COSE 215: Theory of Computation

Lecture 16

Examples of Turing Machines

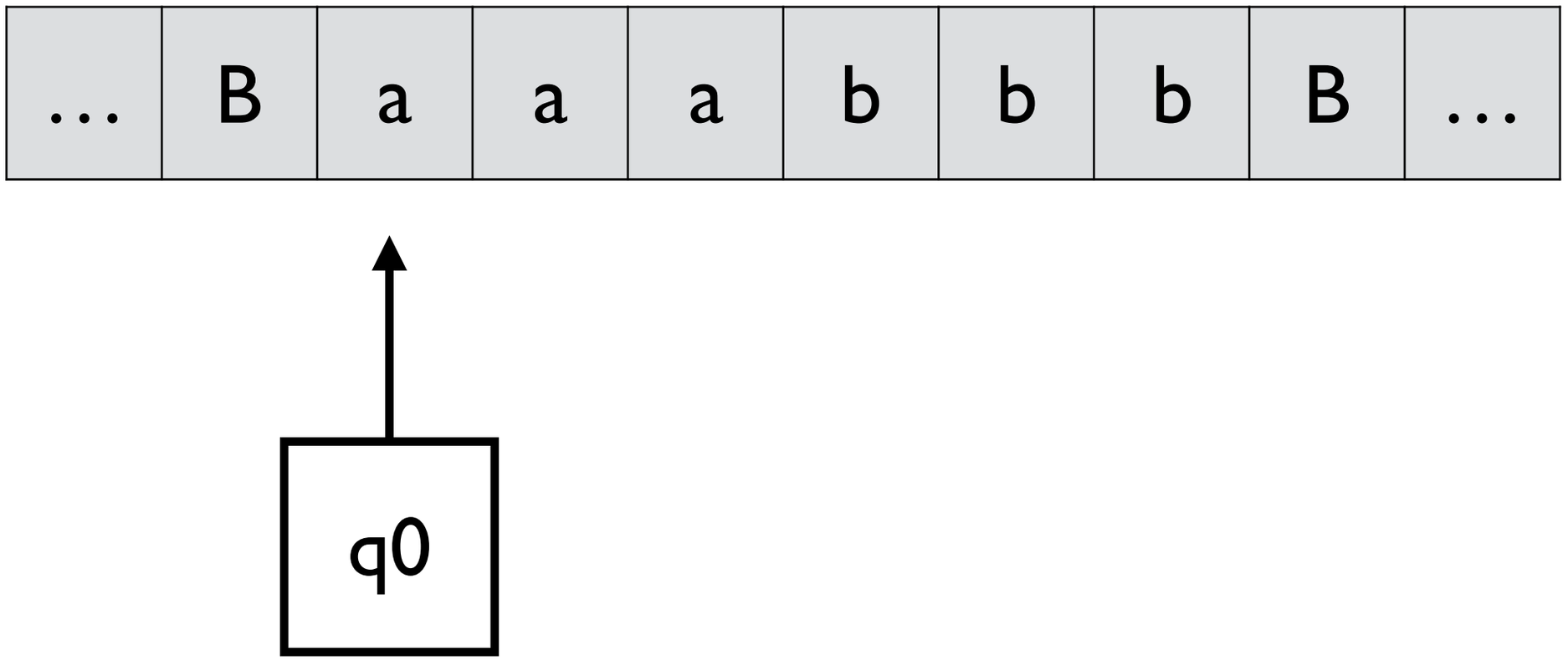
Hakjoo Oh

2018 Spring

Example 1. Design a Turing machine that accepts $L = \{a^n b^n \mid n \geq 1\}$.

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, x, y, B\}, \delta, q_0, B, \{q_4\})$$

$$\begin{array}{lll}
\delta(q_0, a) = (q_1, x, R) & \delta(q_2, y) = (q_2, y, L) & \delta(q_0, y) = (q_3, y, R) \\
\delta(q_1, a) = (q_1, a, R) & \delta(q_2, a) = (q_2, a, L) & \delta(q_3, y) = (q_3, y, R) \\
\delta(q_1, y) = (q_1, y, R) & \delta(q_2, x) = (q_0, x, R) & \delta(q_3, B) = (q_4, B, R) \\
\delta(q_1, b) = (q_2, y, L) & &
\end{array}$$



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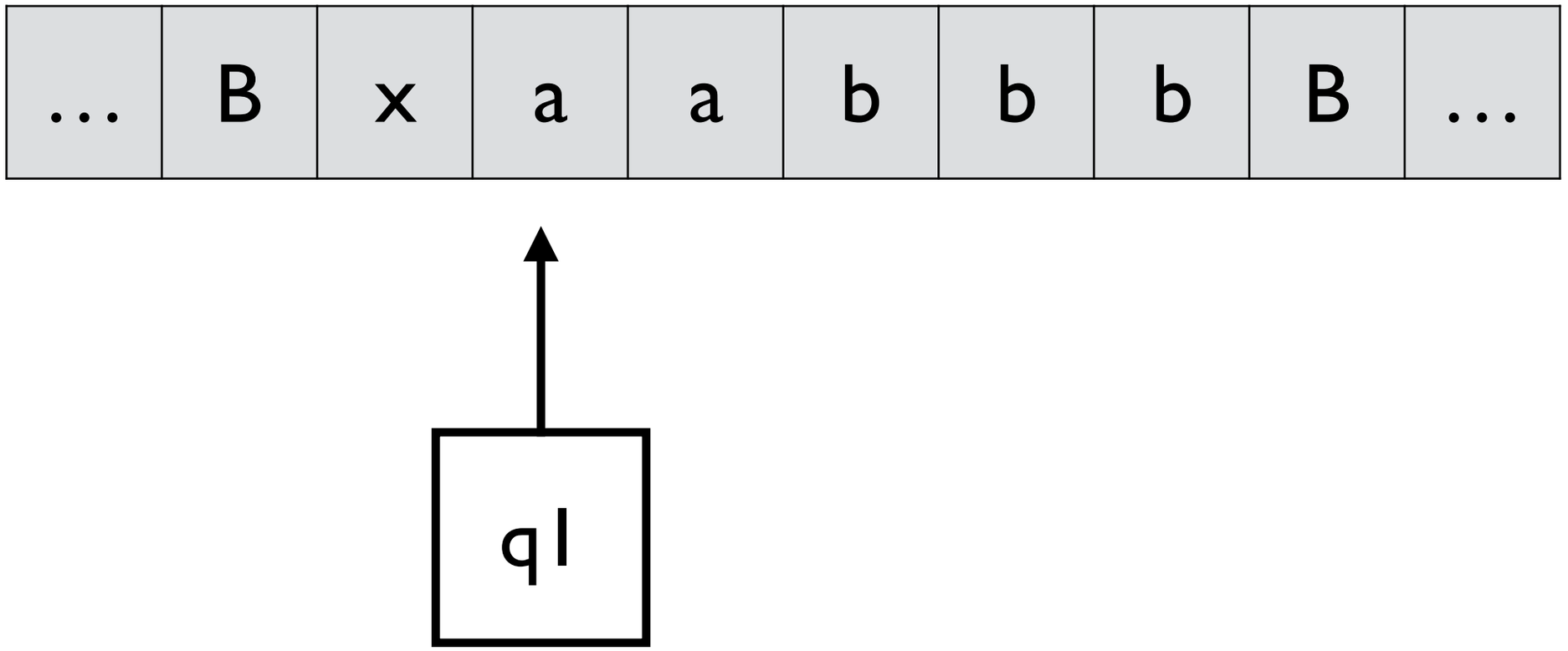

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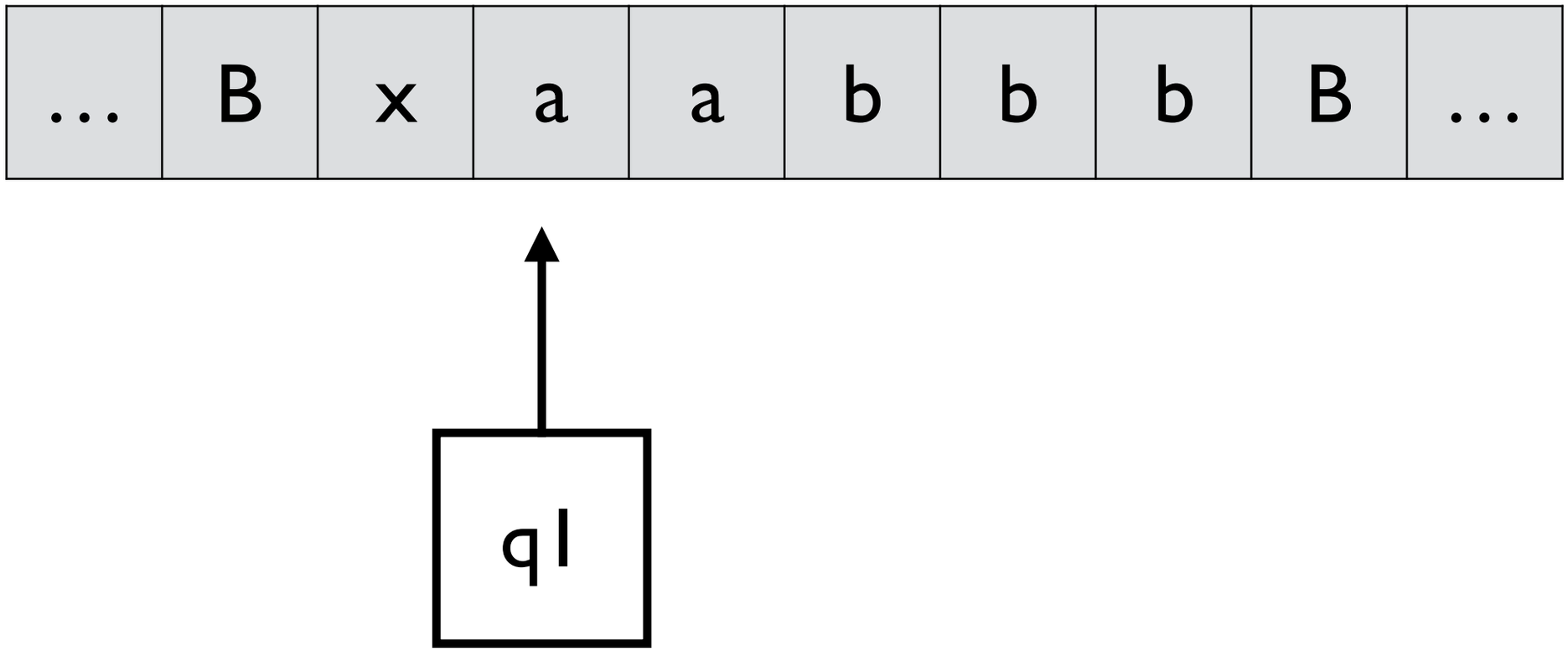

(In q1, move right to search for the leftmost 'b')

$$\delta(q_0, a) = (q_1, x, R)$$
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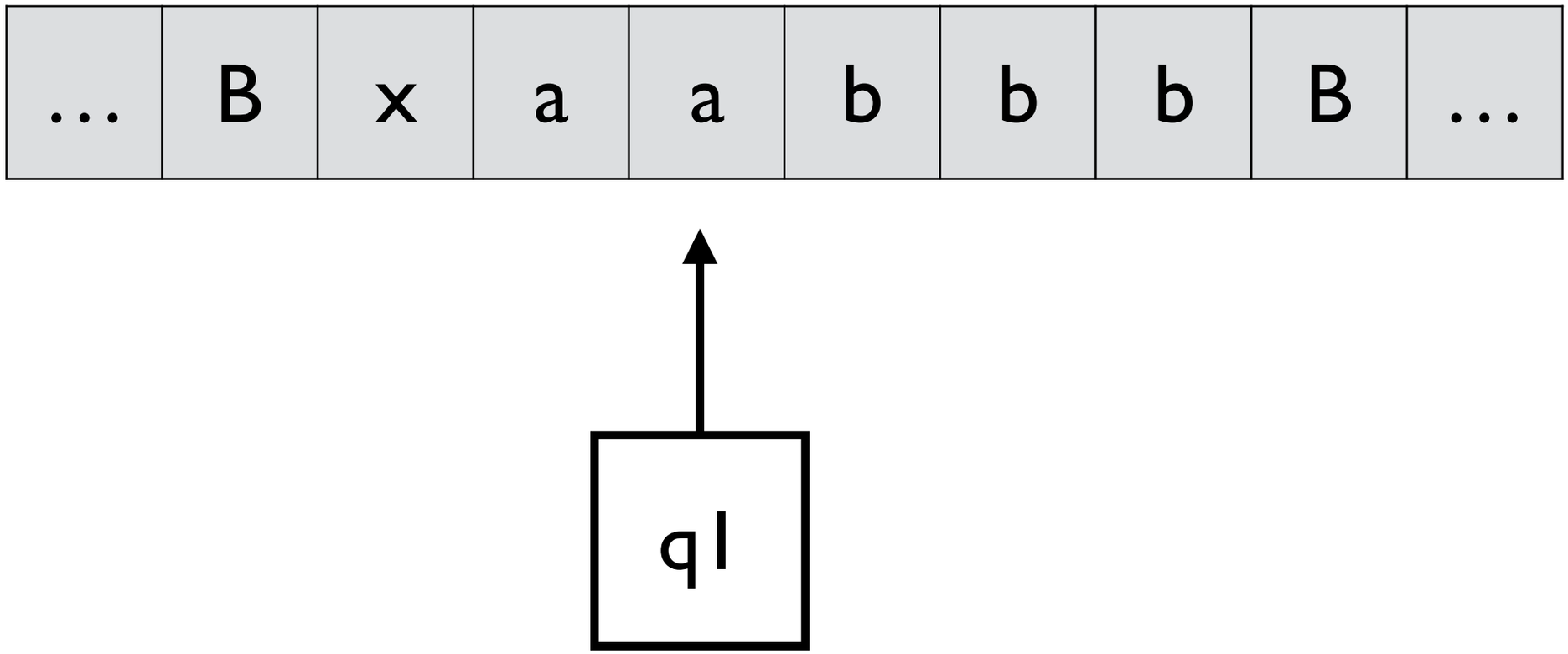

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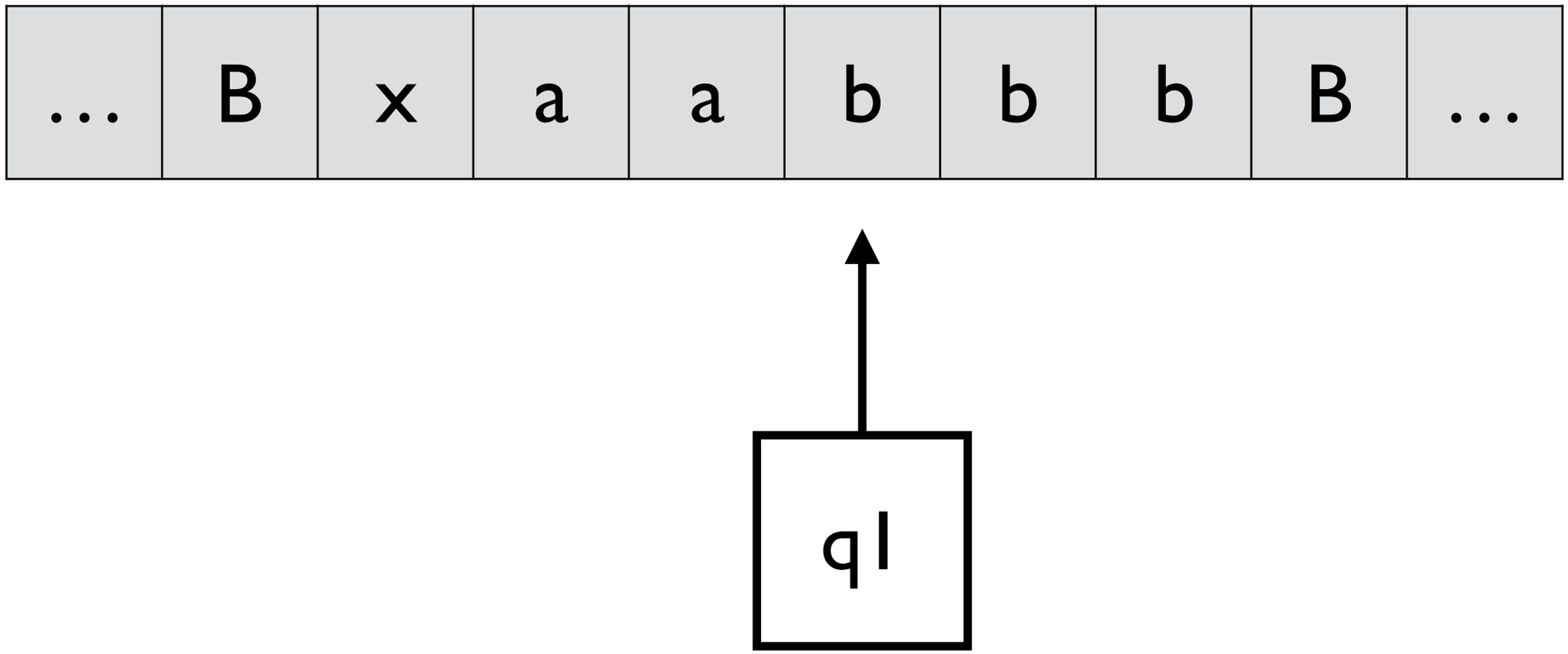

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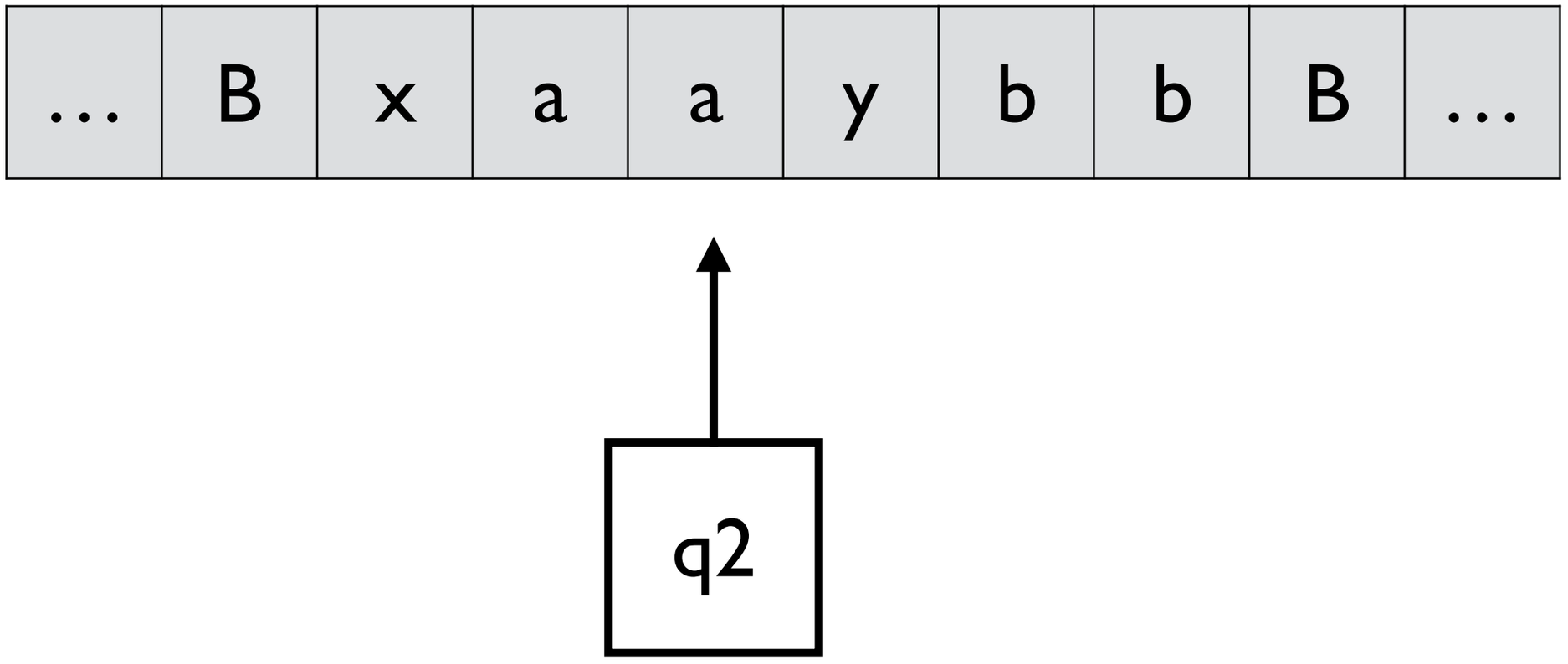

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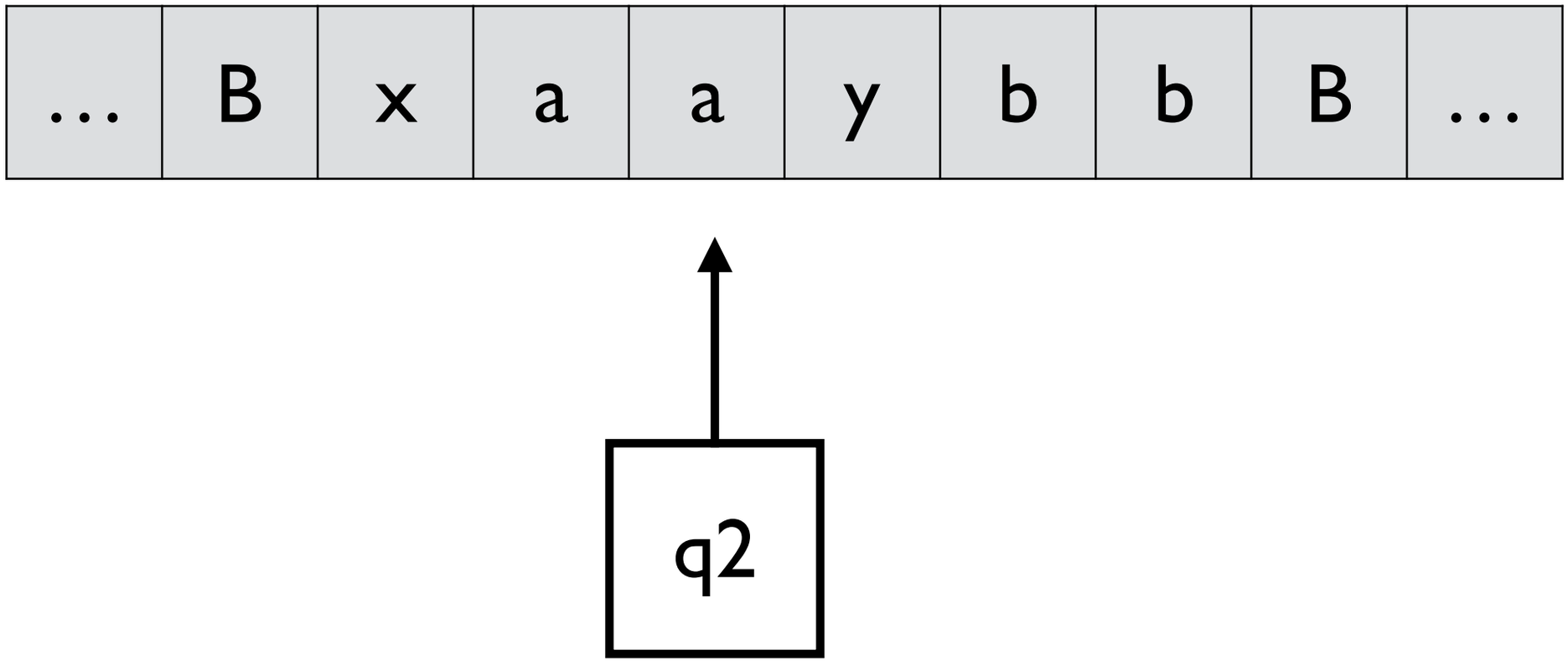

(In q2, move left to search for the leftmost ‘a’)

$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
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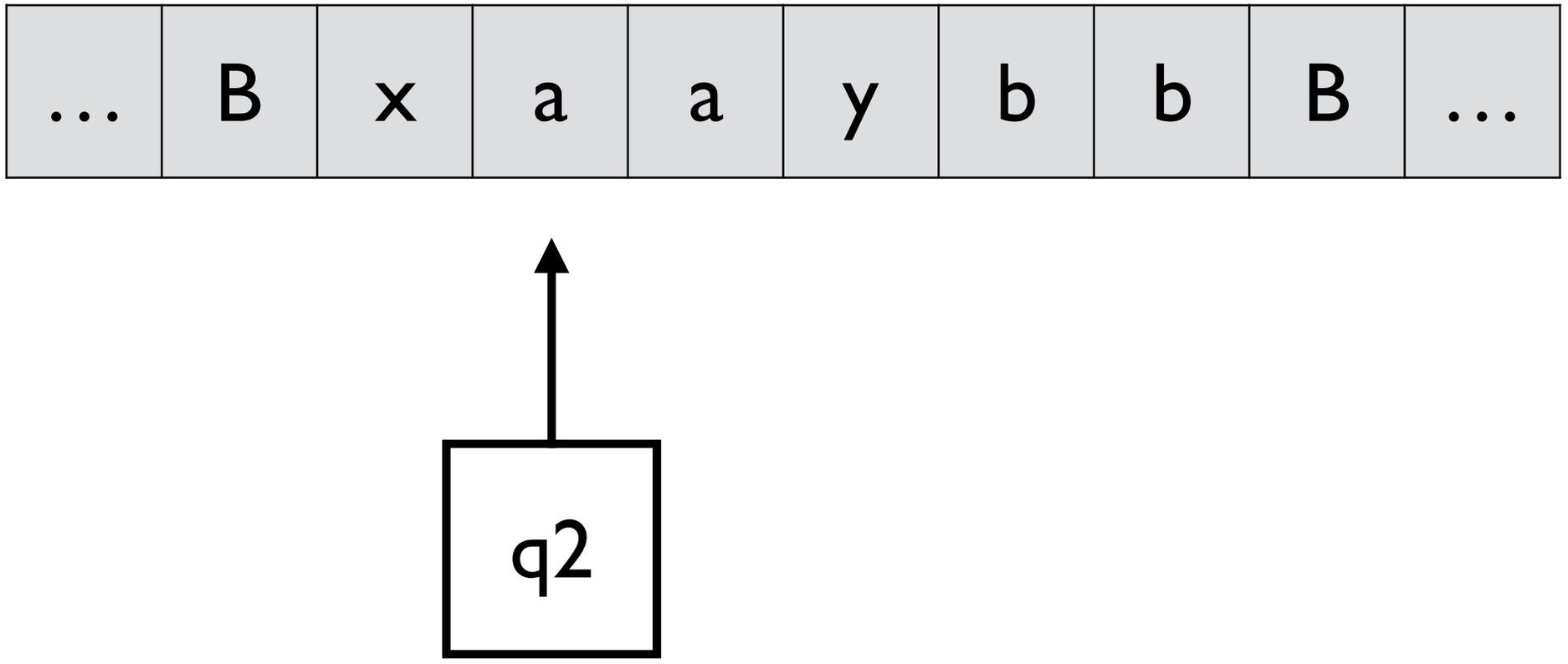

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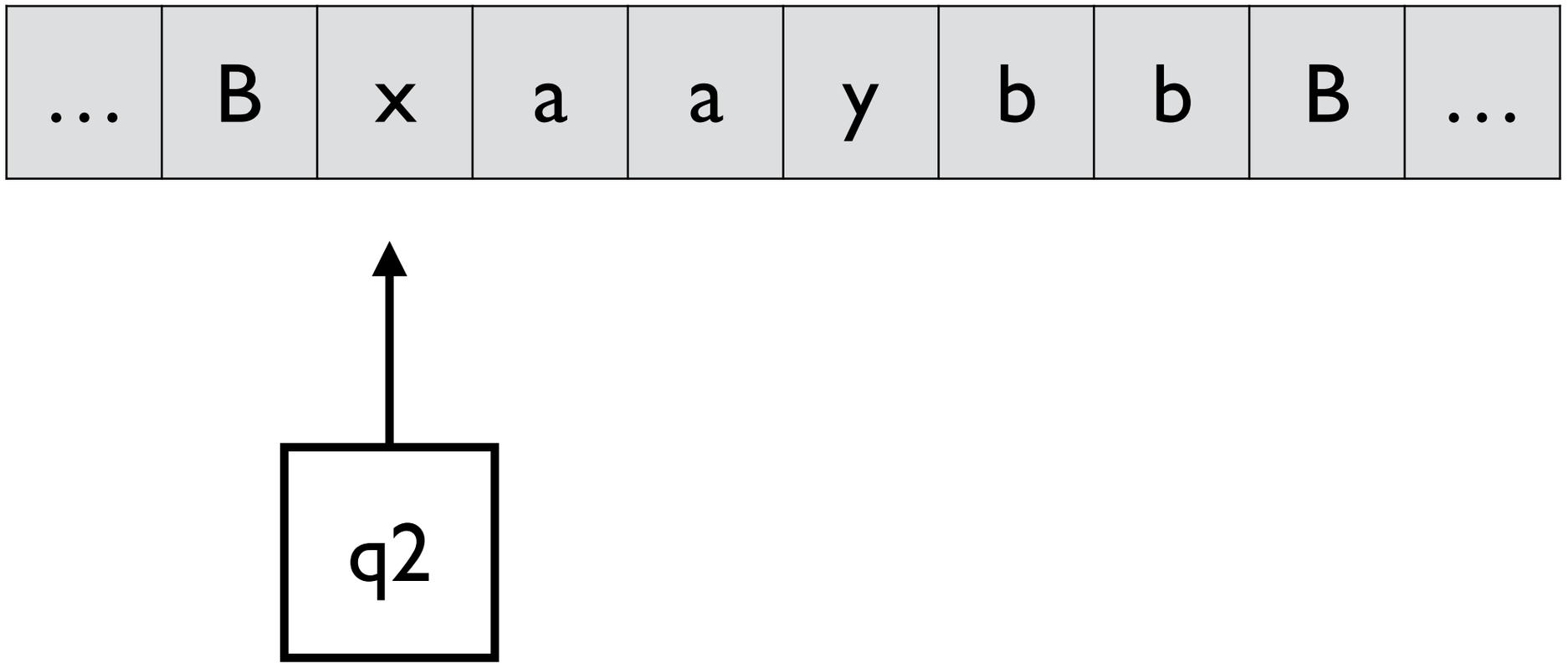

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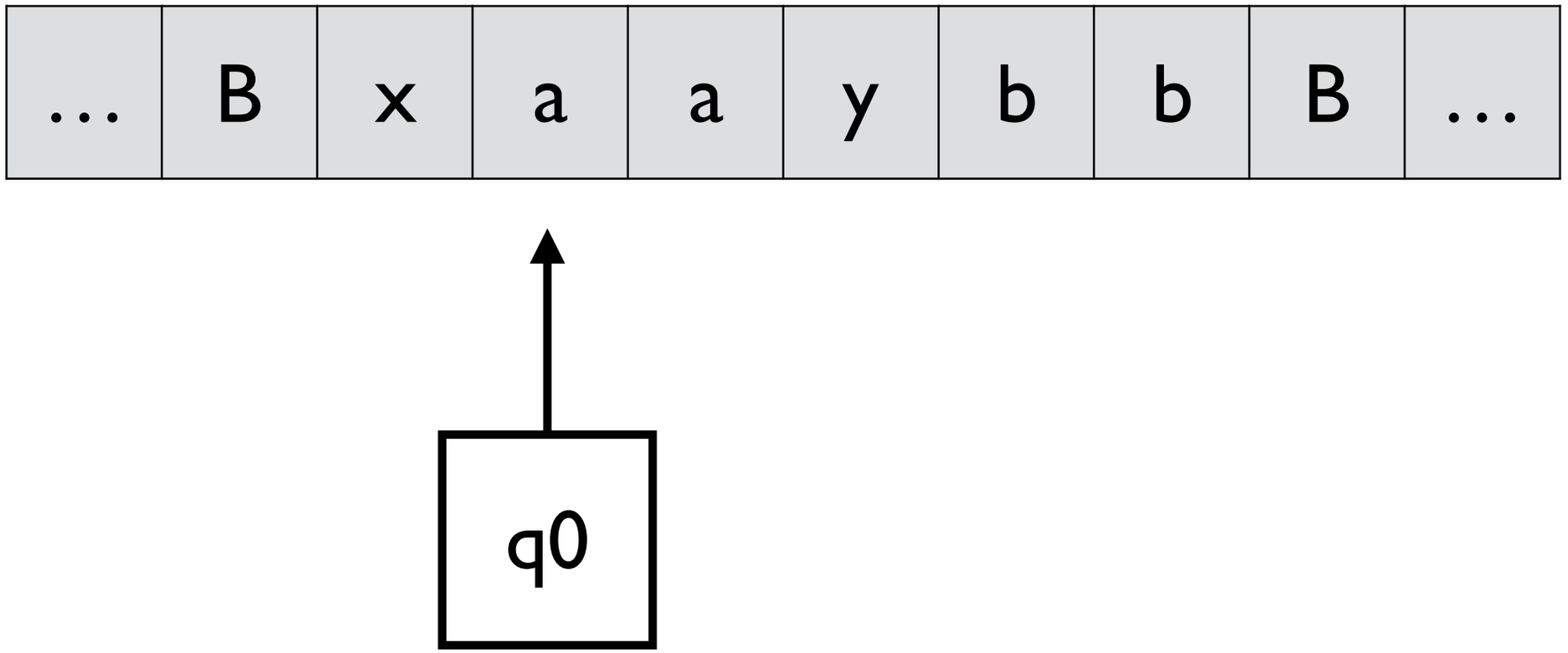

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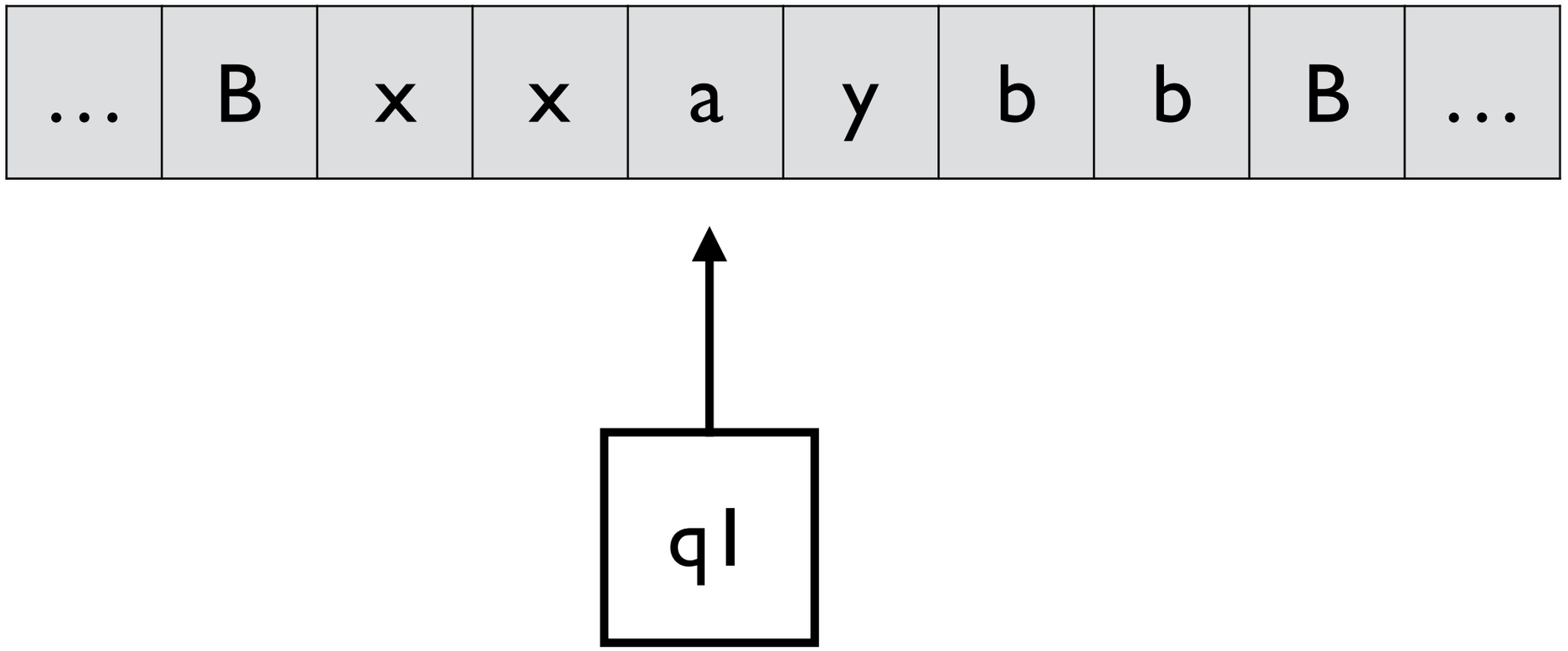

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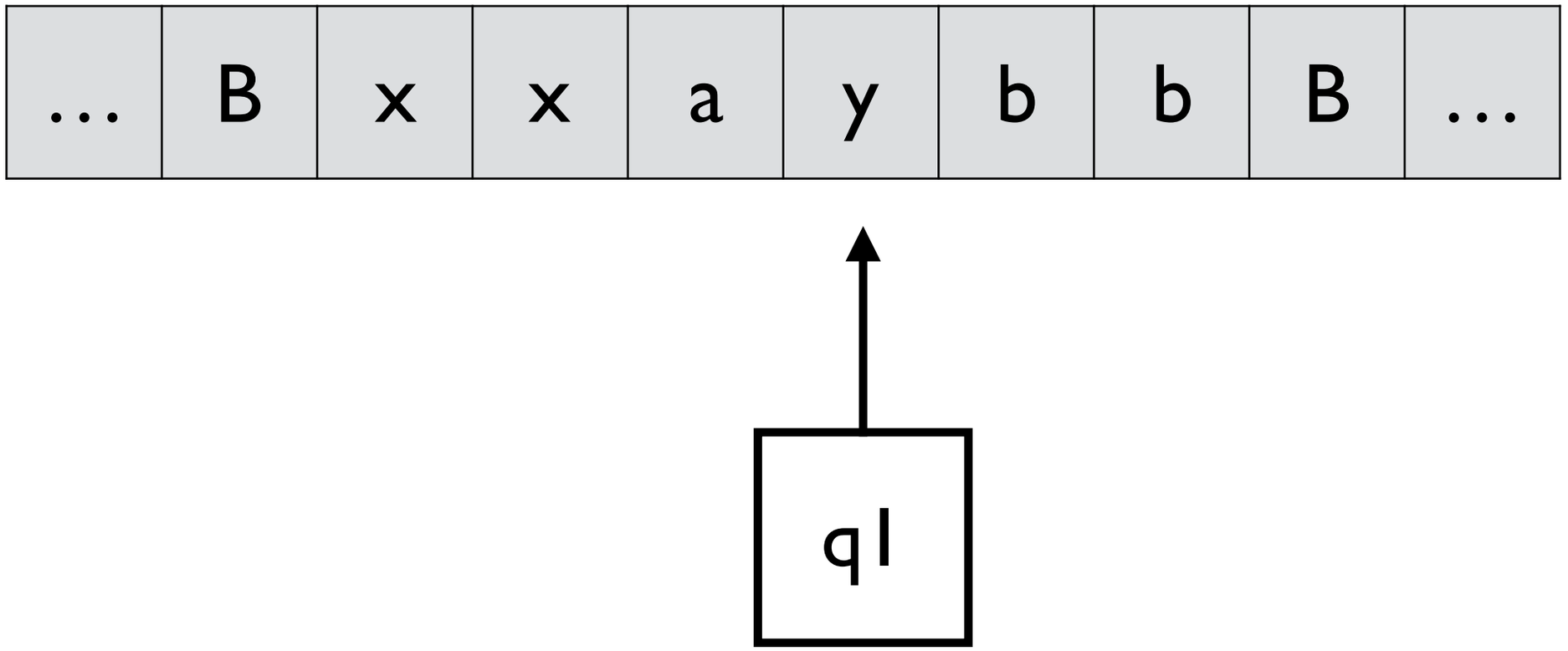

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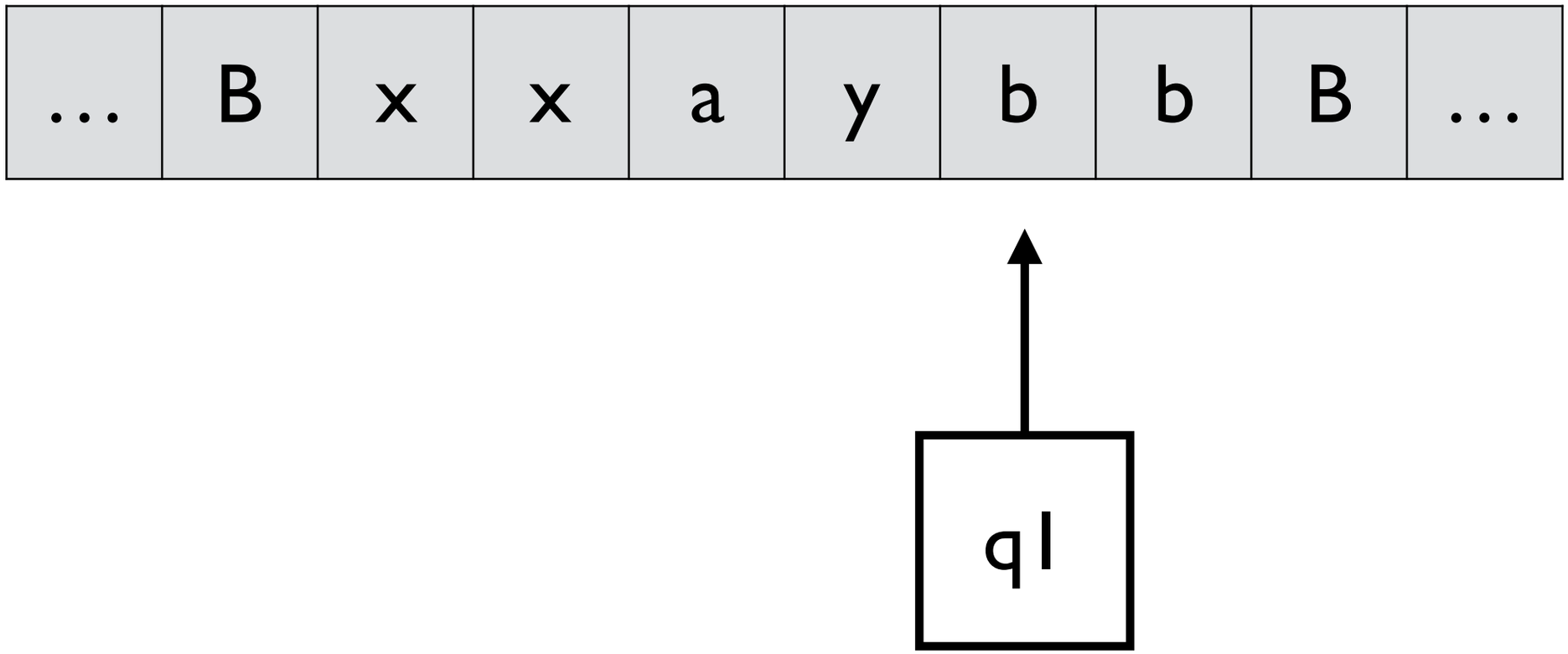

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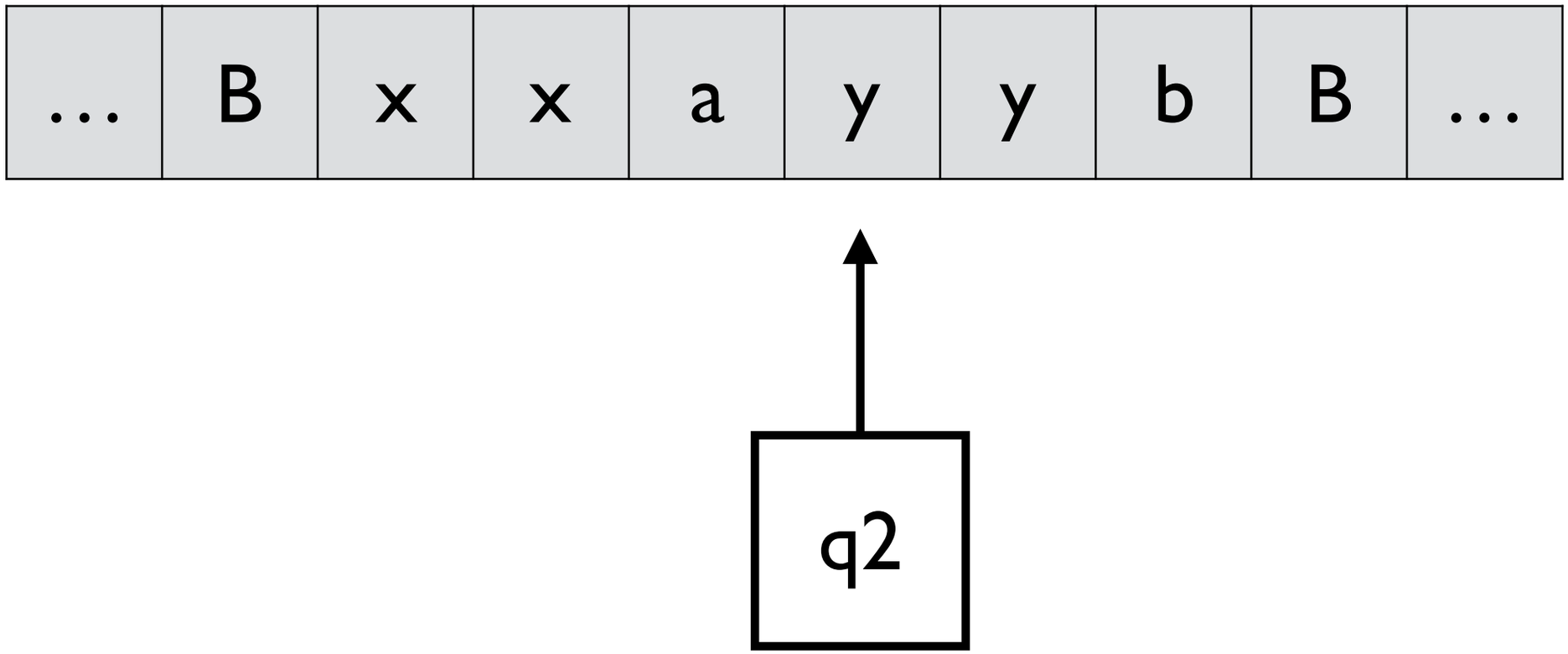

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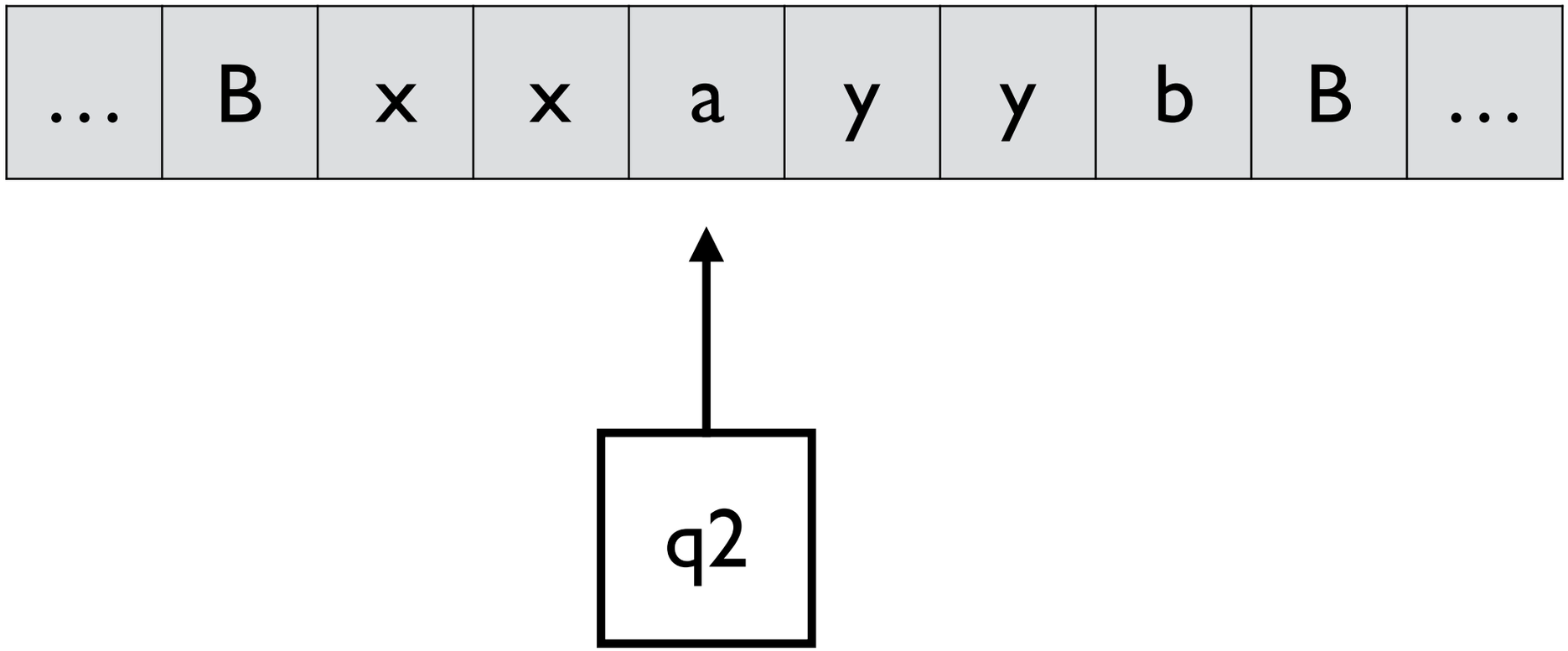

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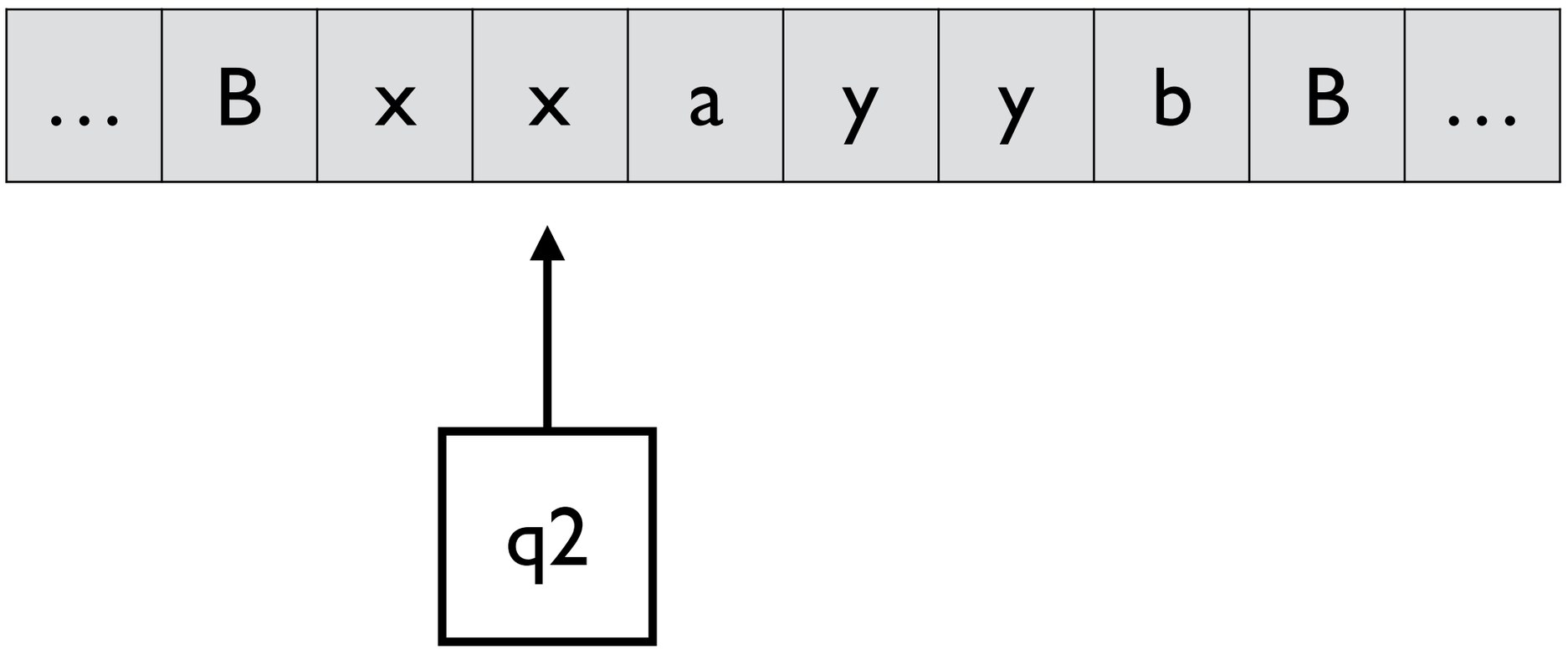

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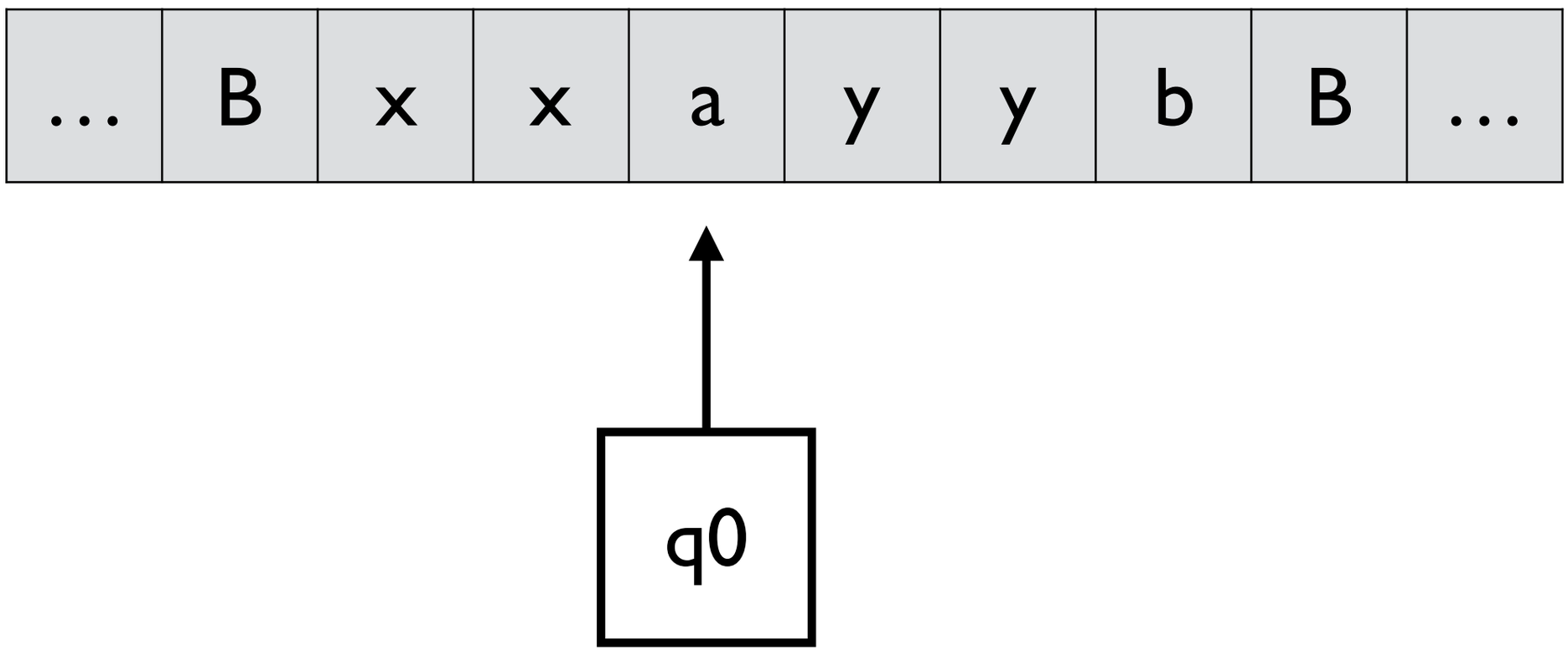

