

Algorithms for Dynamic Right-Sizing in Data Centers

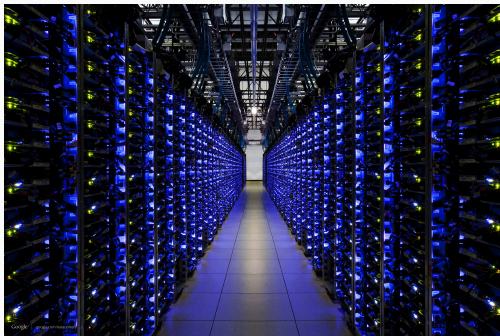
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August 03, 2017

Technical University of Munich

Dynamic Right-Sizing

Dynamically adjust the number of active servers and efficiently distribute the varying workloads



Model and Problem Formulation

Model Description

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- $m \in \mathbb{N} \dots$ Number of homogeneous servers

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- All servers are shut down before and after scheduling process

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Operating costs for one time step t

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$$\text{minimize} \quad \sum_{t=1}^T \underbrace{\left(c_{op}(x_t, \lambda_t) + \beta \max\{0, x_t - x_{t-1}\} \right)}_{c(x_{t-1}, x_t, \lambda_t)}$$

Optimal Offline Algorithm

Pseudo-Linear-Time Algorithm

Fundamental idea: reduce problem to shortest path problem

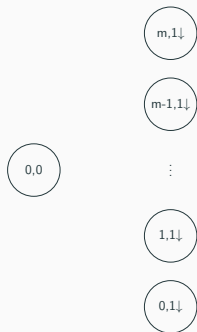
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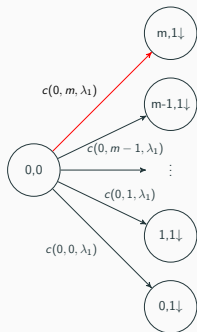
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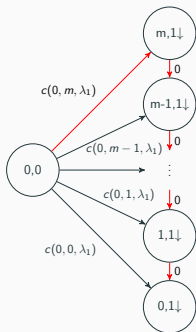
