

OceanAcoustics.jl

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## Chapter 1

# Introduction



# Part I

## Propagation





## Chapter 2

# Introduction to Propagation



## Chapter 3

# Ray Method

The ray method is based upon the following derivation:

1. Start with the wave equation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \vec{v} \quad (3.1)$$

2. to obtain the Helmholtz equation:

$$\text{helmholtz} \quad (3.2)$$

3. Substitute:

- $k^2(\vec{r}) = \omega/c(\vec{r})$
- 

into the Helmholtz equation to obtain:

(TODO)

4. Assume solution of the form:

$$p(\vec{x}) = e^{i\omega\tau(x)} \sum_{j=0}^{\infty} \frac{A_j(\vec{x})}{(i\omega)^j}. \quad (3.3)$$

5. Substitute solution into Helmholtz equation.

6. Equate like terms to obtain  $\mathcal{O}(\omega^2)$  and  $\mathcal{O}(\omega)$  equations respectively as

$$|\nabla \tau|^2 = \frac{1}{c(\vec{x})^2} \quad (\text{eikonal equation})$$

$$2\nabla \tau \cdot \nabla A_0 + (\nabla^2 \tau) A_0 = 0 \quad (\text{transport equation})$$

where the remaining transport equations ( $\mathcal{O}(\omega^{1-j}), j = 1, 2, \dots$ ) are ignored, thus this method provides a second-order approximation, for high frequency (high  $\omega$ ) scenarios.

The eikonal equation is expressed in cylindrical coordinates and written in first-order form as

$$\frac{dr}{ds} = c \xi(s), \quad \frac{dz}{ds} = c \zeta(s), \quad (3.4)$$

$$\frac{d\xi}{ds} = \frac{-1}{c^2} \frac{\partial c}{\partial r}, \quad \frac{d\zeta}{ds} = \frac{-1}{c^2} \frac{\partial c}{\partial z}, \quad (3.5)$$

along with initial conditions

$$r(0) = 0 \quad z(0) = z_0 \quad (3.6)$$

$$\xi(0) = \frac{\cos(\theta_0)}{c(0)} \quad \zeta(0) = \frac{\sin(\theta_0)}{c(0)} \quad (3.7)$$

The phase delay equation is

$$\frac{d\tau}{ds} = \frac{1}{c} \quad (3.8)$$

with initial condition

$$\tau(0) = 0 \quad (3.9)$$

The dynamic ray equations are coupled as

$$\frac{dp}{ds} = \frac{-c_{nn}}{c^2} q(s) \quad \frac{dq}{ds} = c p(s) \quad (3.10)$$

where  $c_{nn}$  is the curvature of the sound speed in a direction normal to the ray path, given by

$$c_{nn} = c^2 \left( \frac{\partial^2 c}{\partial r^2} \zeta^2 - 2 \frac{\partial^2 c}{\partial r \partial z} \xi \zeta + \frac{\partial^2 c}{\partial z^2} \xi^2 \right) \quad (3.11)$$

along with initial conditions

$$p(0) = 1 \quad q(0) = i\pi f W_0^2 \quad (3.12)$$

where  $W_0$  is the initial width of the beam, with value

$$W_0 \in [10, 50]\lambda \quad (3.13)$$

Thus the required inputs from the user are the initial conditions

- $z_0$
- $\theta_0$
- $f$
- $c(r, z)$

## Chapter 4

# Parabolic Equation Method



# Part II

## Detection





## Chapter 5

# Sonar Equations



## Chapter 6

# Detector Decisions