OceanAcoustics.jl

Aaron Kaw

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Ray Method

The ray method is based upon the following derivation:

1. Start with the wave equation:
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \vec{v} \tag{3.1}$$
2. to obtain the Helmholtz equation:
$$helmholtz \tag{3.2}$$
3. Substitute:
   - $k^2(\vec{r}) = \omega / c(\vec{r})$
   -

   into the Helmholtz equation to obtain:
$$\tag{TODO}$$
4. Assume solution of the form:
$$p(\vec{x}) = e^{i\omega\tau(x)} \sum_{j=0}^{\infty} \frac{A_j(\vec{x})}{(i\omega)^j}. \tag{3.3}$$
5. Substitute solution into Helmholtz equation.
6. Equate like terms to obtain $\mathcal{O}(\omega^2)$ and $\mathcal{O}(\omega)$ equations respectively as
$$|\nabla\tau|^2 = \frac{1}{c(\vec{x})^2} \qquad \text{(eikonal equation)}$$
$$2\nabla\tau \cdot \nabla A_0 + (\nabla^2\tau)A_0 = 0 \qquad \text{(transport equation)}$$

where the remaining transport equations ($\mathcal{O}(\omega^{1-j}), j = 1, 2, \dots$) are ignored, thus this method provides a second-order approximation, for high frequency (high ω) scenarios.

The eikonal equation is expressed in cylindrical coordinates and written in first-order form as

$$\frac{\mathrm{d}r}{\mathrm{d}s} = r\ \xi(s), \qquad \frac{\mathrm{d}z}{\mathrm{d}s} = c\ \zeta(s), \tag{3.4}$$

$$\frac{\mathrm{d}\xi}{\mathrm{d}s} = \frac{-1}{c^2}\frac{\partial c}{\partial r}, \qquad \frac{\mathrm{d}\zeta}{\mathrm{d}s} = \frac{-1}{c^2}\frac{\partial c}{\partial z}, \tag{3.5}$$

along with initial conditions

$$r(0) = 0 \qquad z(0) = z_0 \tag{3.6}$$

$$\xi(0) = \frac{\cos(\theta_0)}{c(0)} \qquad \zeta(0) = \frac{\sin(\theta_0)}{c(0)} \tag{3.7}$$

The phase delay equation is

$$\frac{\mathrm{d}\tau}{\mathrm{d}s} = \frac{1}{c} \tag{3.8}$$

with initial condition

$$\tau(0) = 0 \tag{3.9}$$

The dynamic ray equations are coupled as

$$\frac{\mathrm{d}p}{\mathrm{d}s} = \frac{-c_{nn}}{c^2} \qquad \frac{\mathrm{d}q}{\mathrm{d}s} = c\ p(s) \tag{3.10}$$

where c_{nn} is the curvature of the sound speed in a direction normal to the ray path, given by

$$c_{nn} = c^2\left(\frac{\partial^2 c}{\partial r^2}\zeta^2 - 2\frac{\partial^2 c}{\partial r\partial z}\ \xi\zeta + \frac{\partial^2 c}{\partial z^2}\xi^2\right) \tag{3.11}$$

along with initial conditions

$$p(0) = 1 \qquad q(0) = i\pi f W_0^2 \tag{3.12}$$

where W_0 is the initial width of the beam, with value

$$W_0 \in [10, 50]\lambda \tag{3.13}$$

Thus the required inputs from the user are the initial conditions

- z_0
- θ_0
- f
- $c(r, z)$

Chapter 4

Parabolic Equation Method

Part II

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