### 1. Ocean Sonar

## 1.1. Sonar Oceanography

### 1.2. Ocean Acoustics

# 1.3. Sonar Signal Processing

# 1.4. Statistical Detection Theory

#### 1.4.1. Probability Metrics

The vector of received acoustic data is modelled as  $\vec{x}$ . The two possibilities to be discerned upon receiving acoustic data are:

- · Noise only
- · Signal present in the noise

The hypothesis that the acoustic data contains only noise is labelled  $H_0$ , and the hypothesis that the acoustic data contains a signal with the noise is labelled  $H_1$ .

The distributions of the acoustic data under the noise-only hypothesis  $X_0 = T(\vec{N})$  and the signal-present hypothesis  $X_1 = T(\vec{S} + \vec{N})$  depend on your noise  $\vec{N}$  and signal  $\vec{S}$  distributions and the choice of data processor T for enhancing the signal-to-noise ratio.

The probability of false alarm  $P_f$  is the metric for an incorrect decision that a signal is present when the true scenario is that the signal is absent. It is thus defined as

$$P_f = \Pr \left\{ \vec{X}_0 \ge h \mid H_0 \right\} \tag{1}$$

Similarly, the probability of detection  $P_d$  is the metric for a correct decision that a signal is present. It is defined as

$$P_f = \Pr\{\vec{X}_1 \ge h \mid H_1\} \tag{2}$$

The threshold value h is termed the *detector threshold*, to be distinguished from the detection threshold.

The values of both probabilities need to be under consideration when making a decision as to the presence of a signal.

#### 1.4.1.1. Gaussian Noise with Gaussian Signal

The following assumptions are made:

- Noise is white, bandpass, wide-sense-stationary, ergodic, Gaussian.
- Noise spectral density is  $\vec{N} \sim \mathcal{CN}\left(\vec{0}, \lambda_0 \vec{I}\right)$