Foundations of Mathematics
and the Foundational Crisis

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- Can we trust a computer?

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    - ⇒ Controversies
- But it is not the first controversy in mathematical history. . .

Overview

1+1=2

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- Math is about abstraction.

Math as an Applied Tool

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Infinitesimal Controversy

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- Math as an applied tool, rather than

I Think I Broke Math

The End