

Foundations of Mathematics and the Foundational Crisis

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- Can we trust each other?
- Can we trust a computer?

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- But it is not the first controversy in mathematical history...

Overview

1 History of Mathematics

- Ancient Mathematics
- Infinitesimal Controversy

2 Foundational Crisis

- Causes
- Logicism
- Formalism
- Intuitionism
- Peak and End

3 Aftermath and Prospects

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- Math is about abstraction.

I can draw you the world

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 - Geometry!

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- What also seems quite intuitive?
 - Geometry!
- Arithmetic and geometry were the first branches of mathematics.

I Think I Broke Math

The End