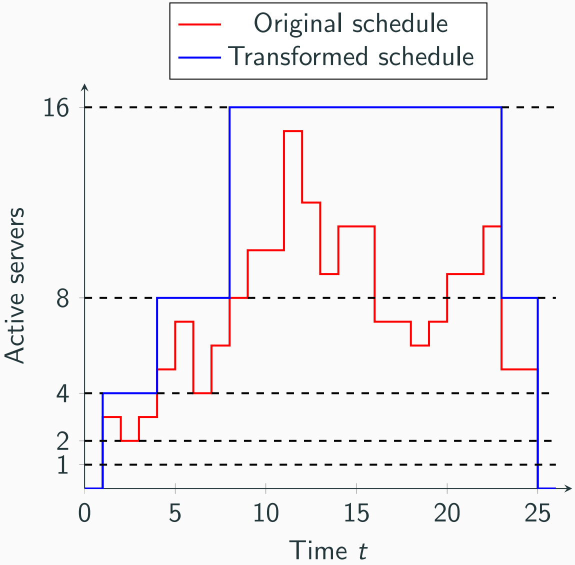

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Approximation for arbitrary precision $\varepsilon > 0$ with
time complexity $\Theta\big(T \log_{1+\varepsilon}(m)\big)$

# Summary and Prospects

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$(1+\varepsilon)$-Optimal Offline Algorithm with linear runtime

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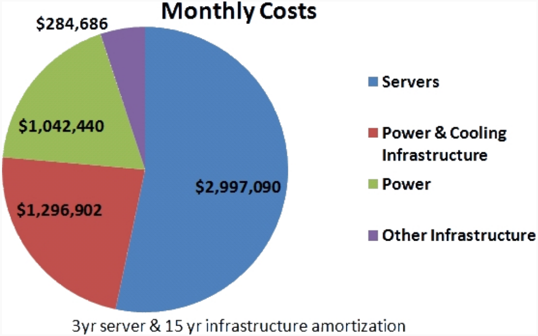
Open question: Is there a polynomial-time optimal algorithm or is it an NP-hard problem?

**Thanks for your attention!**

**Any questions?**

# Data Centers' Costs

Fast-growing demand for data collection, processing, and storage



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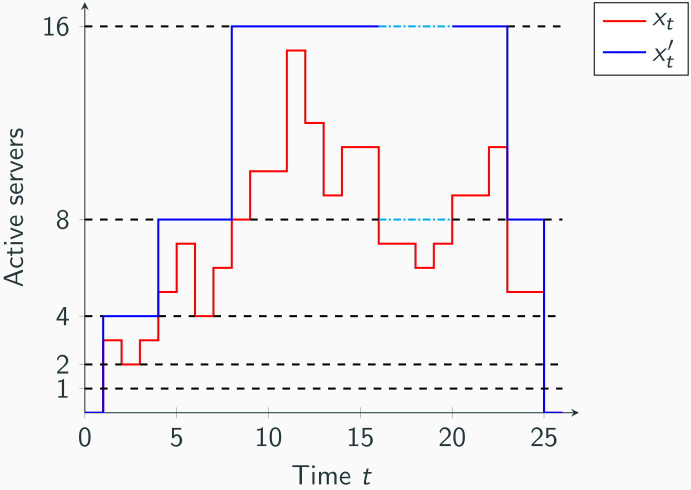
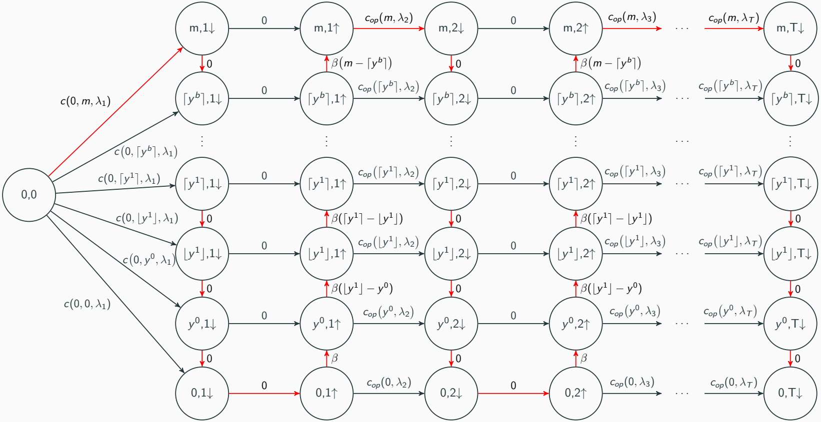


# $(1+\varepsilon)$-Optimal Offline Algorithm




where $y := 1+\varepsilon$, $b := \lfloor \log_y(m) \rfloor$

# Image Sources I

- Data center: `datacentervoice.com/wp-content/uploads/2015/12/data-center.jpg`
- Data center costs: `perspectives.mvdirona.com/2008/11/cost-of-power-in-large-scale-data-centers/`