Algorithms for Dynamic Right-Sizing in Data Centers

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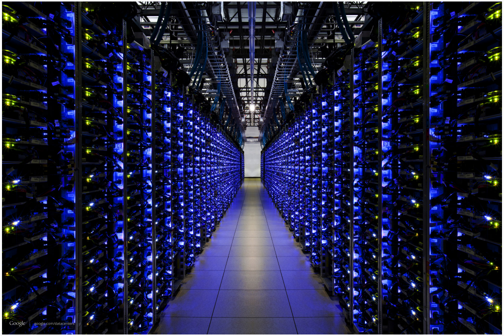
Dynamic Right-Sizing

Dynamically adjust the number of active servers and efficiently distribute the varying workloads



Model and Problem Formulation

Model Description

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Notation:

- $x_1, \ldots, x_T \in \{0, \ldots, m\} \ldots$ Numbers of active servers

Problem Statement

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Goal: Minimize total costs

$$\text{minimize} \quad \sum_{t=1}^{T} \Big(\underbrace{c_{op}(x_t, \lambda_t) + \beta \max\{0, x_t - x_{t-1}\}}_{c(x_{t-1}, x_t, \lambda_t)} \Big)$$

Optimal Offline Algorithm

Pseudo-Linear-Time Algorithm

Fundamental idea: reduce problem to shortest path problem

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Time complexity: $\Theta(Tm)$

$\ldots$only $\log_2(m)$ bits required to encode m.

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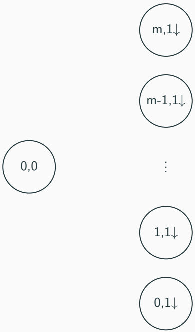

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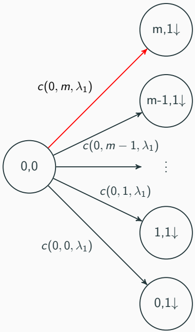

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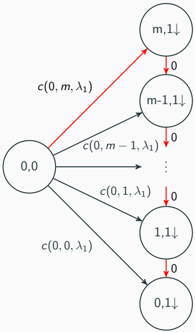

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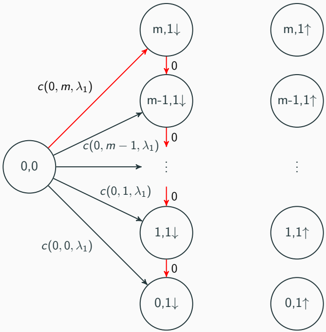

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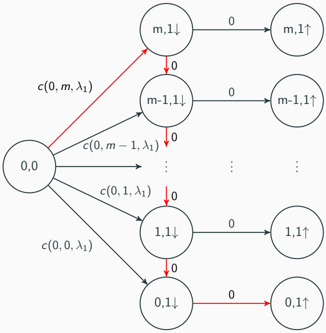

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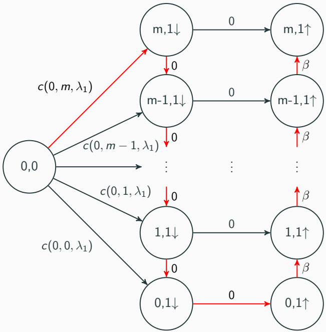

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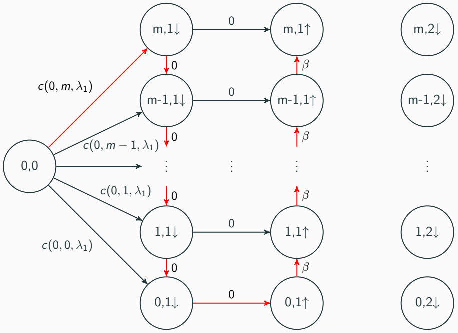