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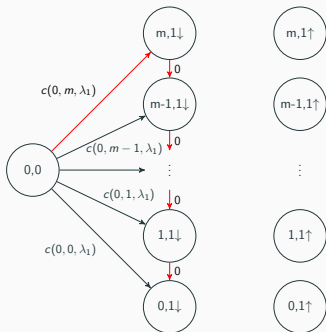
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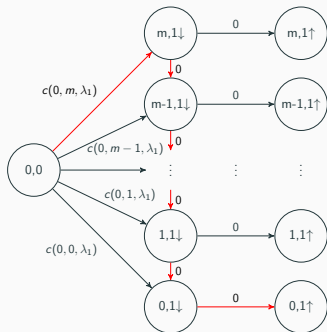
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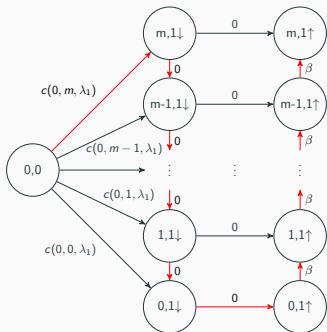
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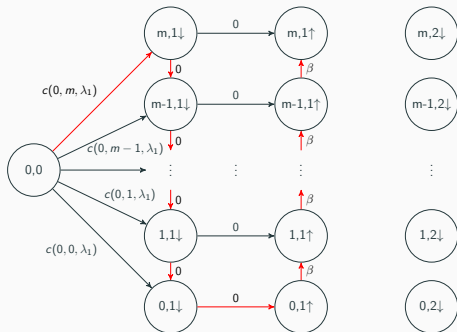
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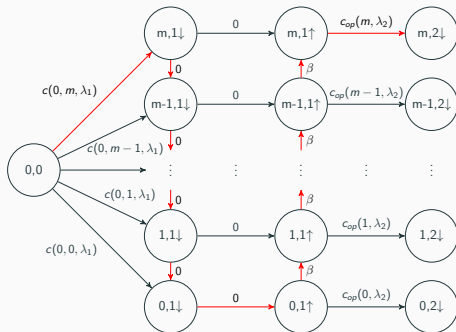
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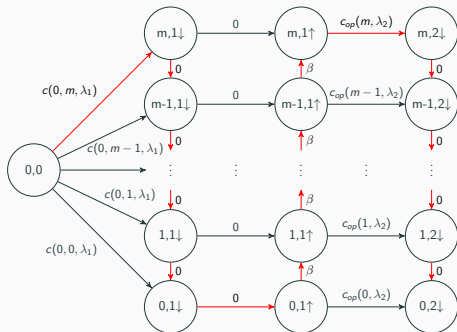
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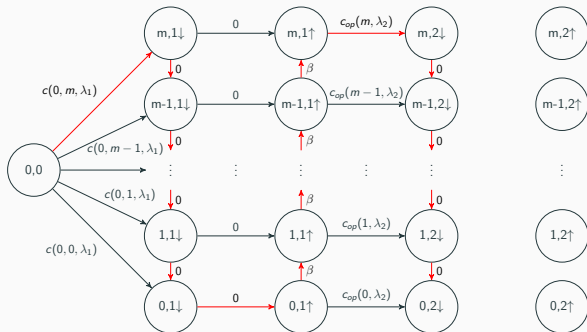
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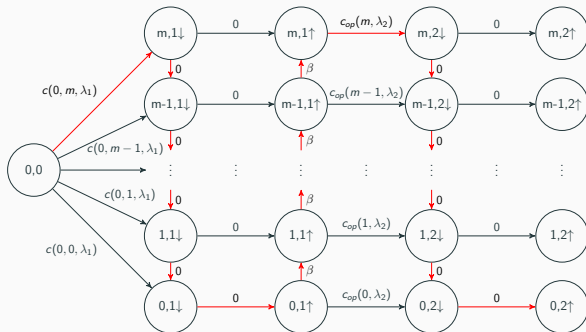
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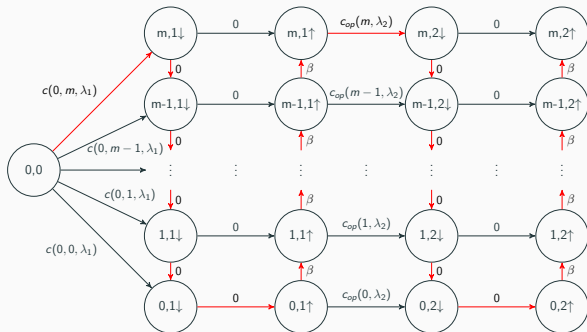
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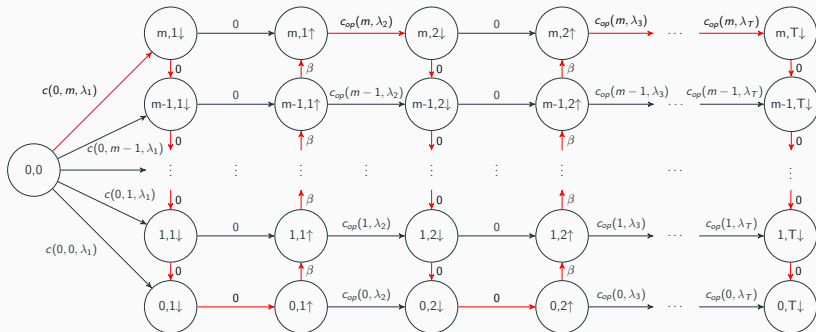
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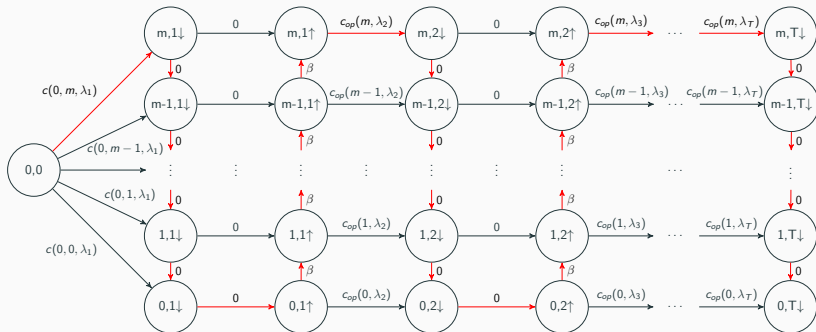
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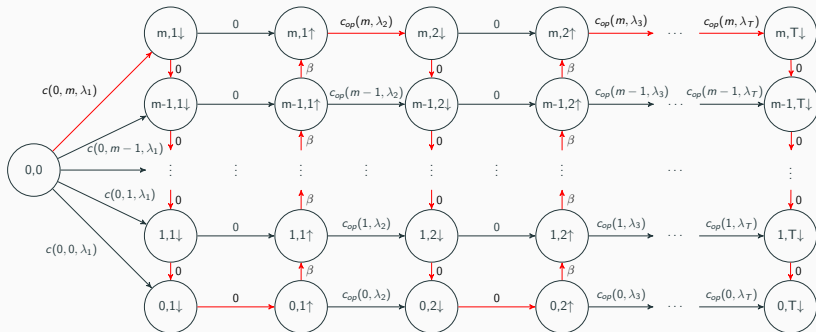
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Time complexity: $\Theta(Tm)$