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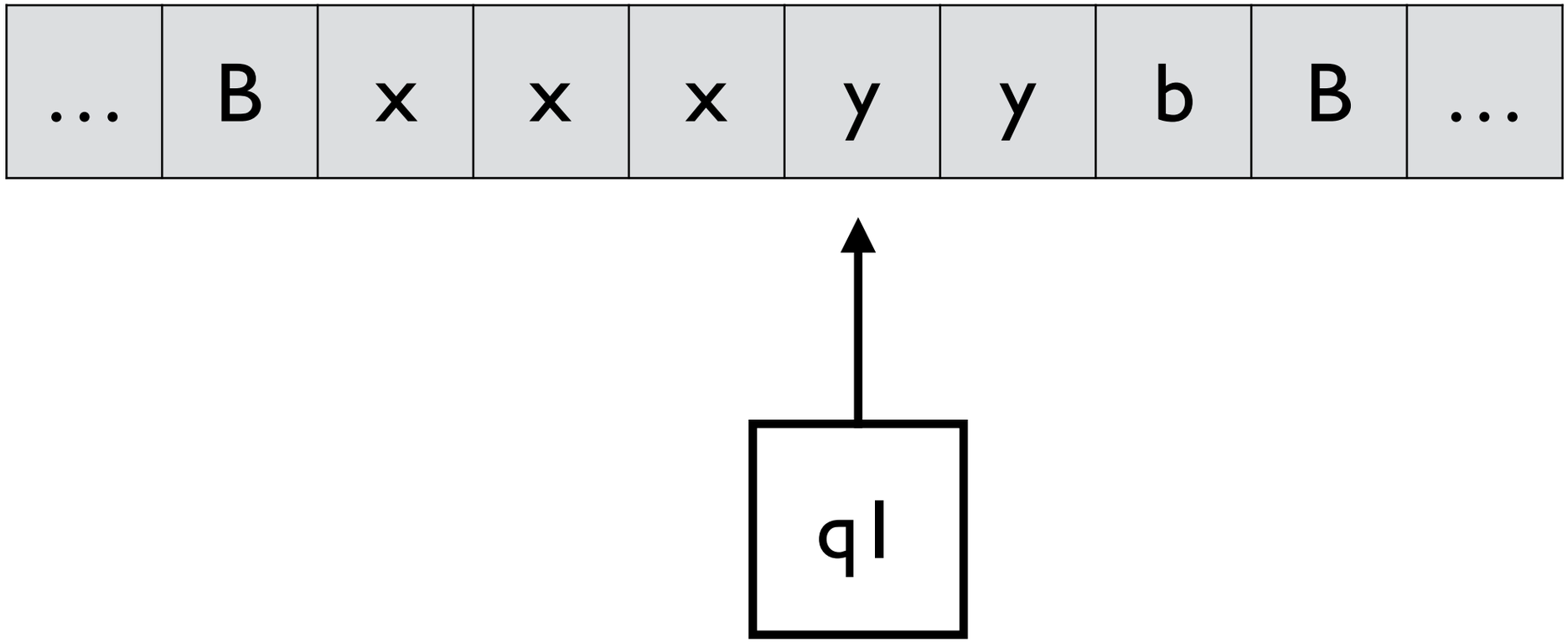

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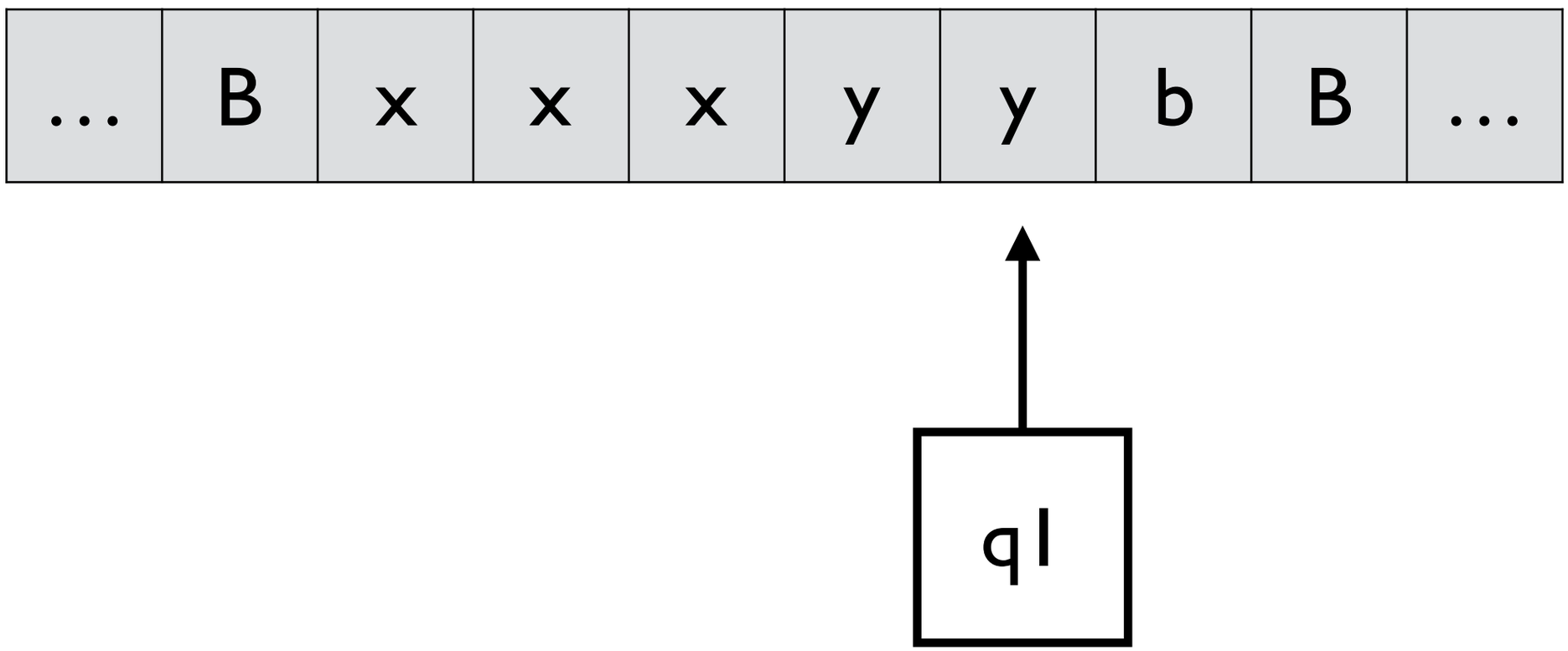

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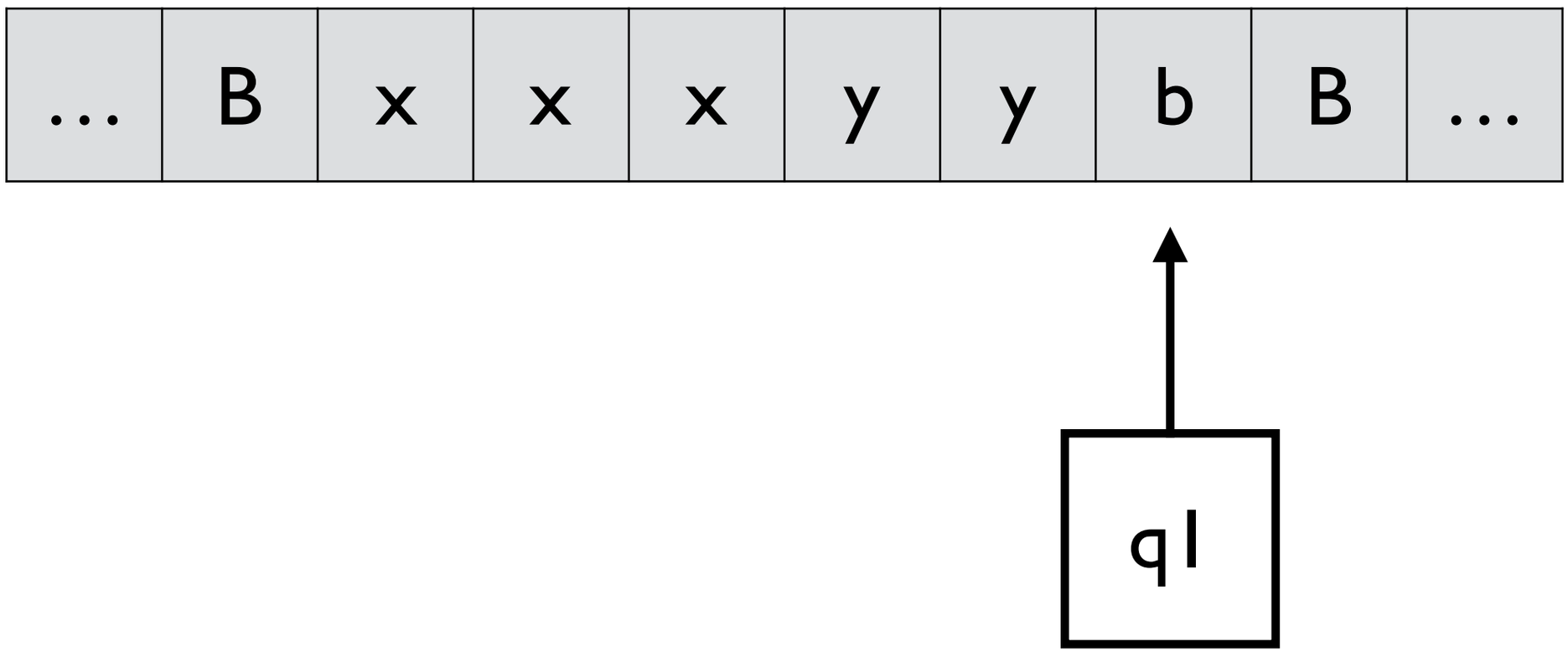

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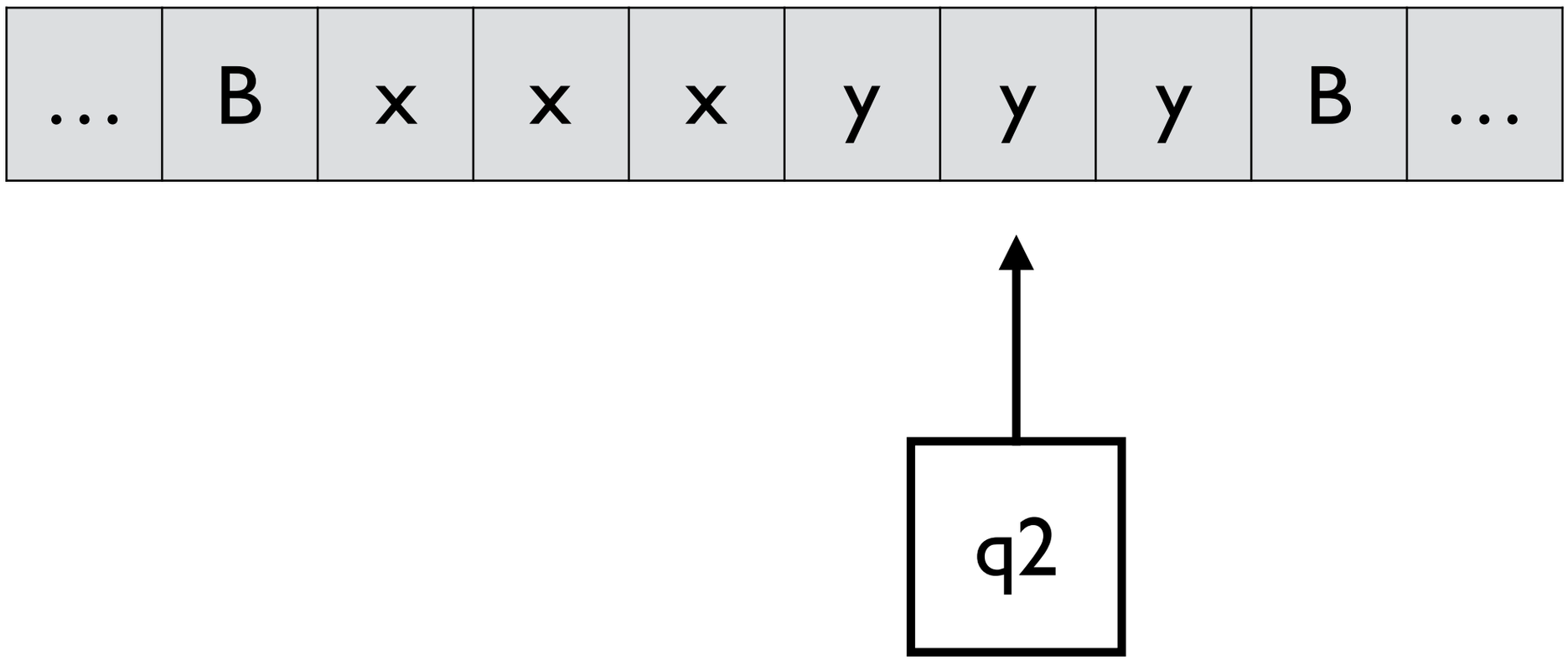

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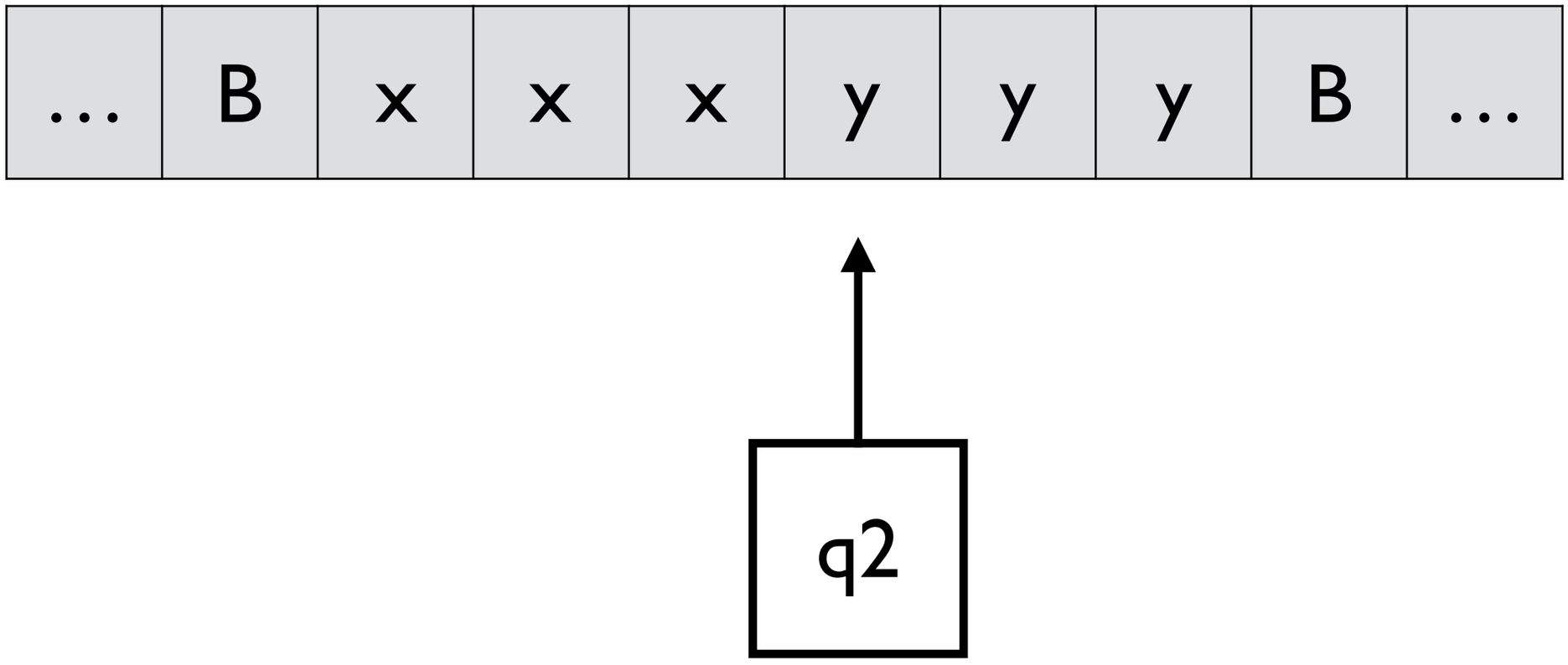

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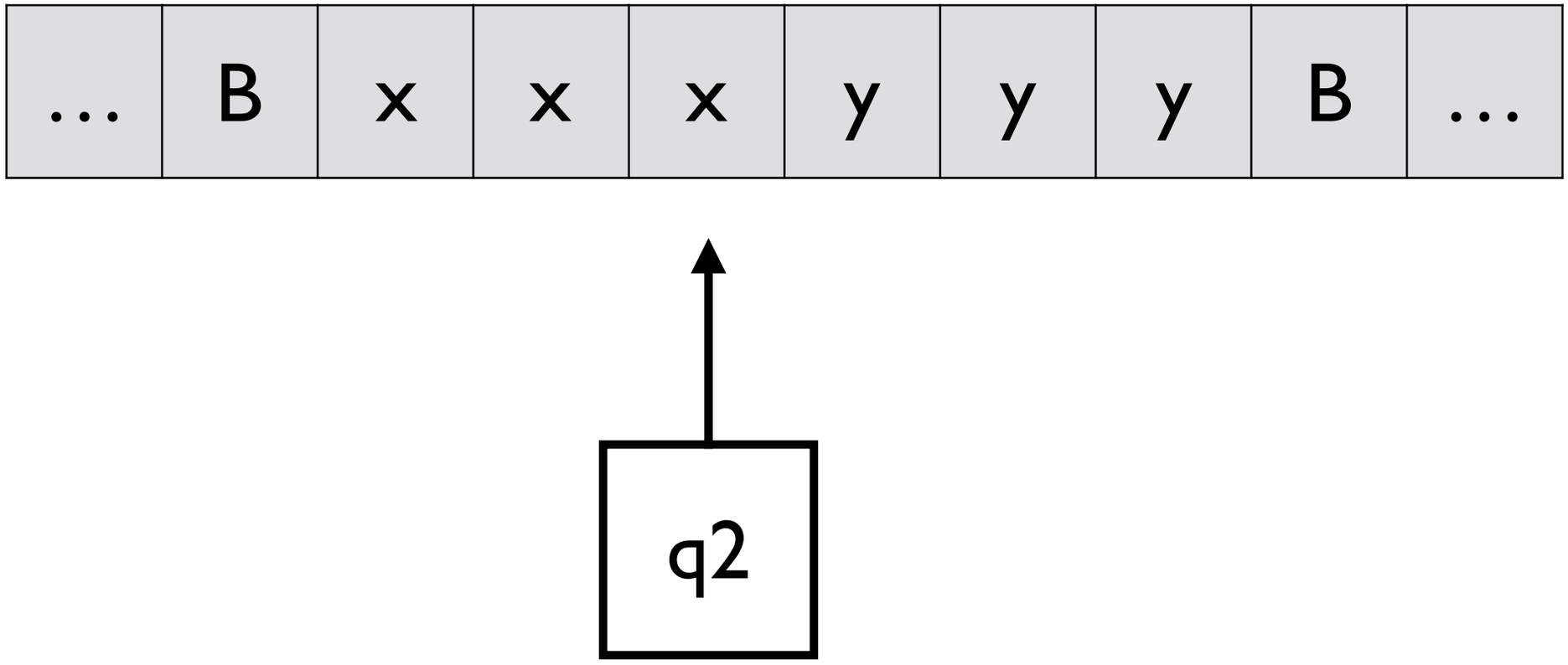
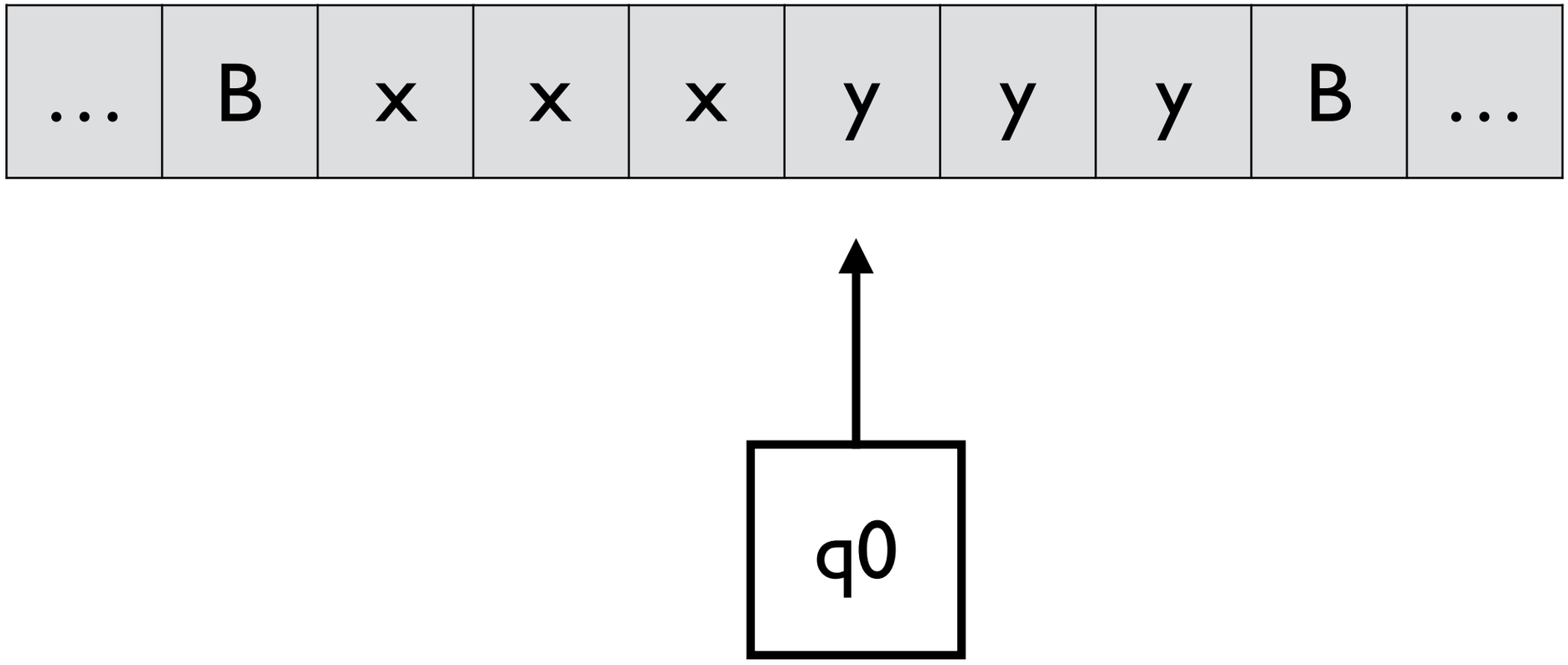
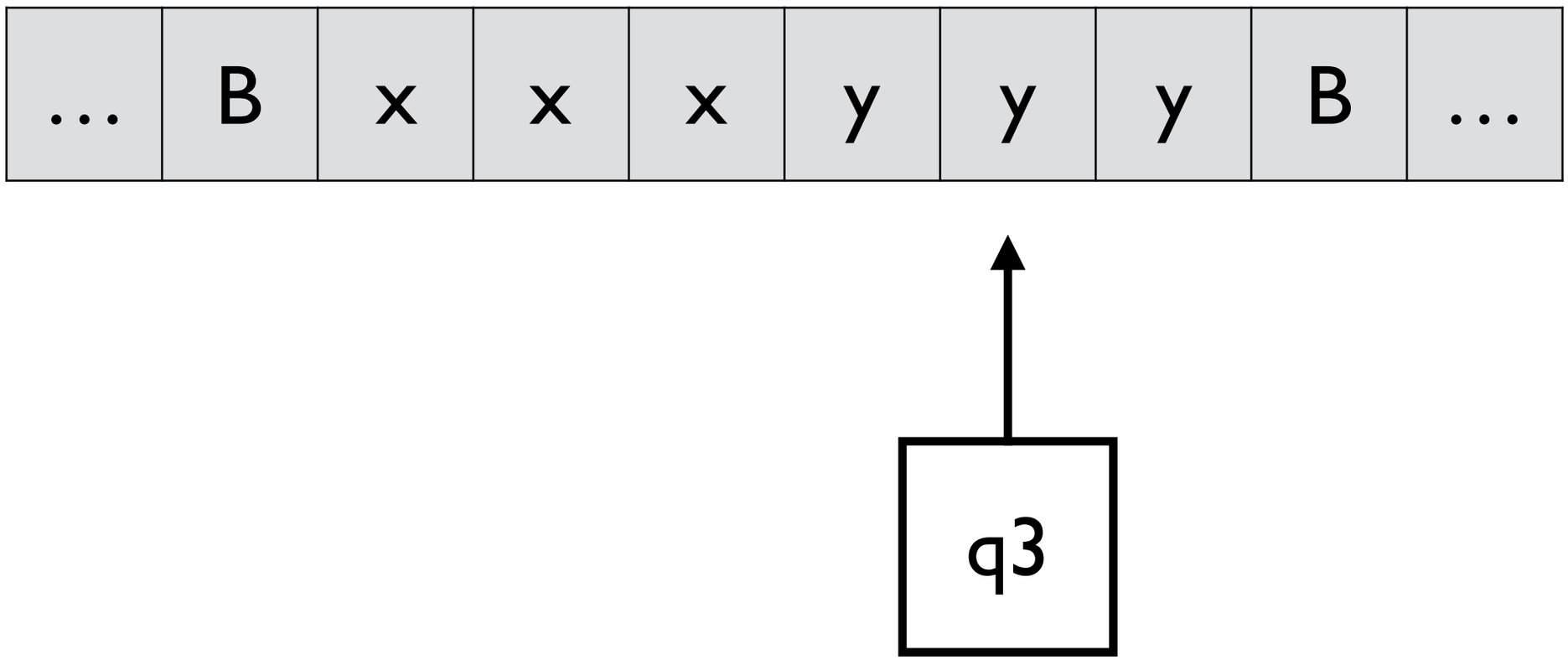
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



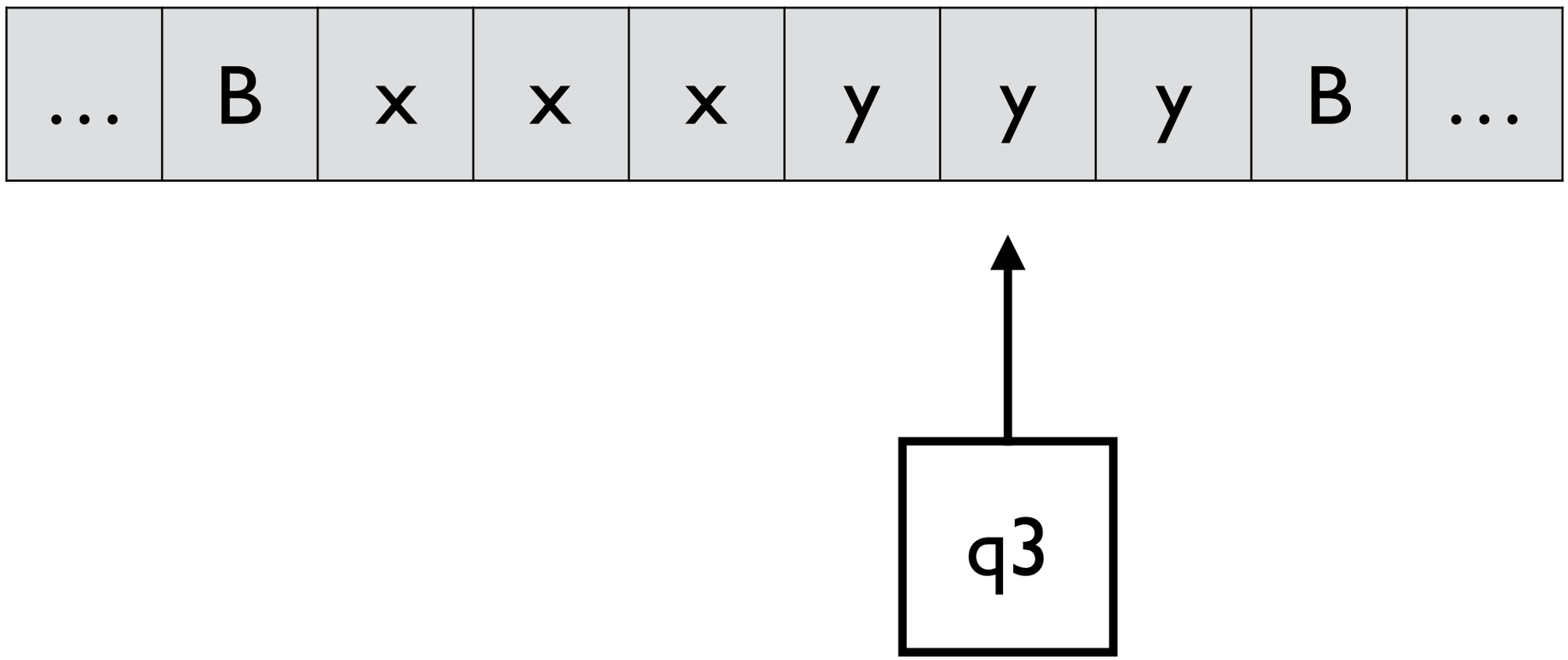
$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$



$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

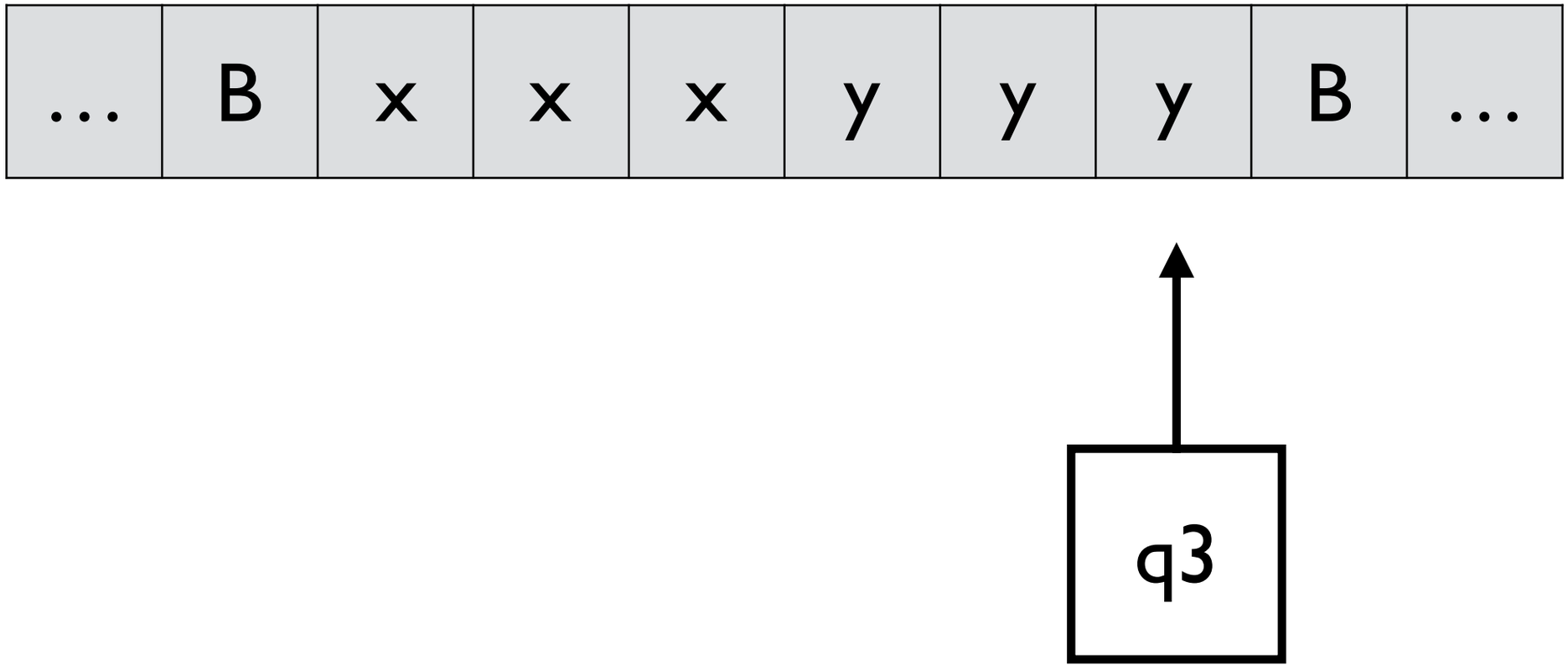


$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

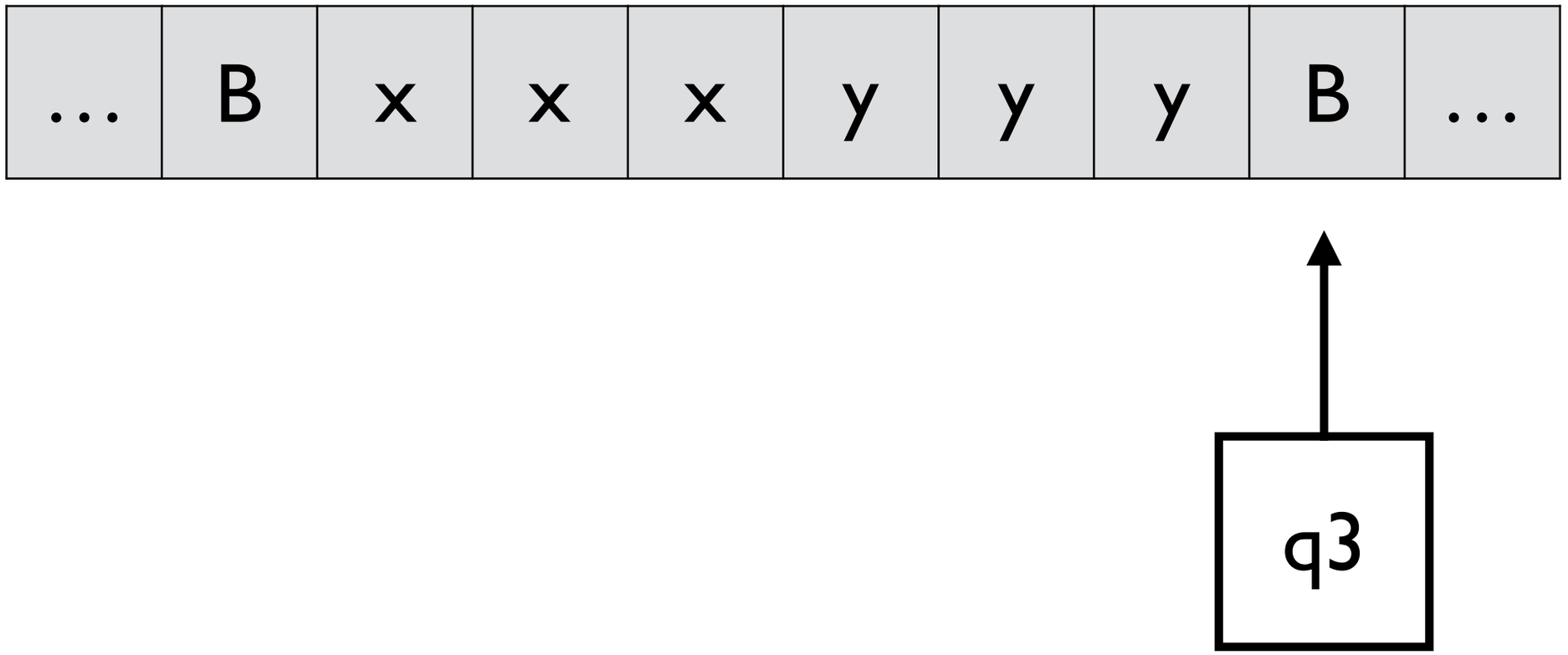


(In q3, move right to check that there are no more b's)

$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

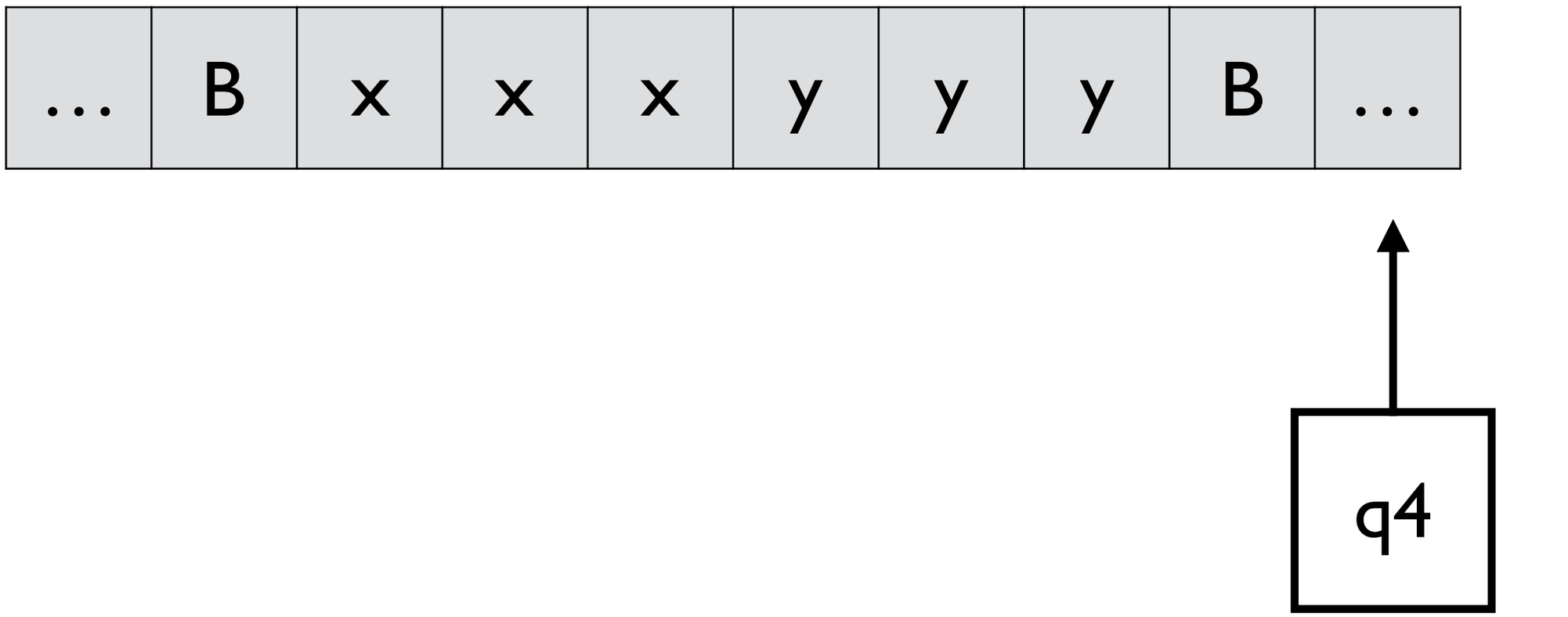


$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



(no more b's)

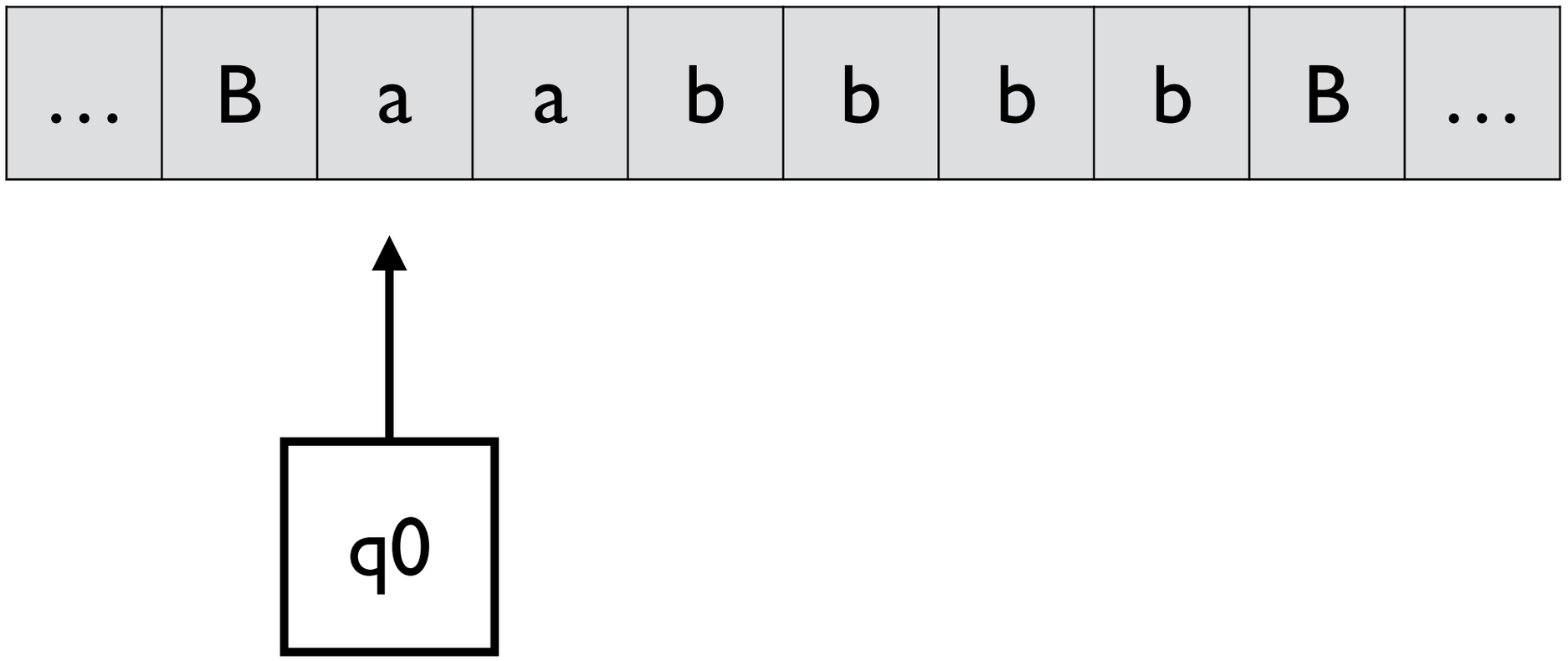
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



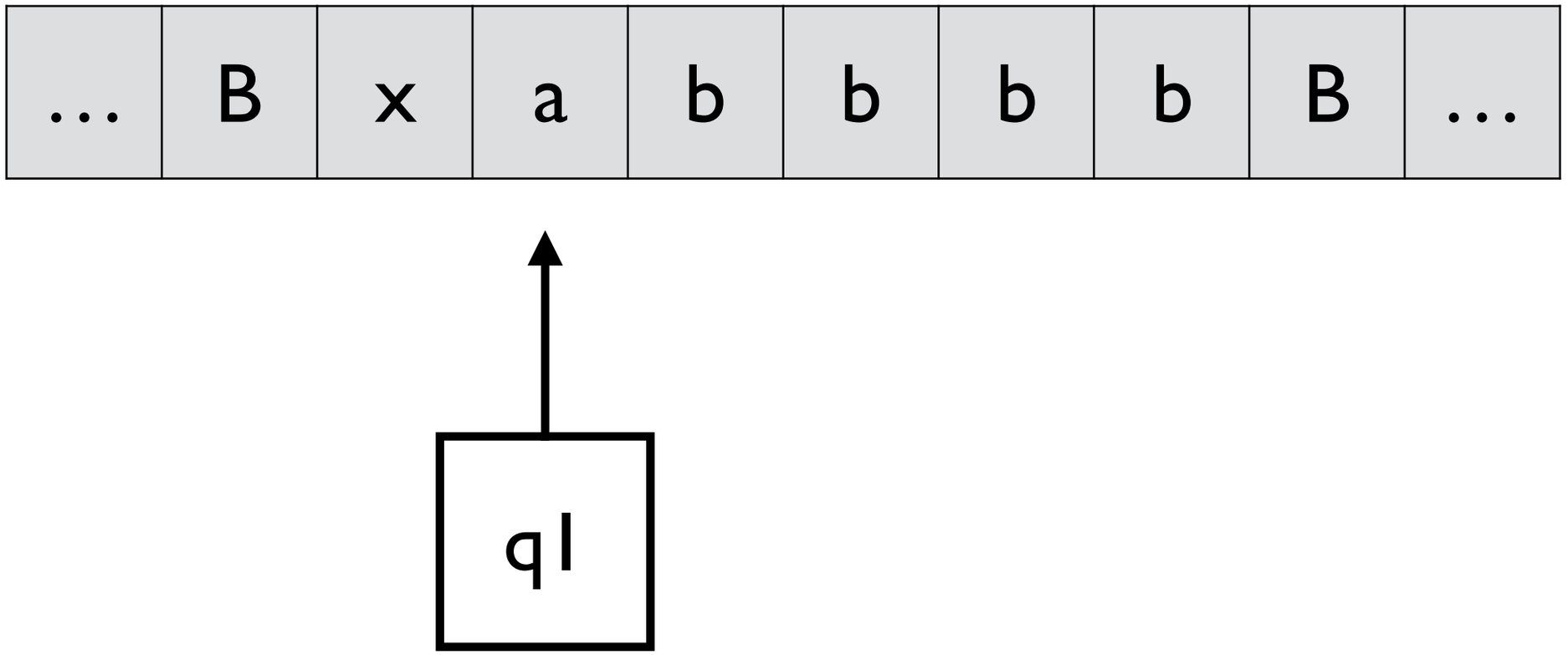
"final state"

$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

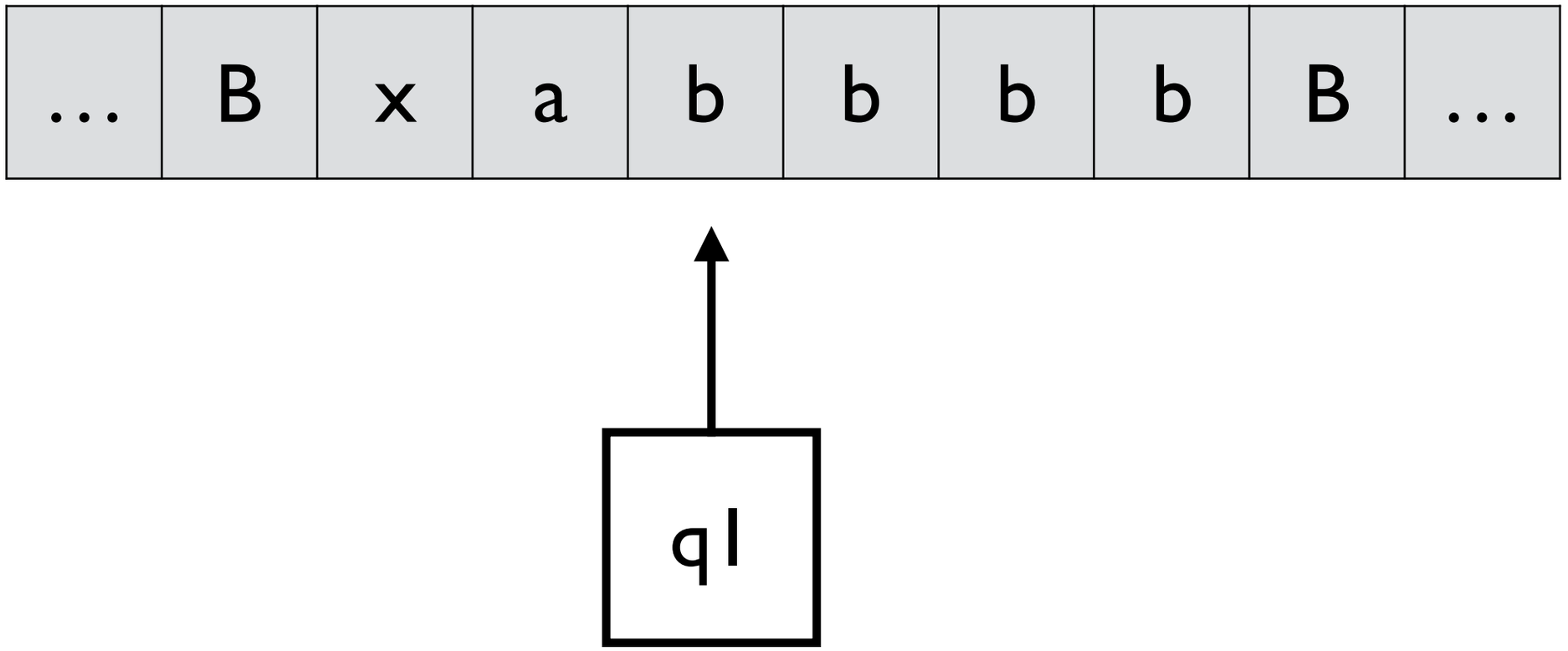
When the input string is not in the language:



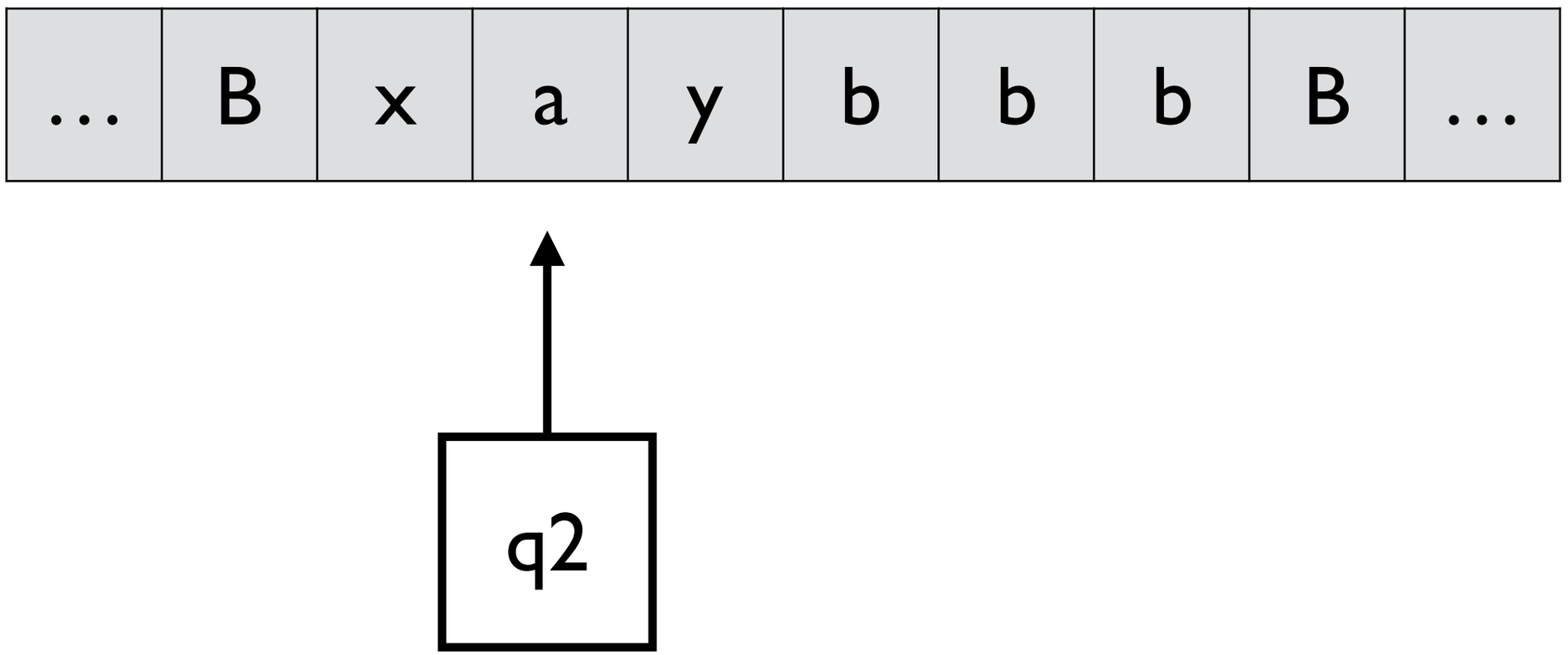
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



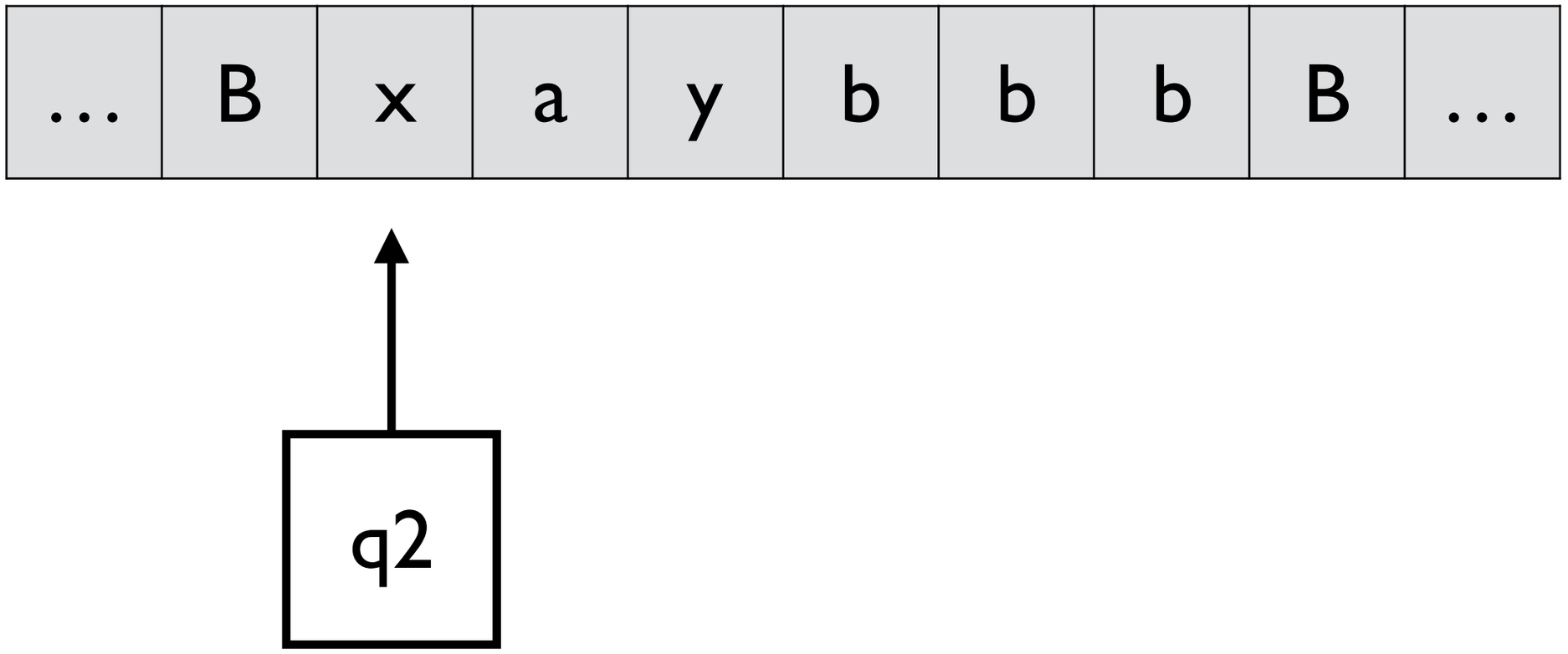
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



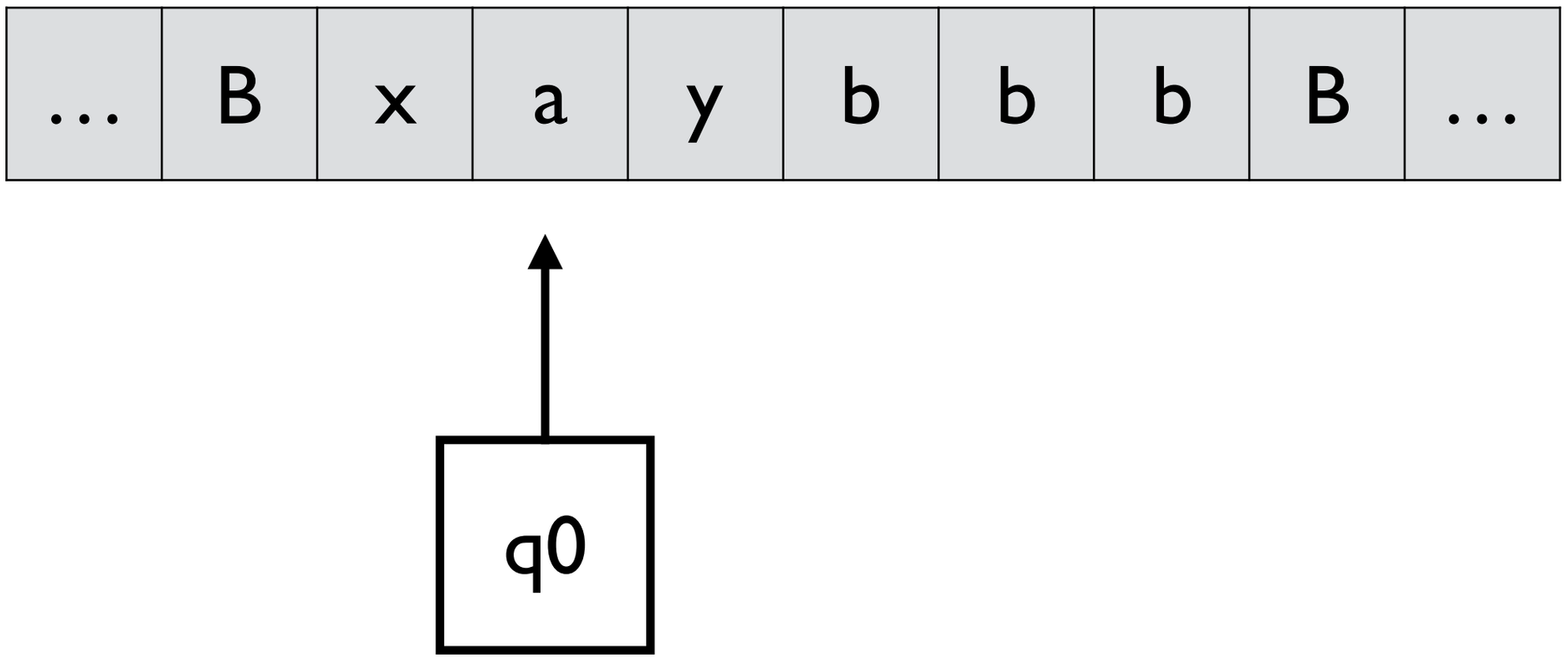
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