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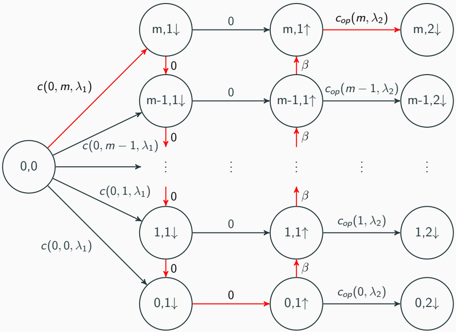

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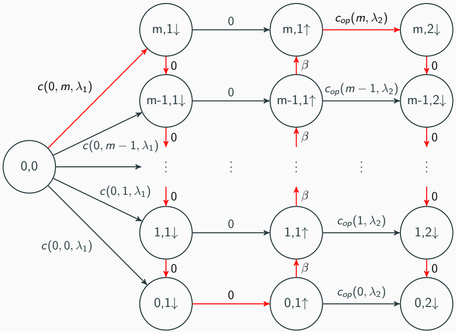

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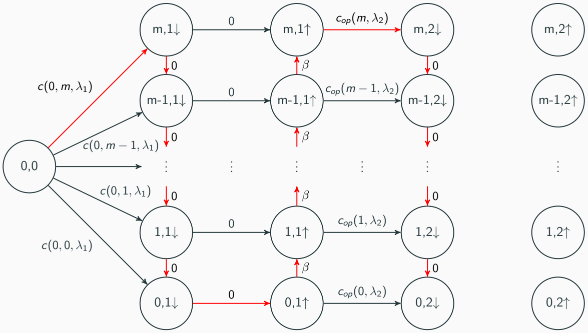

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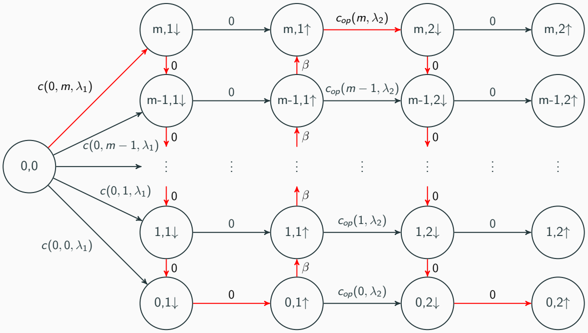

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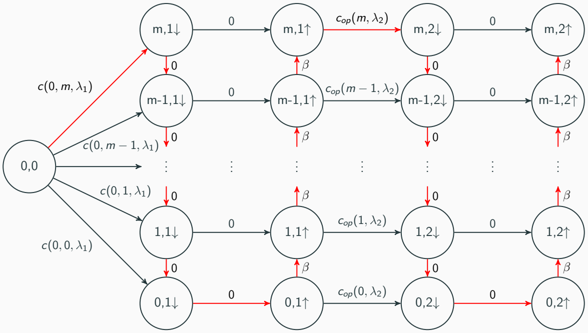

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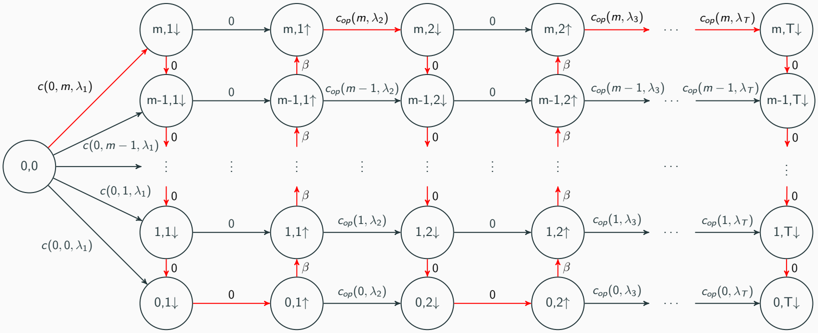