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... only $\log_2(m)$ bits required to encode m .

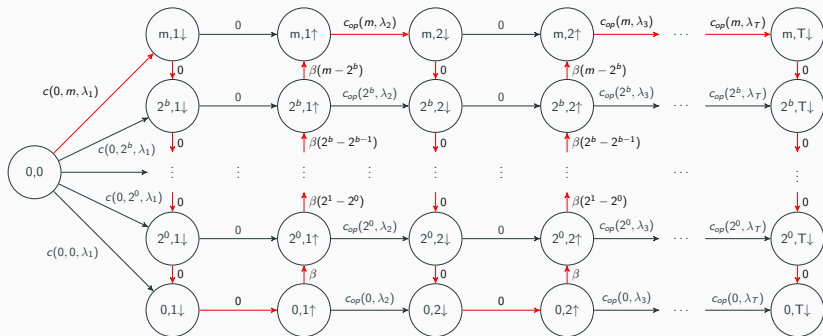
Offline Approximation Algorithm

2-Optimal Linear-Time Algorithm

Use logarithmic steps to reduce number of nodes

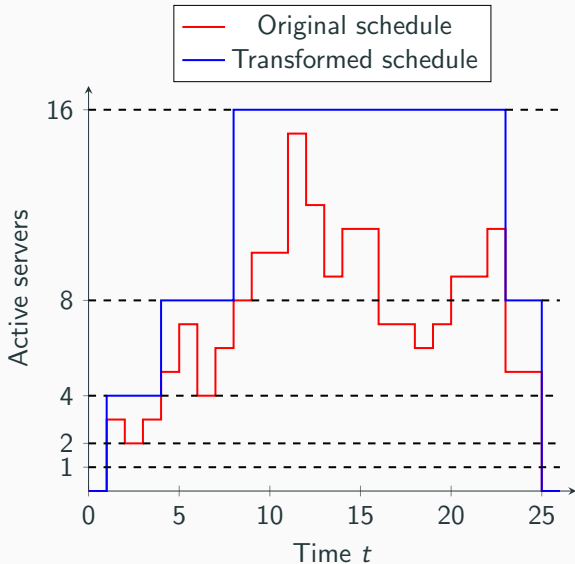
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where $b := \lfloor \log_2(m) \rfloor$

2-Optimal Linear-Time Algorithm



Approximative Scheduling

Shortest Path in graph corresponds to 2-optimal schedule.

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Can we generalize to allow for arbitrary precisions? Yes!