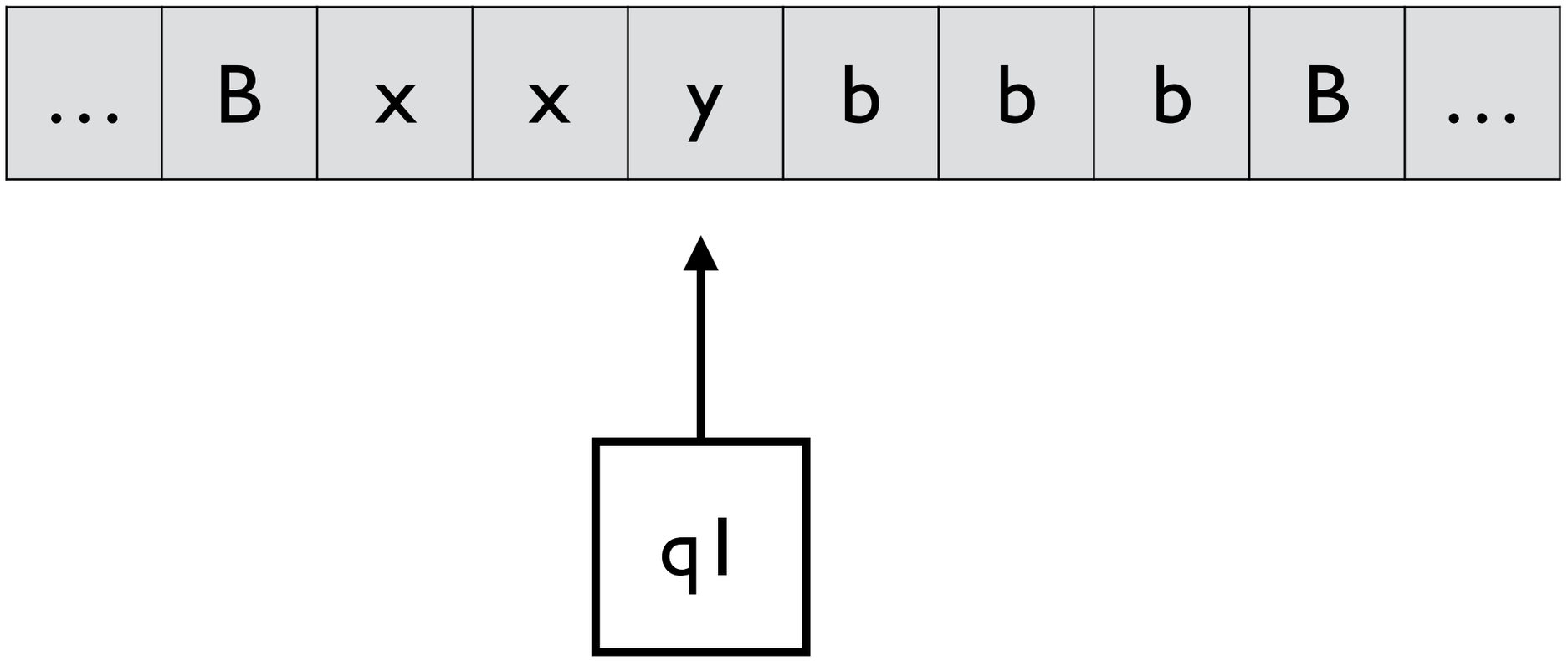
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



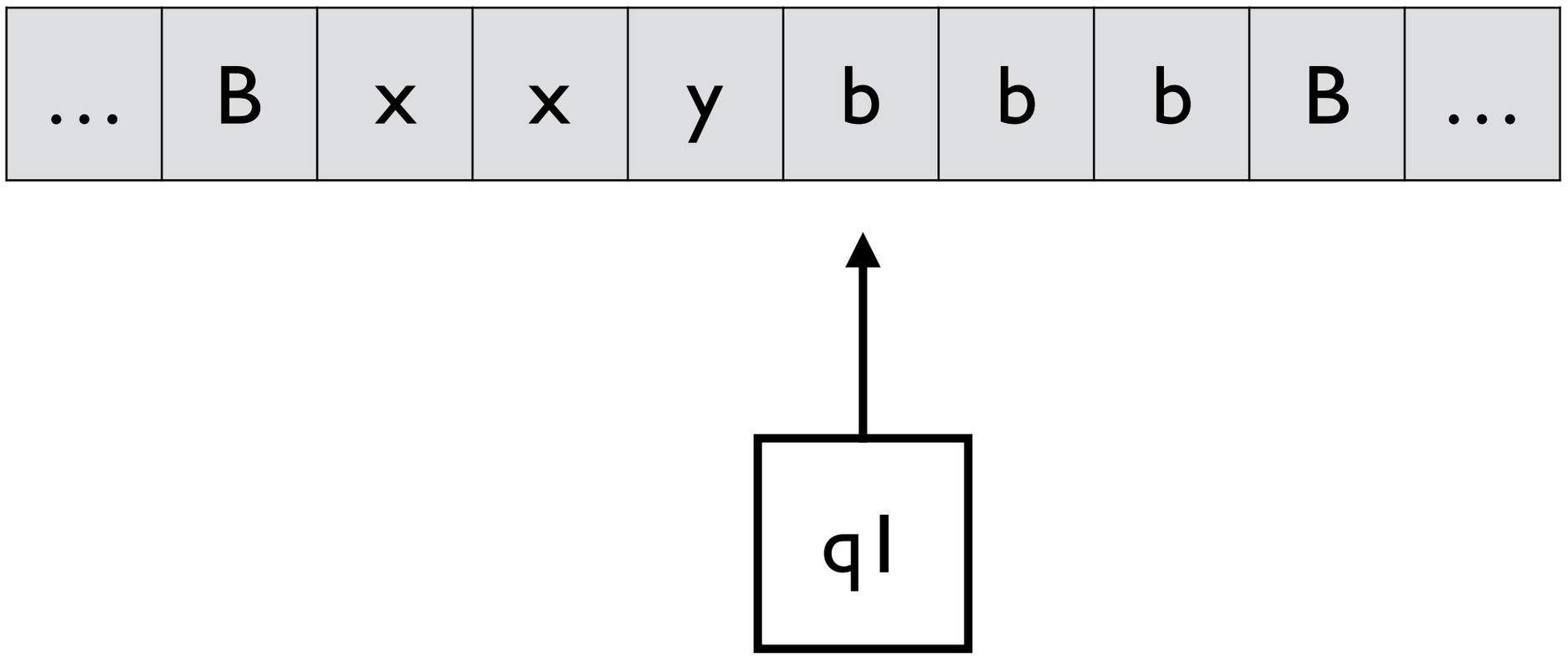
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



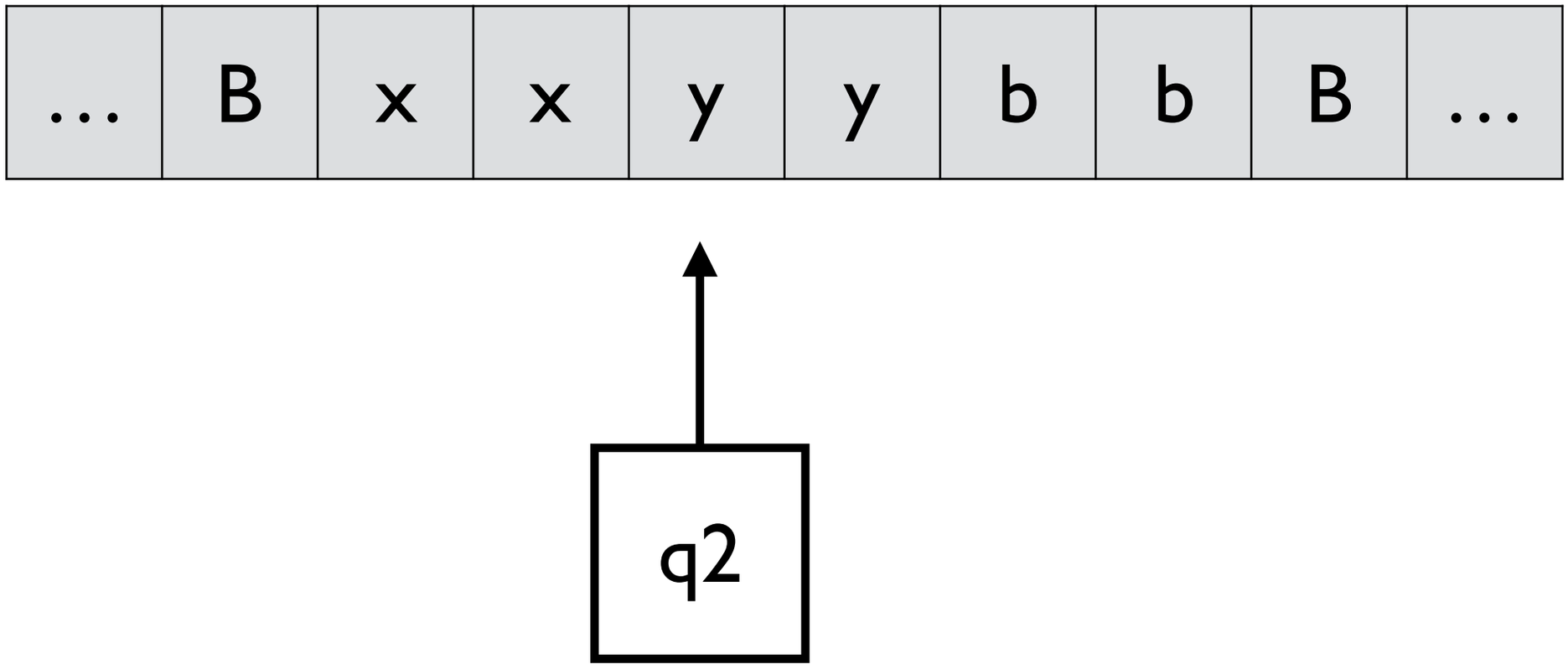
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



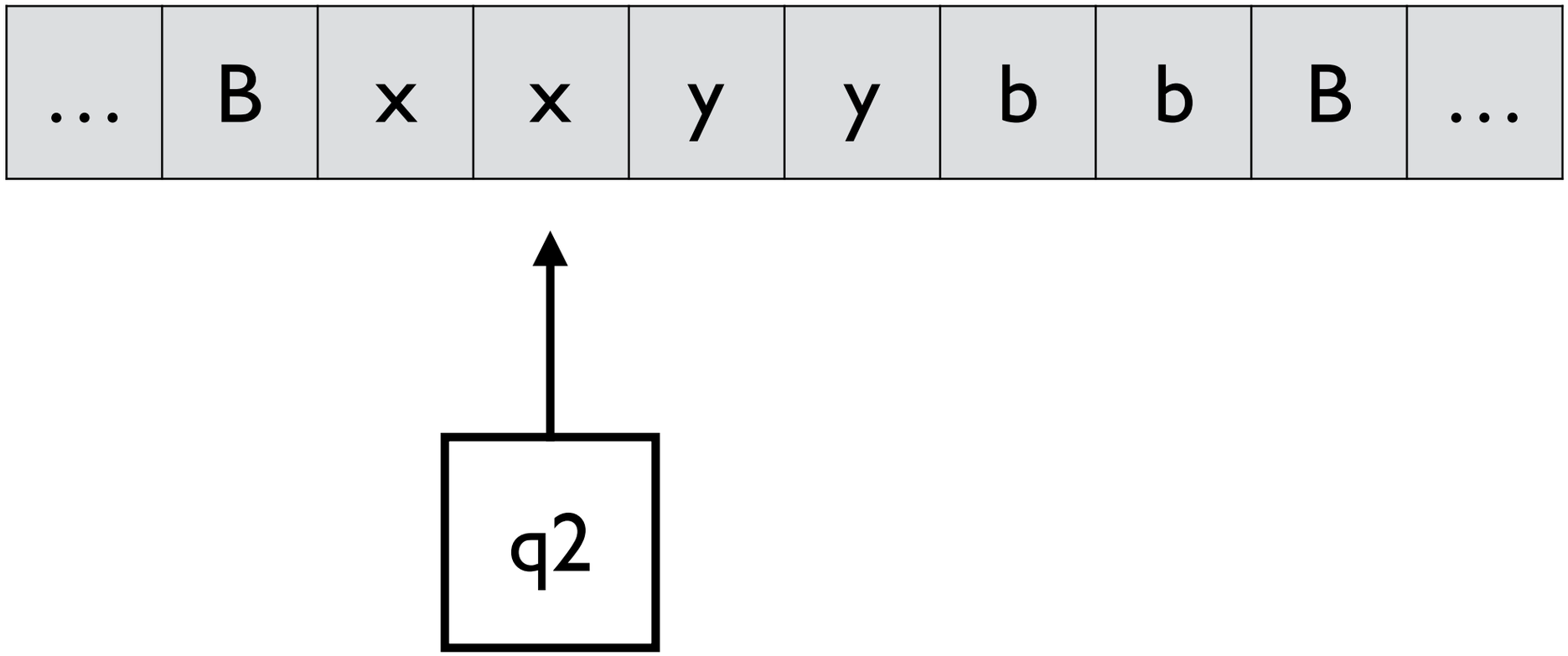
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



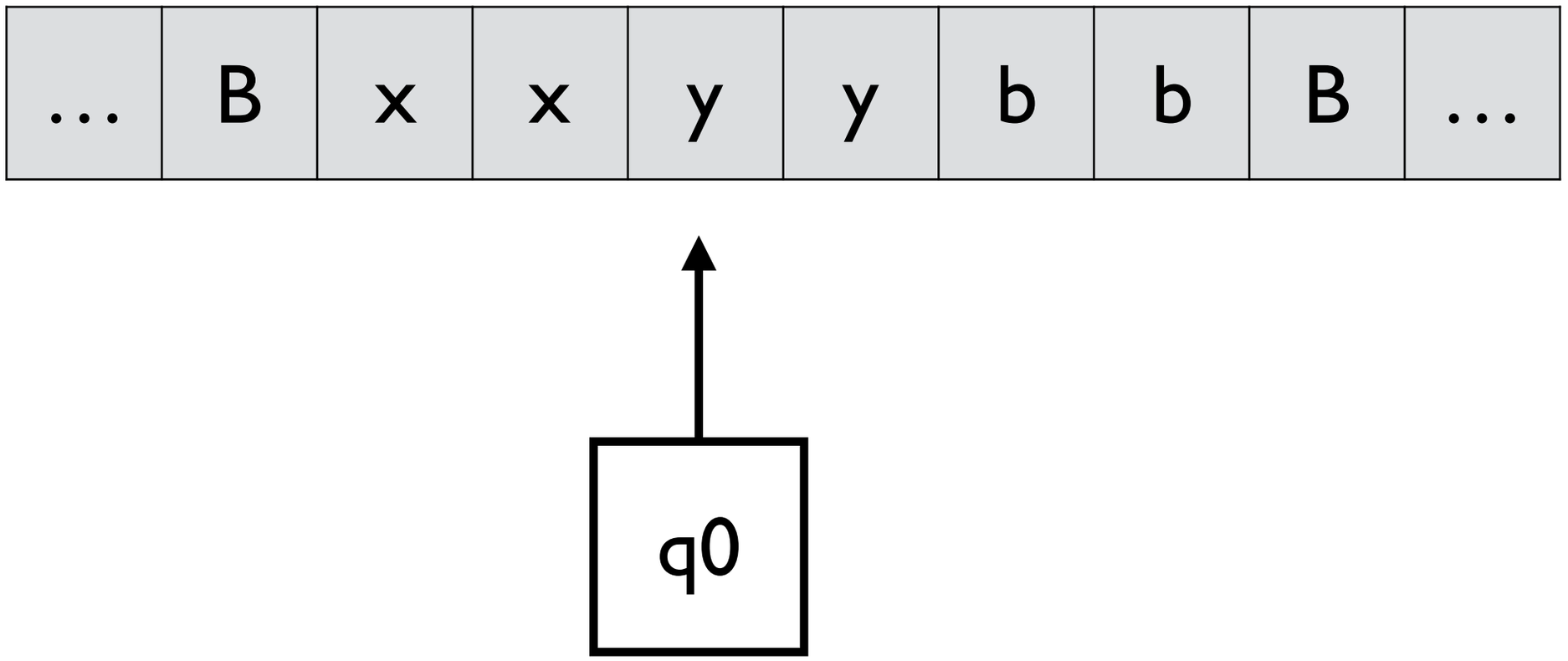
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



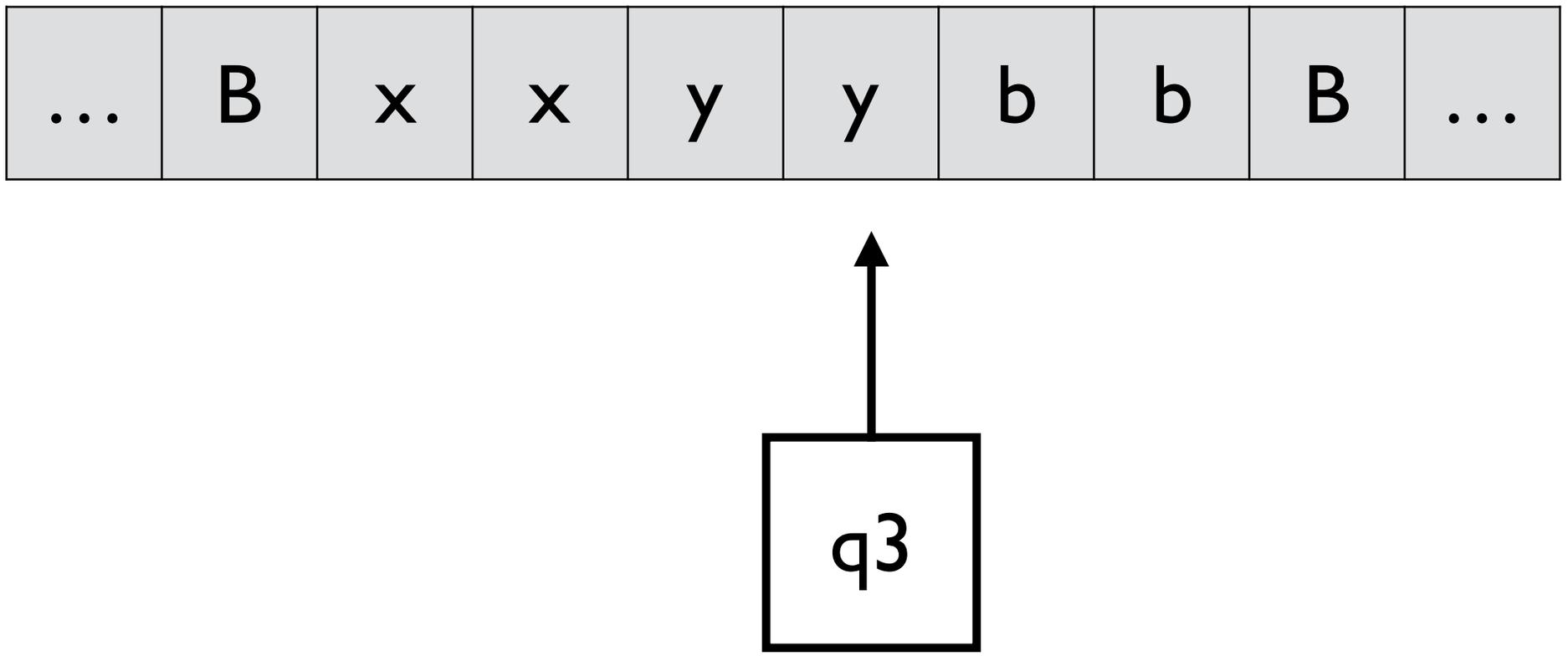
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



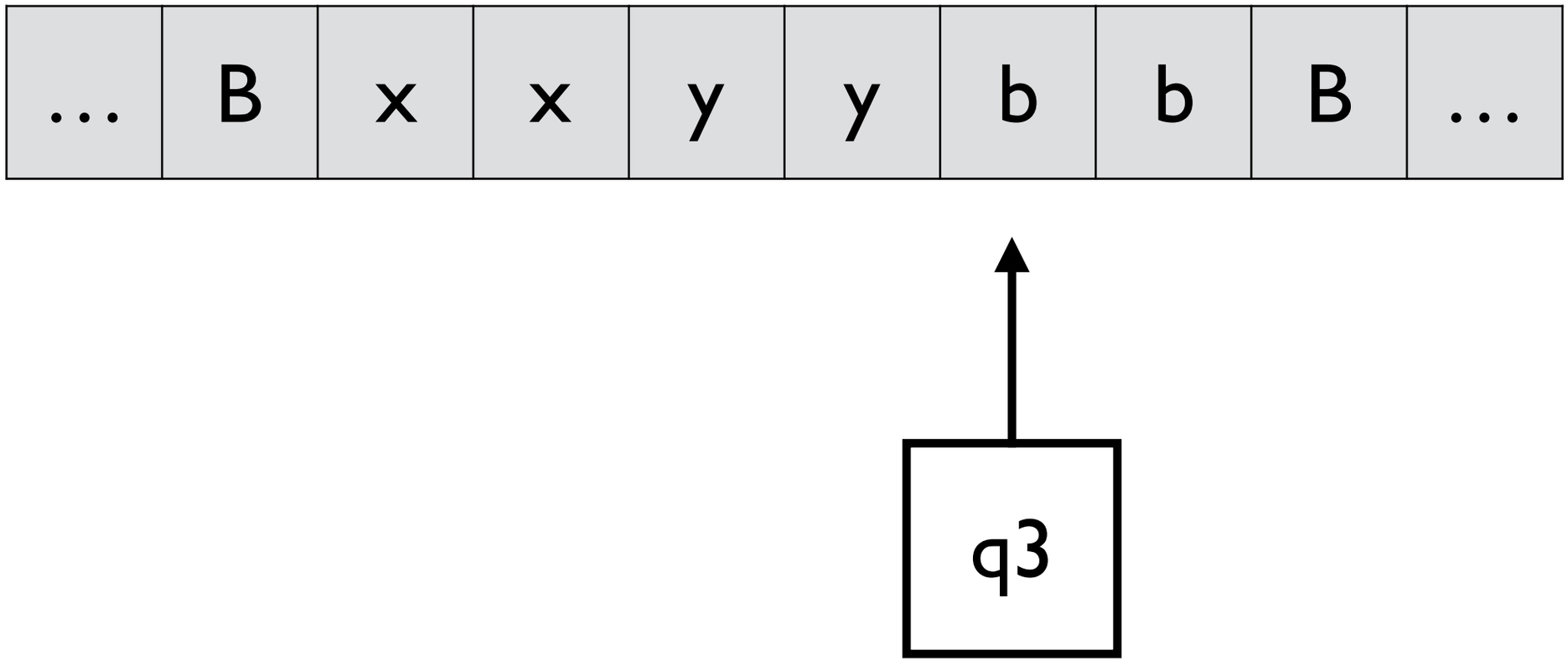
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



(undefined, halt)

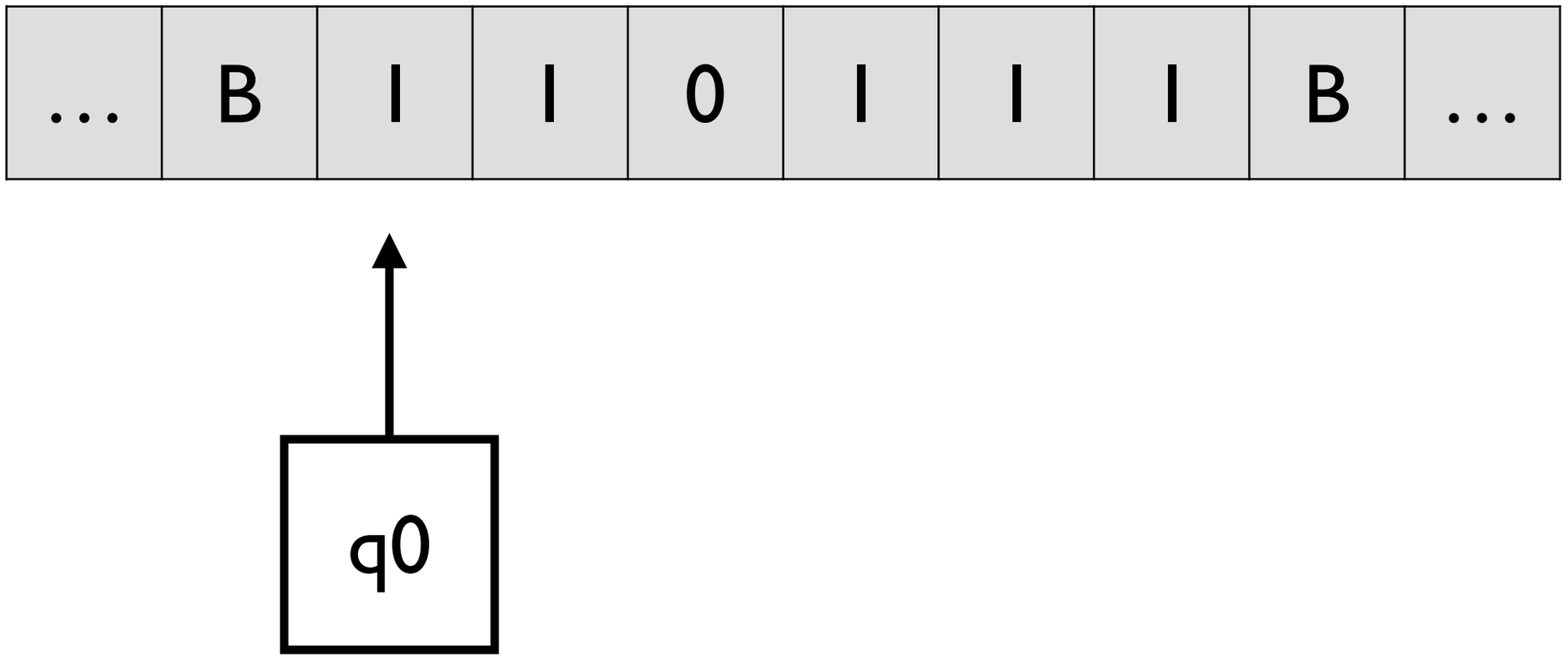
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

Example 2. Given x and y, design a Turing machine that computes $x + y$.

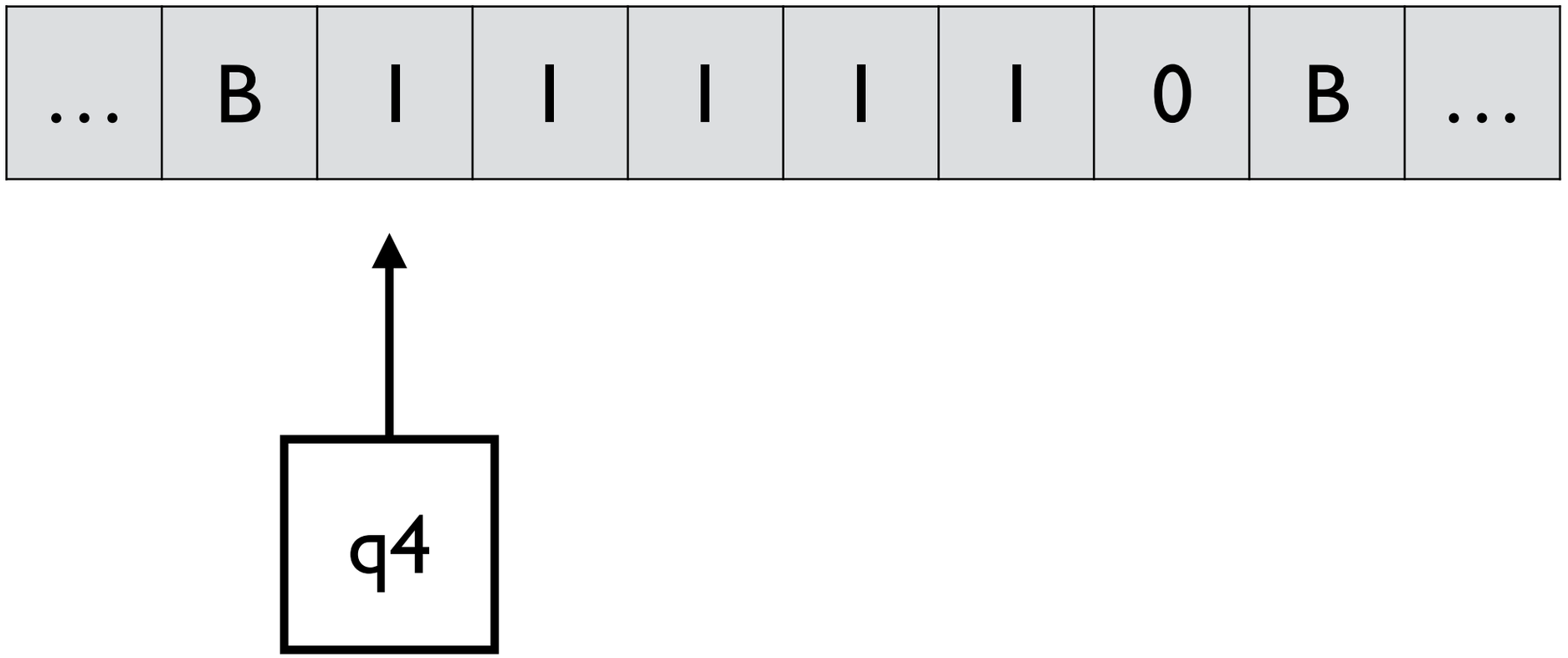
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_4\})$$

$$\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}$$

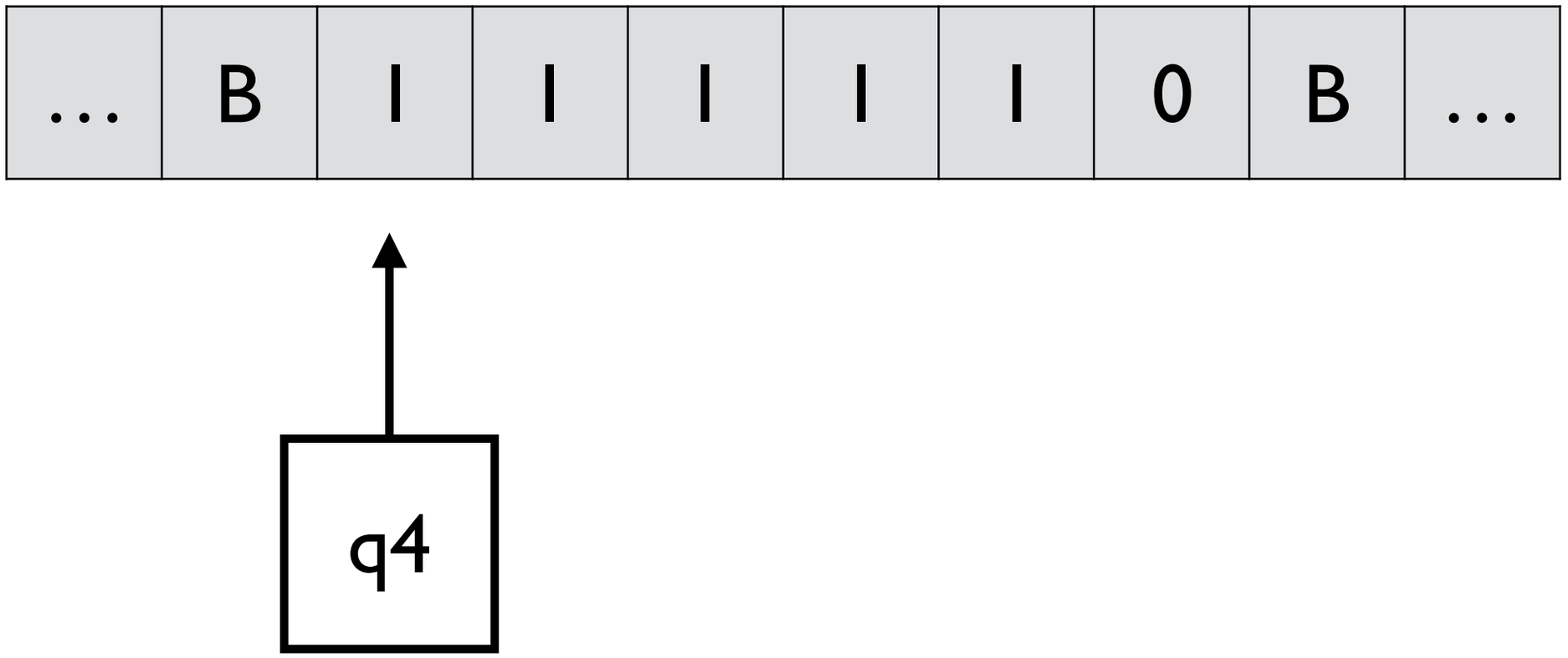
Initial machine configuration:



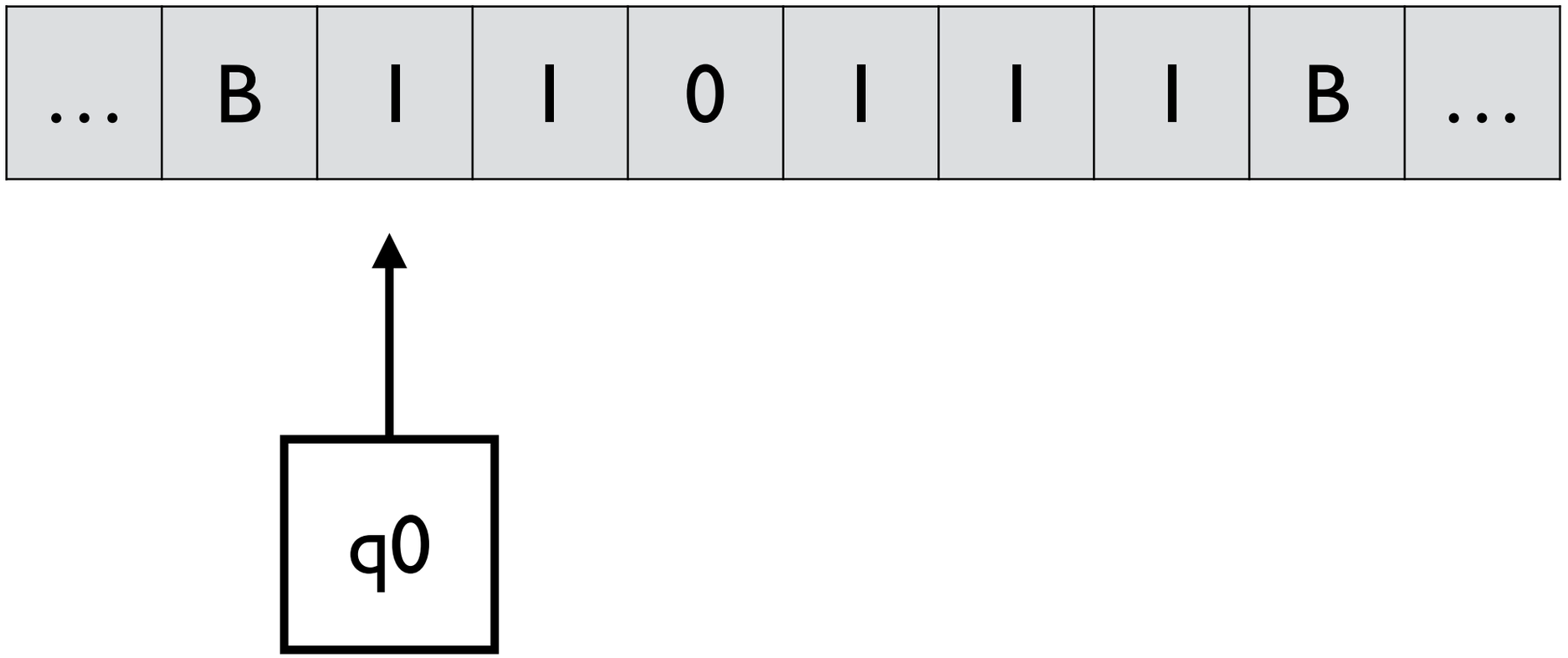
Final machine configuration:



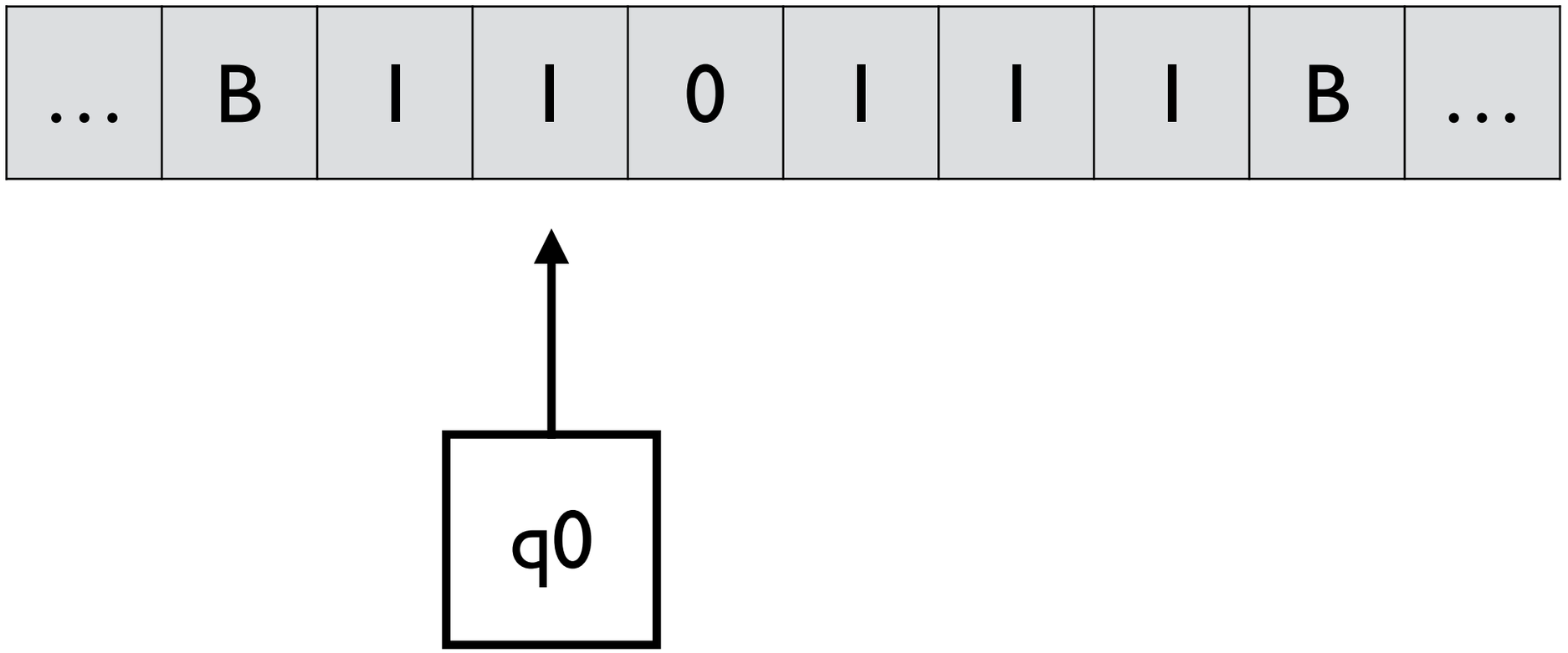
Final machine configuration:



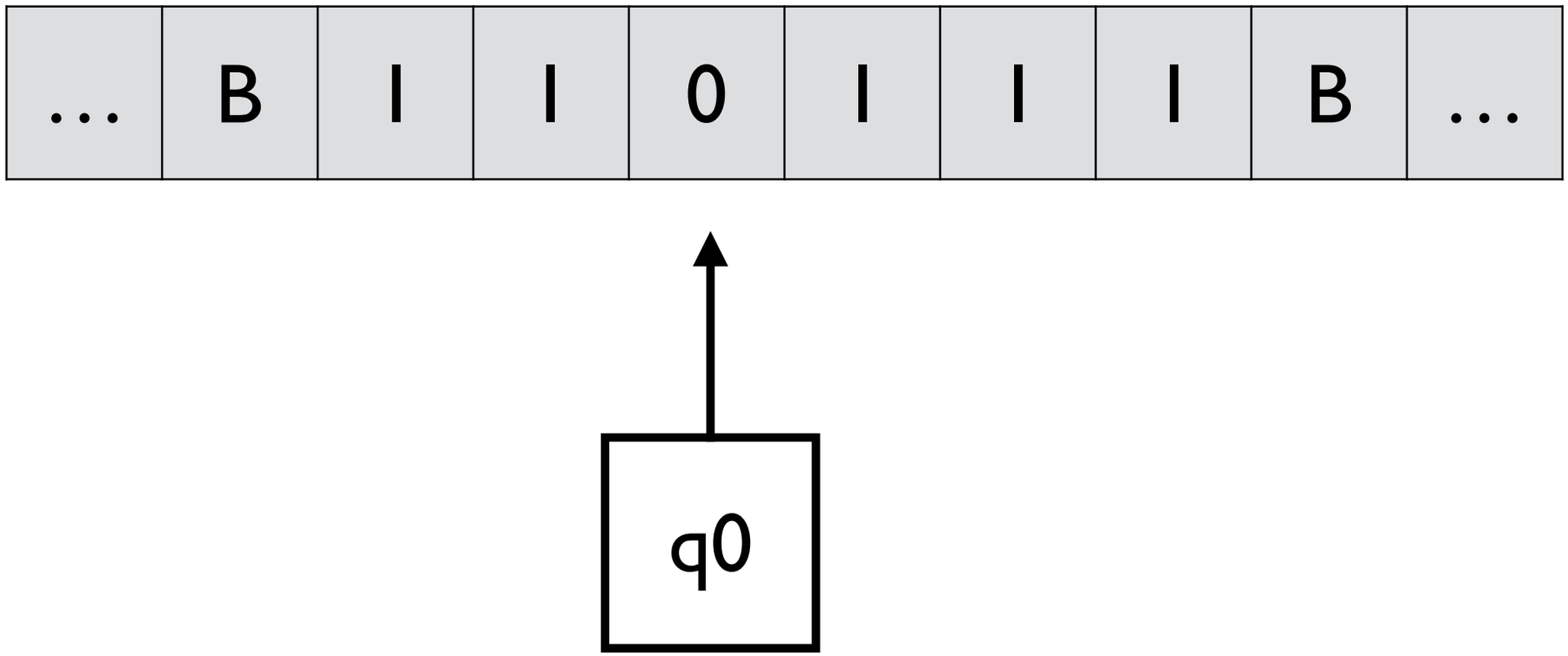
Addition in math is to move 0 to the right end



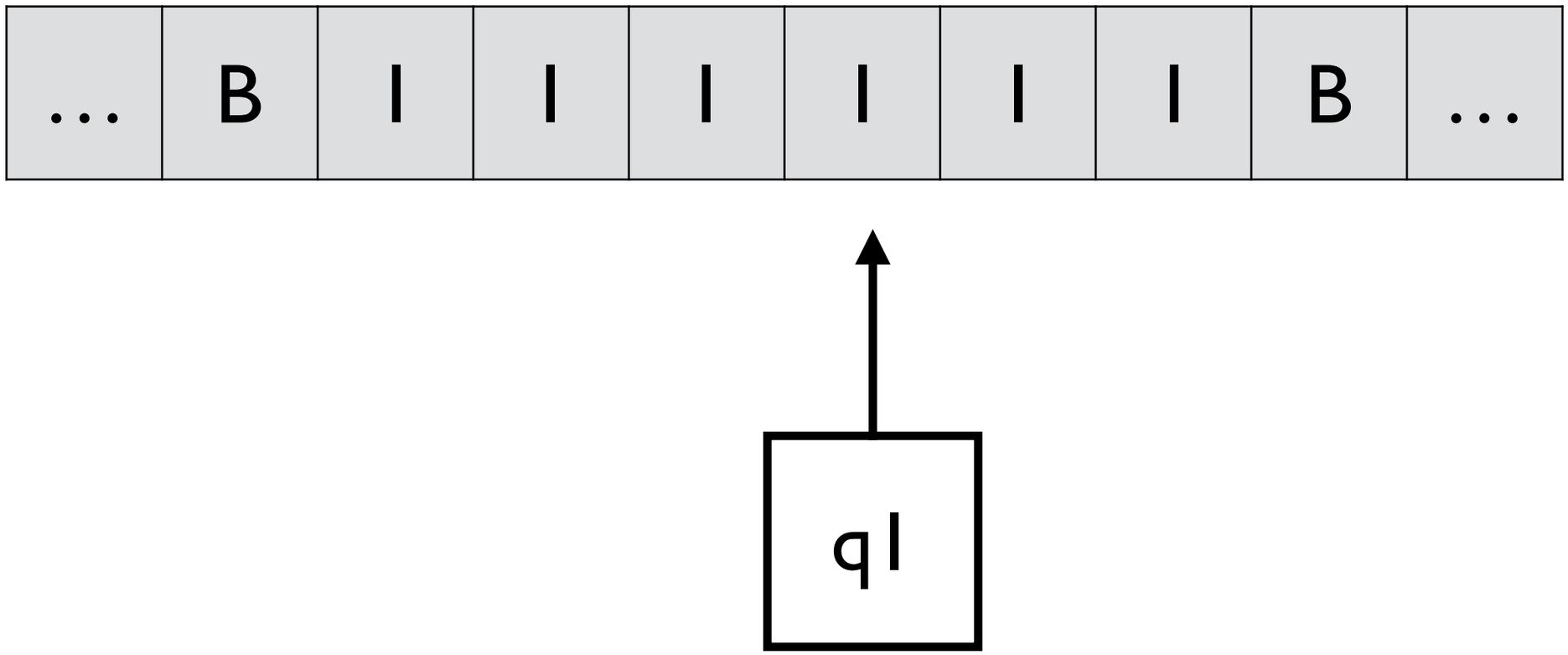
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$

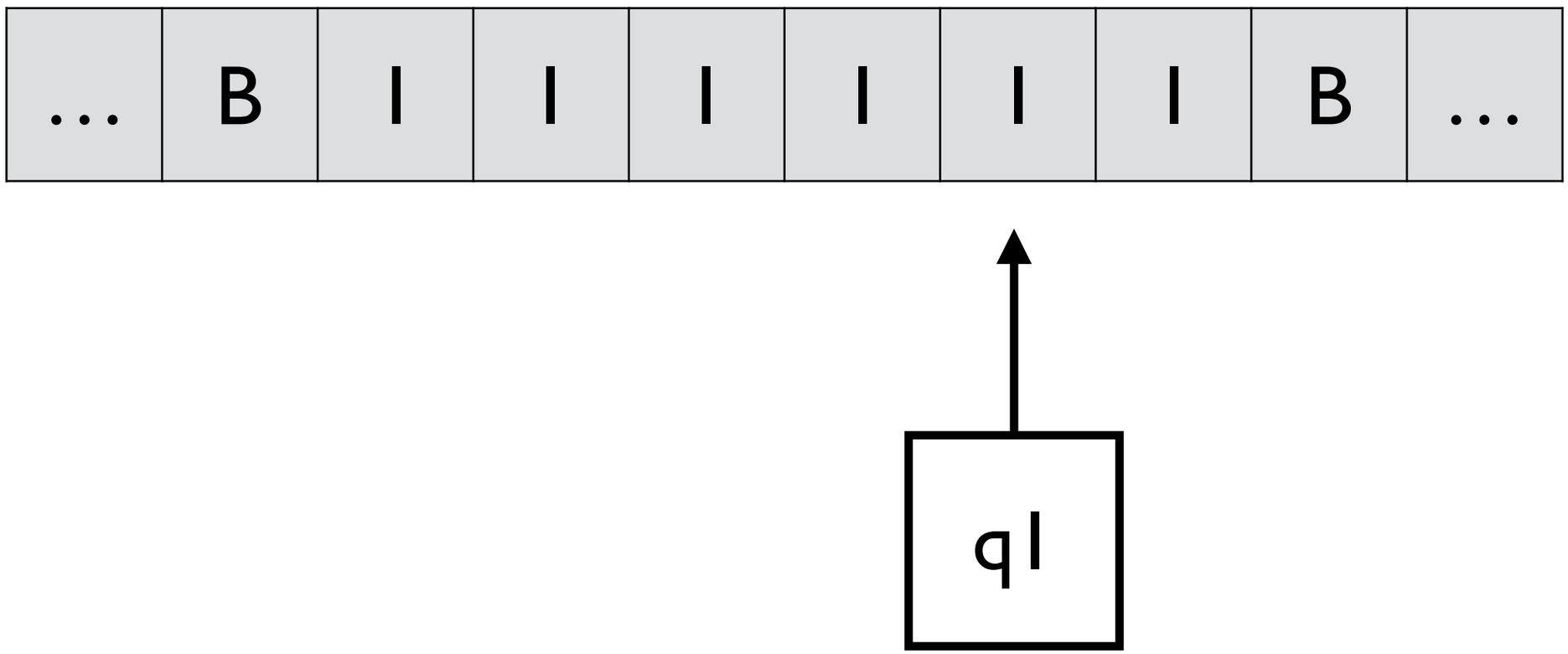


$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$

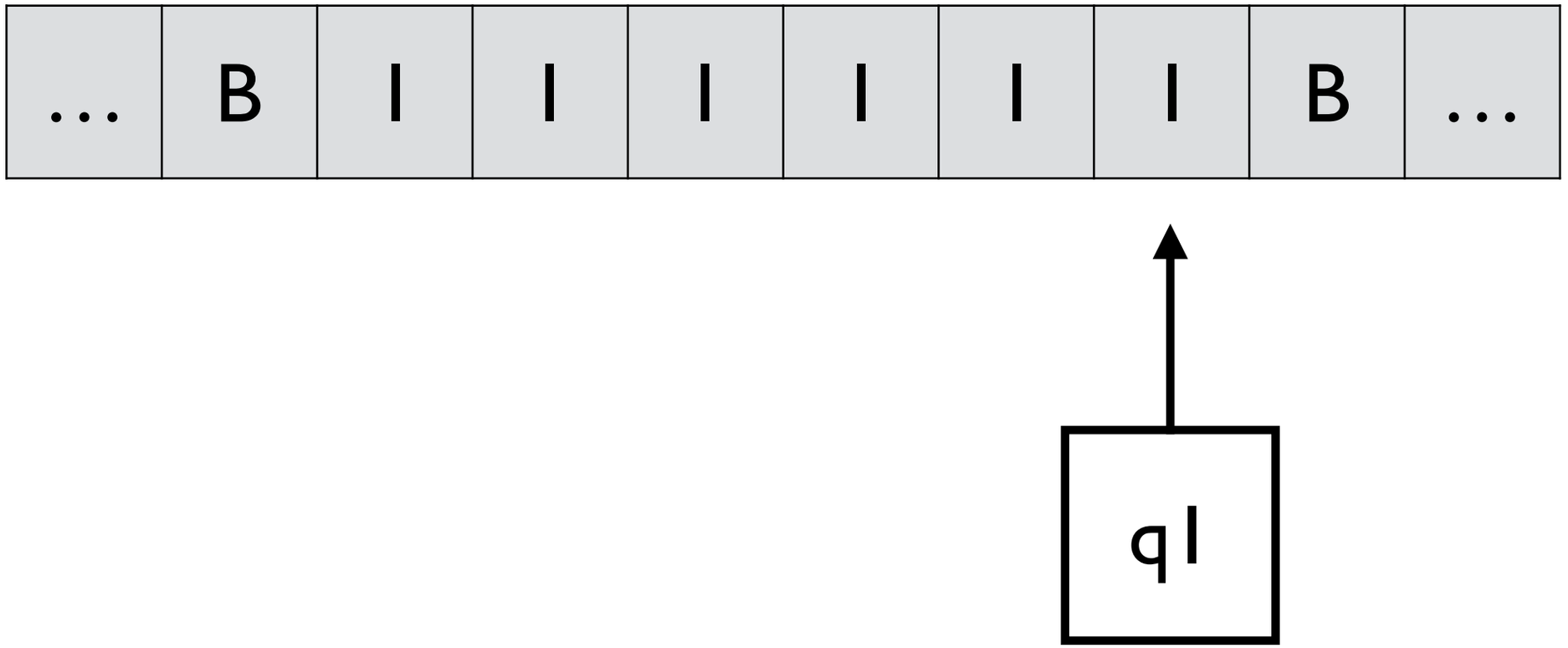


(In q1, search for the right end of y)

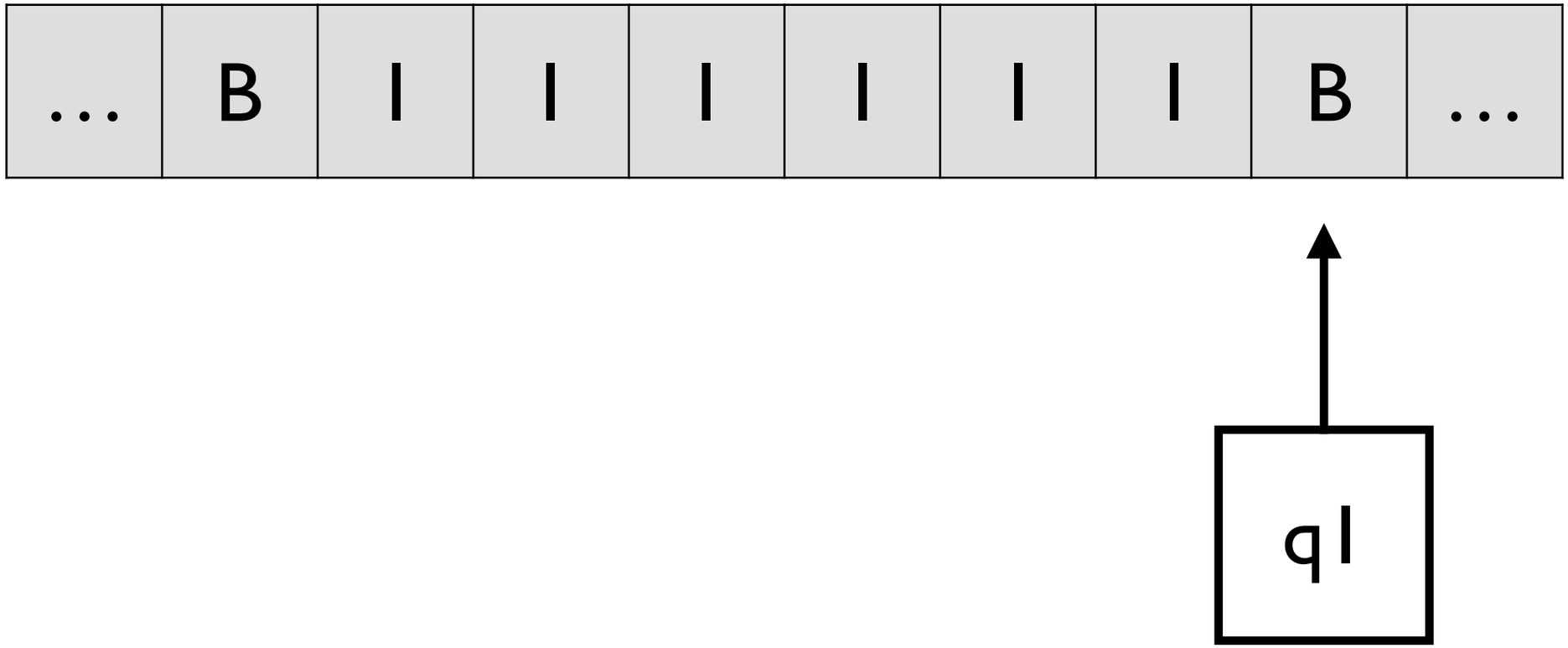
$$\delta(q_0, 1) = (q_0, 1, R)$$
$$\delta(q_0, 0) = (q_1, 1, R)$$
$$\delta(q_1, 1) = (q_1, 1, R)$$
$$\delta(q_1, B) = (q_2, B, L)$$
$$\delta(q_2, 1) = (q_3, 0, L)$$
$$\delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, B) = (q_4, B, R)$$