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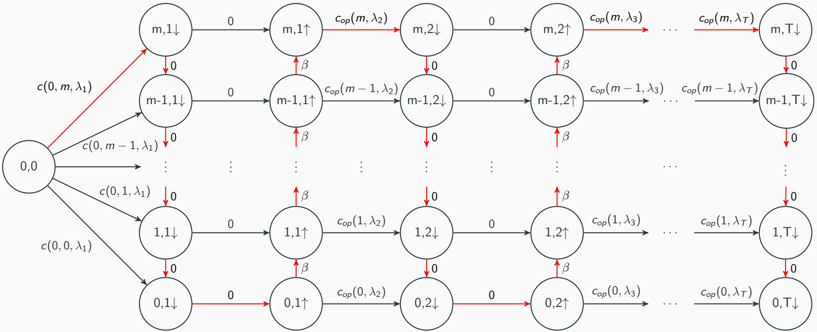

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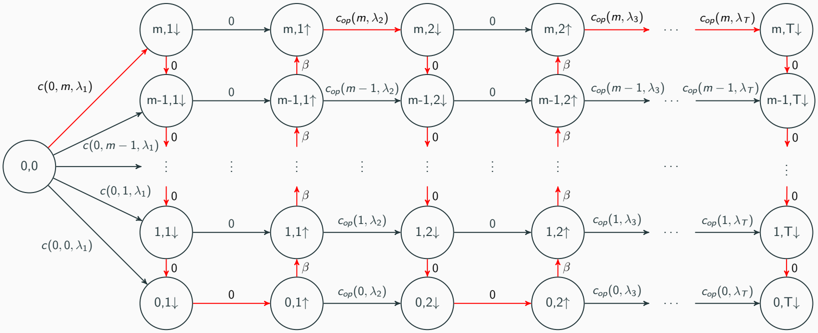

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Offline Approximation Algorithm

2-Optimal Linear-Time Algorithm

Use logarithmic steps to reduce number of nodes

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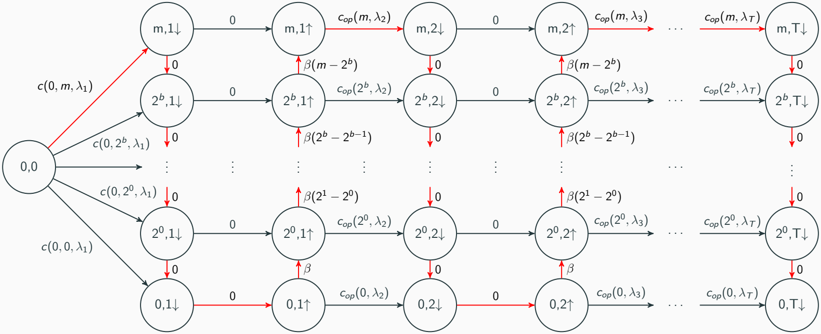

where $b := \lfloor \log_2(m) \rfloor$

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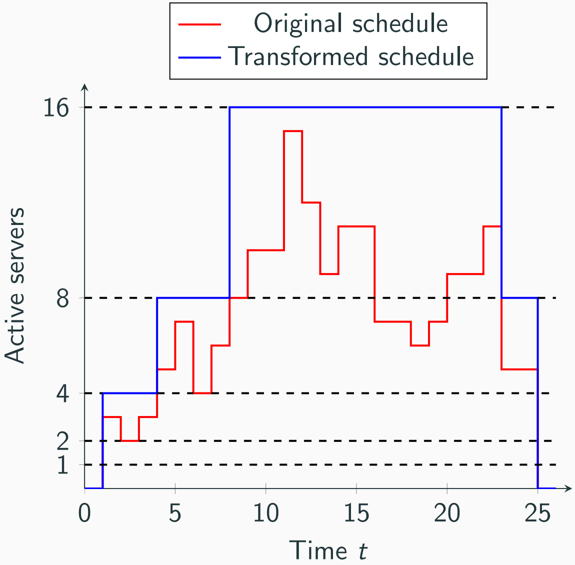