Approximation for arbitrary precision $\epsilon > 0$ with
time complexity $\Theta(T \log_{1+\epsilon}(m))$

Summary and Prospects

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We reduced the scheduling problem to the shortest path problem of acyclic graphs.

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$(1 + \epsilon)$ -Optimal Offline Algorithm with linear runtime
 $\Theta(T \log_{1+\epsilon}(m))$

Prospects

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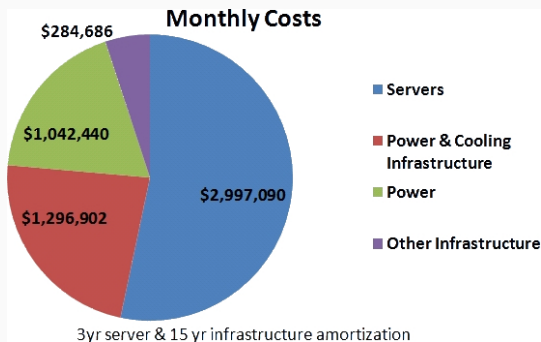
Open question: Is there a polynomial optimal algorithm or is it an NP-hard problem?

Thanks for your attention!

Any questions?

Data Centers' Costs

Fast-growing demand for data collection, processing, and storage

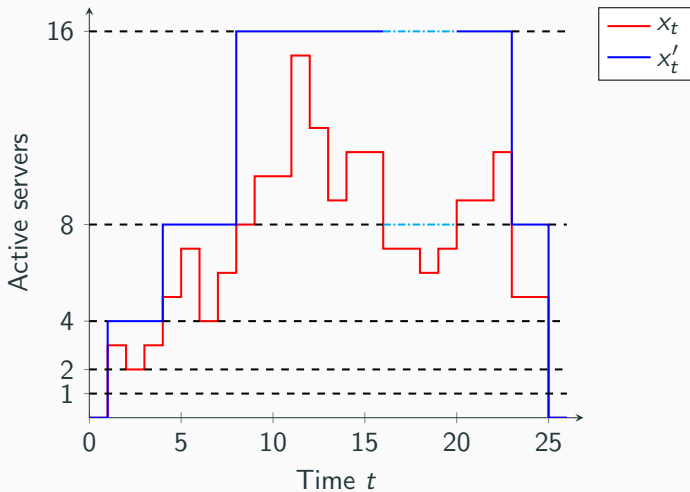


Proof Idea

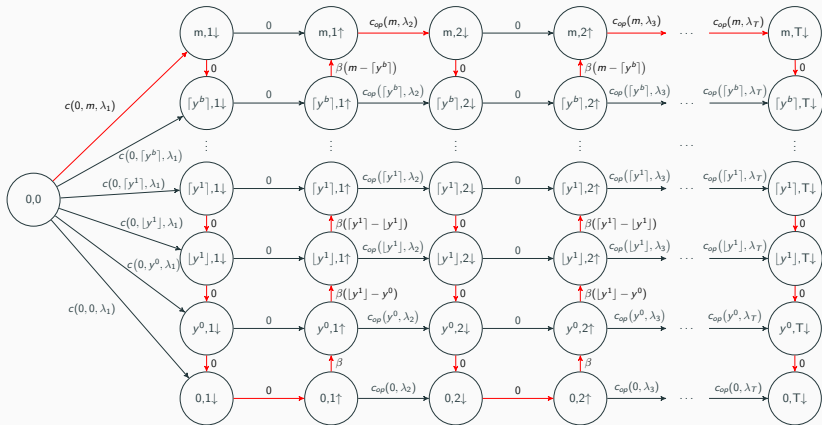
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$(1 + \varepsilon)$ -Optimal Offline Algorithm



where $y := 1 + \varepsilon$, $b := \lfloor \log_y(m) \rfloor$

Image Sources I

- Data center: datacentervoice.com/wp-content/uploads/2015/12/data-center.jpg
- Data center costs: perspectives.mvdirona.com/2008/11/cost-of-power-in-large-scale-data-centers/