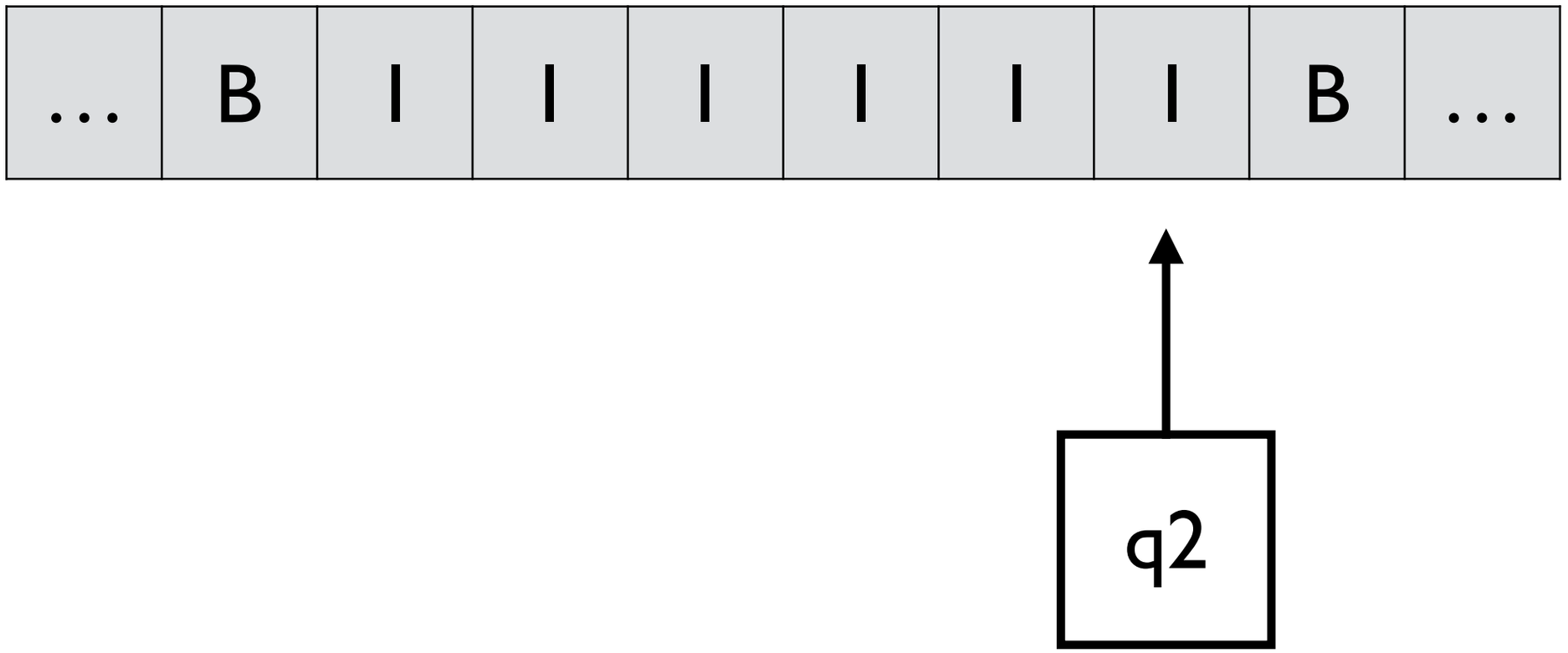
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$

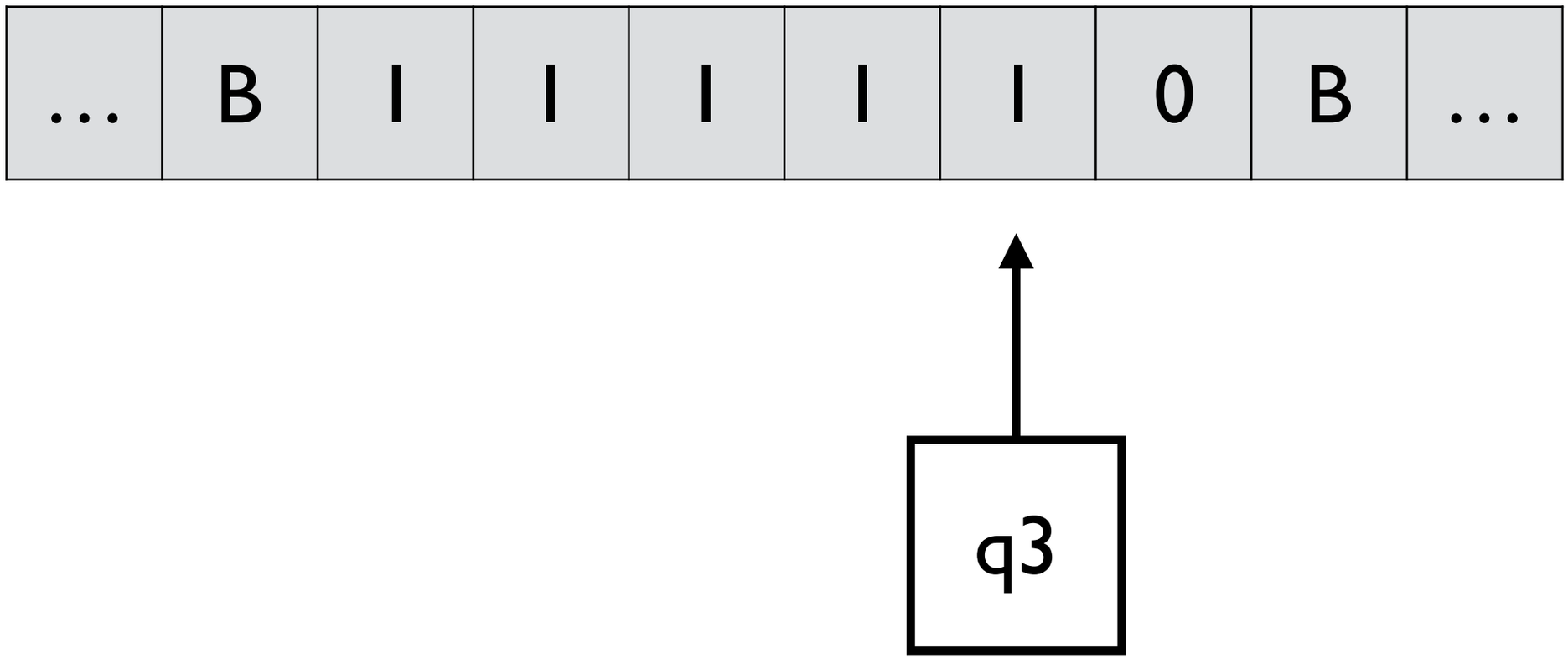


$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$



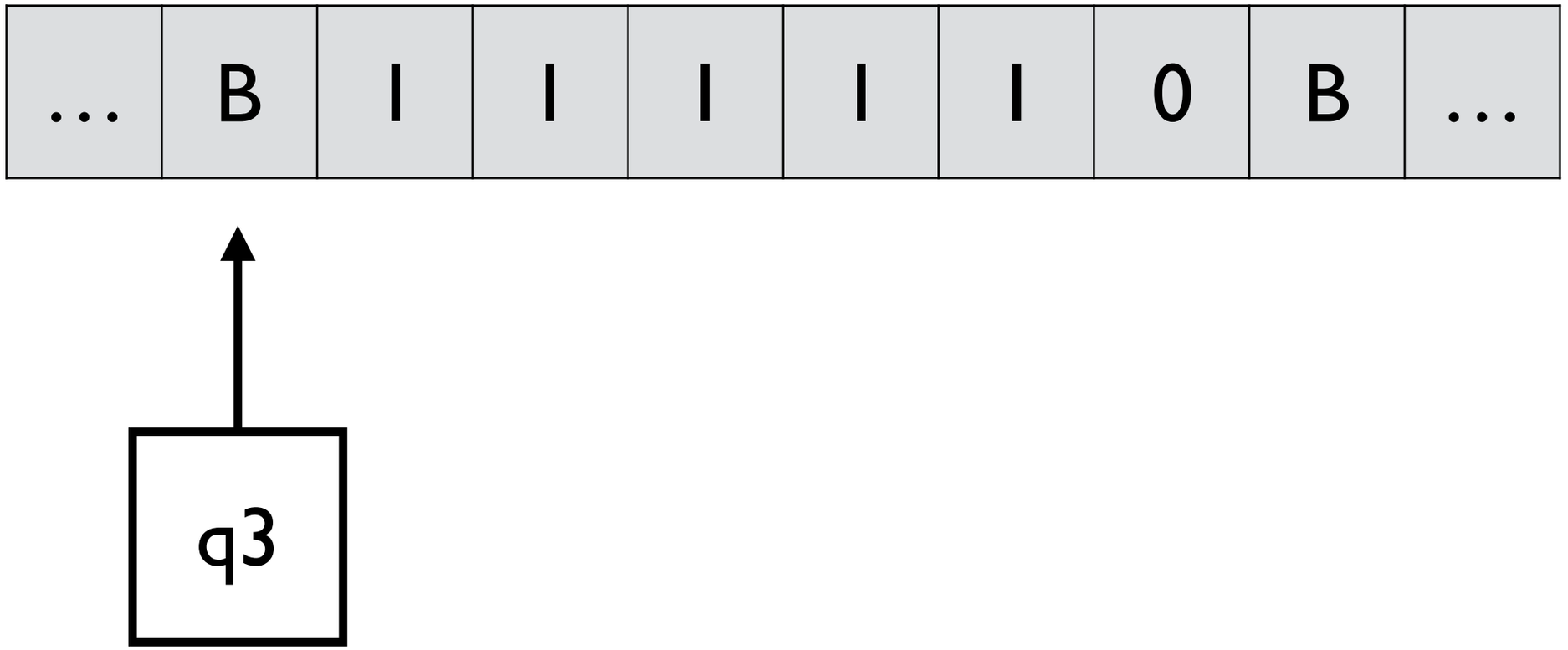
(In q2, replace the rightmost 1 by 0)

$$\delta(q_0, 1) = (q_0, 1, R)$$
$$\delta(q_0, 0) = (q_1, 1, R)$$
$$\delta(q_1, 1) = (q_1, 1, R)$$
$$\delta(q_1, B) = (q_2, B, L)$$
$$\delta(q_2, 1) = (q_3, 0, L)$$
$$\delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, B) = (q_4, B, R)$$

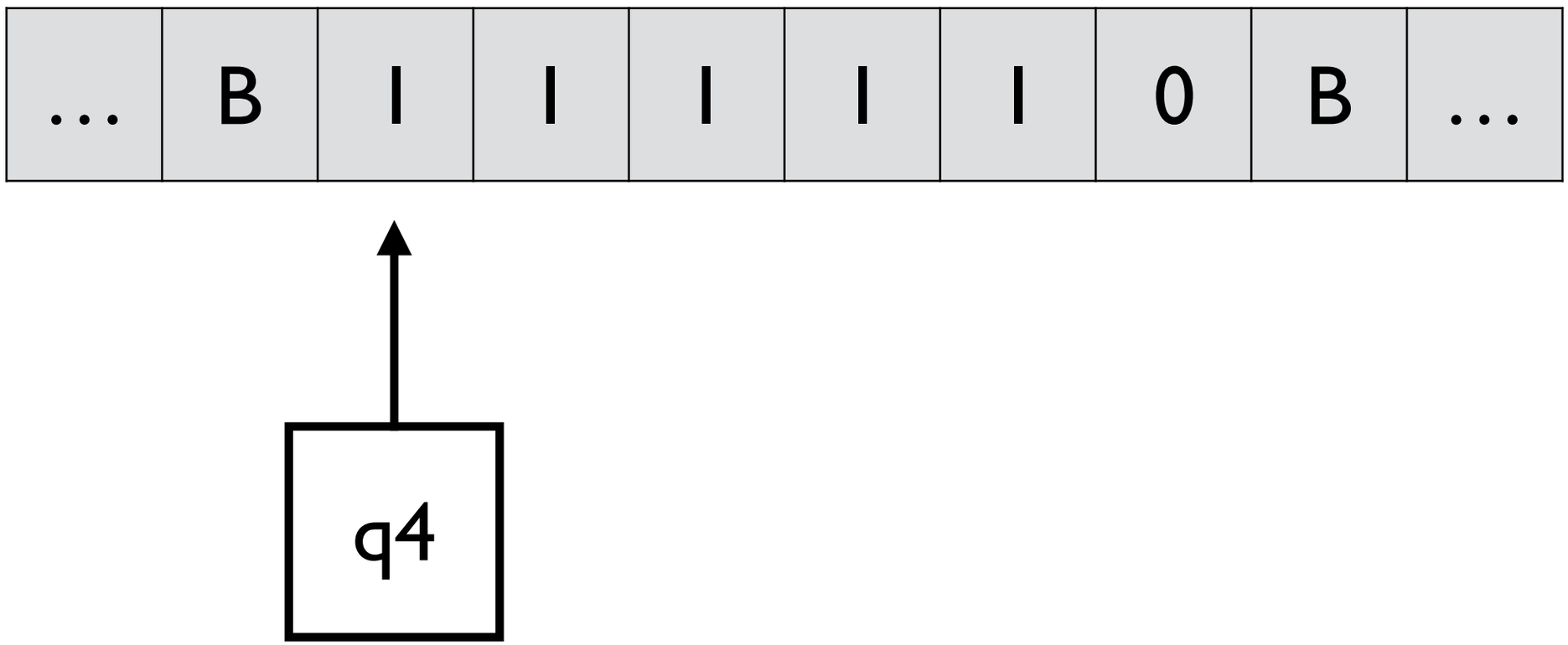


(In q3, look for the leftmost 1)

$$\delta(q_0, 1) = (q_0, 1, R)$$
$$\delta(q_0, 0) = (q_1, 1, R)$$
$$\delta(q_1, 1) = (q_1, 1, R)$$
$$\delta(q_1, B) = (q_2, B, L)$$
$$\delta(q_2, 1) = (q_3, 0, L)$$
$$\delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$



"final state"

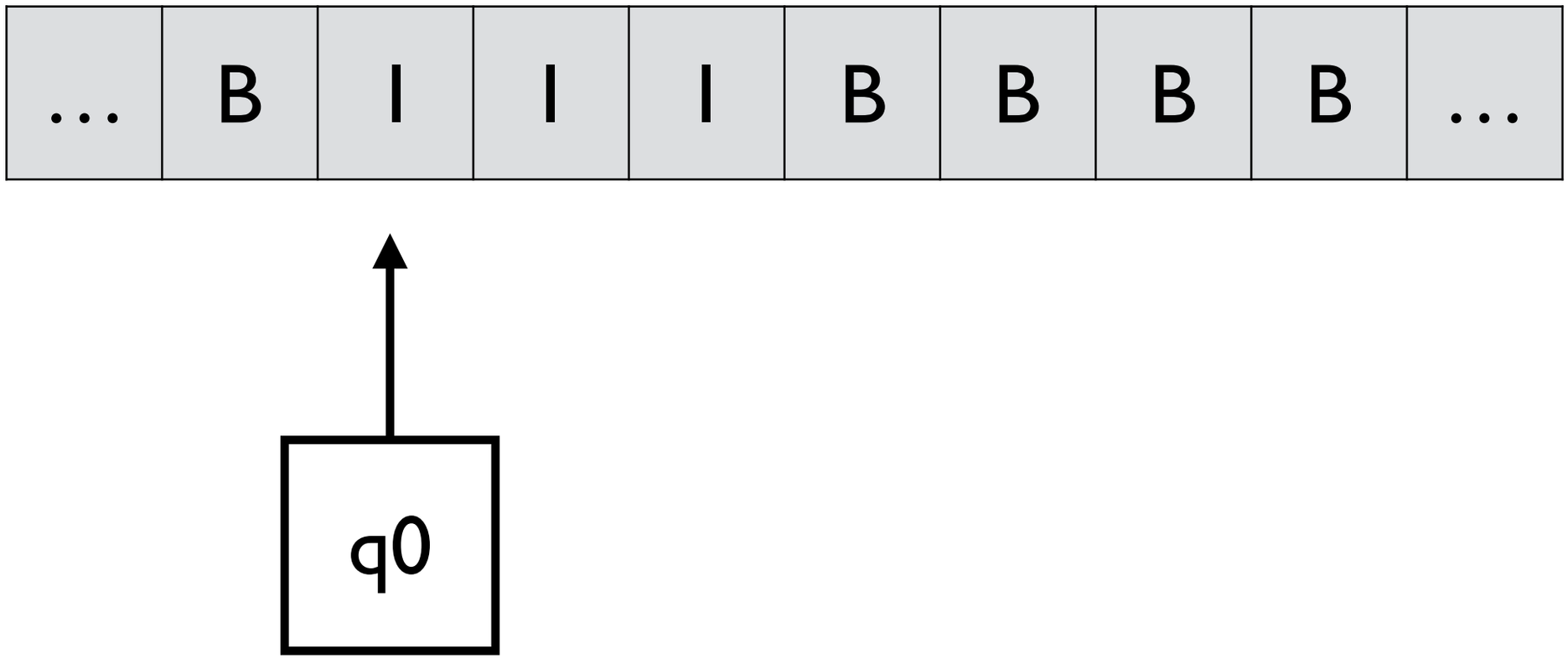
$$\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}$$

Example 3. Design a Turing machine that transforms w into ww.

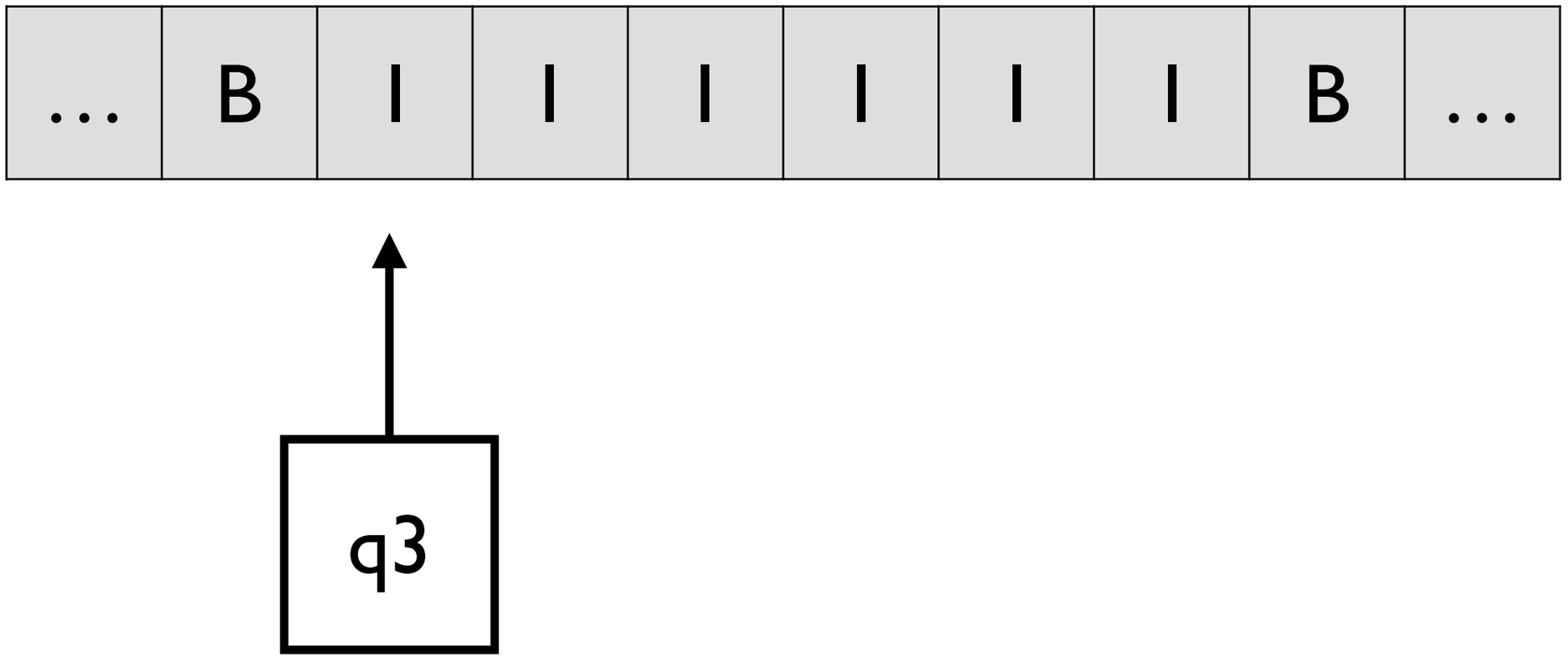
$$M = (\{q_0, q_1, q_2, q_3\}, \{1\}, \{1, x, B\}, \delta, q_0, B, \{q_3\})$$

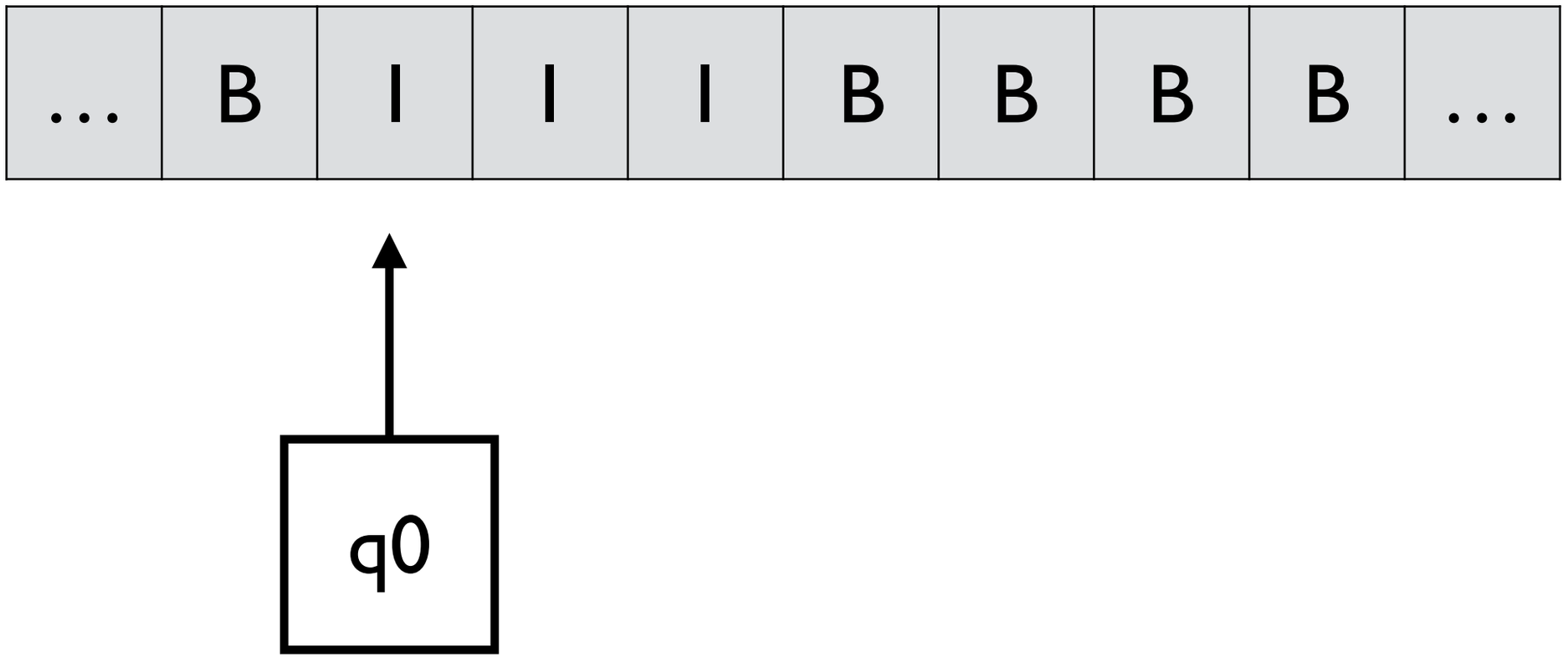
$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$

Initial machine configuration:

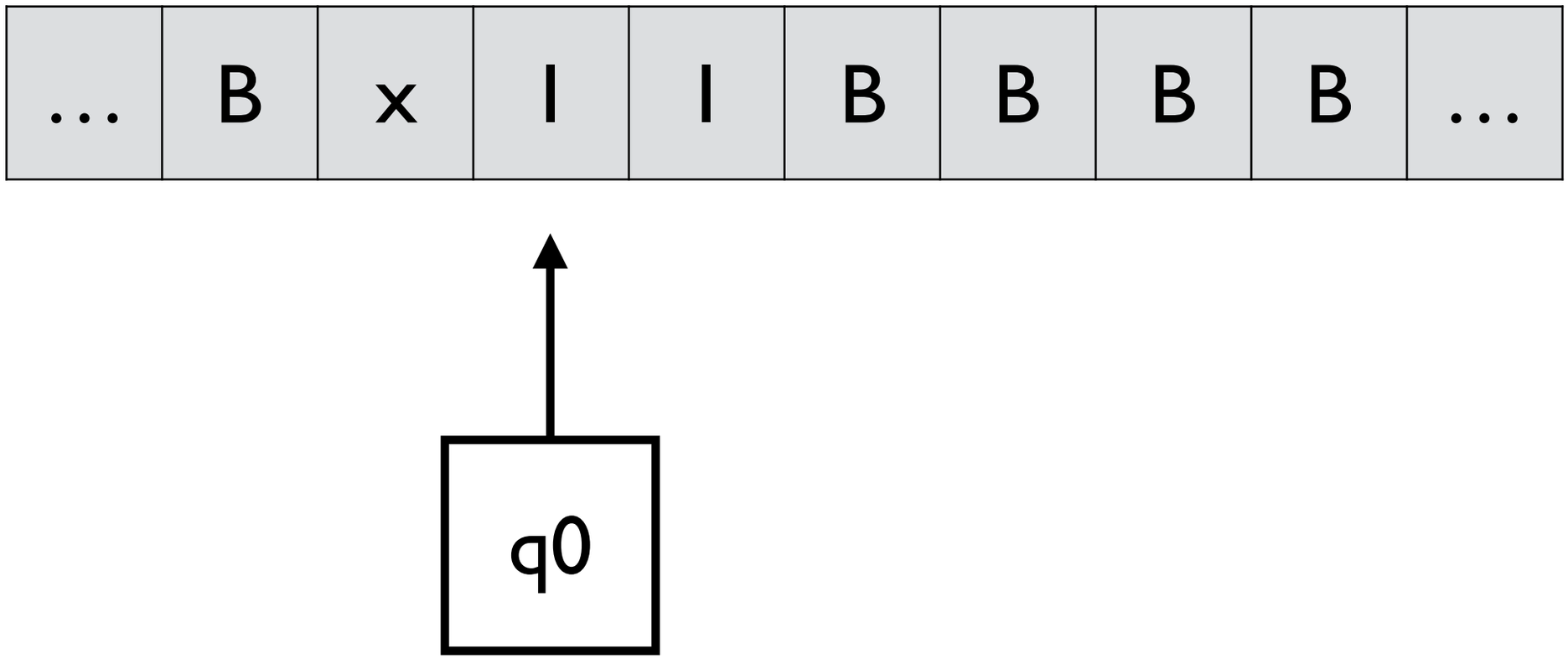


Final machine configuration:

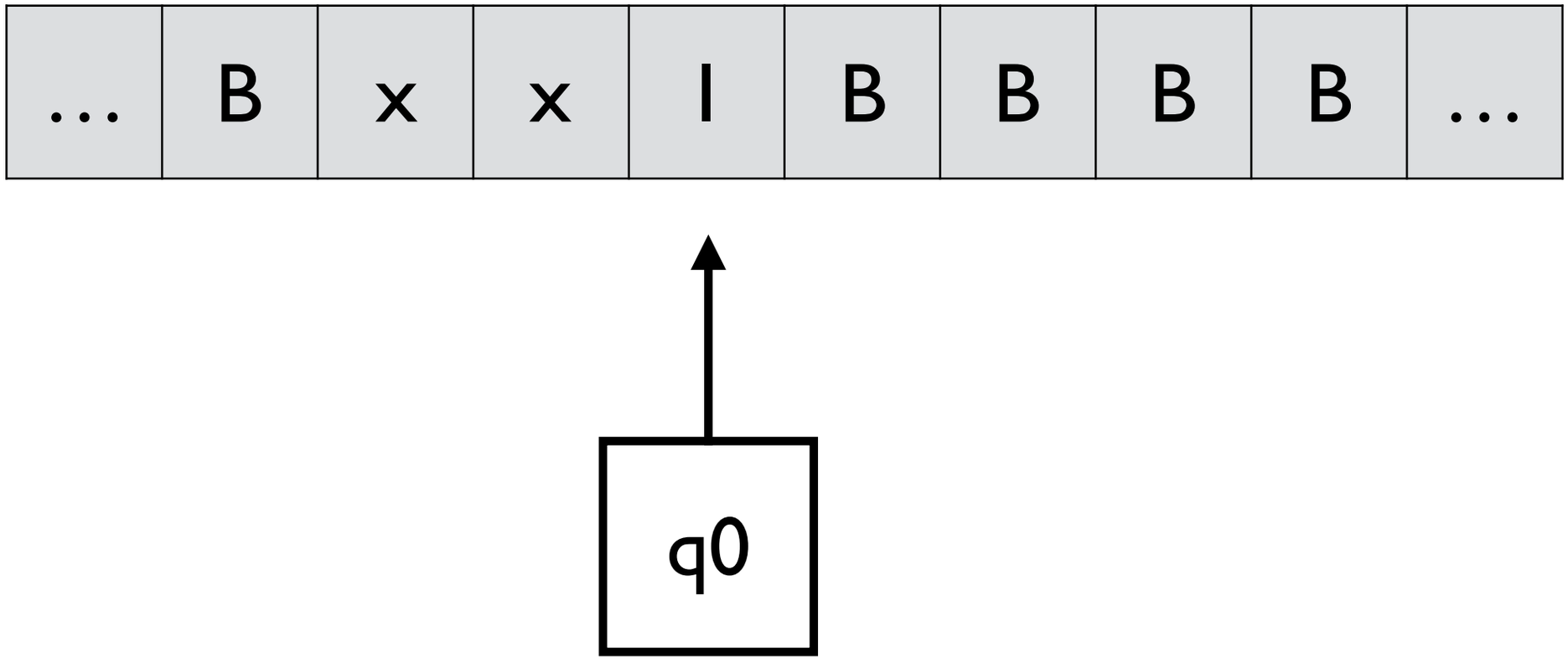




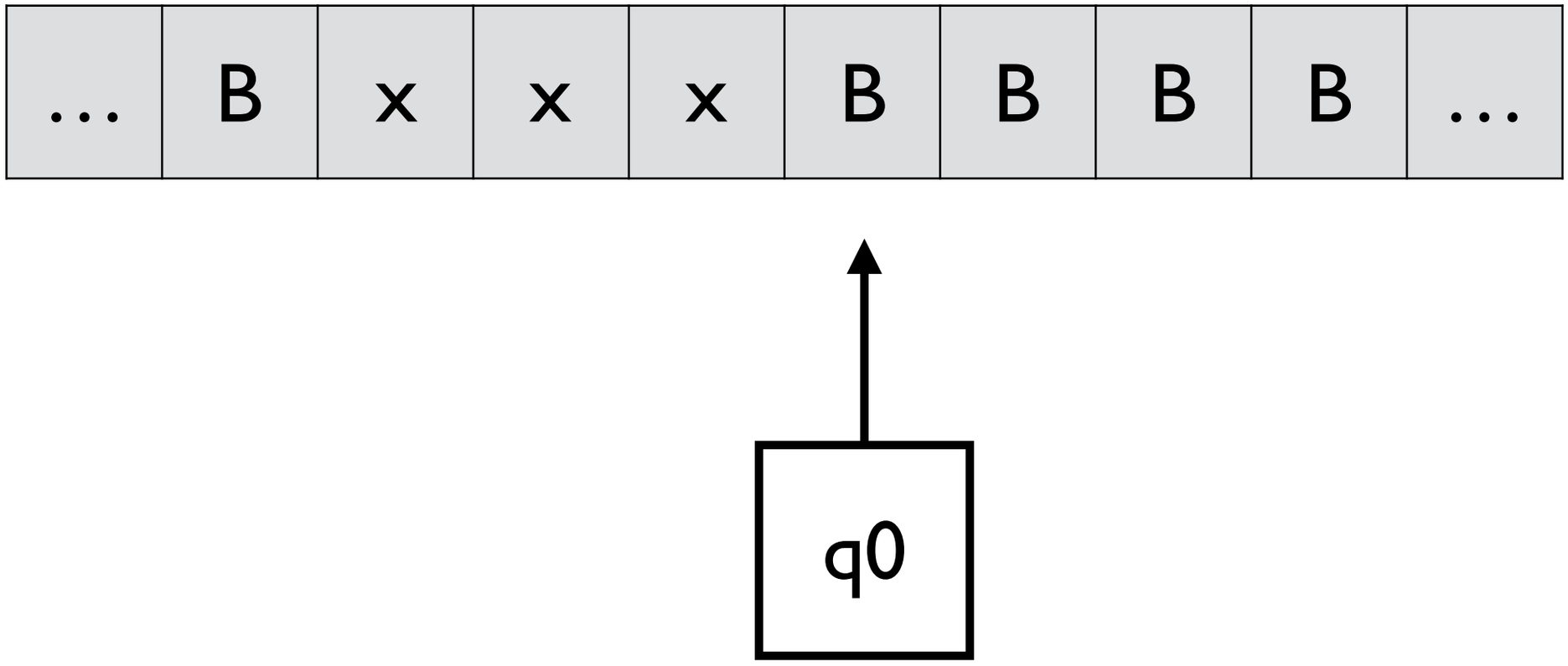
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$

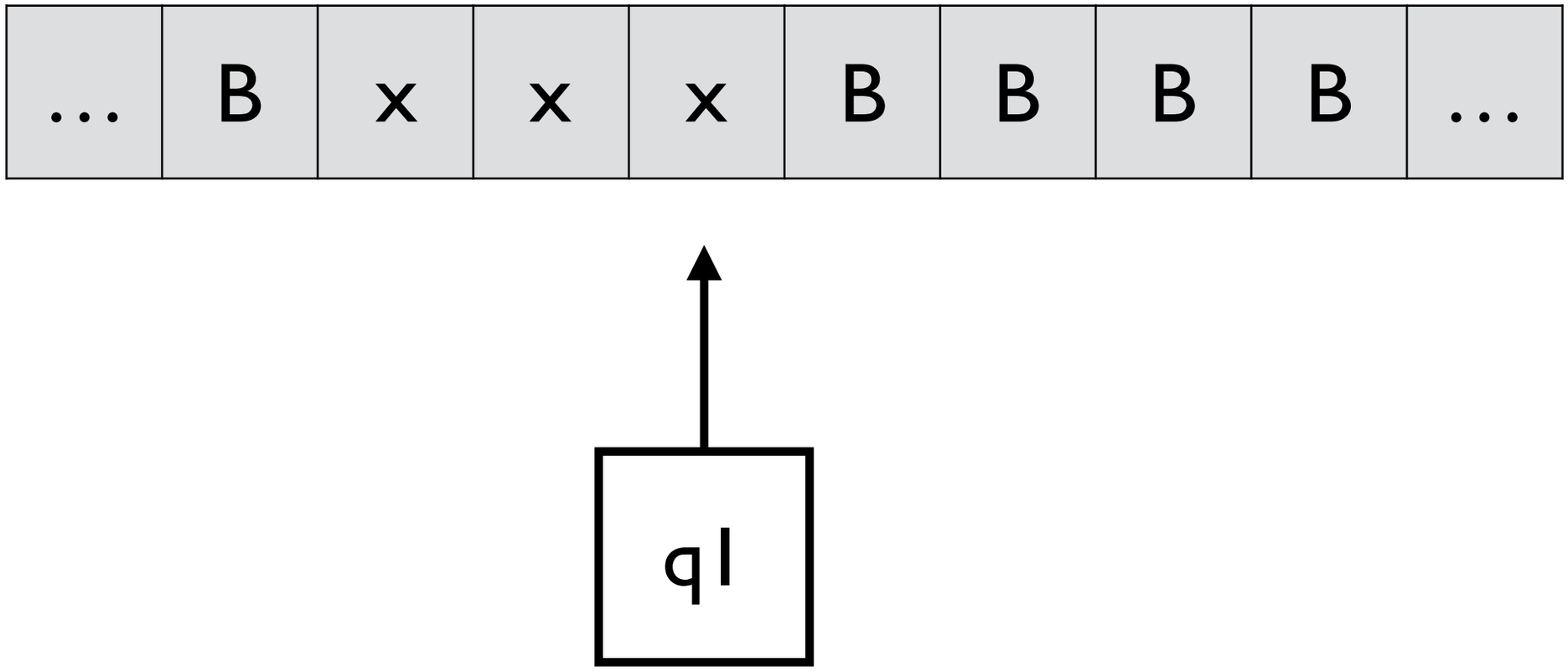


$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



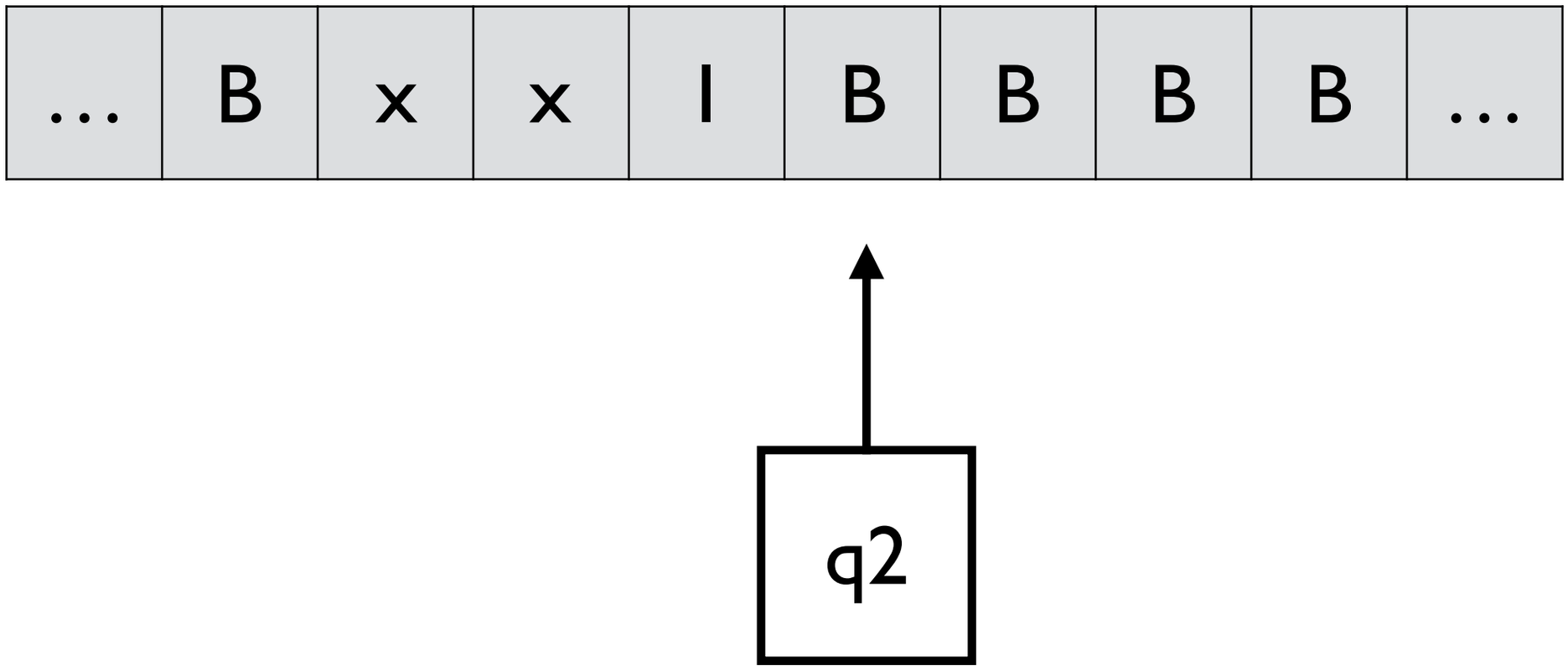
Initially, replace every 1 by x

$$\delta(q_0, 1) = (q_0, x, R)$$
$$\delta(q_0, B) = (q_1, B, L)$$
$$\delta(q_1, 1) = (q_1, 1, L)$$
$$\delta(q_1, x) = (q_2, 1, R)$$
$$\delta(q_2, 1) = (q_2, 1, R)$$
$$\delta(q_2, B) = (q_1, 1, L)$$
$$\delta(q_1, B) = (q_3, B, R)$$



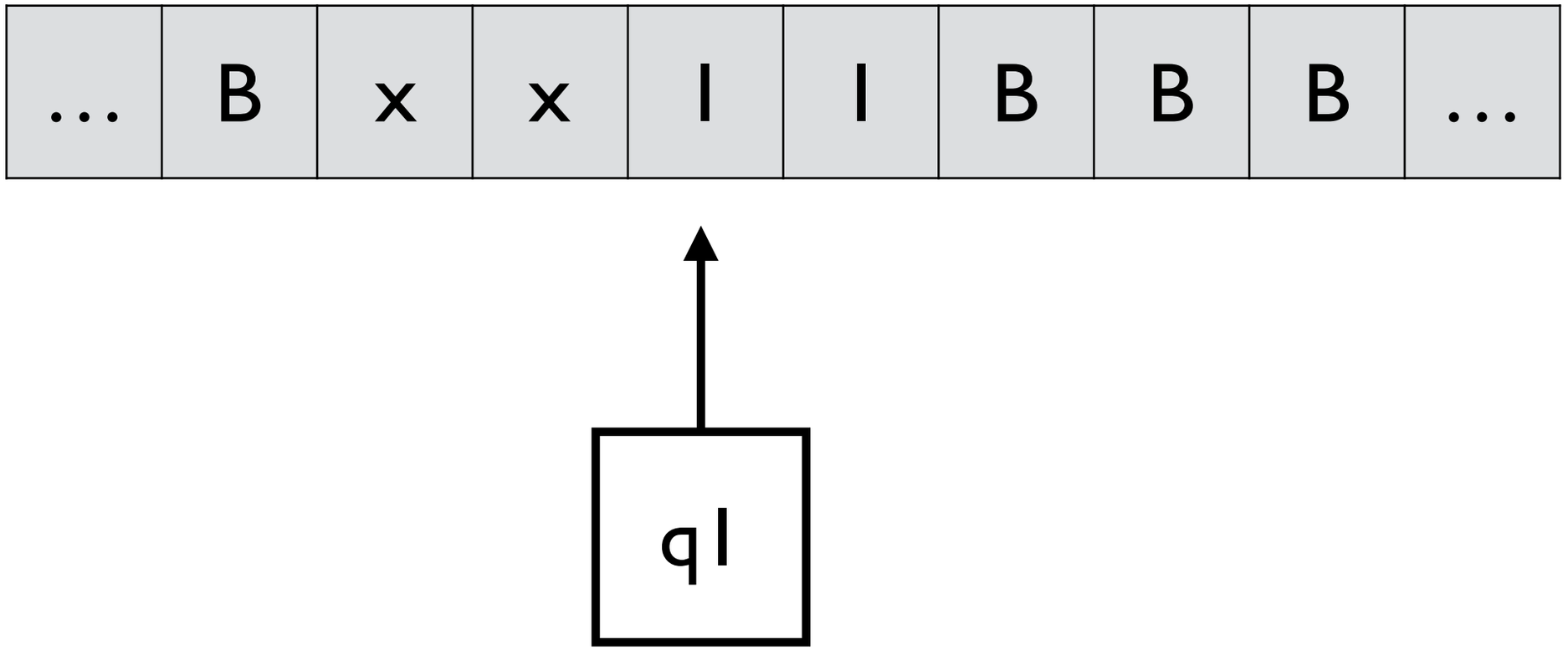
In q1, look for the rightmost x

$$\delta(q_0, 1) = (q_0, x, R)$$
$$\delta(q_0, B) = (q_1, B, L)$$
$$\delta(q_1, 1) = (q_1, 1, L)$$
$$\delta(q_1, x) = (q_2, 1, R)$$
$$\delta(q_2, 1) = (q_2, 1, R)$$
$$\delta(q_2, B) = (q_1, 1, L)$$
$$\delta(q_1, B) = (q_3, B, R)$$

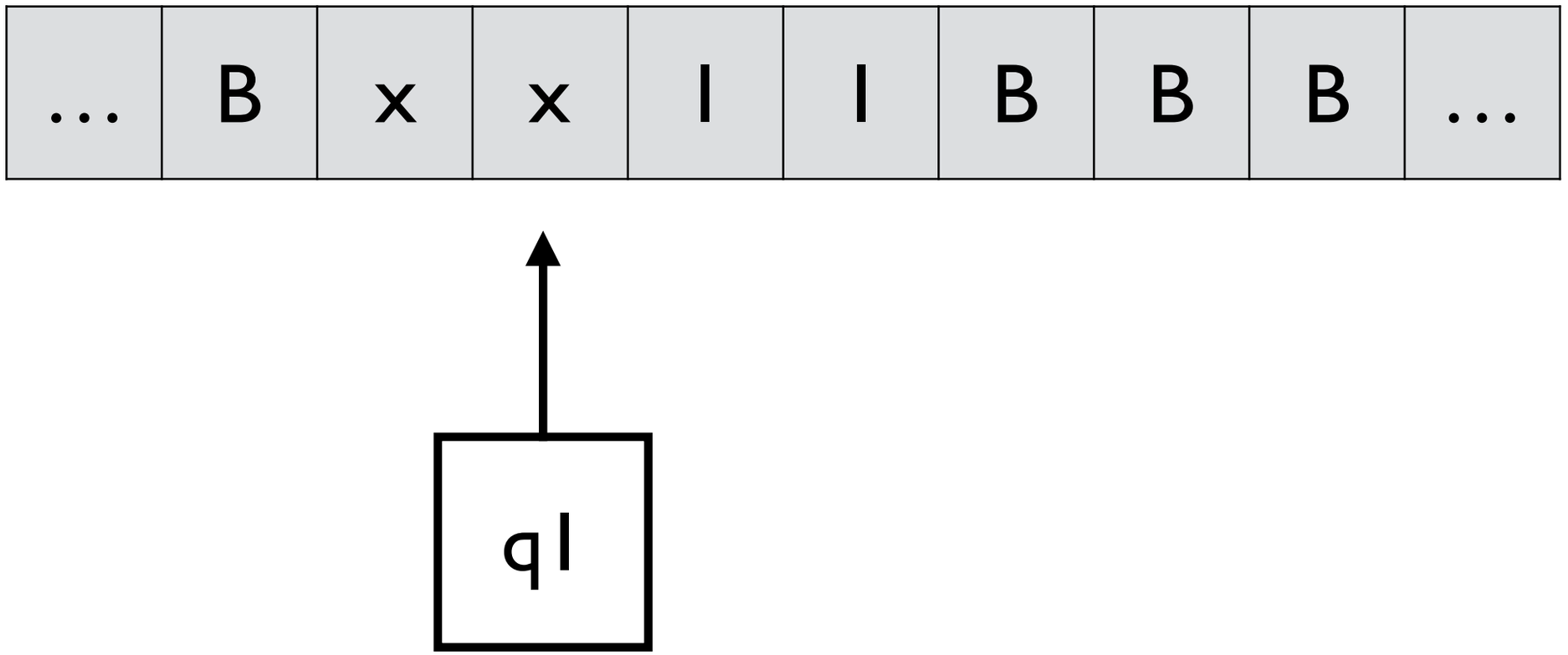


In q2, look for the first blank and write 1

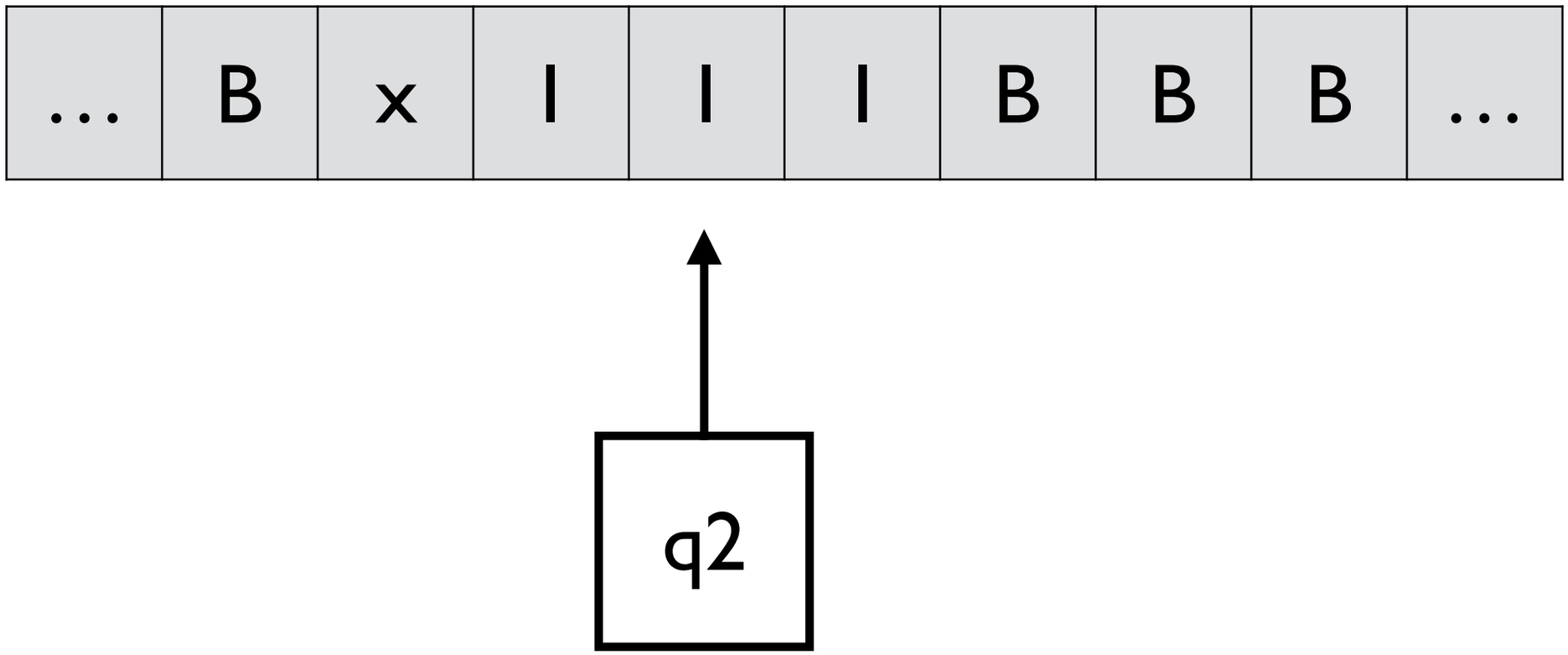
$$\delta(q_0, 1) = (q_0, x, R)$$
$$\delta(q_0, B) = (q_1, B, L)$$
$$\delta(q_1, 1) = (q_1, 1, L)$$
$$\delta(q_1, x) = (q_2, 1, R)$$
$$\delta(q_2, 1) = (q_2, 1, R)$$
$$\delta(q_2, B) = (q_1, 1, L)$$
$$\delta(q_1, B) = (q_3, B, R)$$



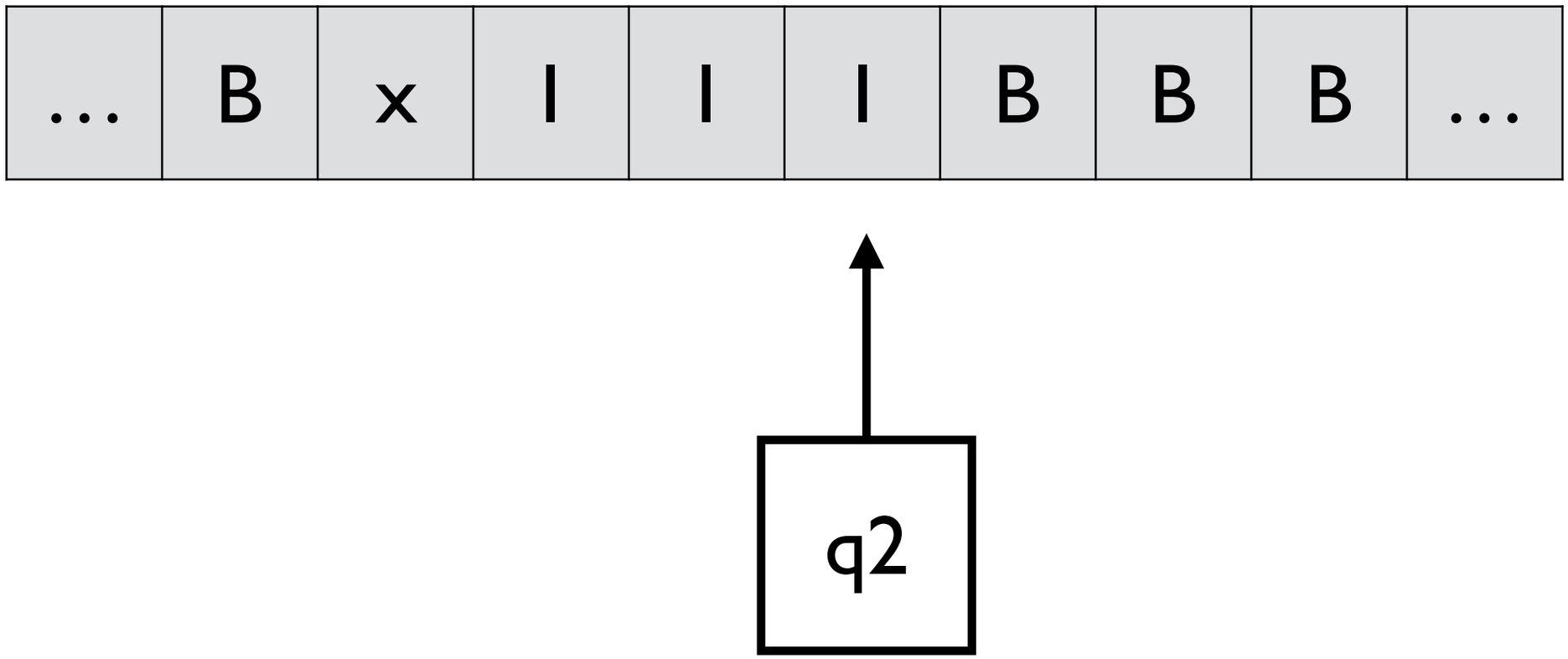
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



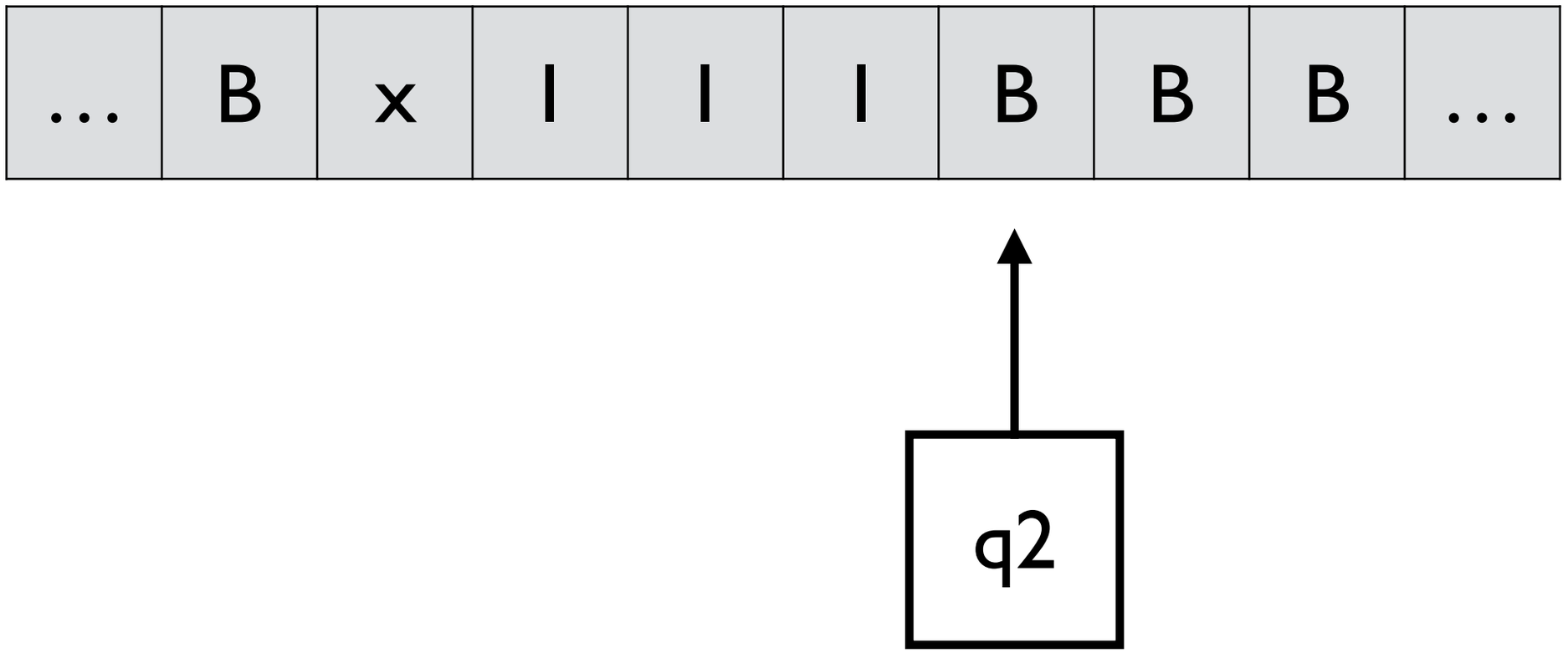
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



