

Foundations of Mathematics and the Foundational Crisis

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- But it is not the first controversy in mathematical history...

Overview

- 1 History of Mathematics
- 2 Foundational Crisis
 - Causes
 - Logicism
 - Formalism
 - Intuitionism
 - Peak and End
- 3 Aftermath and Prospects

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- Maths is about abstraction.
- Numbers were the first abstraction.

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- Maths served as an applied tool.
 - No proofs, but heuristics and pictures

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 - ...at least so had been thought

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- 4 That all right angles are equal to one another.
- 5 That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.”

I Think I Broke Math

The End