Approximative Scheduling

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Approximation for arbitrary precision $\varepsilon > 0$ with time complexity $\Theta\big(T \log_{1+\varepsilon}(m)\big)$

Summary and Prospects

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$(1+\varepsilon)$-Optimal Offline Algorithm with linear runtime

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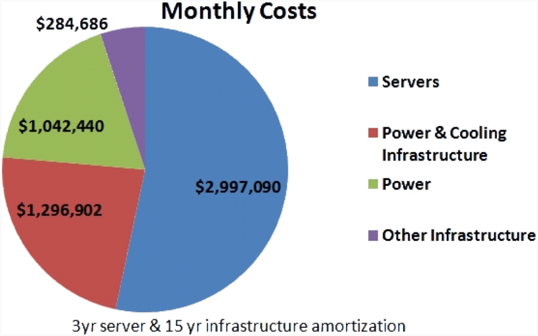
Open question: Is there a polynomial optimal algorithm or is it an NP-hard problem?

Thanks for your attention!

Any questions?

Data Centers' Costs

Fast-growing demand for data collection, processing, and storage



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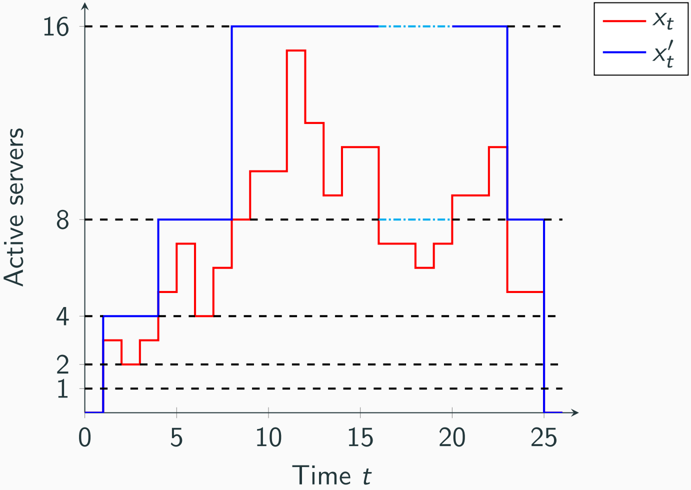
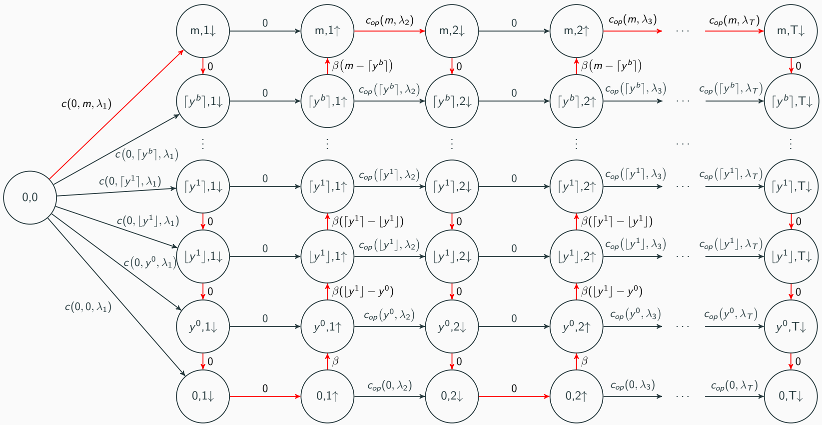


$(1+\varepsilon)$-Optimal Offline Algorithm



where $y := 1 + \varepsilon$, $b := \lfloor \log_y(m) \rfloor$

Image Sources I

- Data center: `datacentervoice.com/wp-content/uploads/2015/12/data-center.jpg`
- Data center costs: `perspectives.mvdirona.com/2008/11/cost-of-power-in-large-scale-data-centers/`