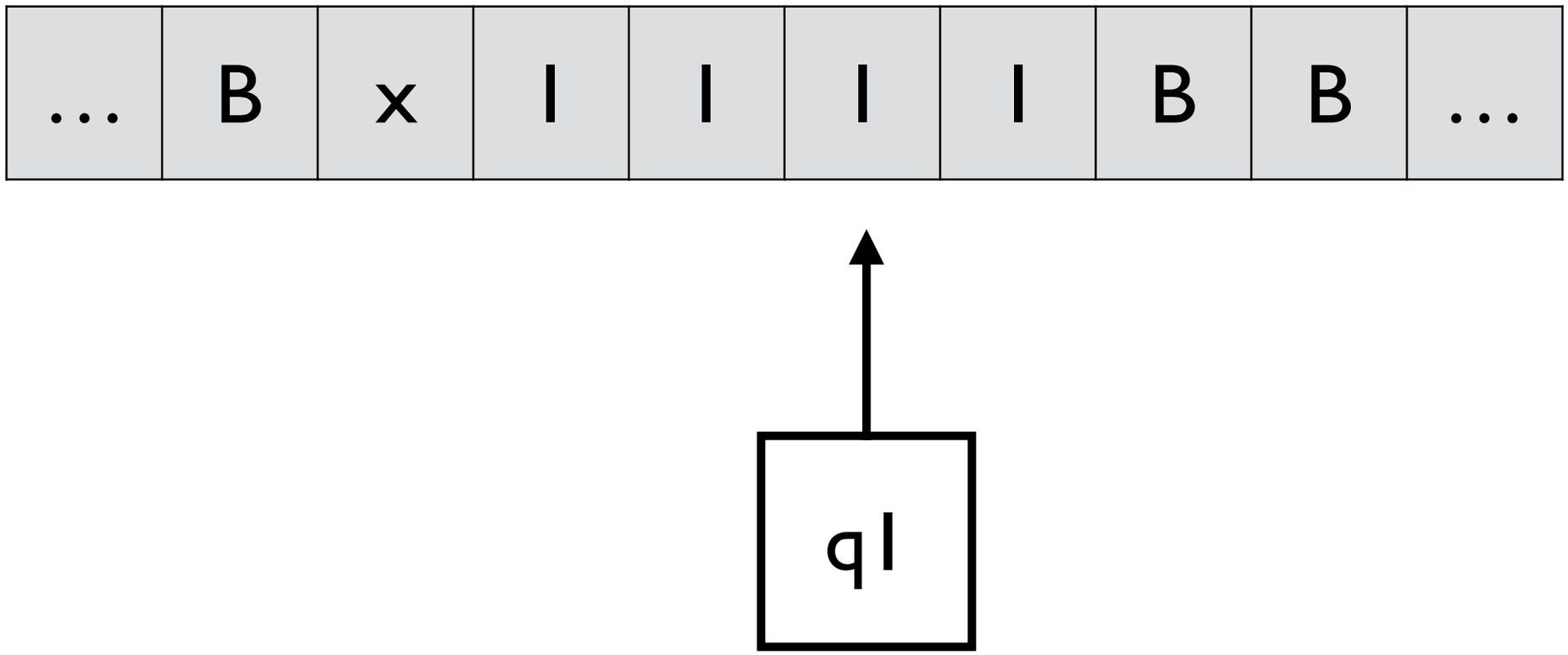
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



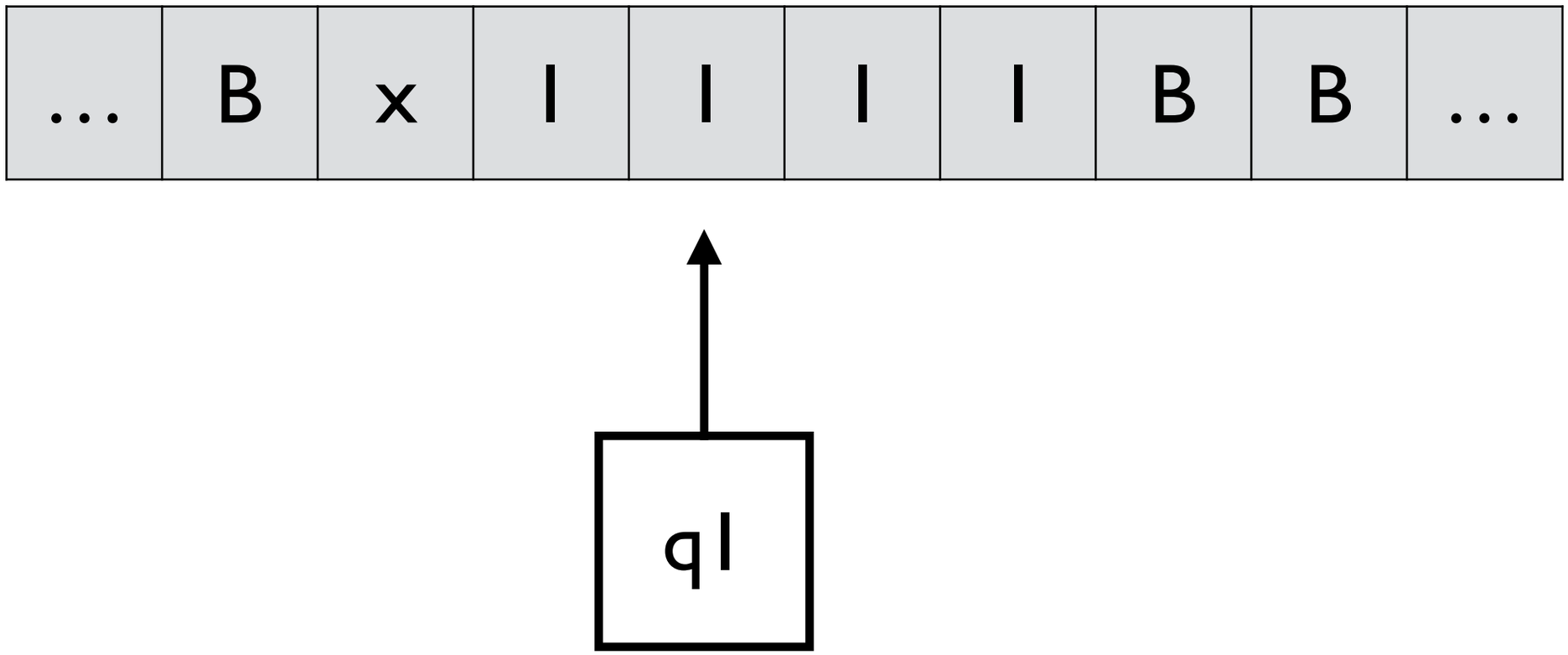
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



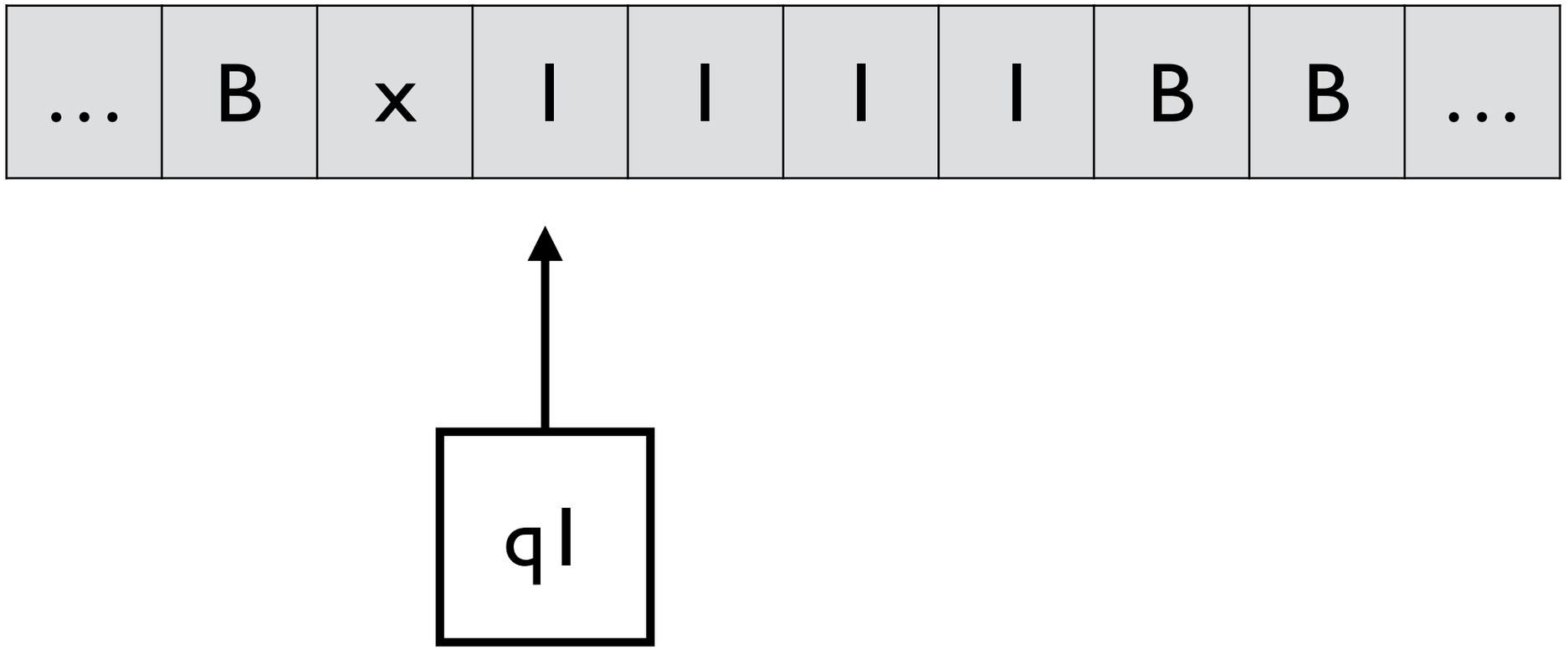
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



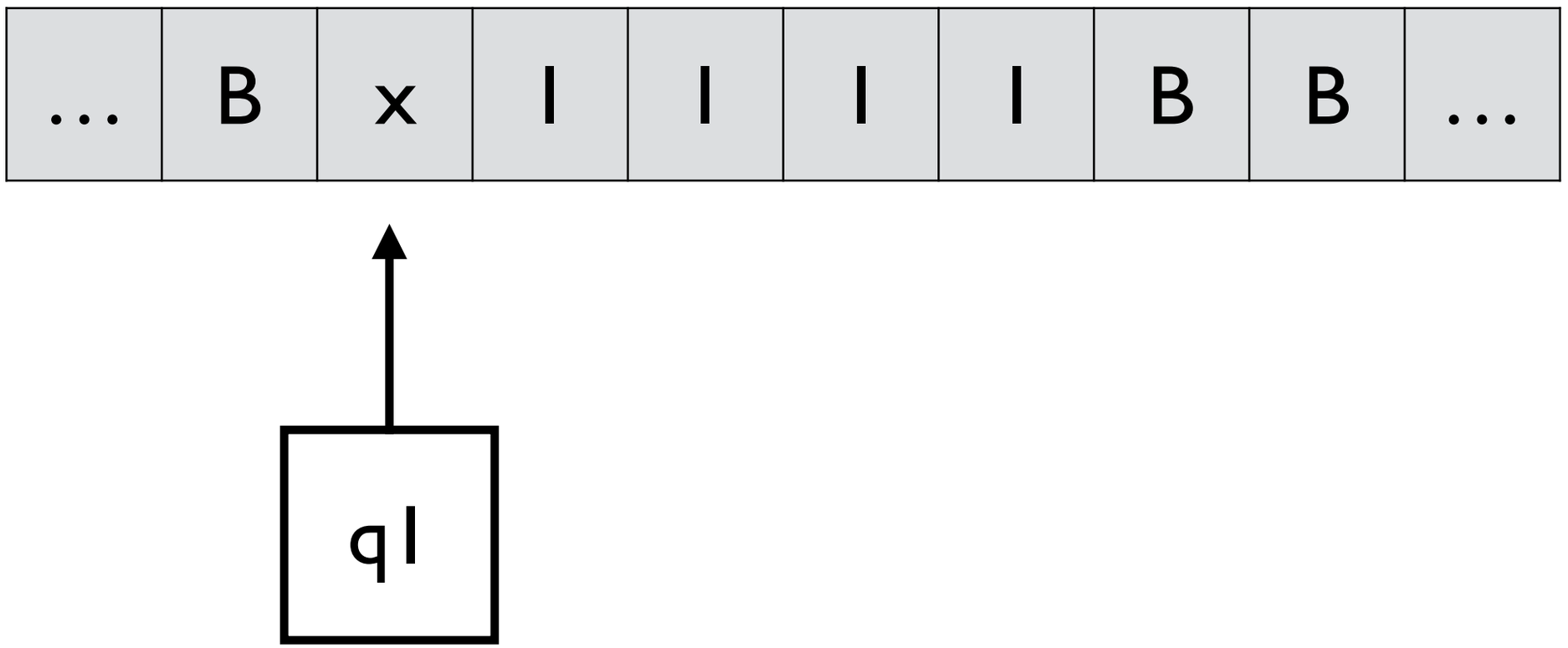
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



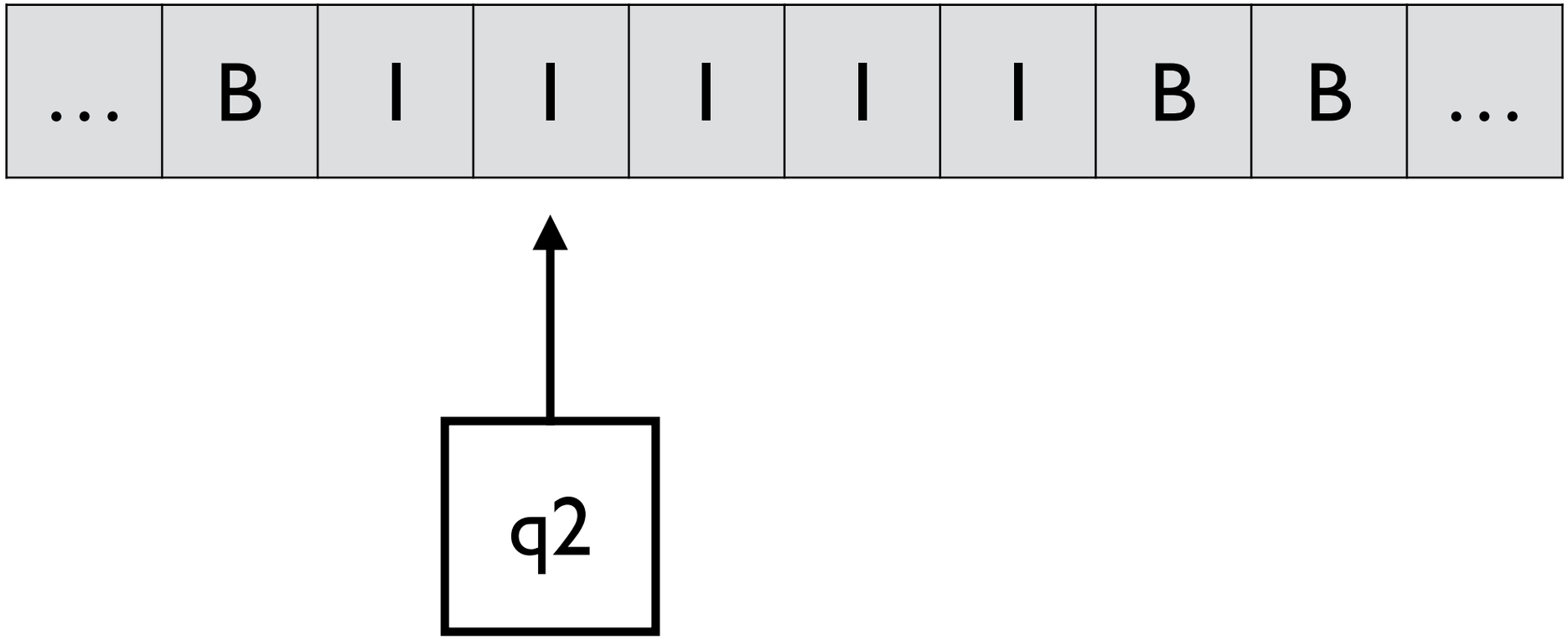
$$\delta(q_0, 1) = (q_0, x, R)$$
$$\delta(q_0, B) = (q_1, B, L)$$
$$\delta(q_1, 1) = (q_1, 1, L)$$
$$\delta(q_1, x) = (q_2, 1, R)$$
$$\delta(q_2, 1) = (q_2, 1, R)$$
$$\delta(q_2, B) = (q_1, 1, L)$$
$$\delta(q_1, B) = (q_3, B, R)$$



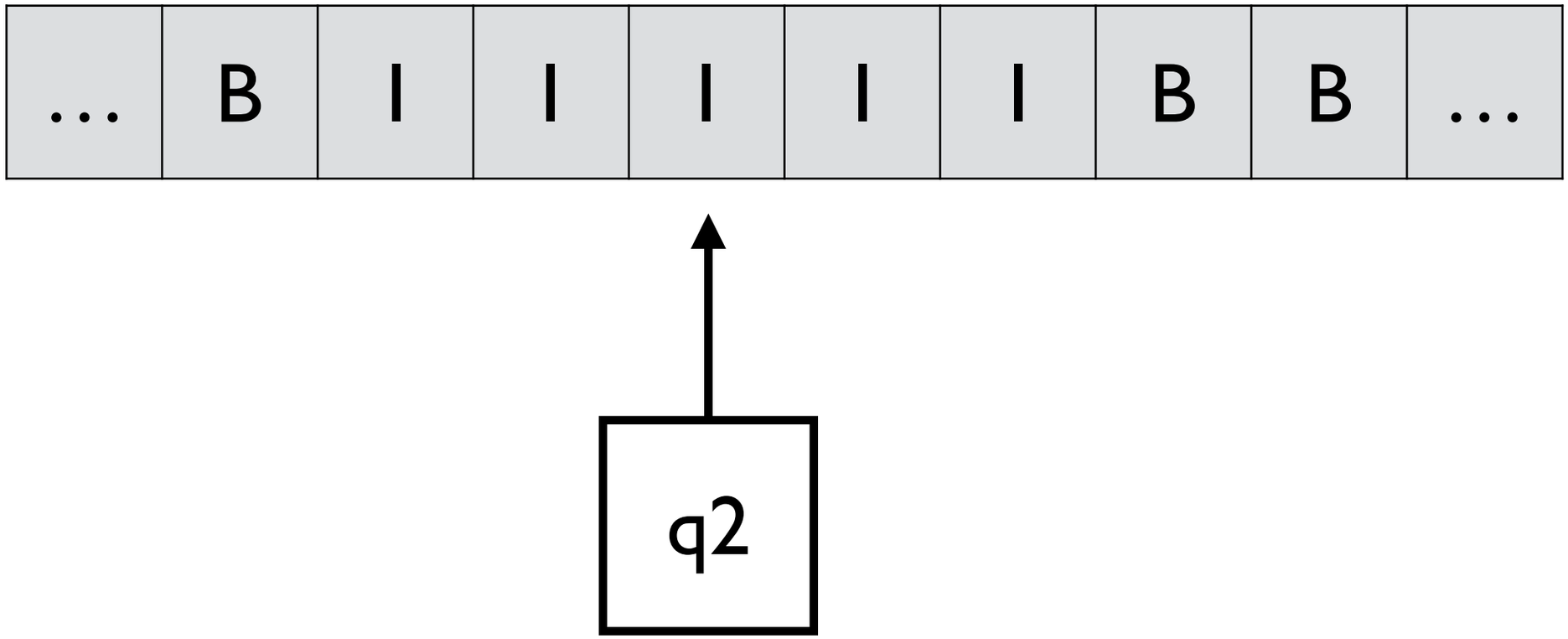
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



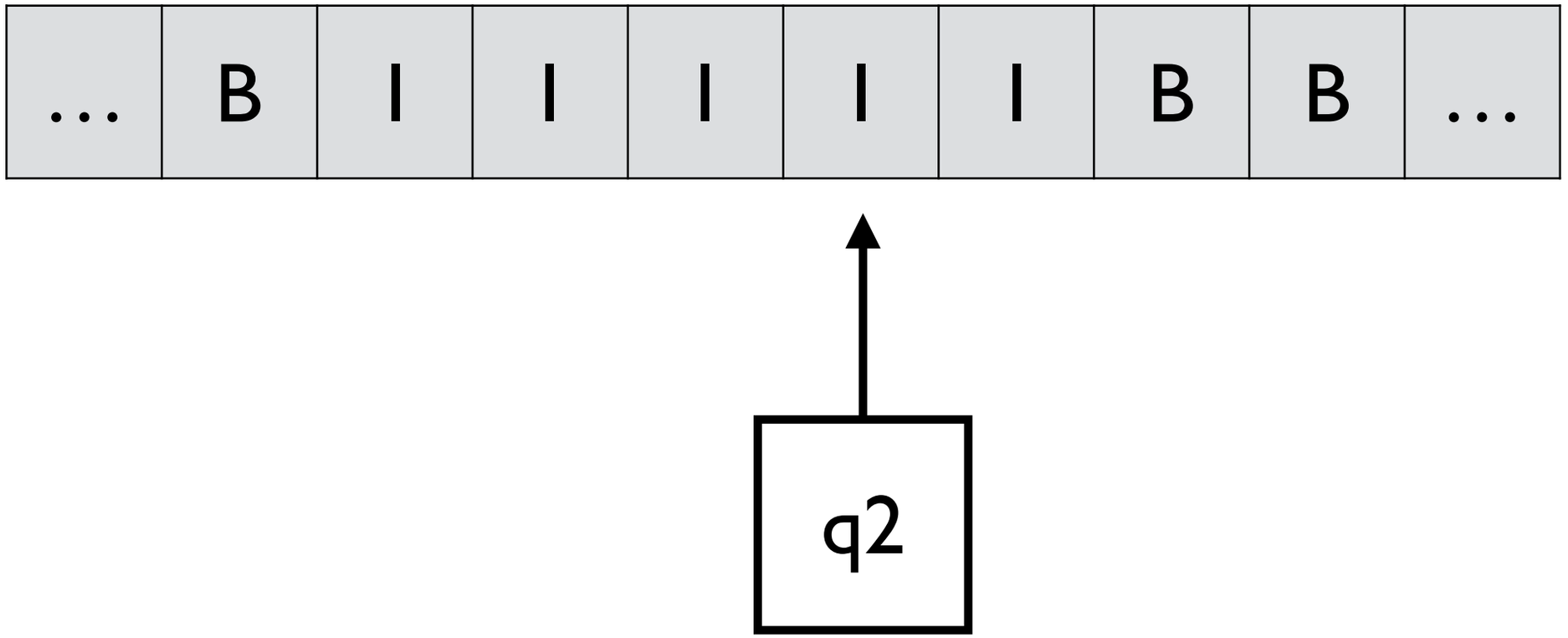
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



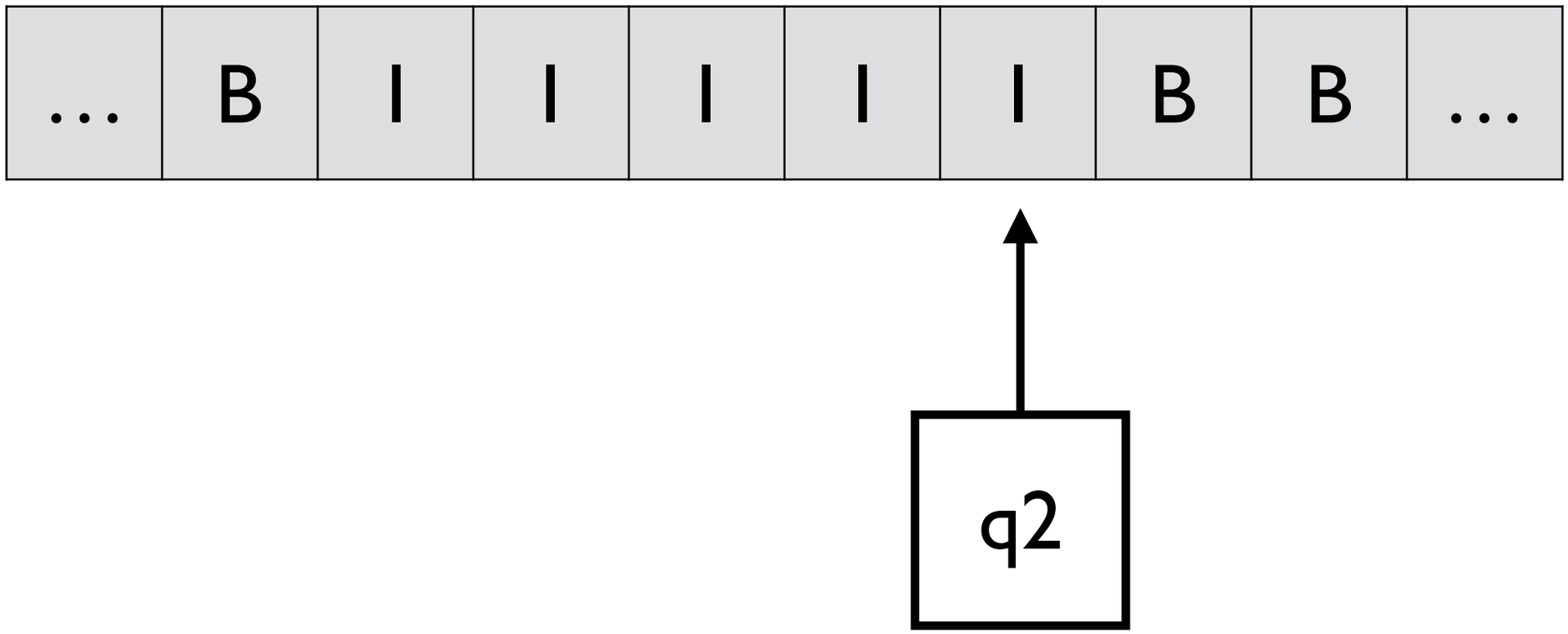
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



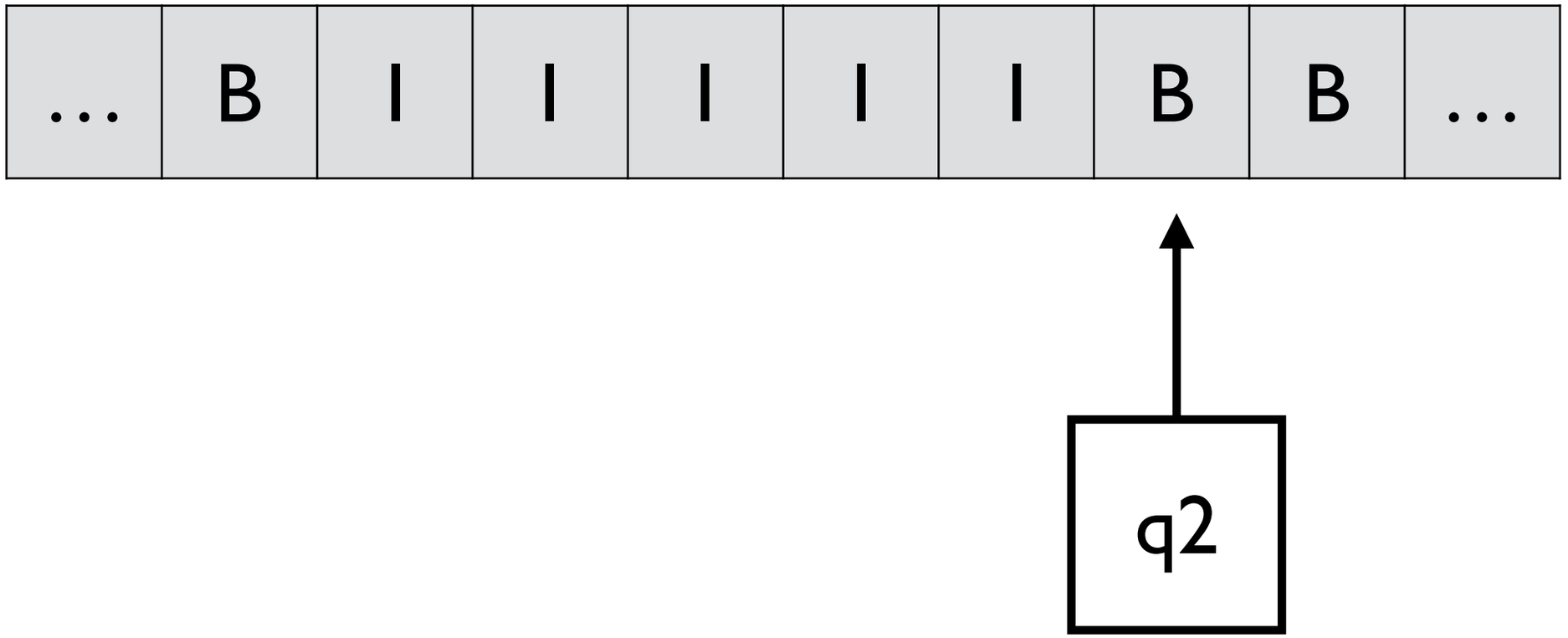
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



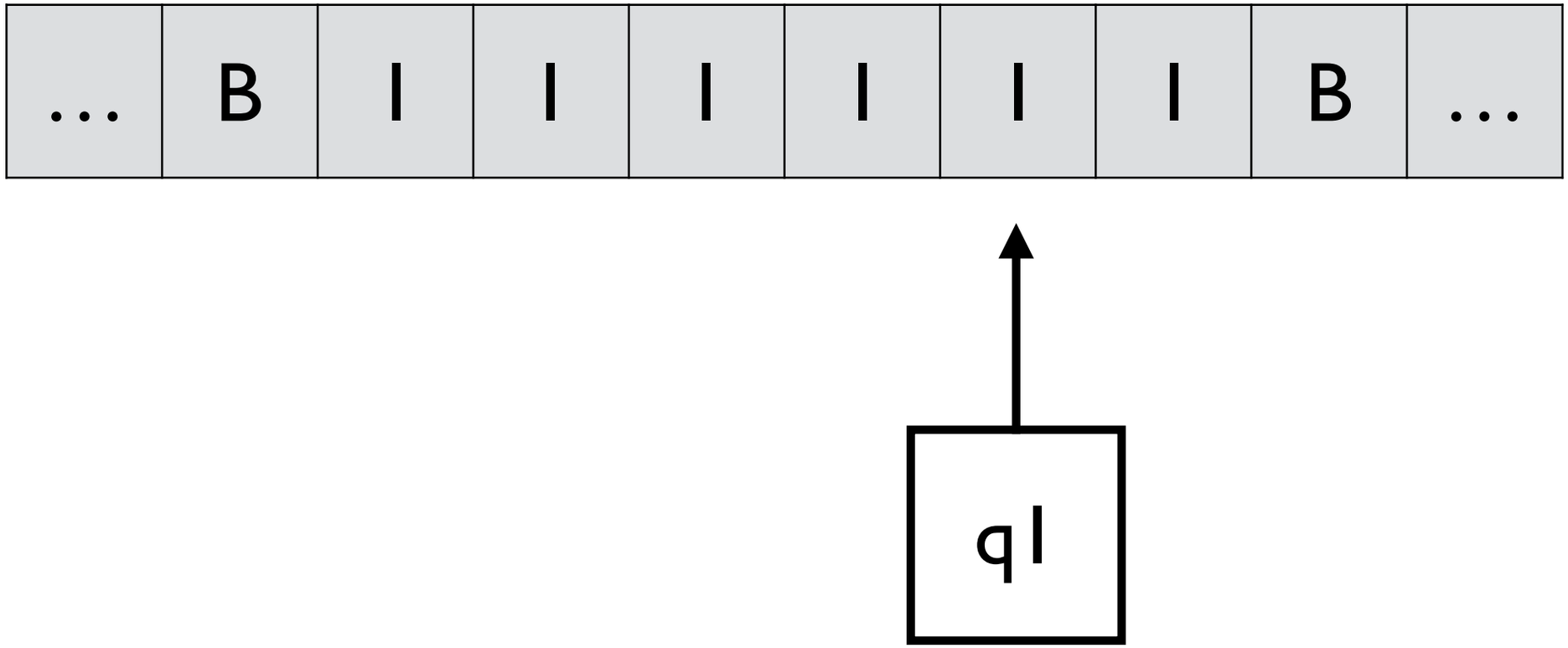
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



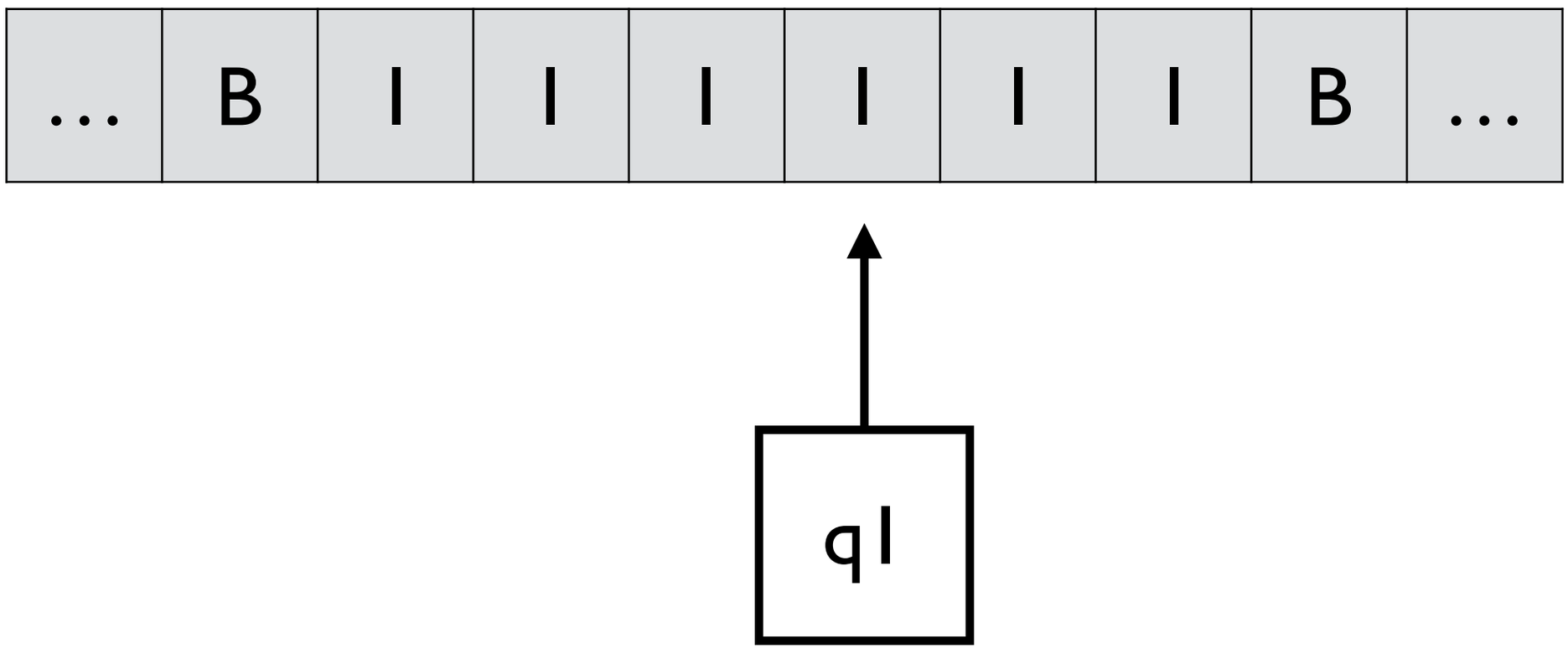
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



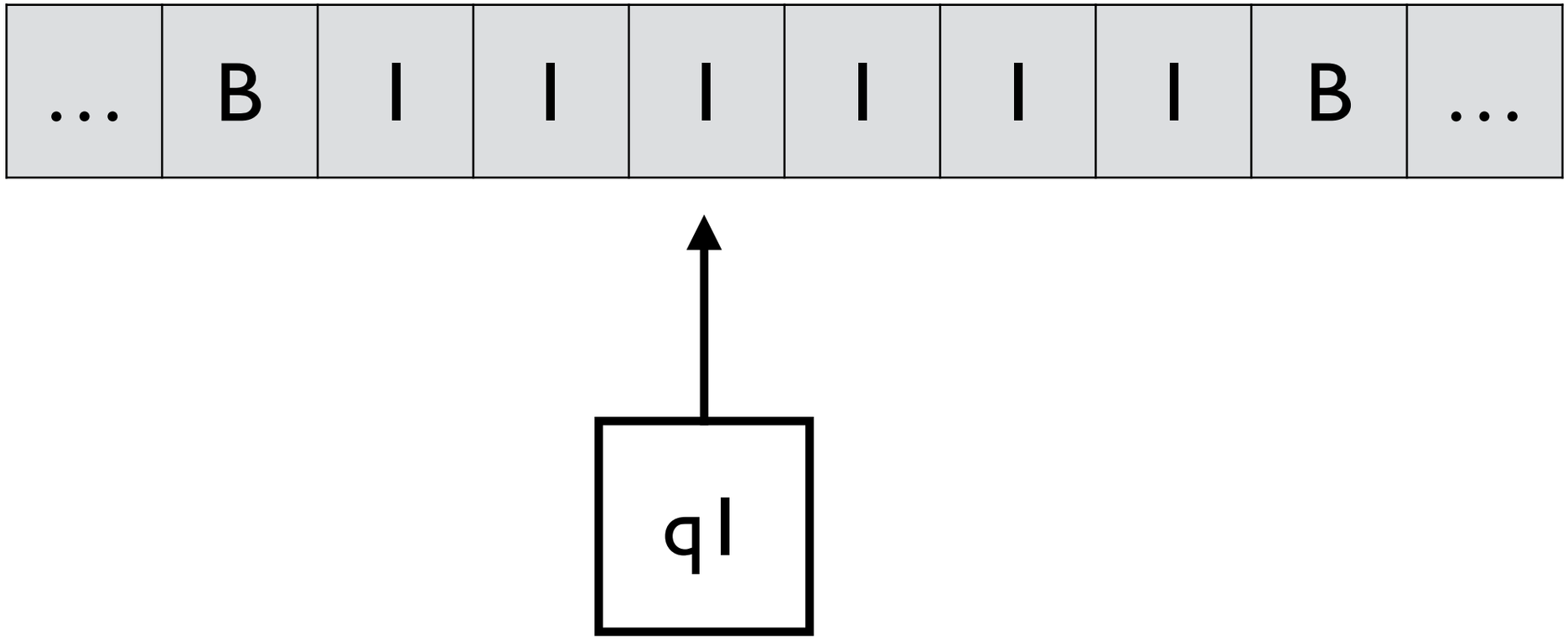
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



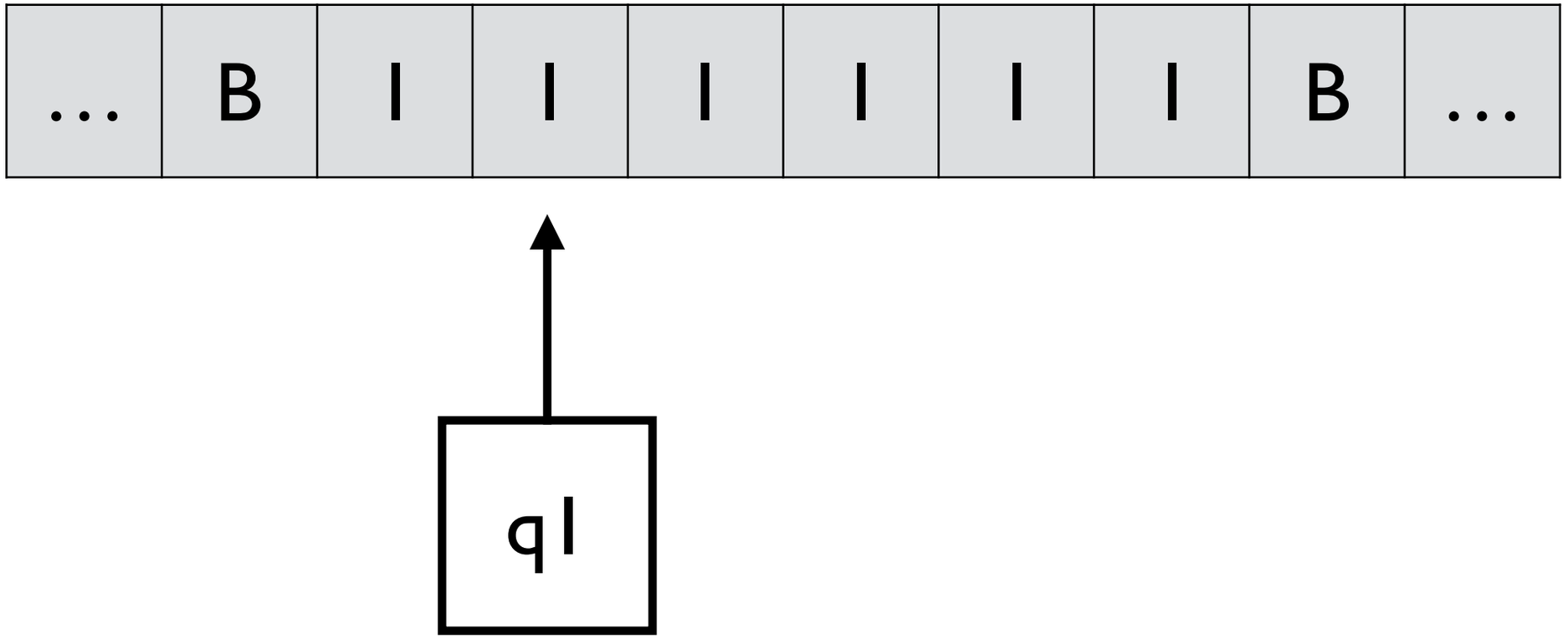
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



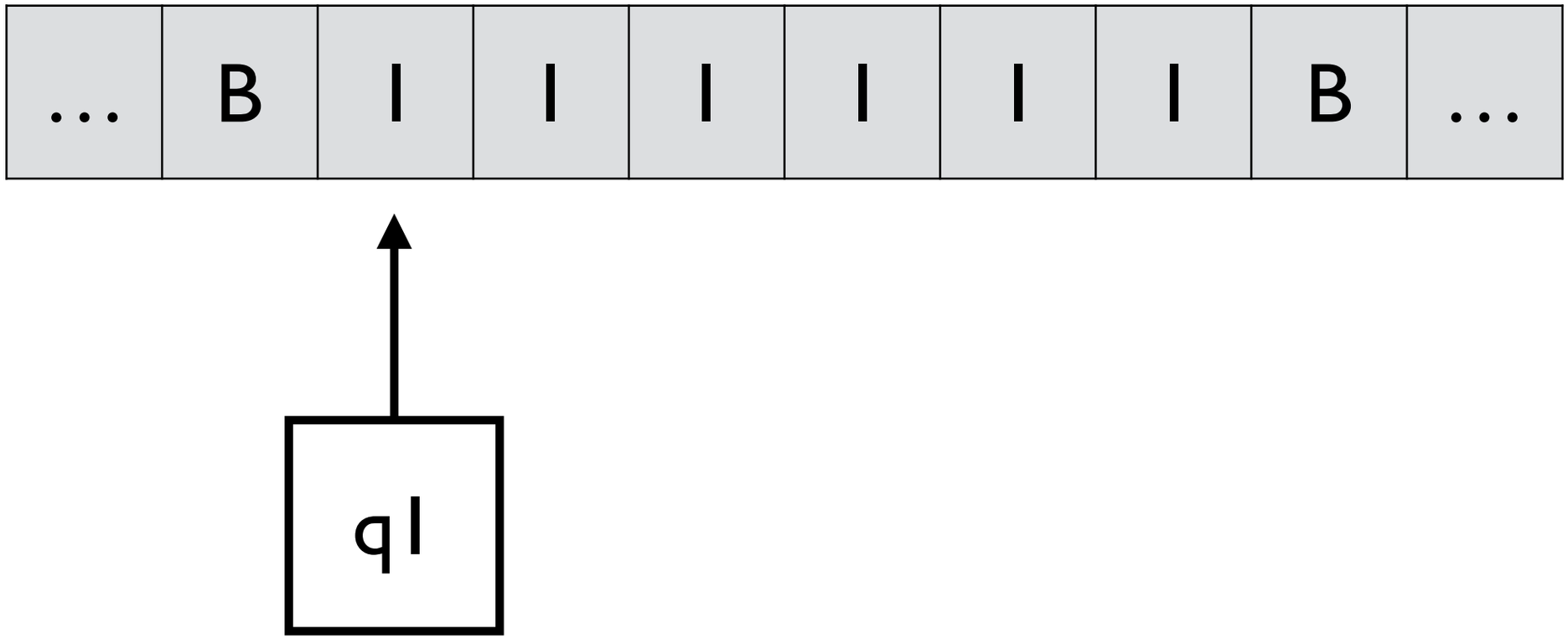
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



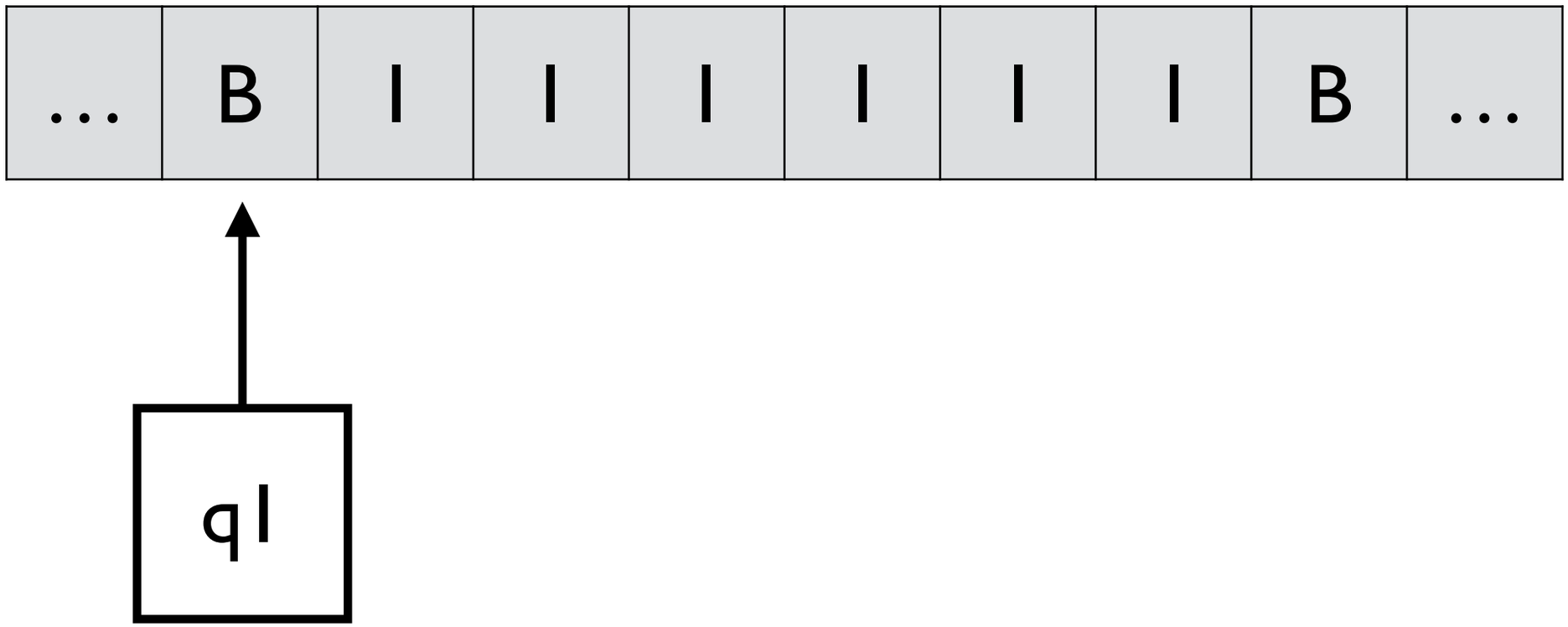
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



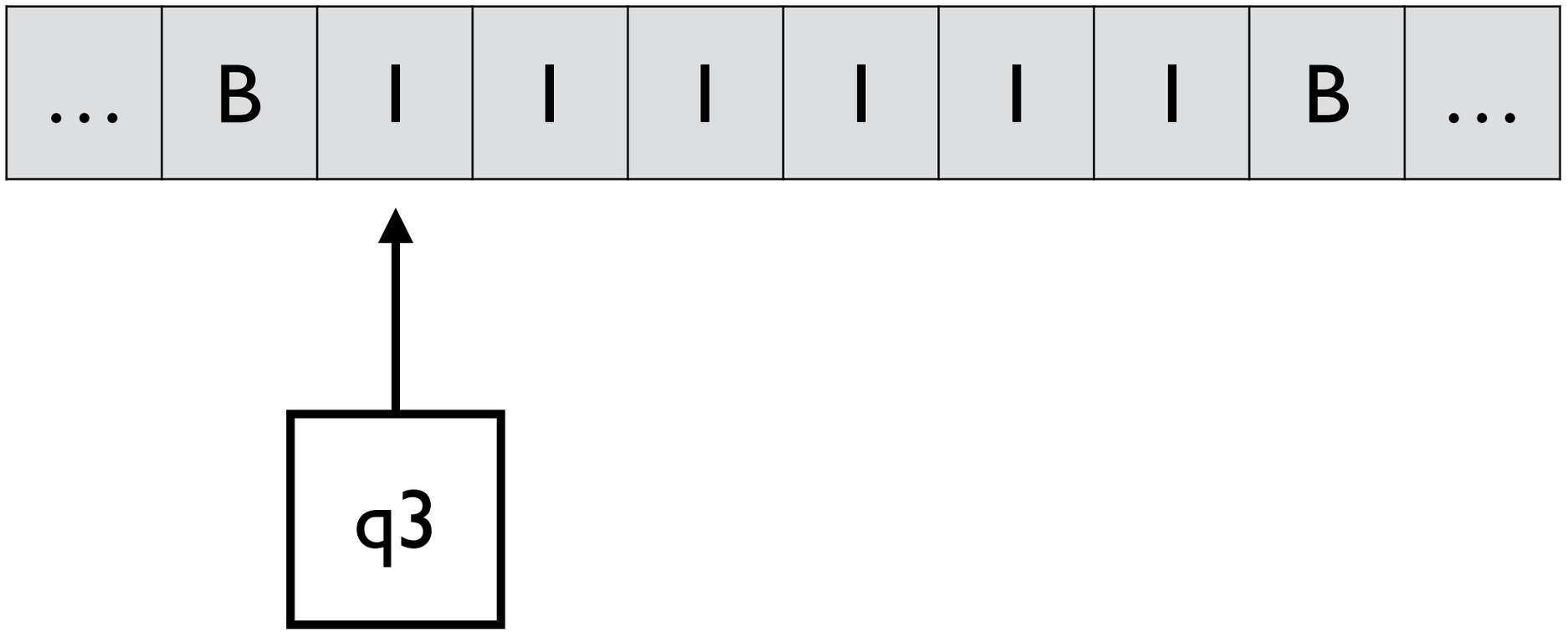
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



“final state”

$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$

Example 4. Design a Turing machine that computes $f(m, n)$.

$$f(m, n) = max(m - n, 0) = \text{if } m \geq n \text{ then } m - n \text{ else } 0$$

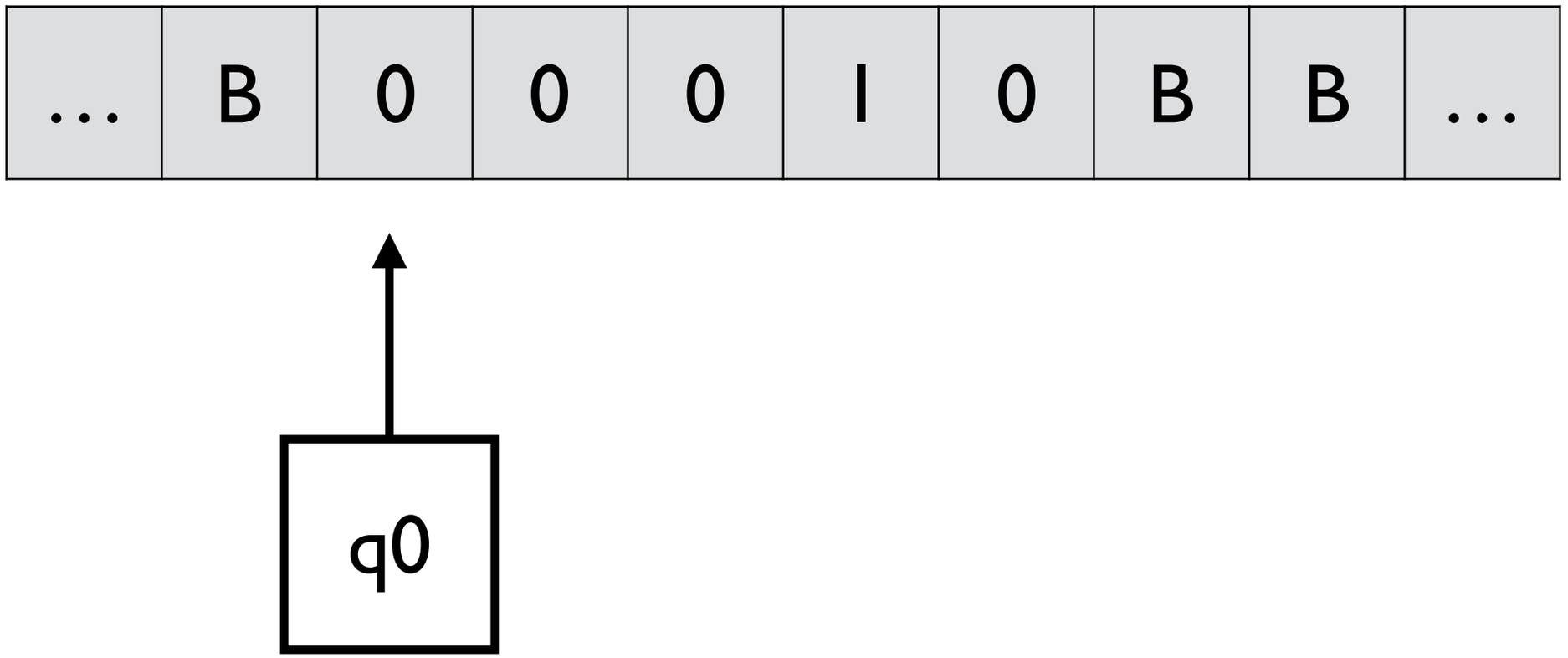
Example 4. Design a Turing machine that computes $f(m, n)$.

$$f(m, n) = max(m - n, 0) = \text{if } m \geq n \text{ then } m - n \text{ else } 0$$

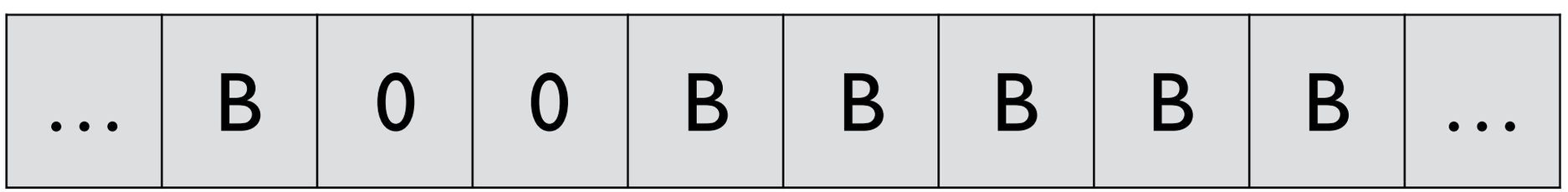
$$M = (\{q_0, q_1, \ldots, q_6\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_6\})$$

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

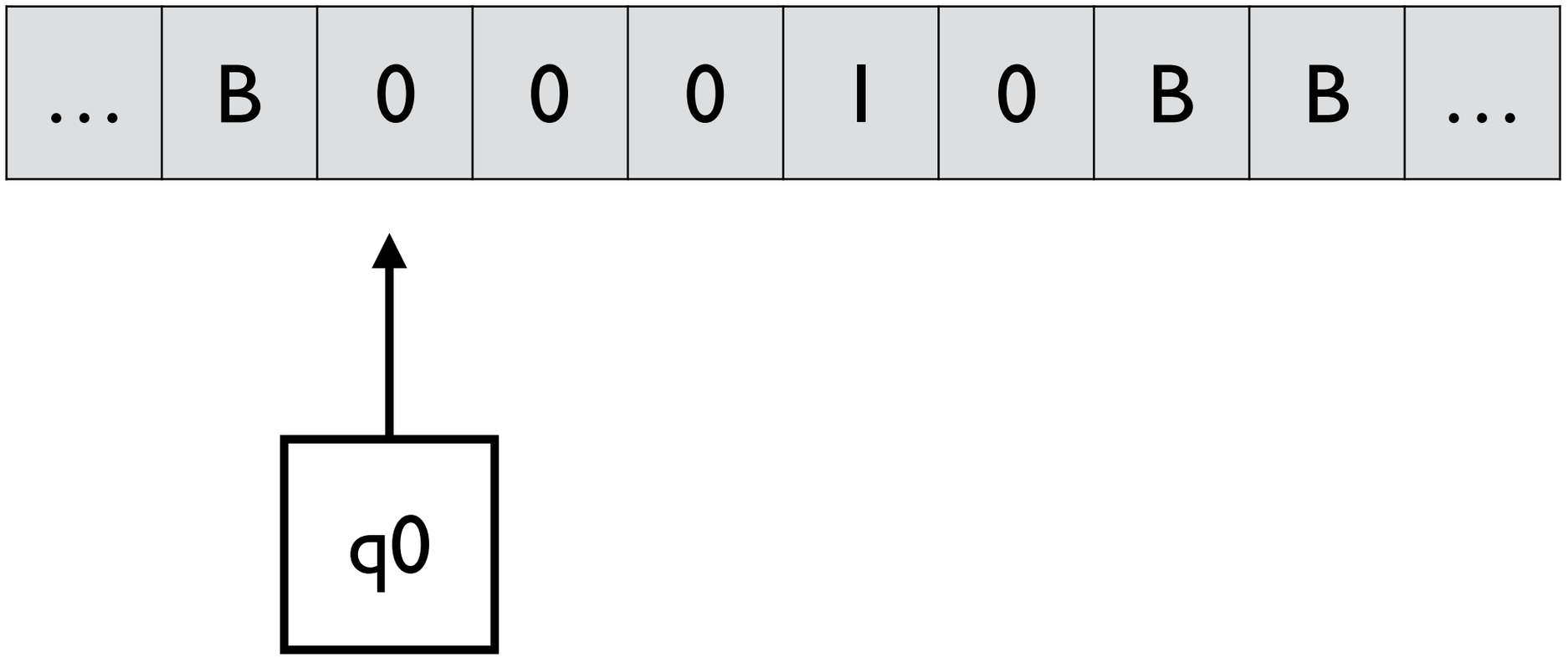
Initial machine configuration: e.g., f(3,1)



When the machine halts:

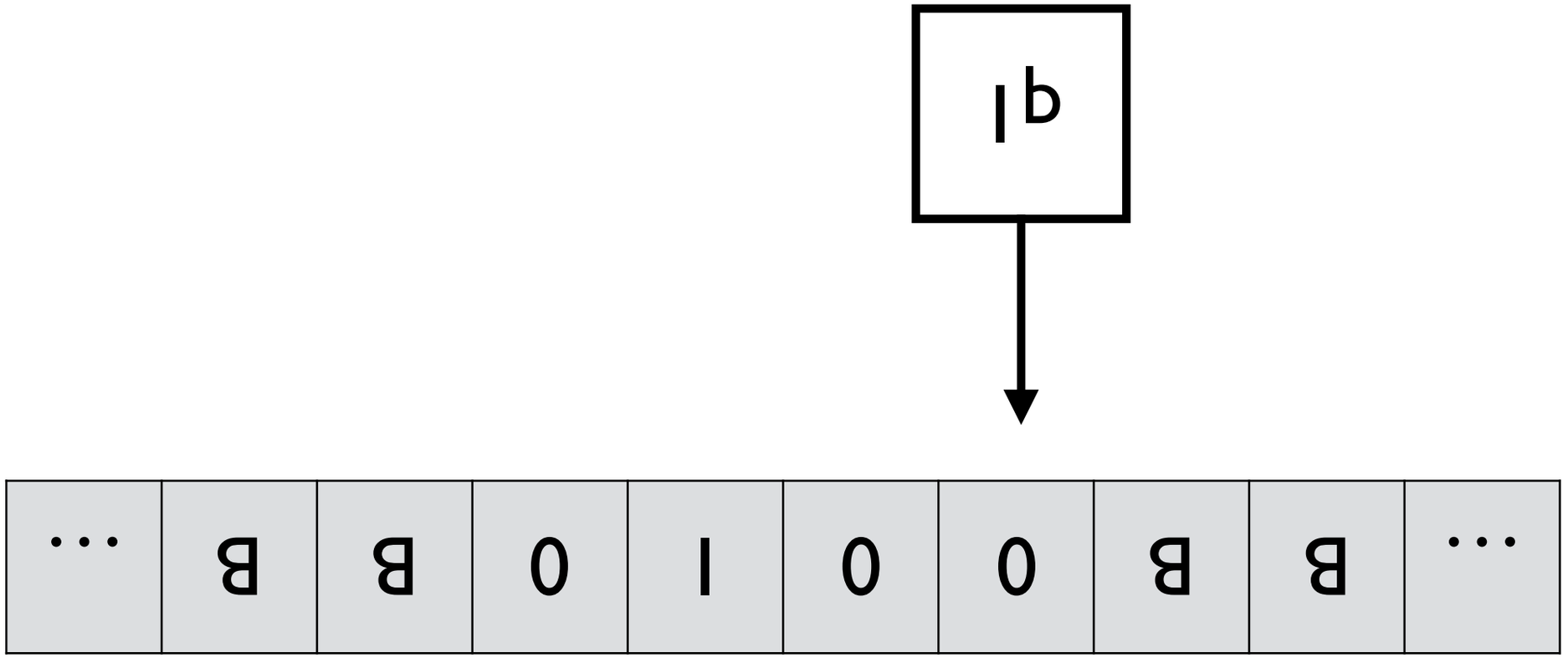


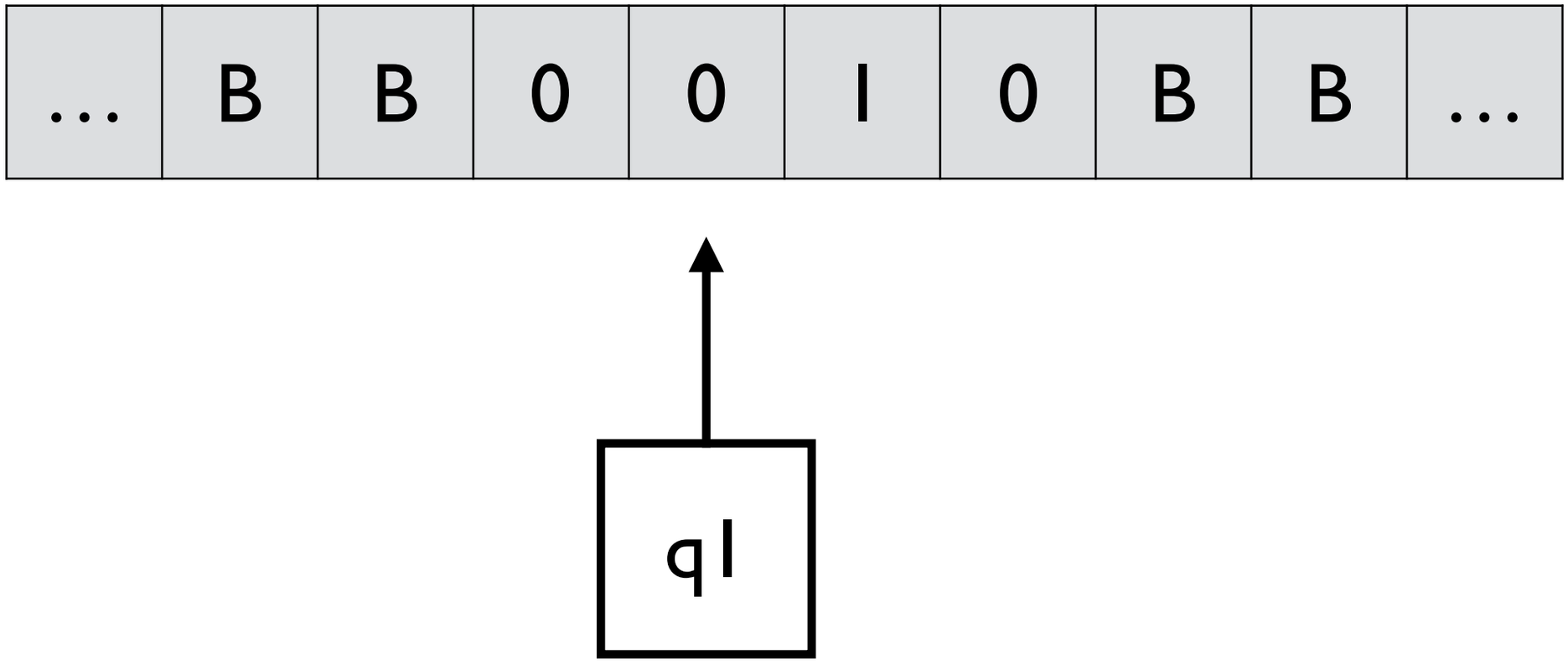
e.g., f(3,1)



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

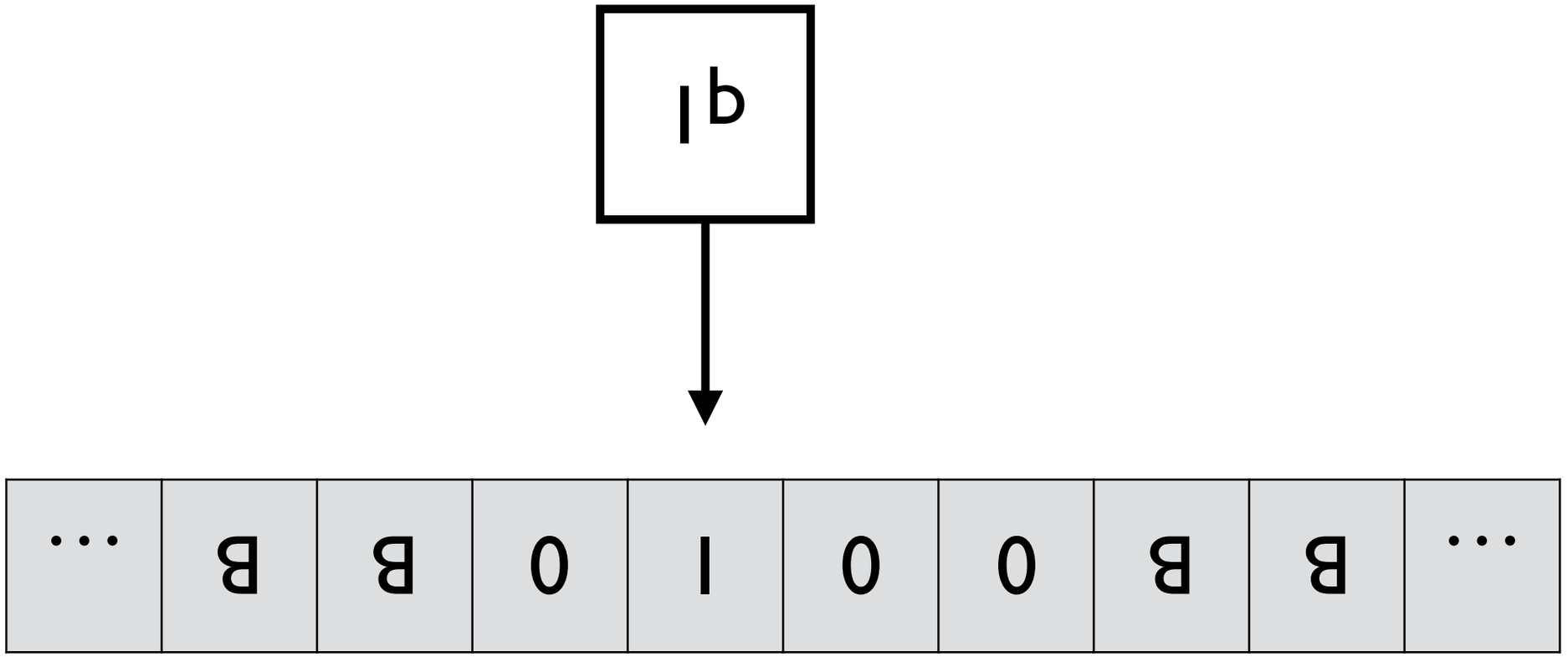
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

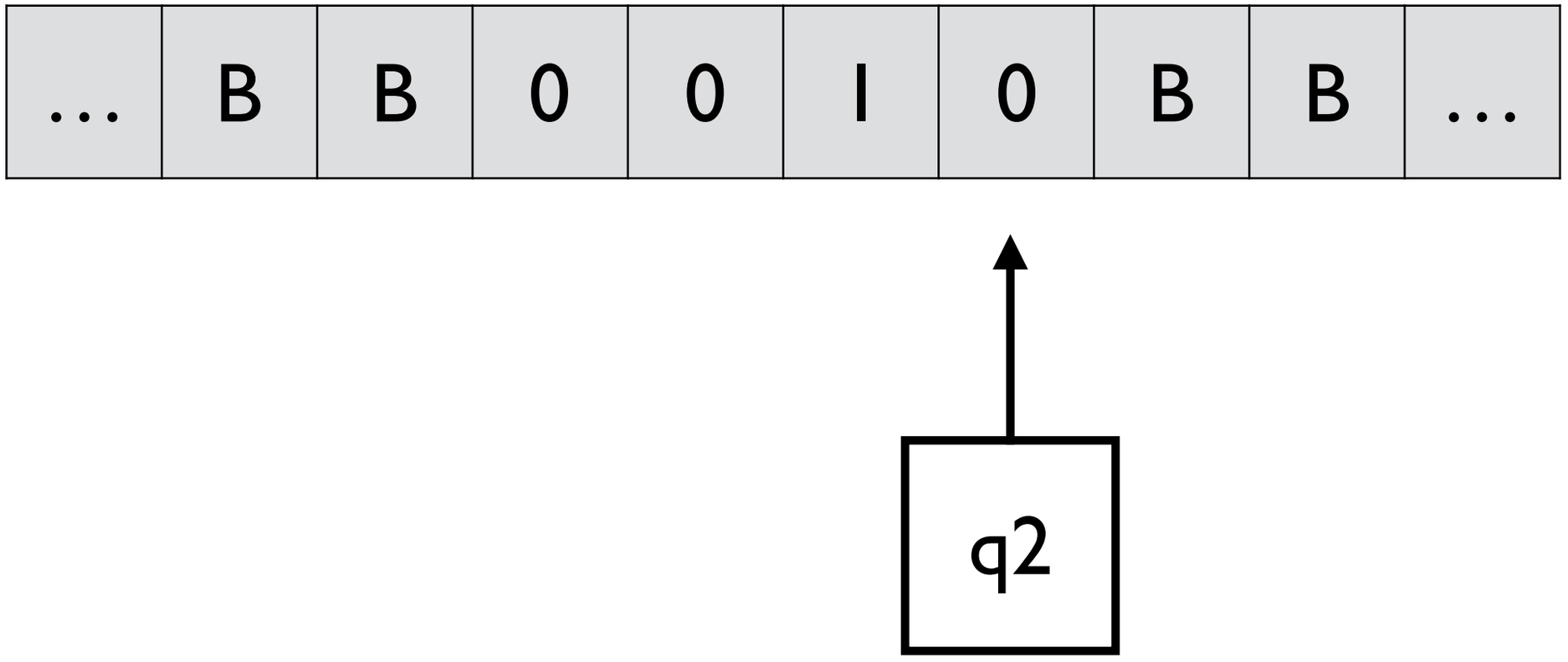




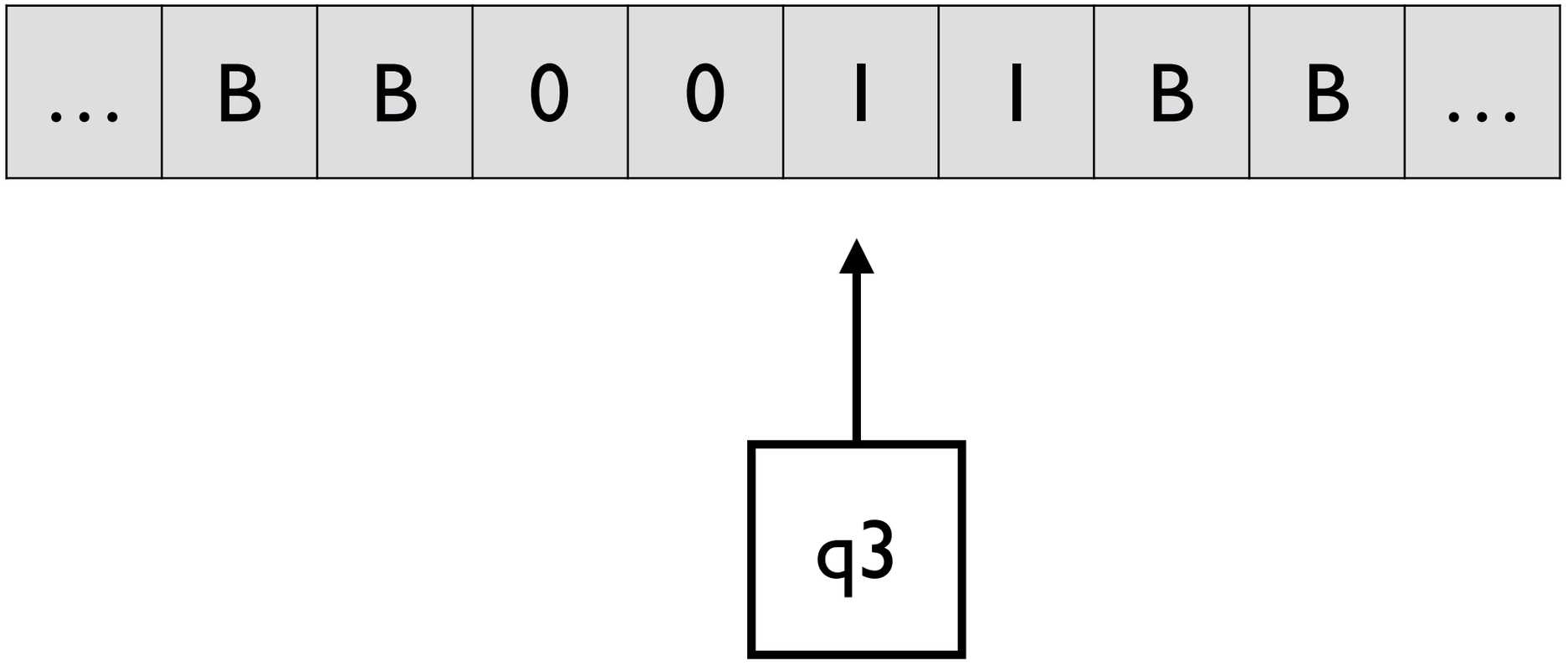
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

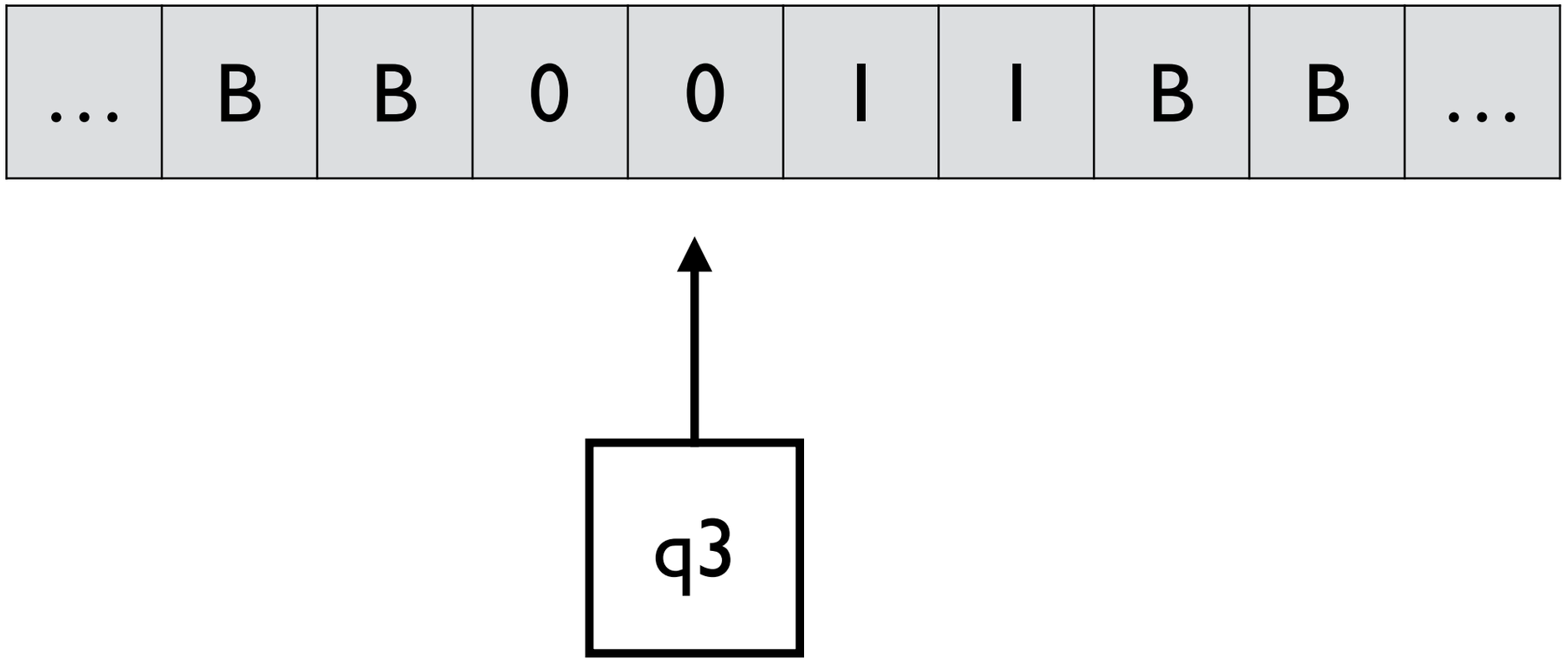




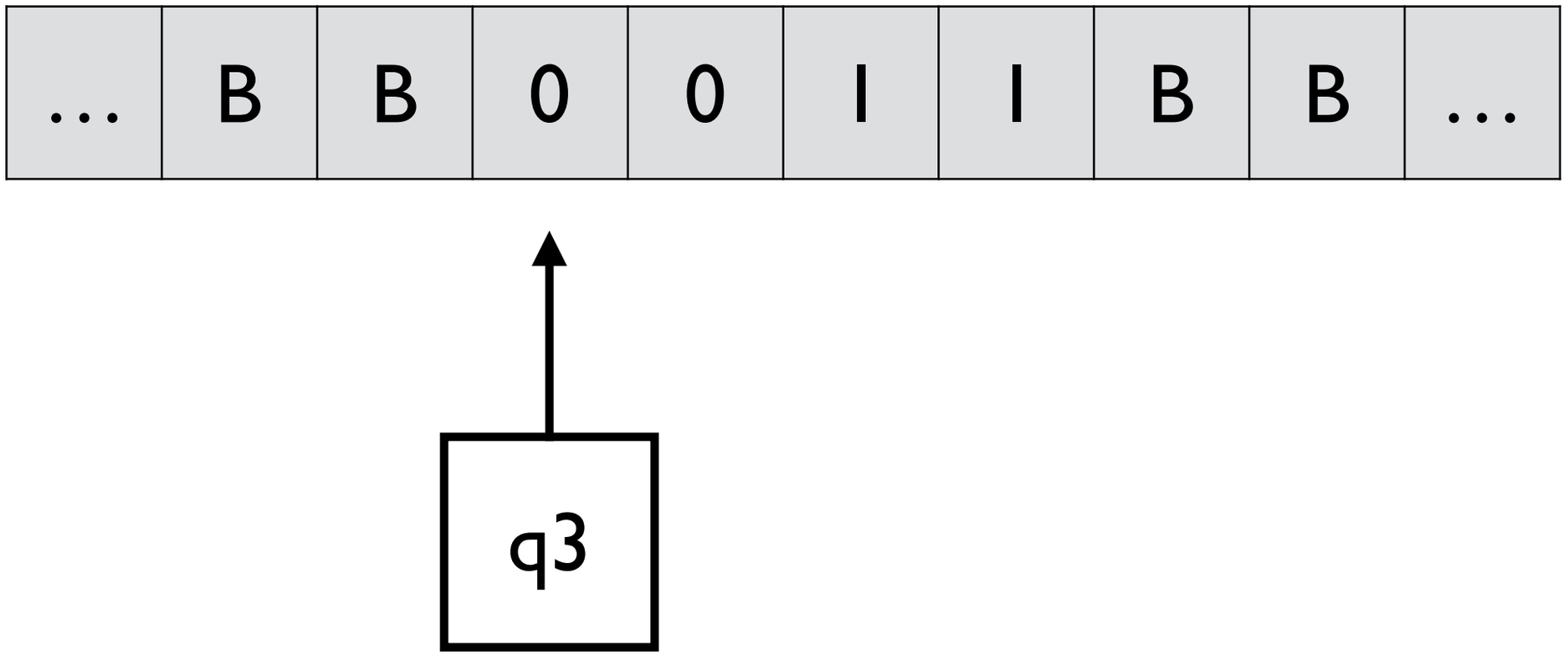
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



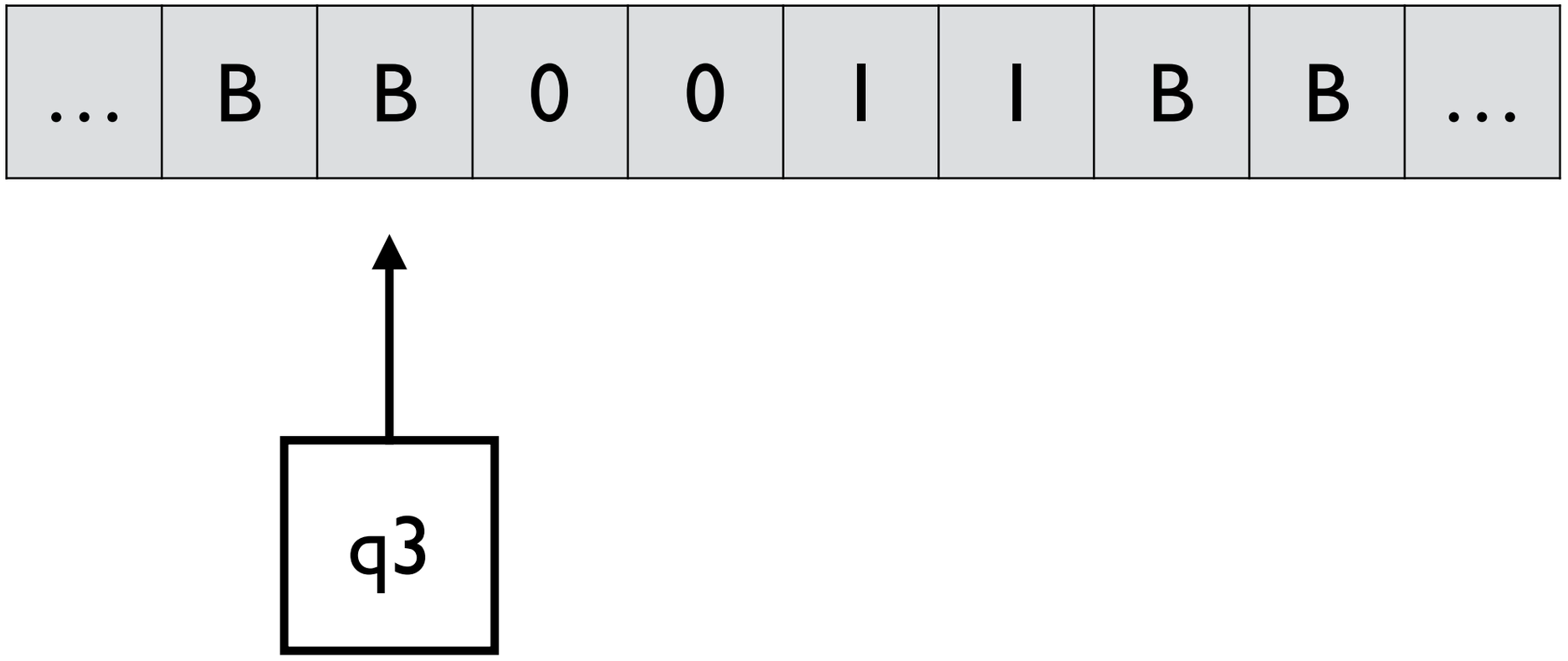
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



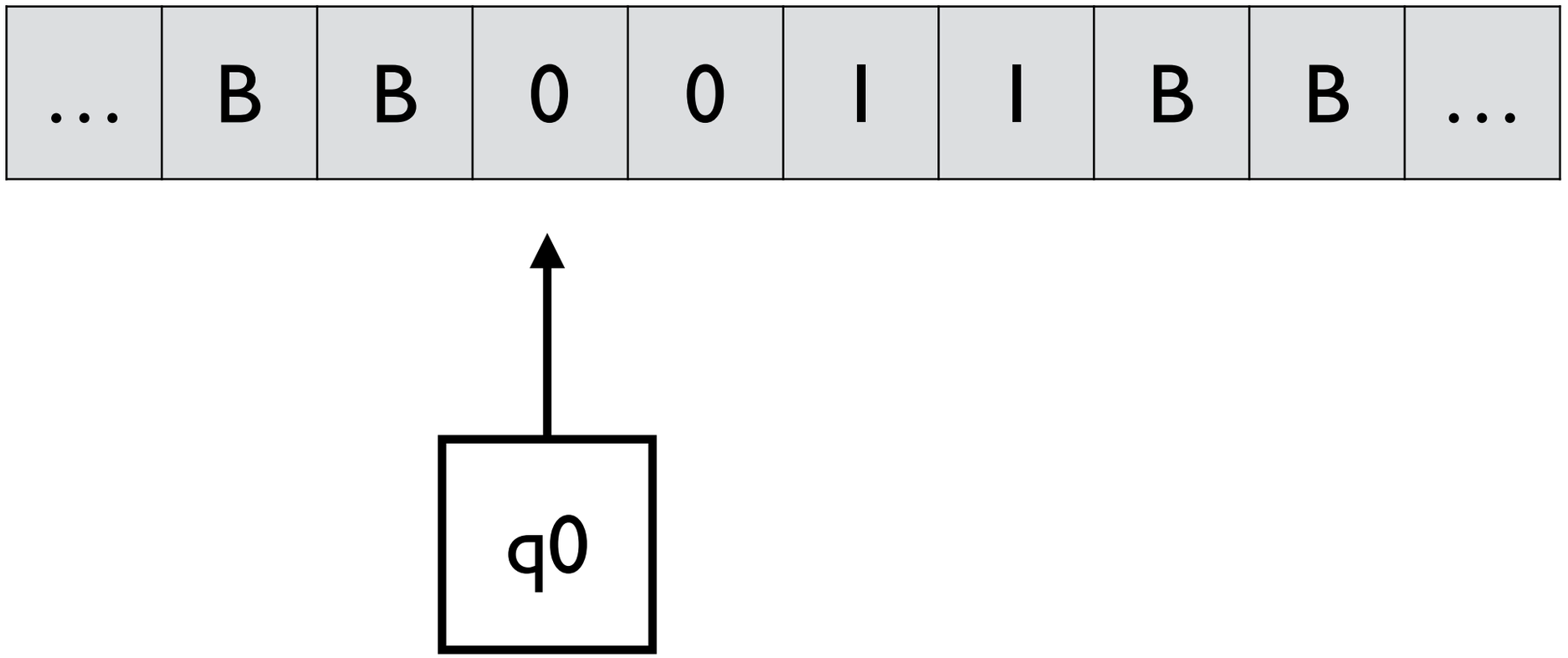
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

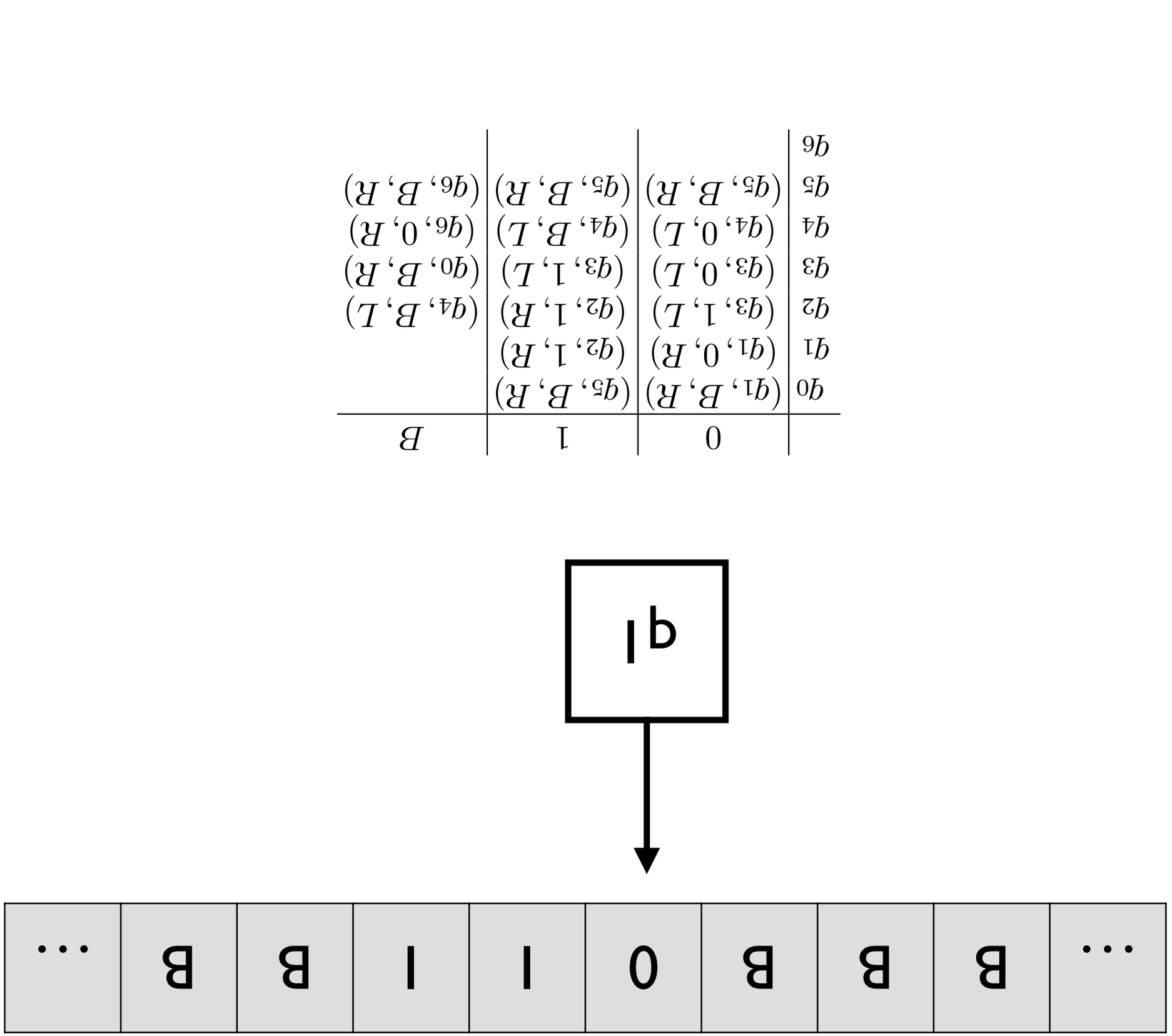


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

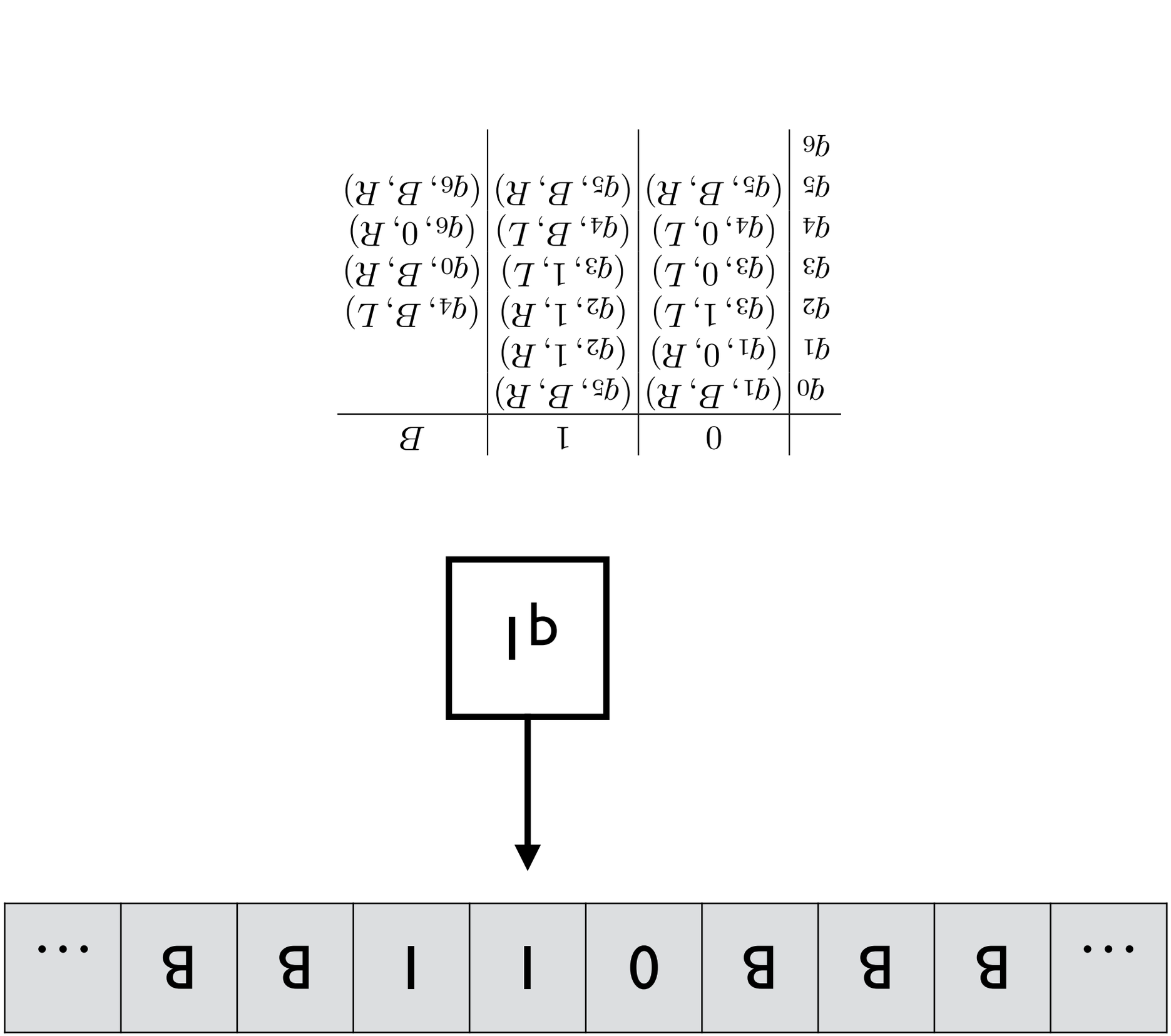


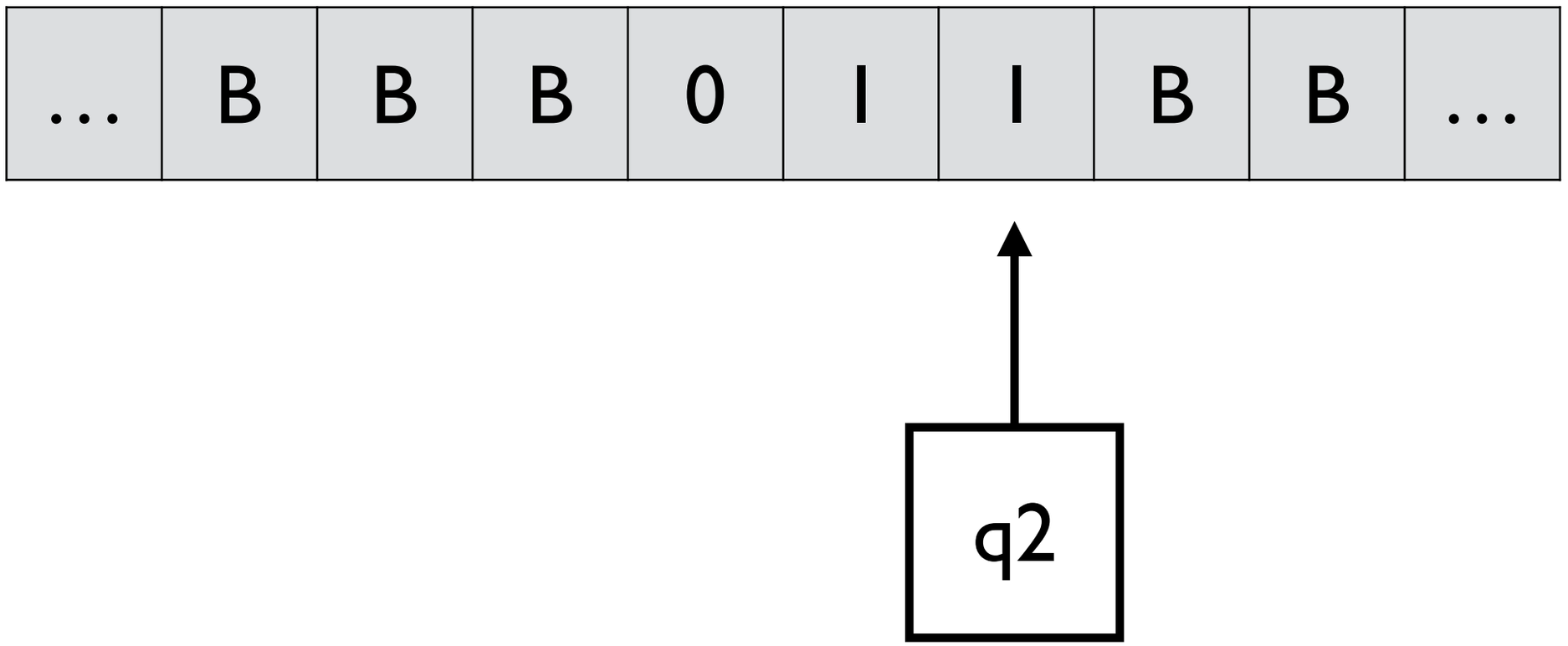
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

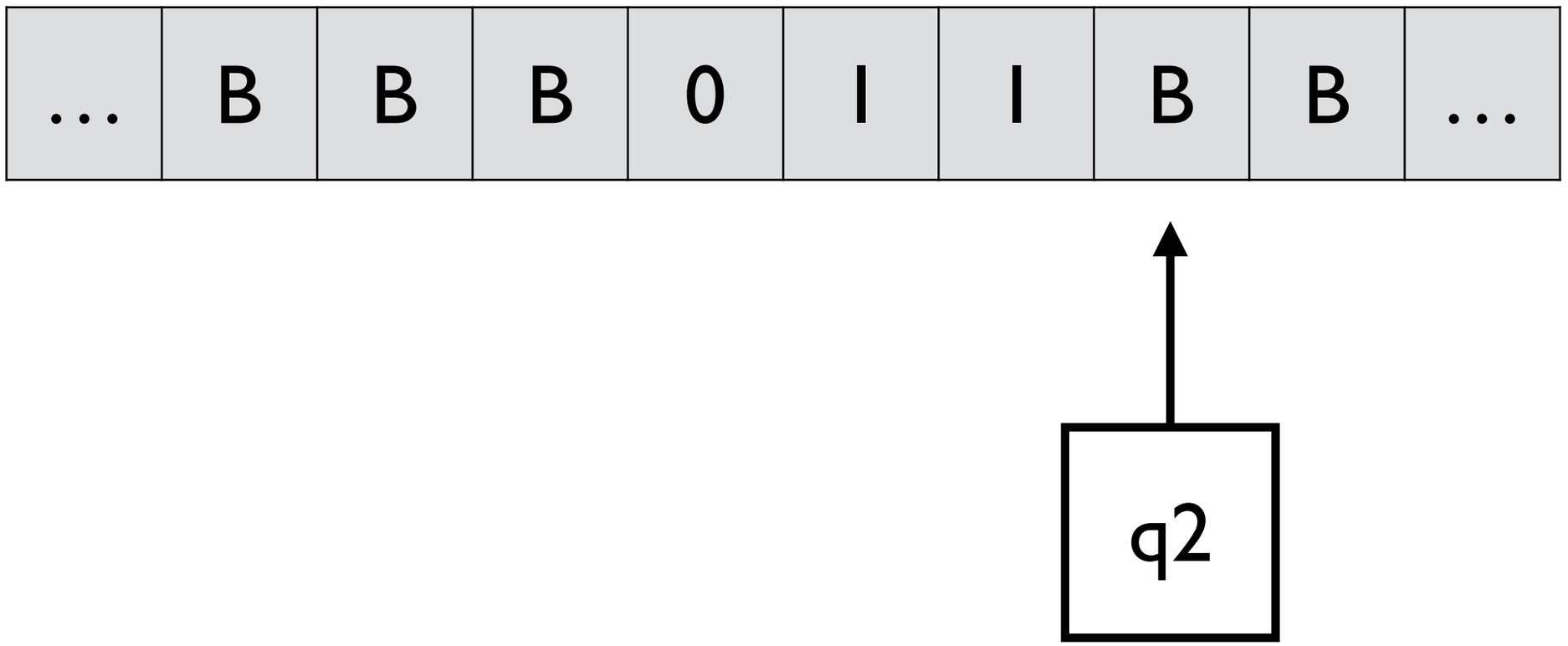


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

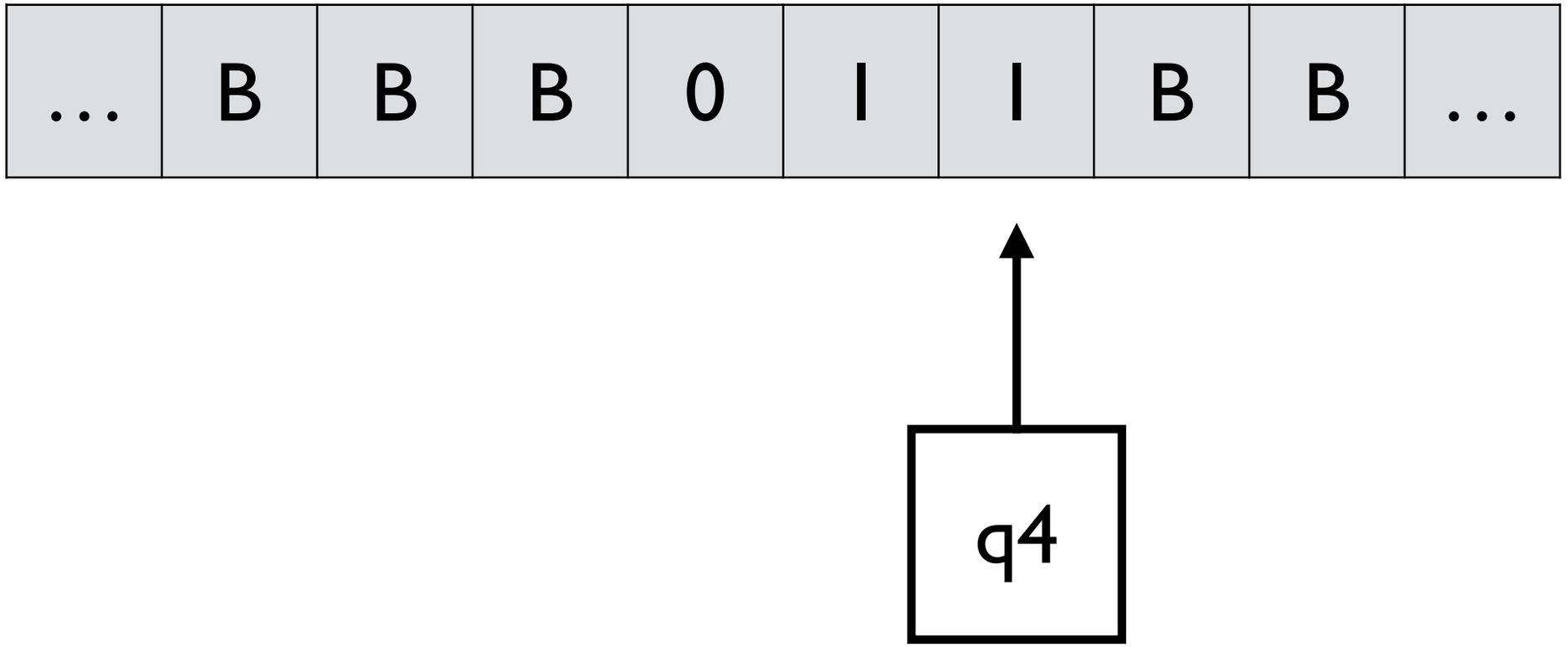




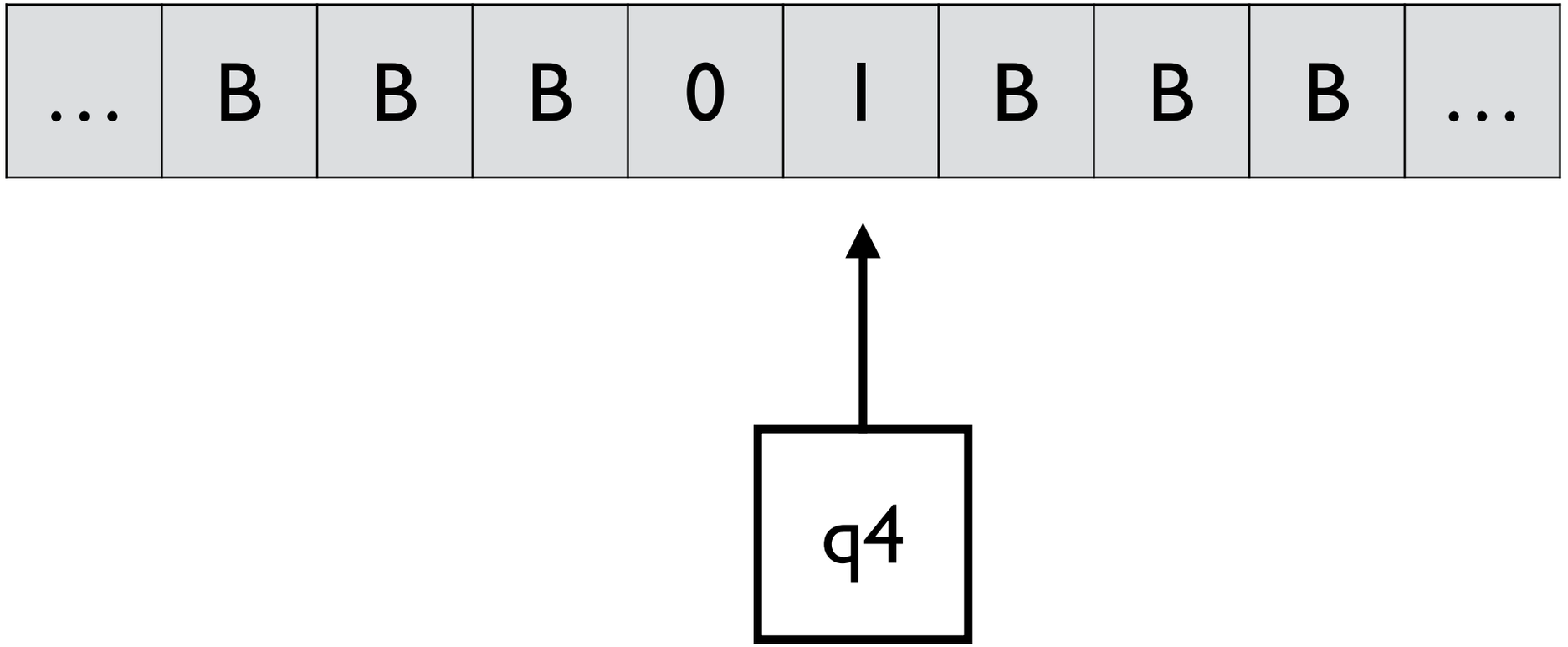
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



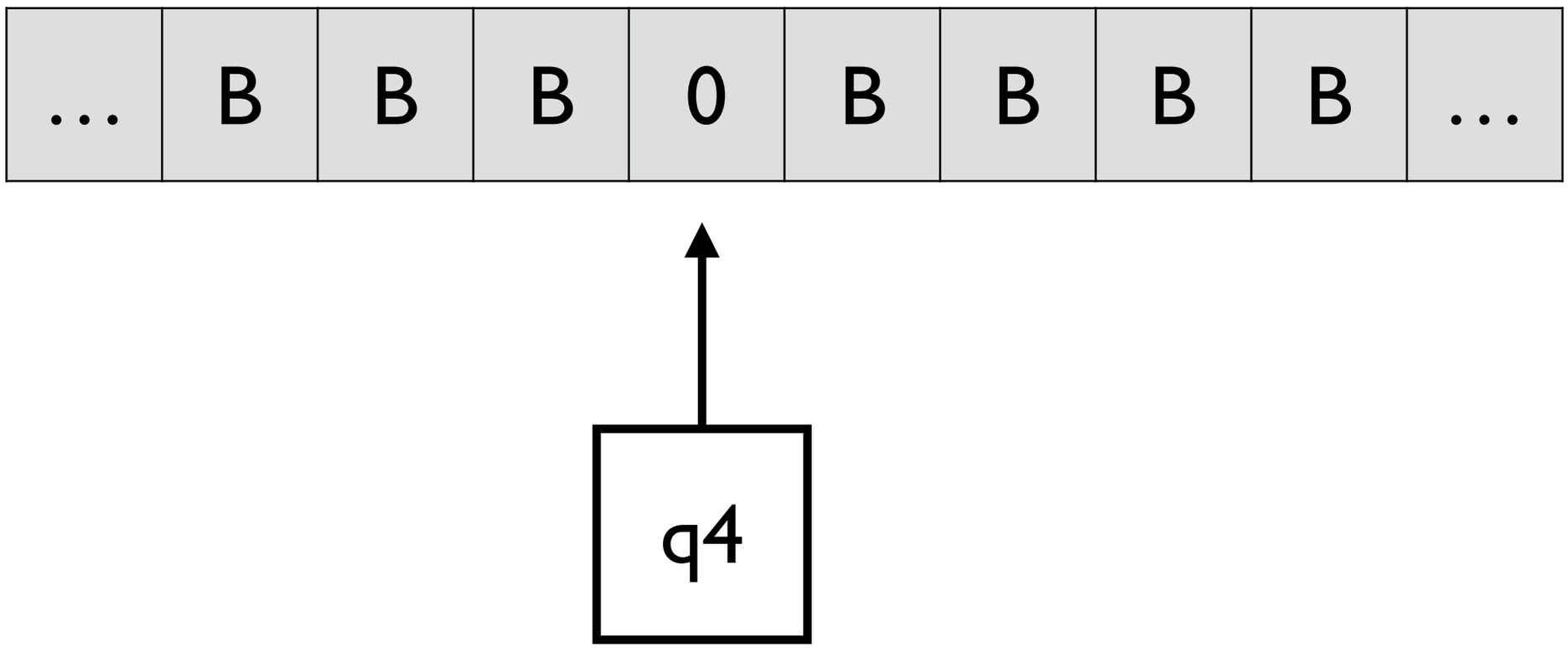
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



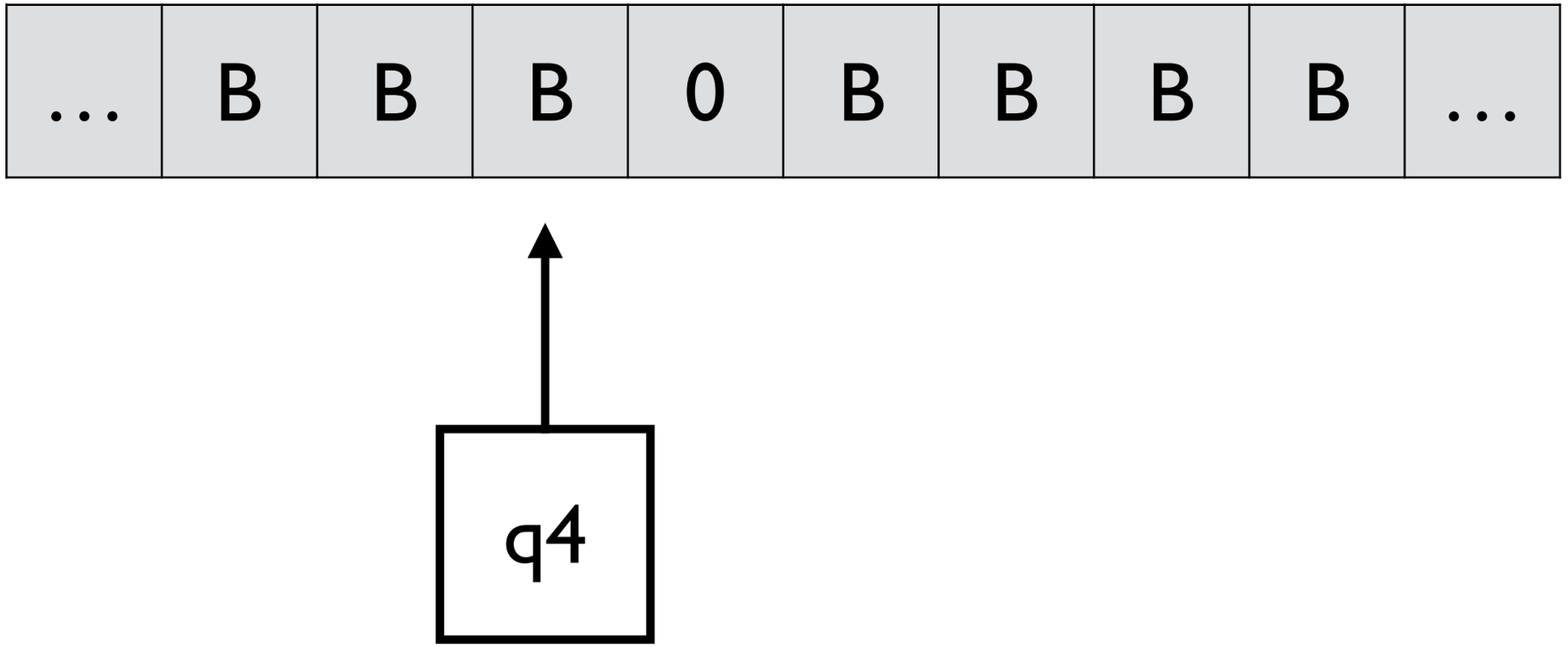
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



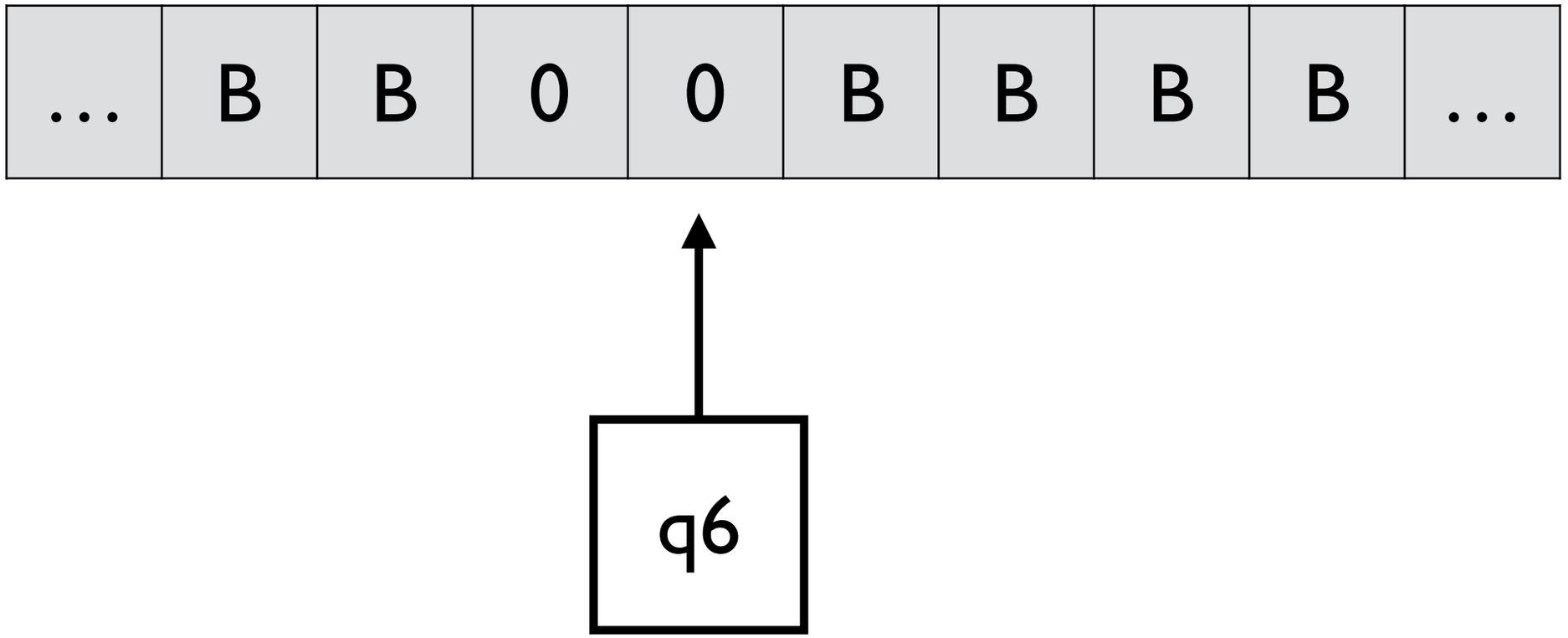
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

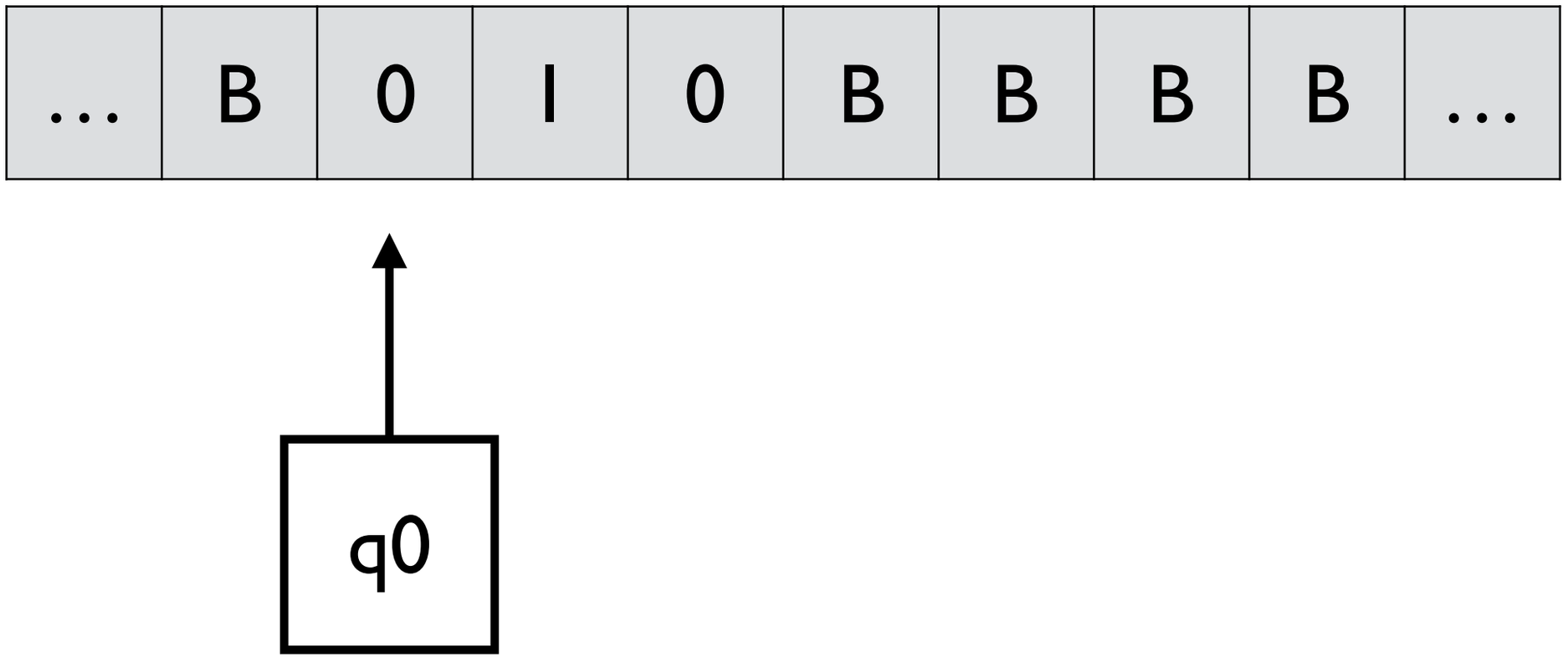


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

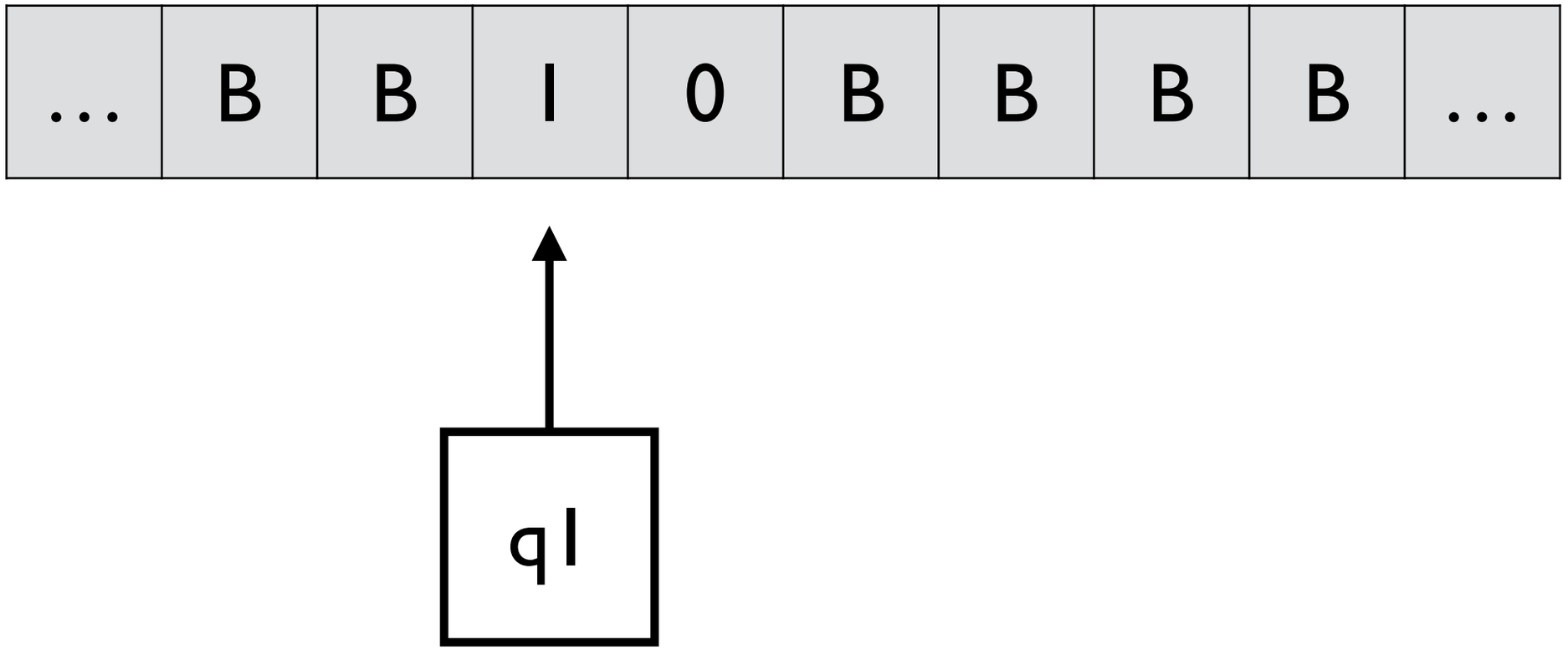


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

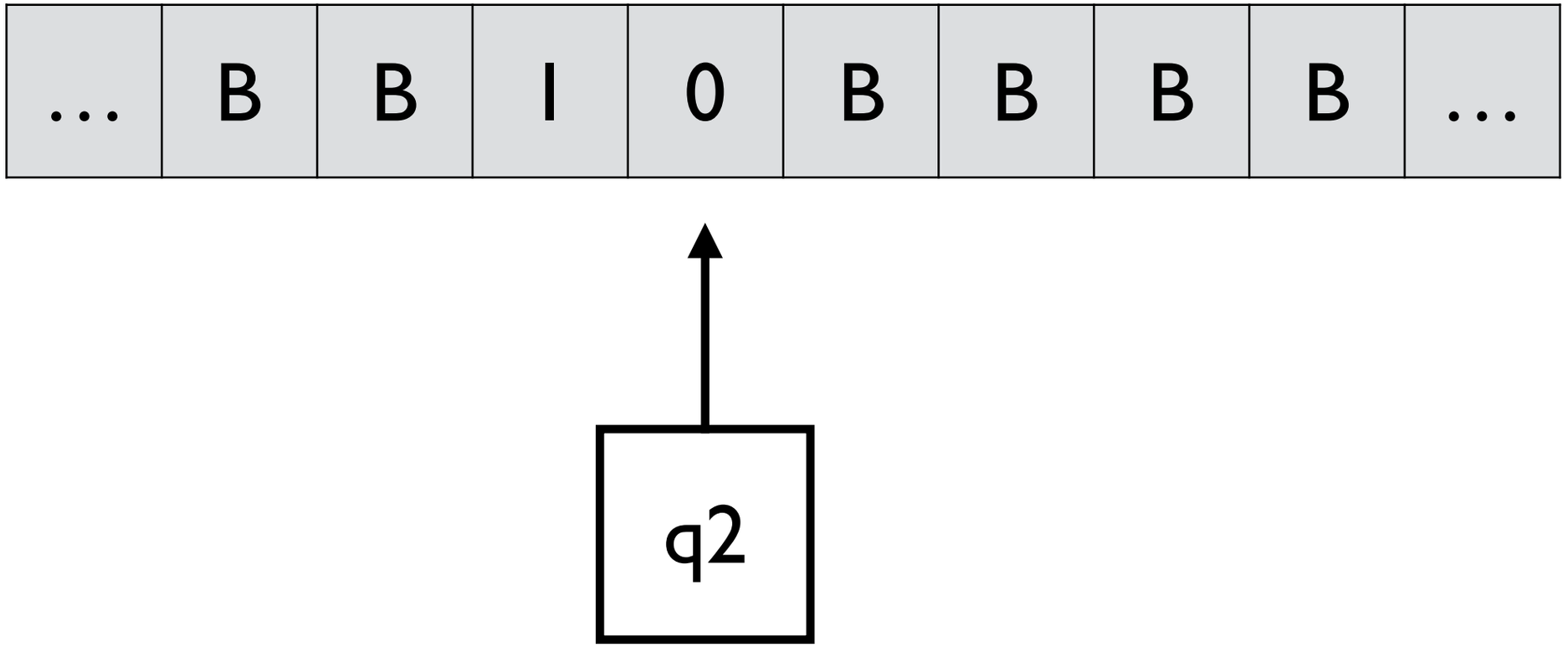
e.g., f(1,1)



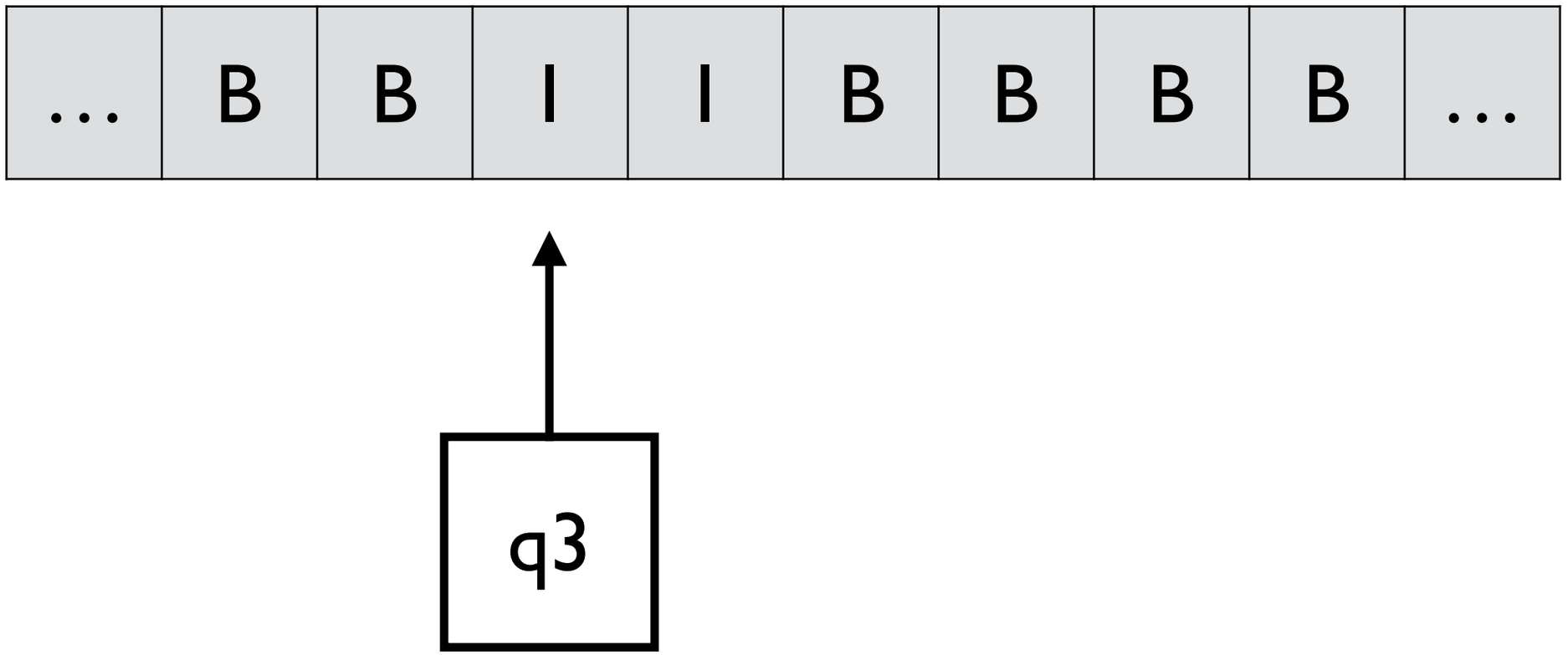
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



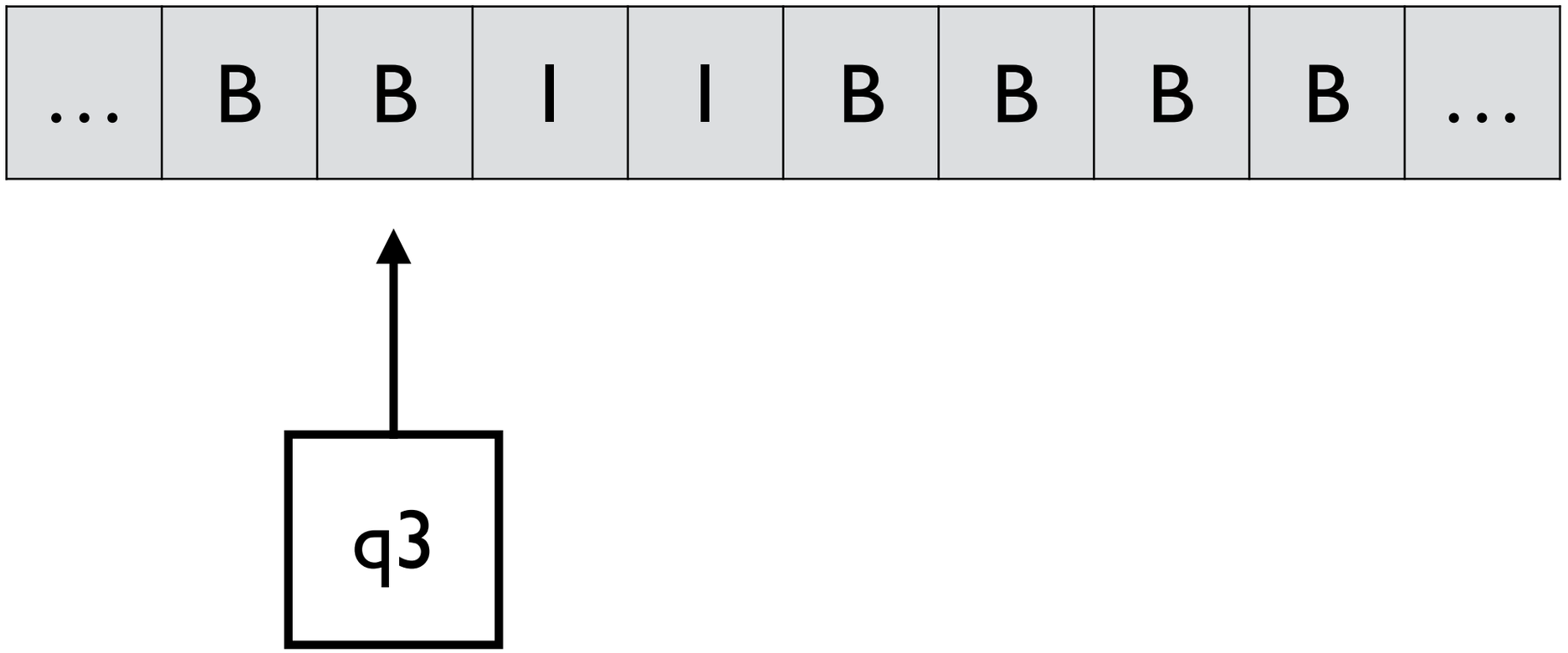
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



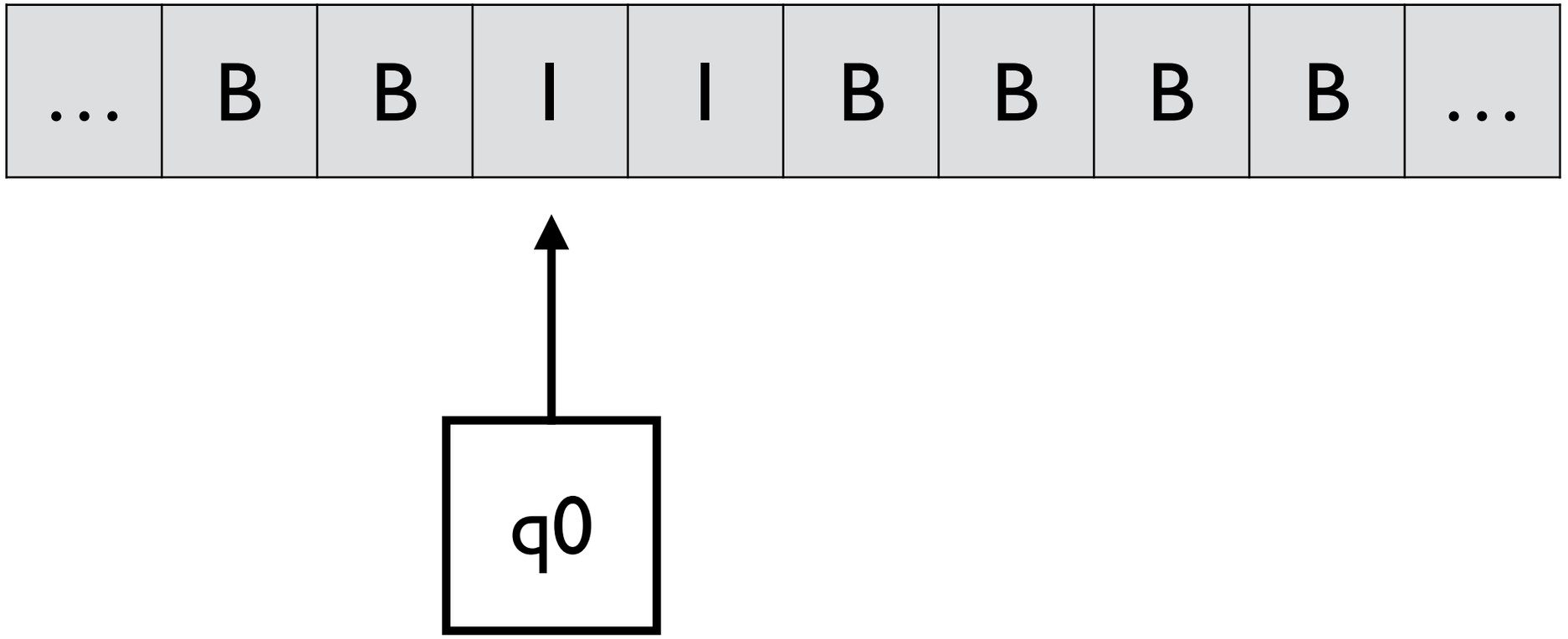
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



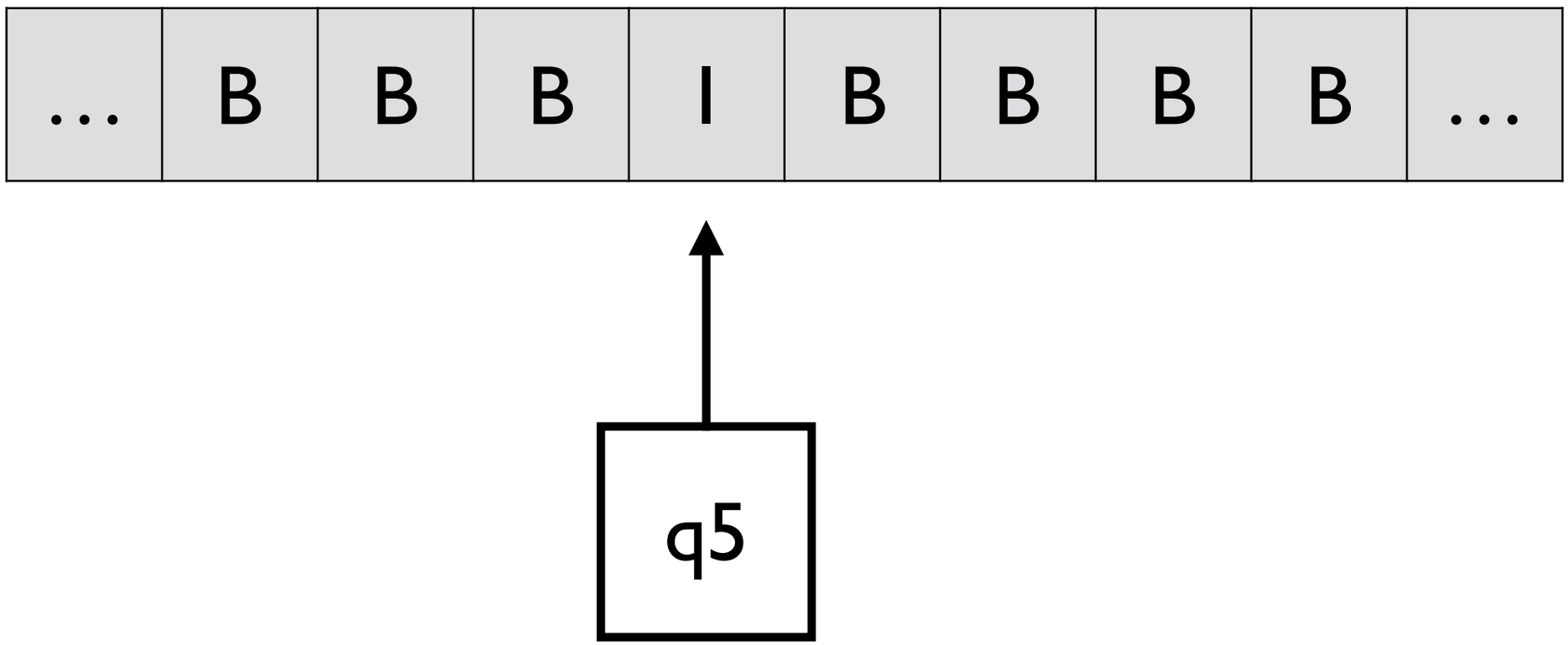
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



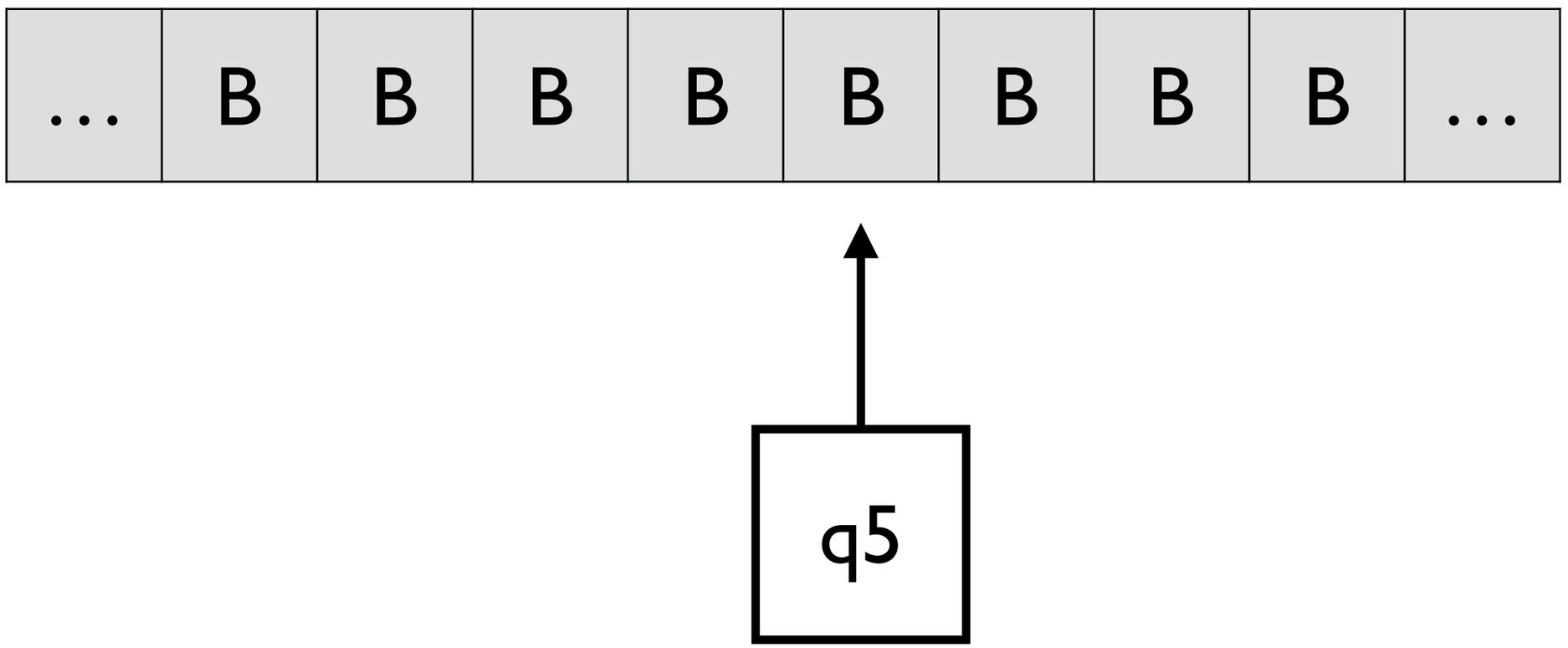
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



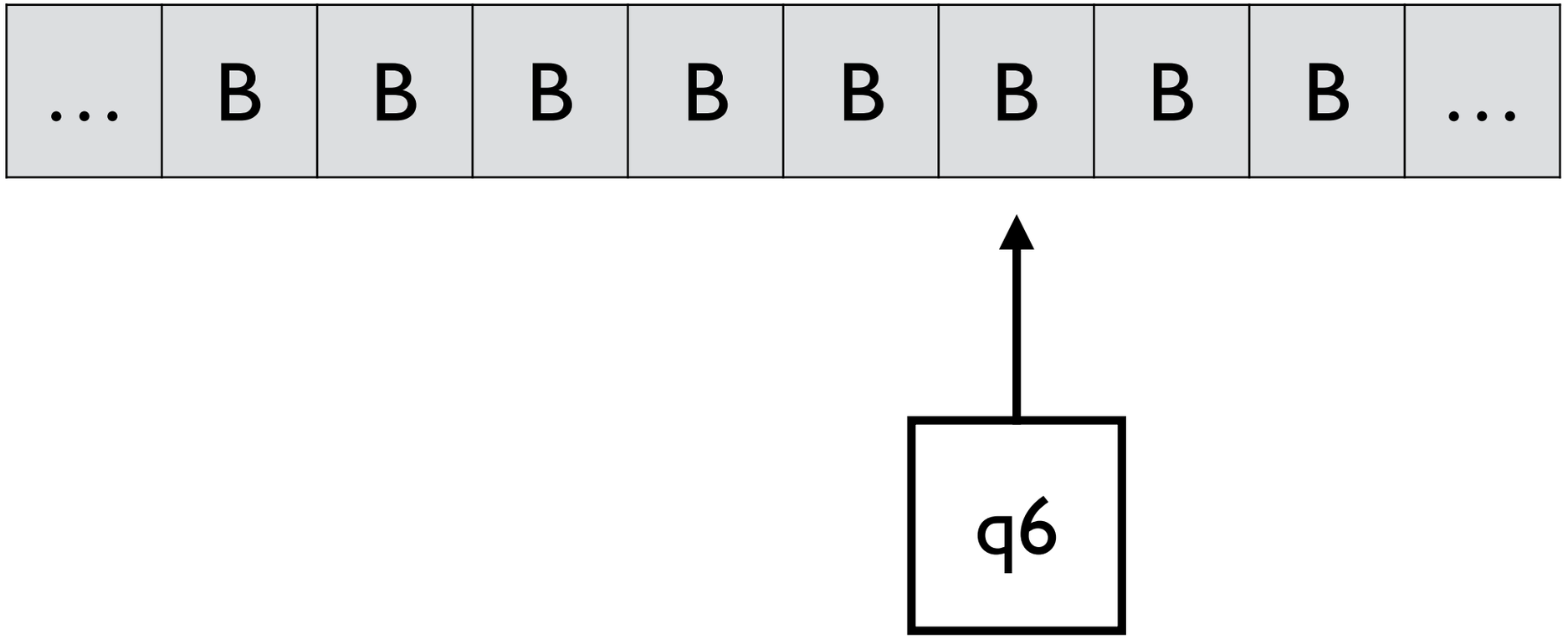
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

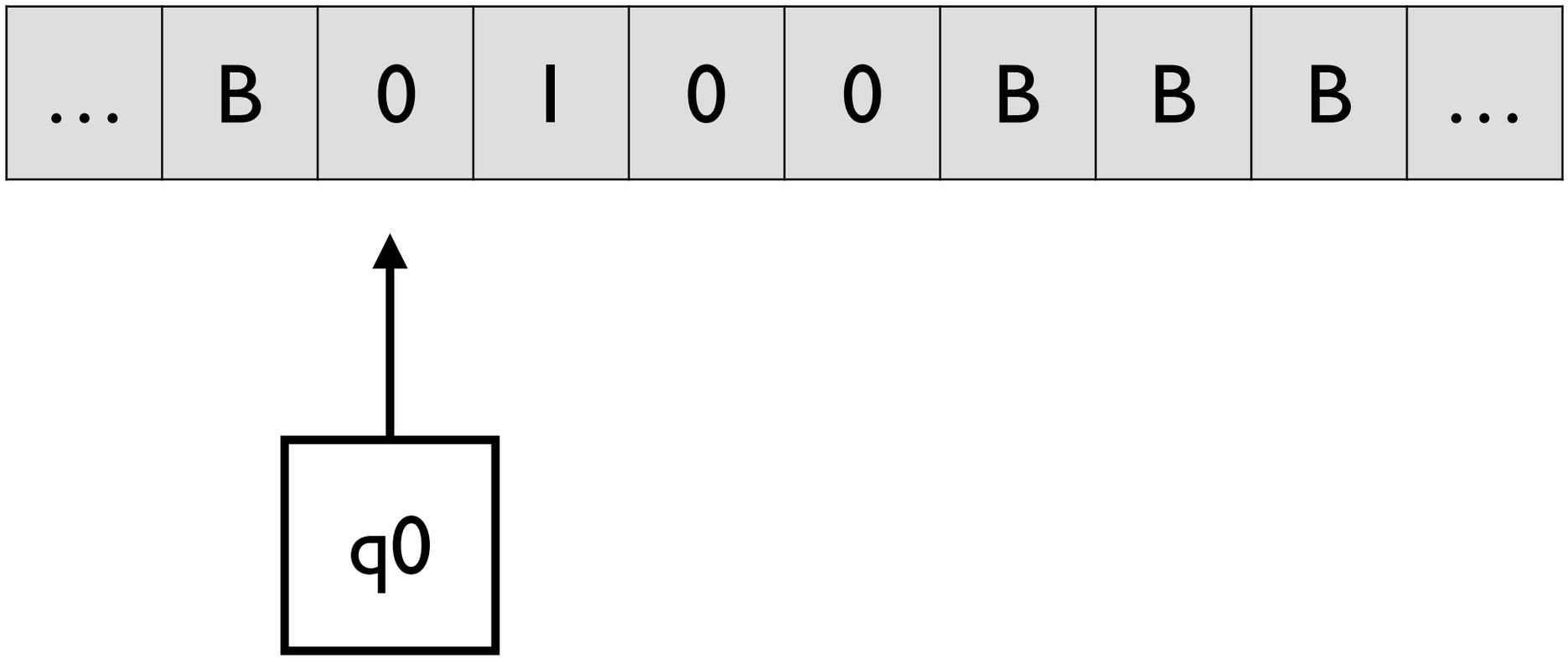


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

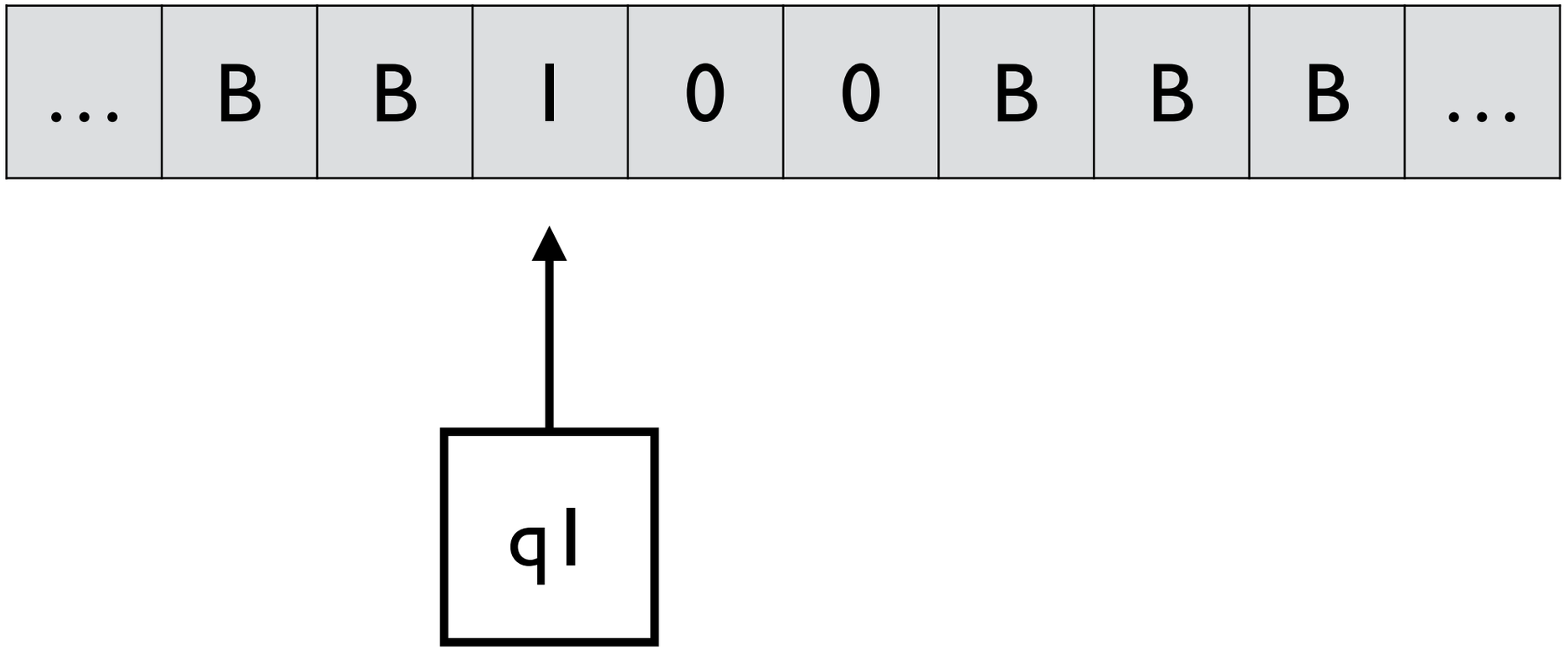


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

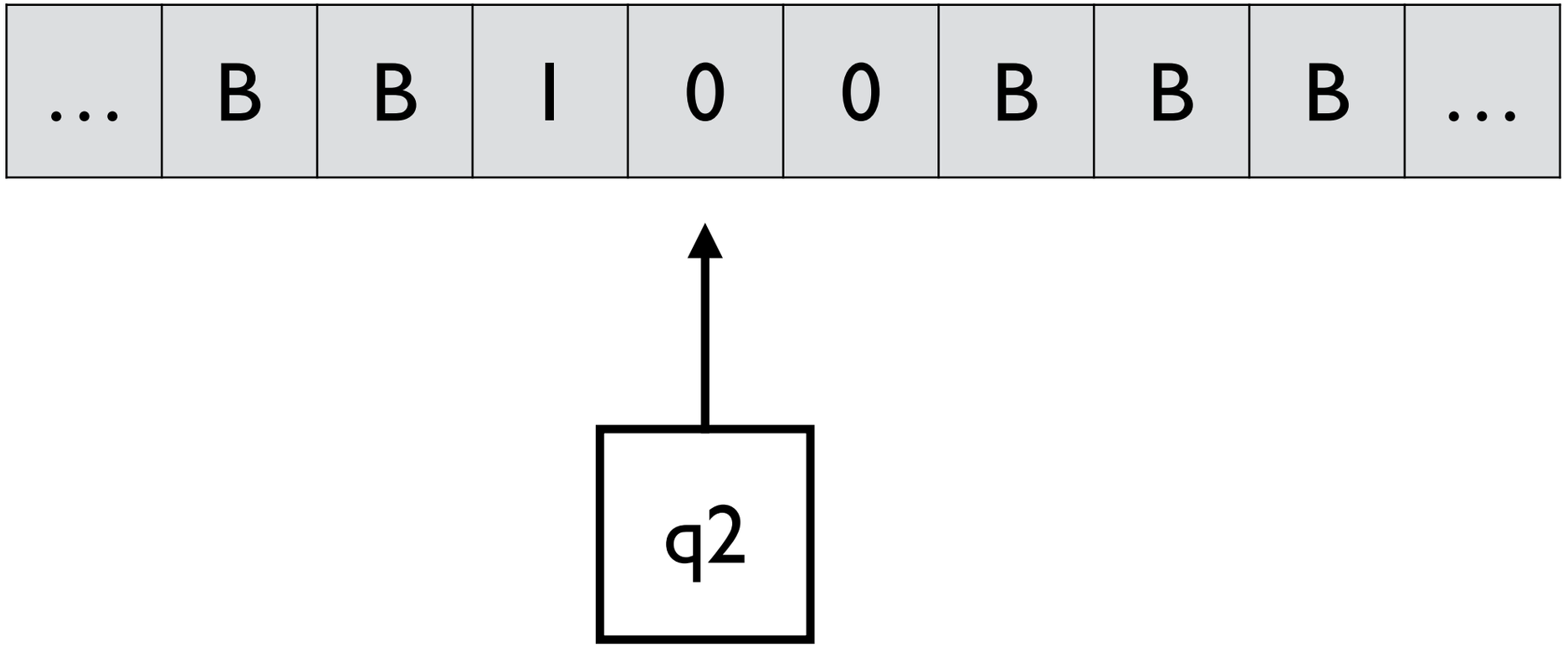
e.g., f(1,2)



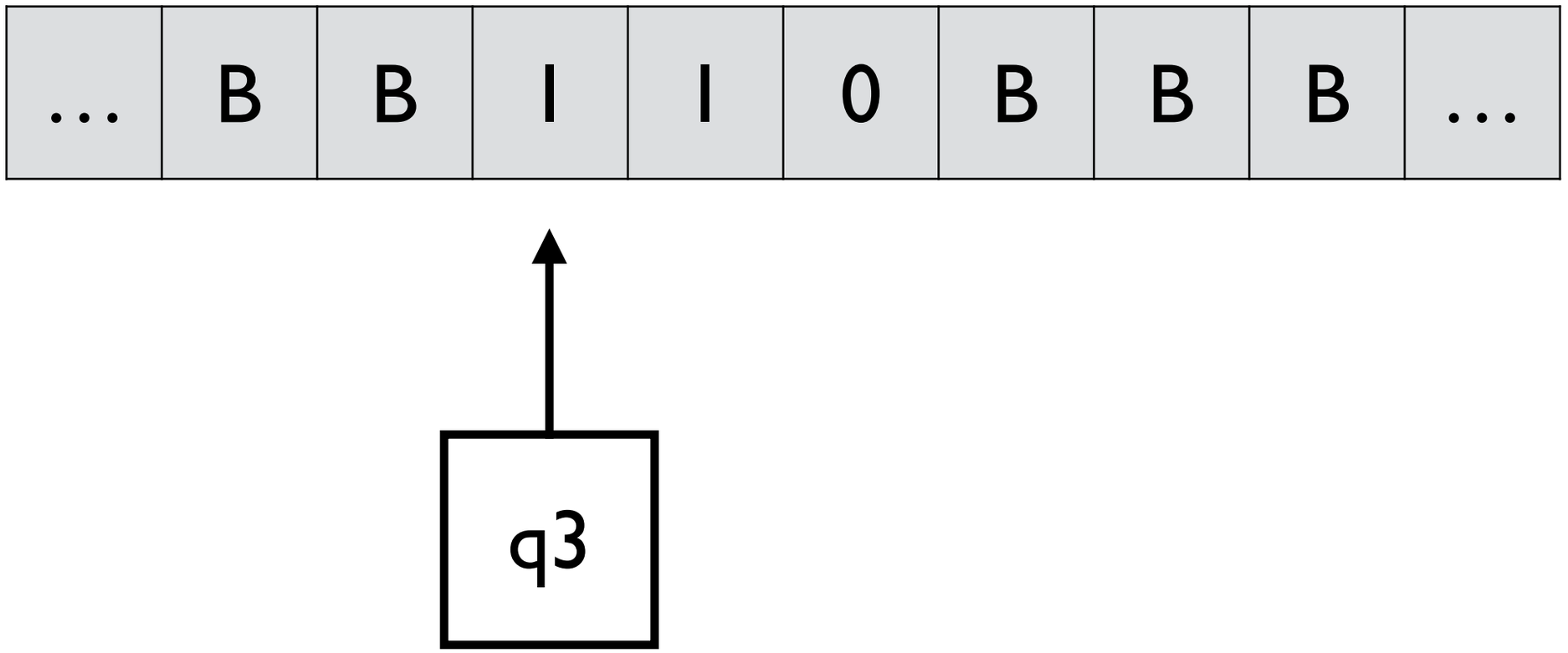
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



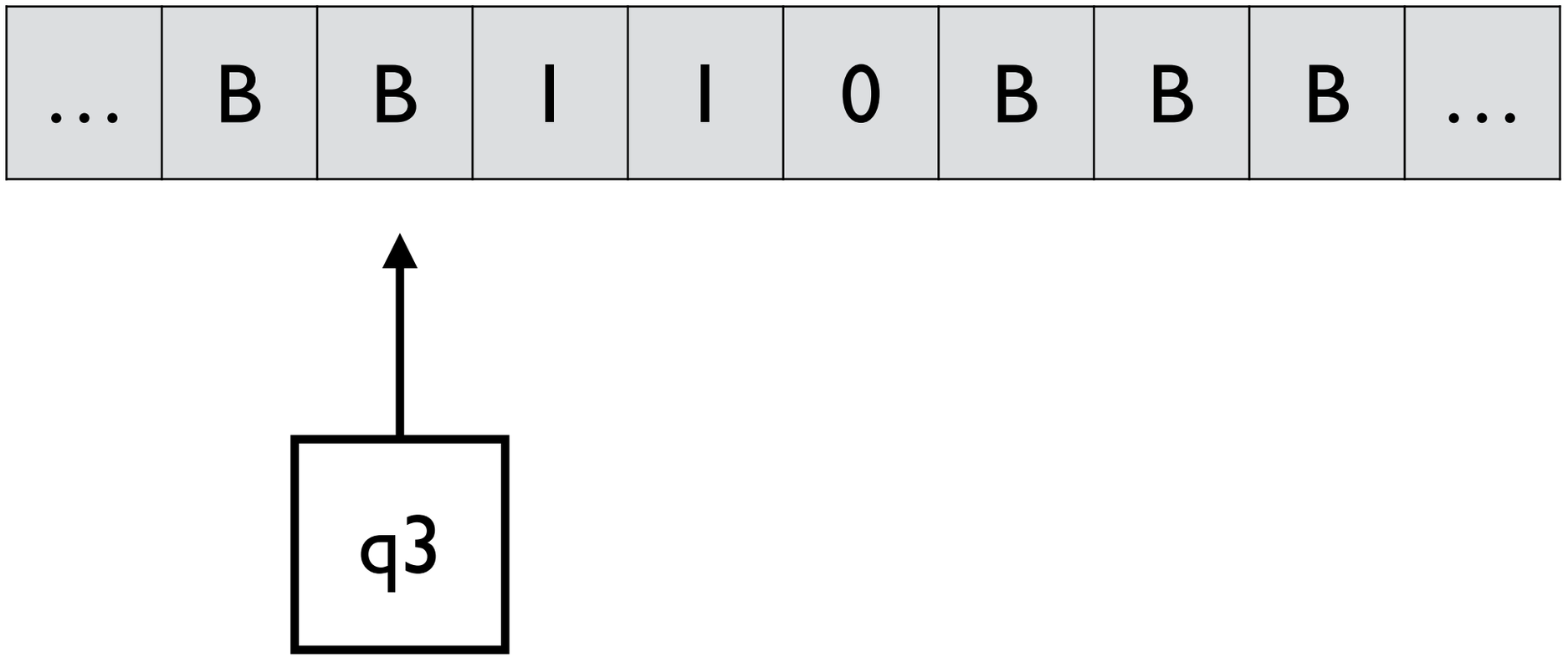
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



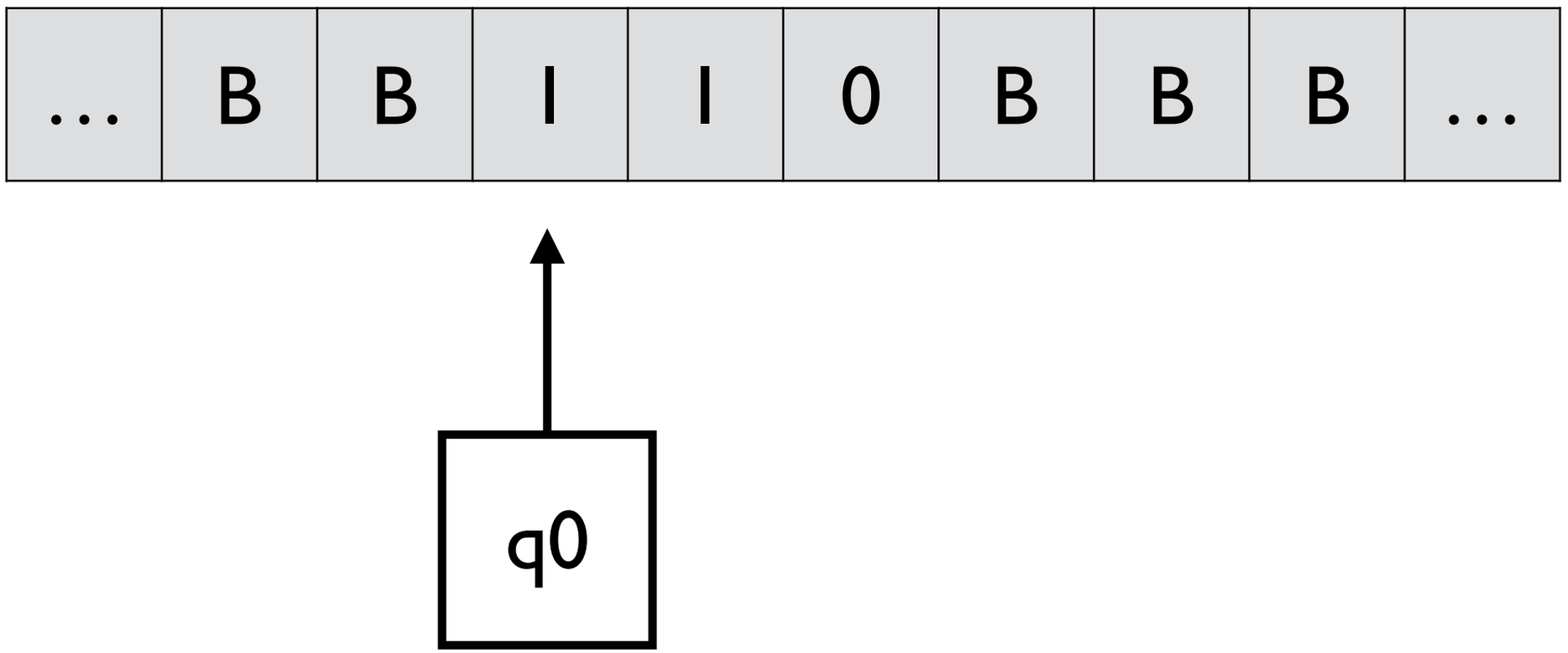
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



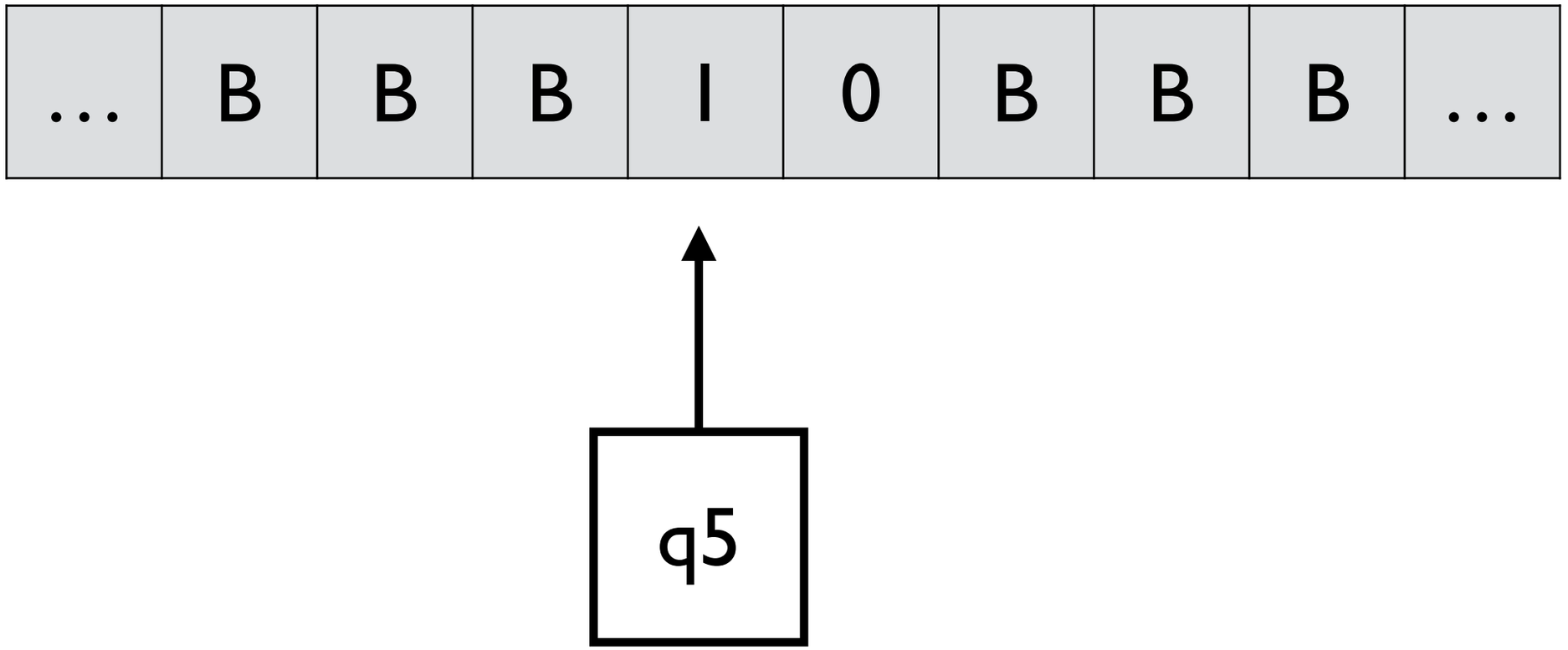
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



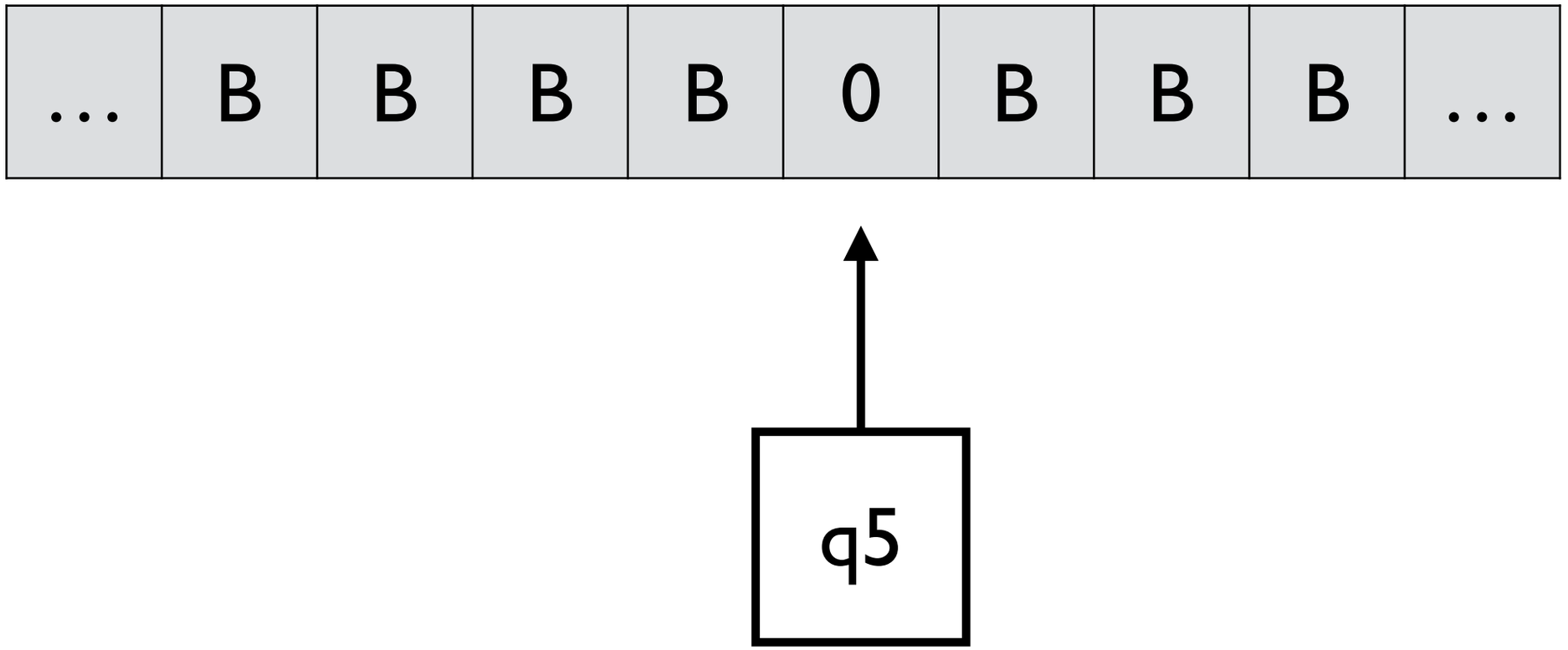
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



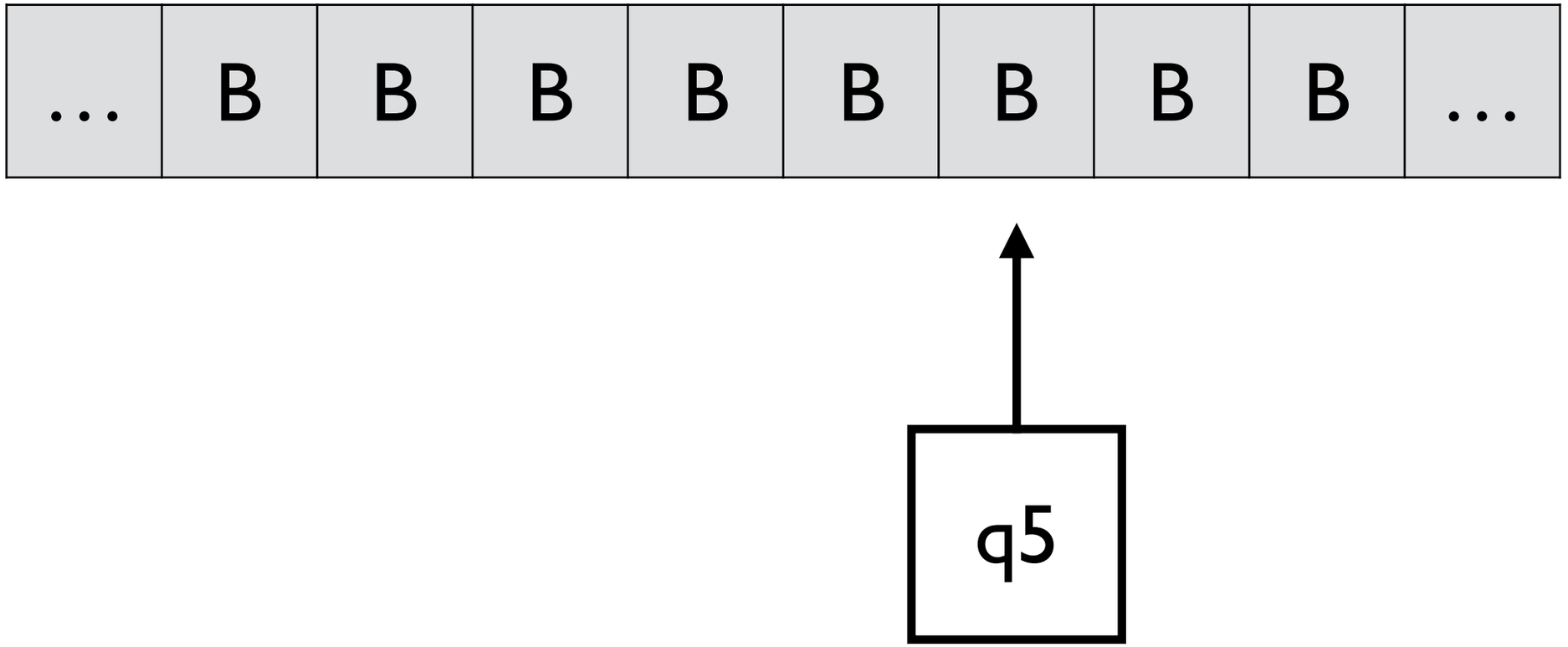
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



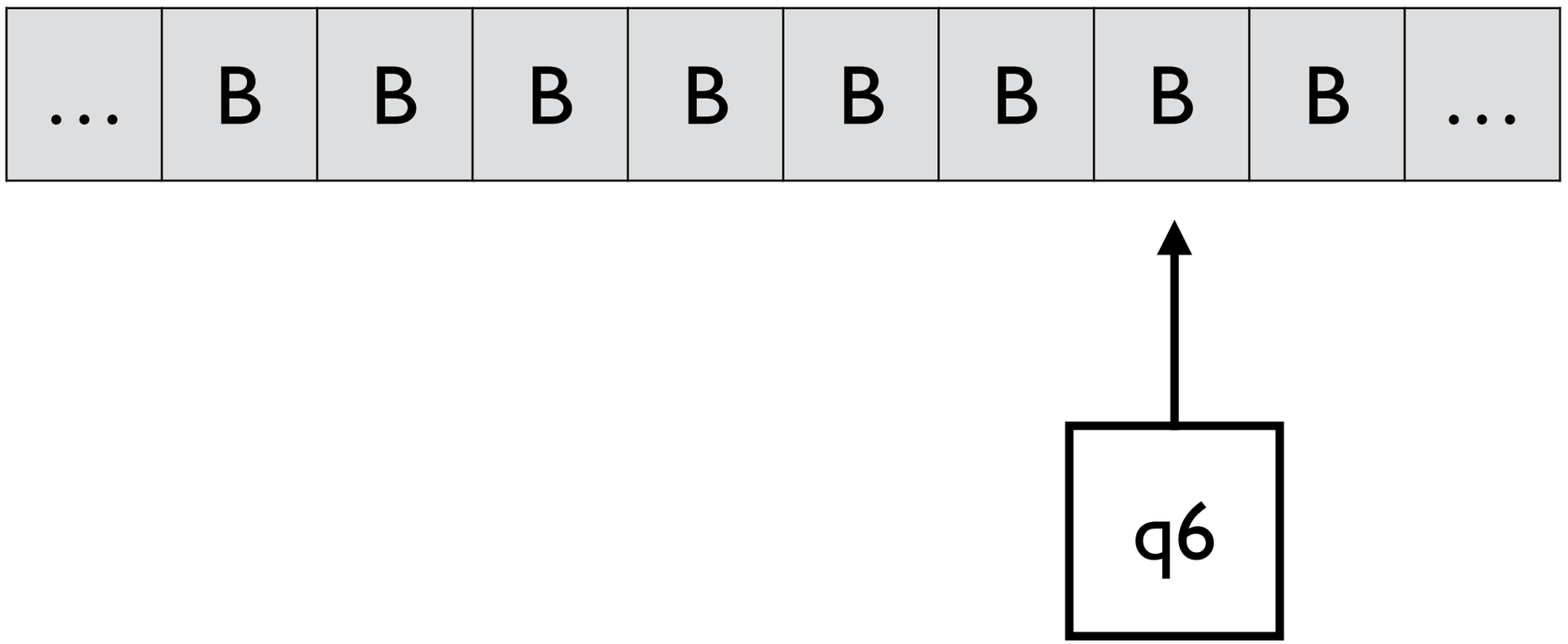
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



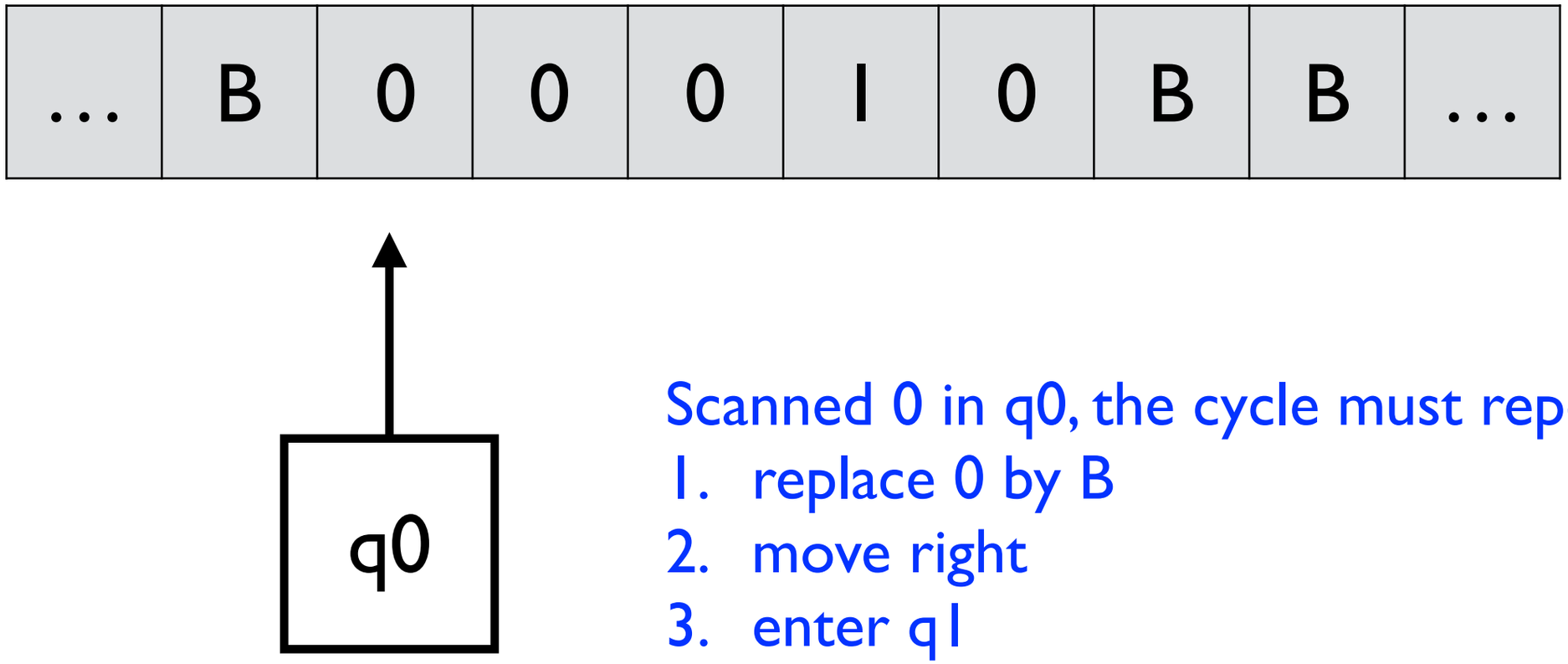
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

Example, revisited

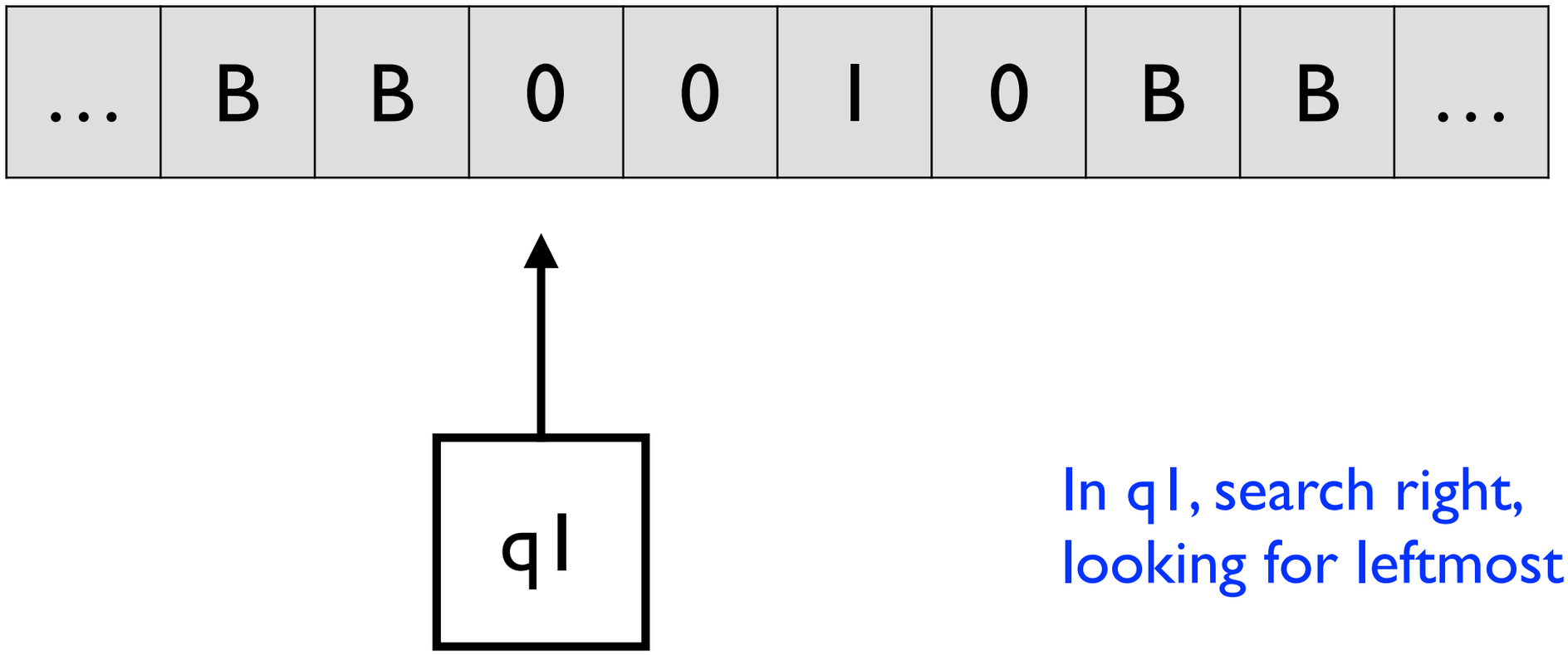
e.g., f(3,1)



Scanned 0 in q0, the cycle must repeat:

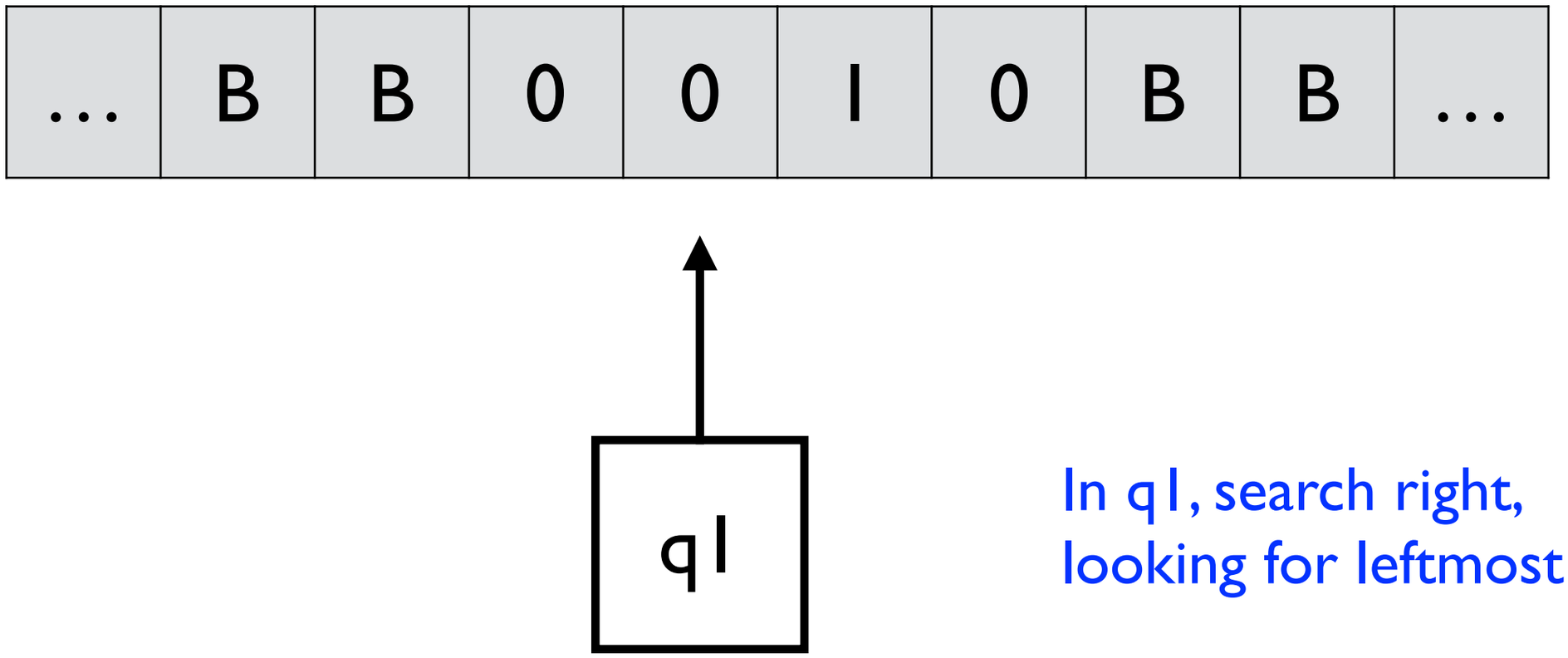
1. replace 0 by B
2. move right
3. enter q1

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



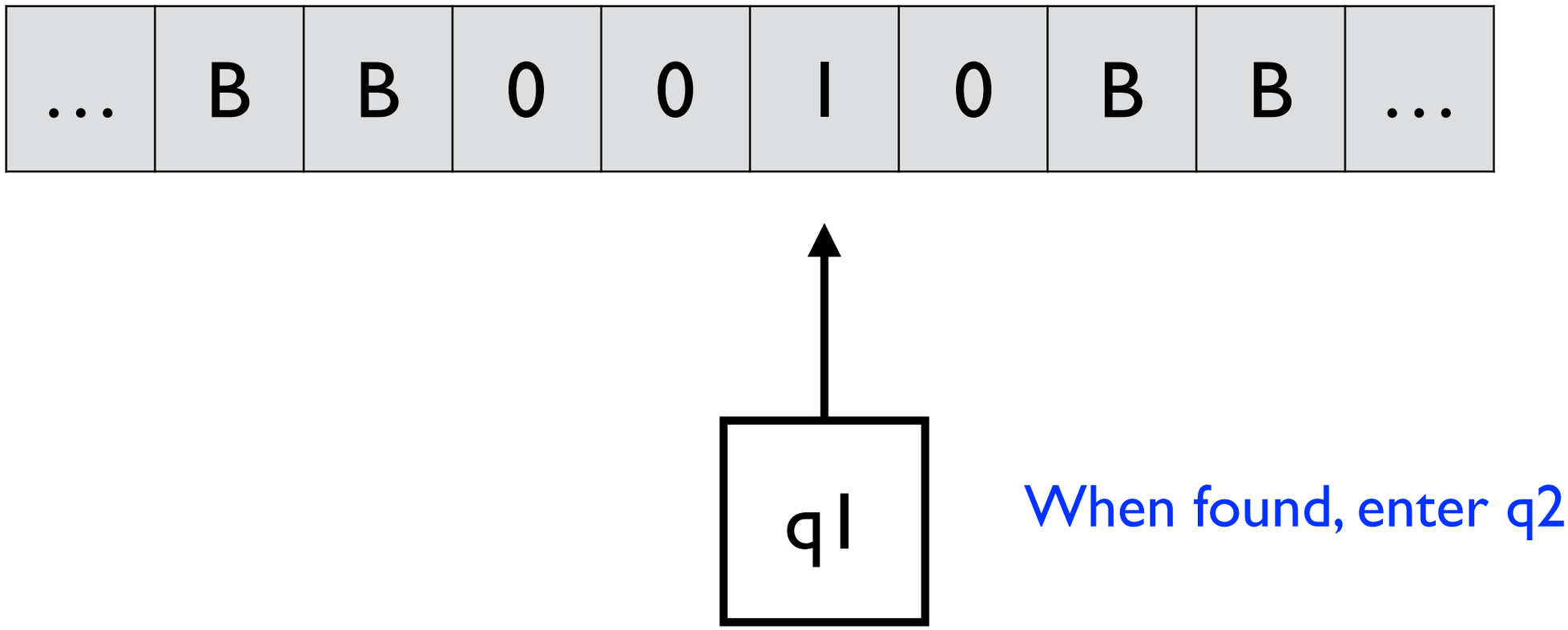
In q1, search right,
looking for leftmost 1

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

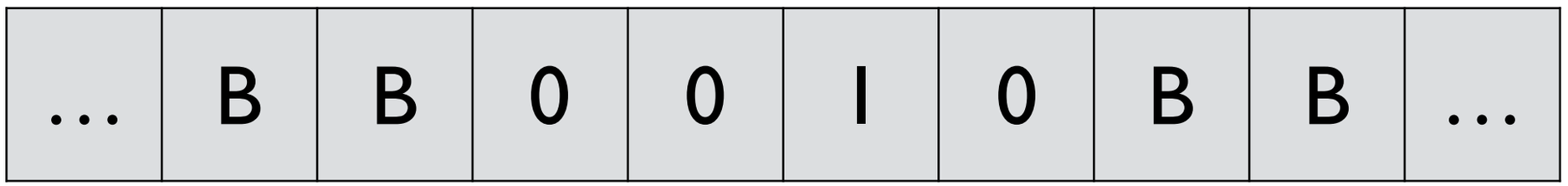


In q1, search right,
looking for leftmost 1

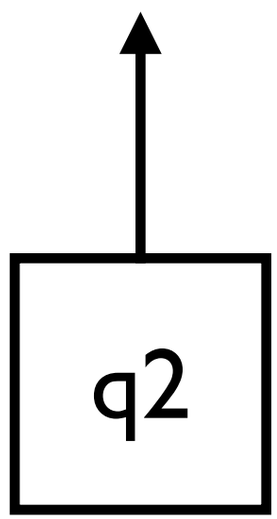
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



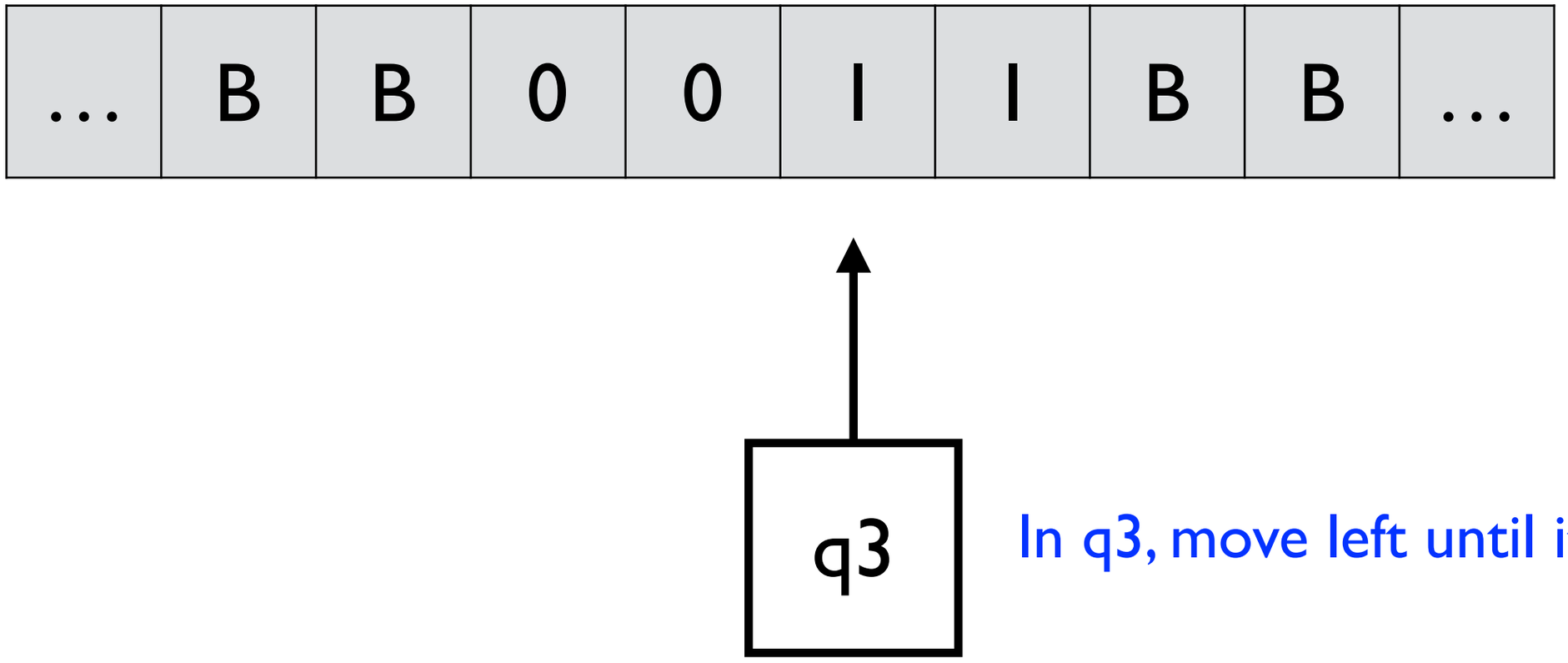
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



- In q2, move right until it finds 0
- When found, change it by 1
- enter q3

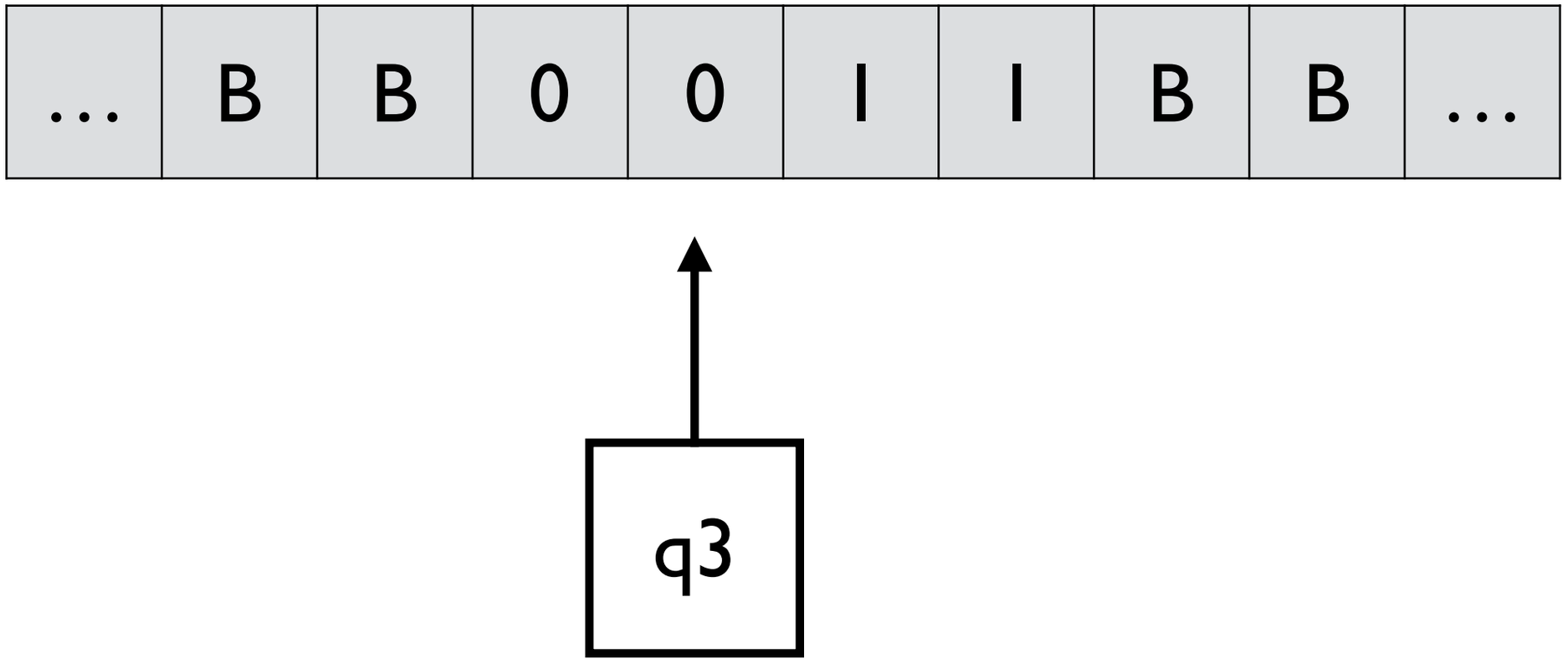


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

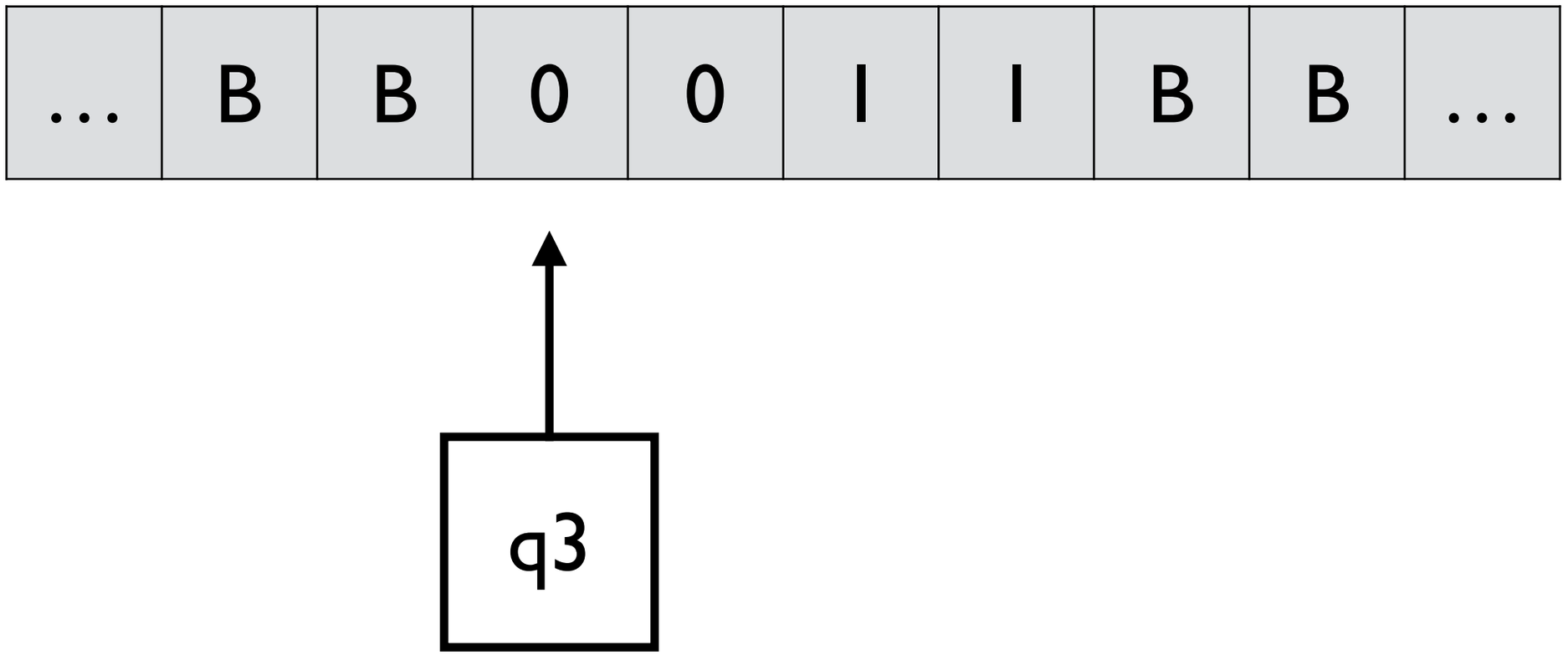


In q3, move left until it find B

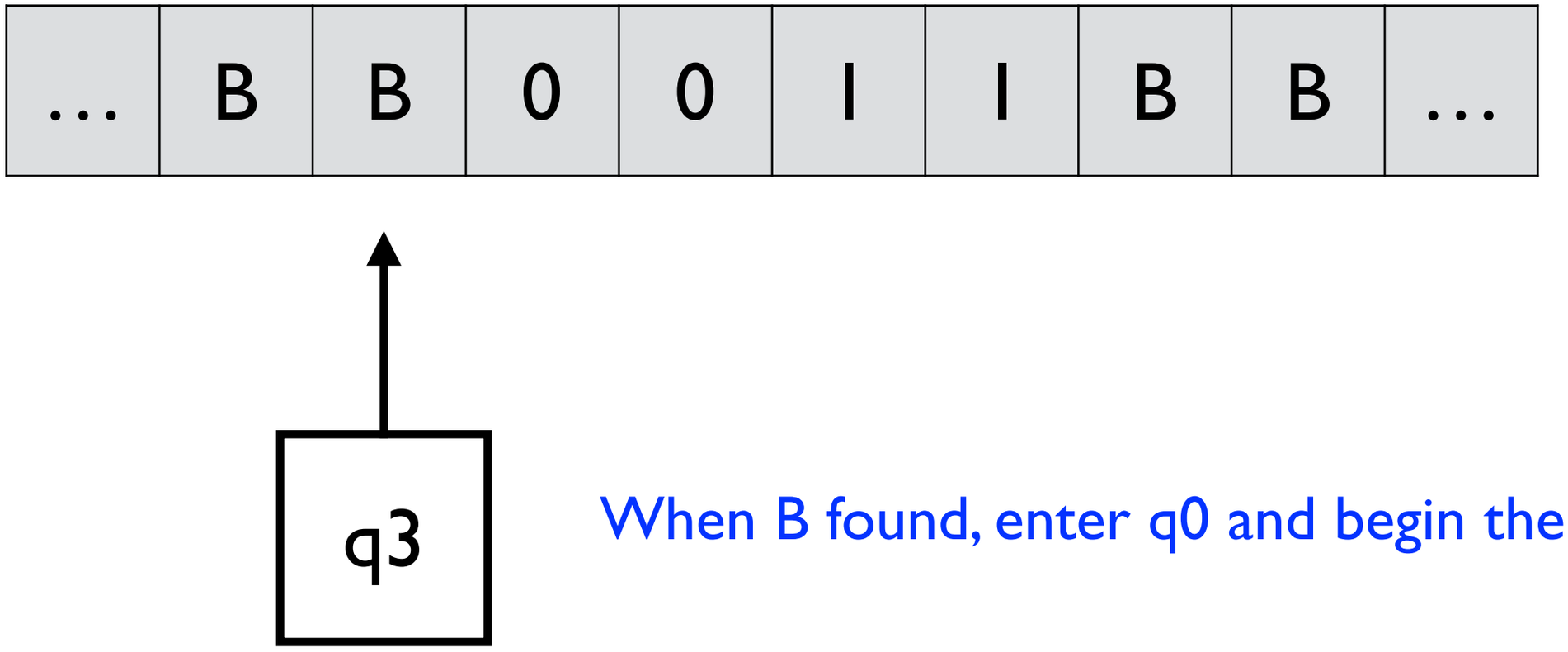
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

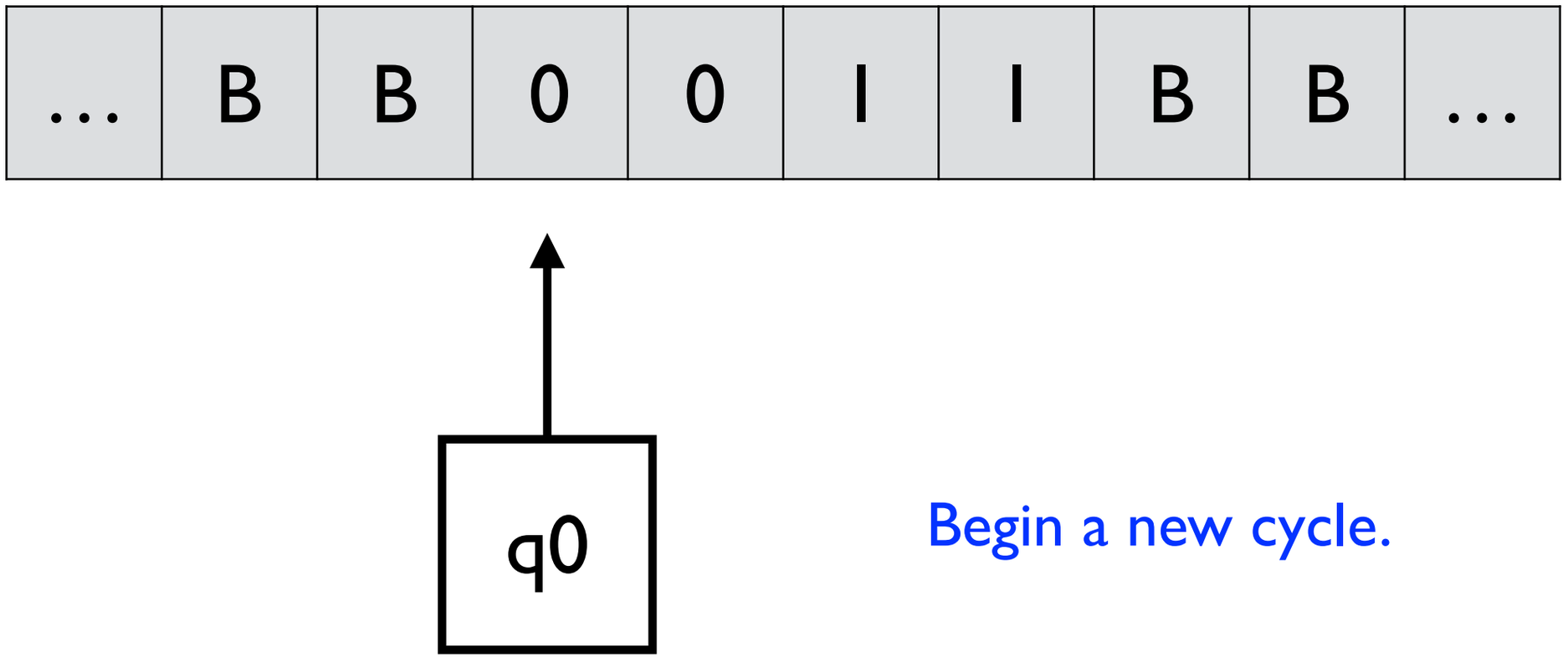


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



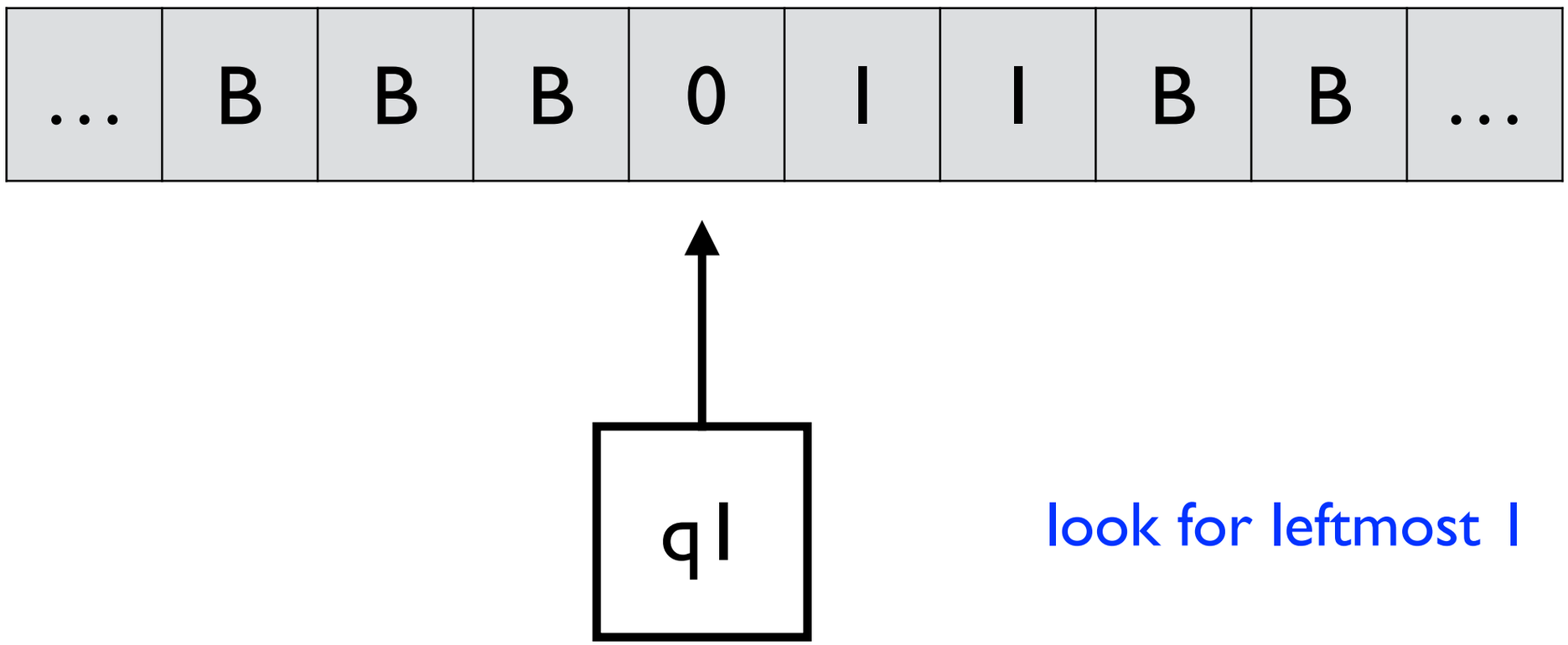
When B found, enter q0 and begin the cycle

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

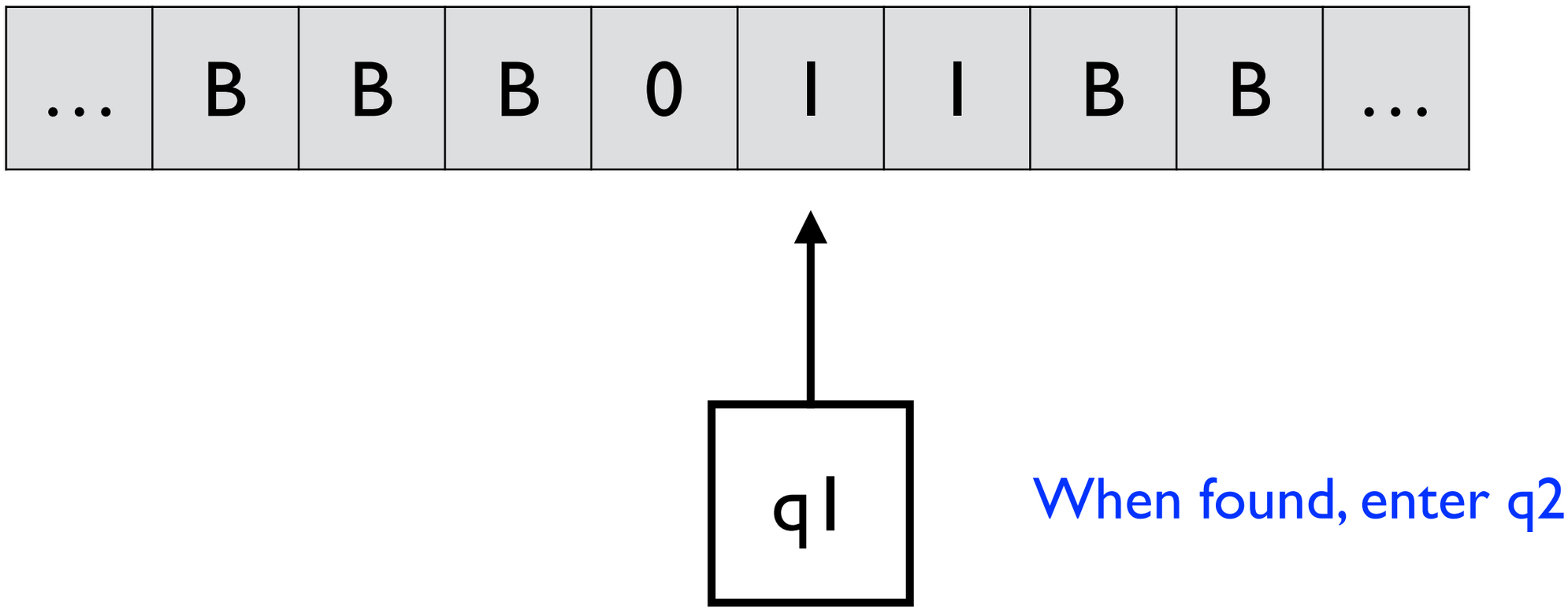


Begin a new cycle.

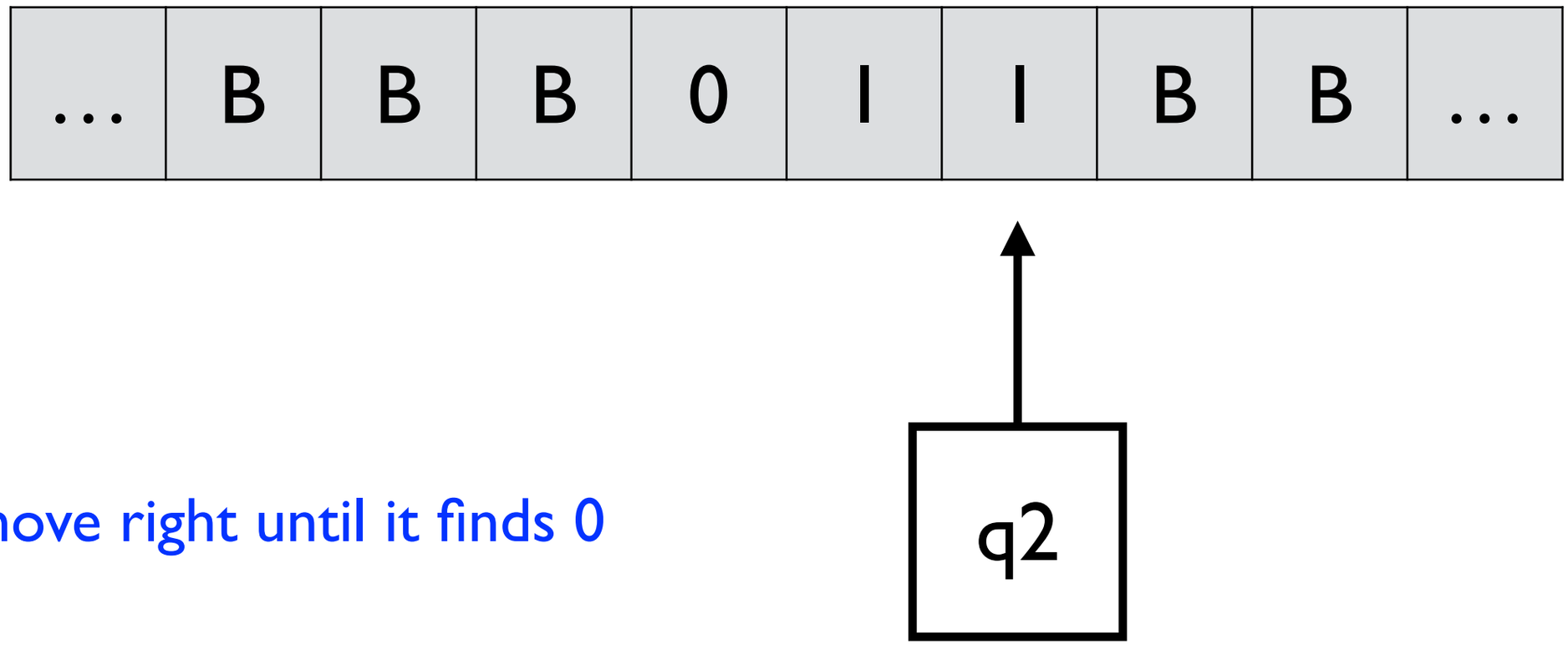
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

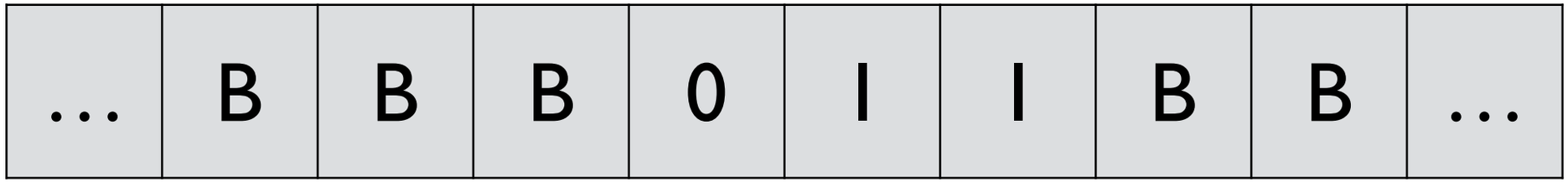


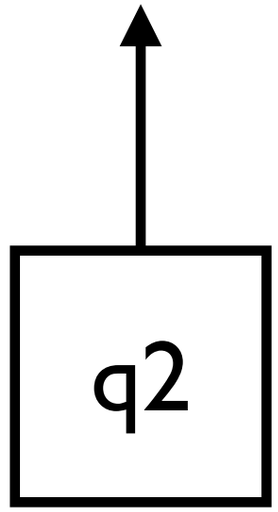
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



move right until it finds 0

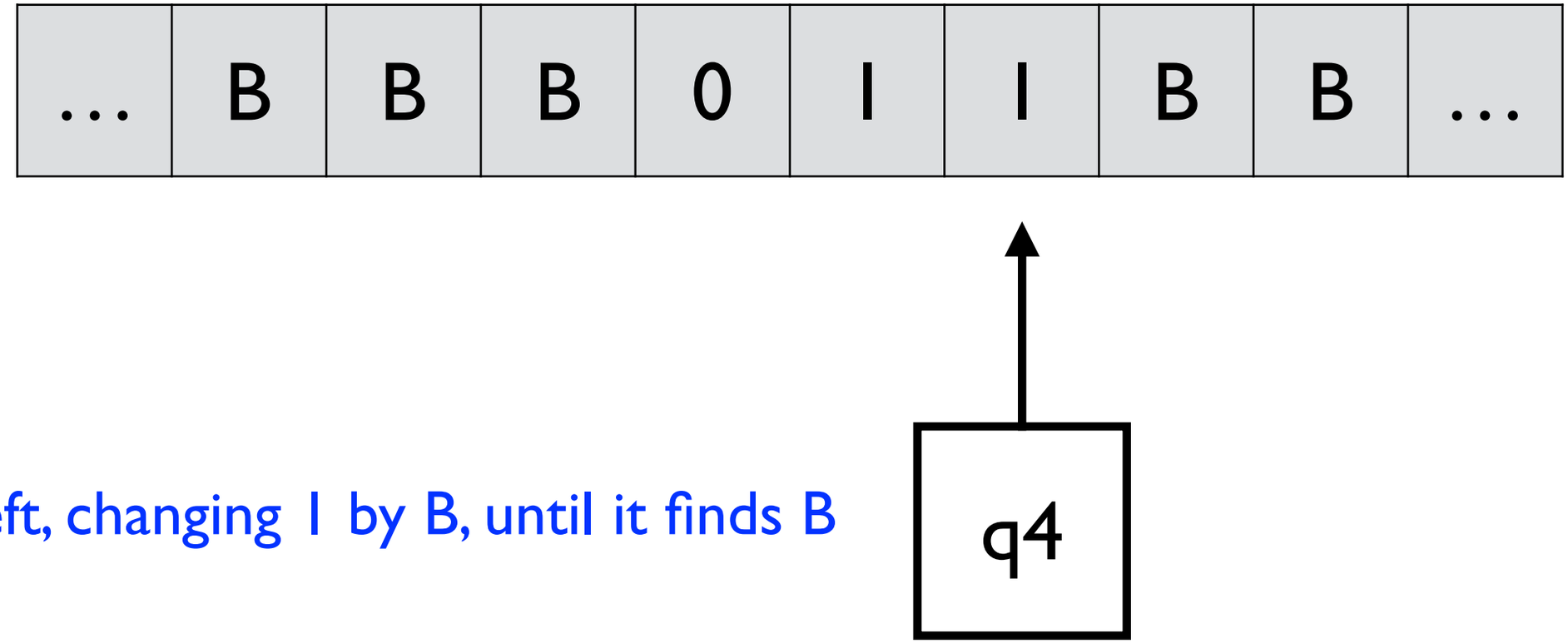
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			





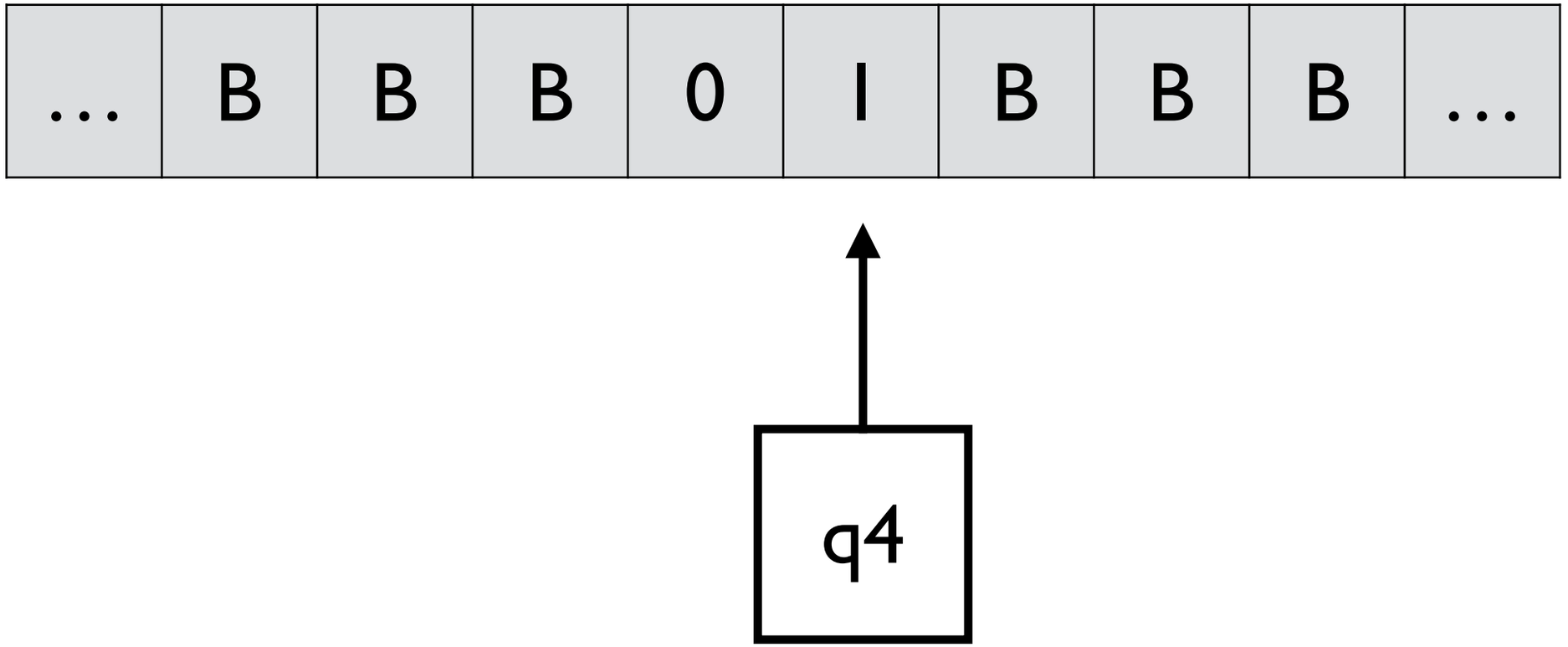
- No more 0's.
- All n 0's changed to 1
- n+1 0's changed to B
- m-(n+1) 0's on the tape
- replace all 1's by B and put one 0

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

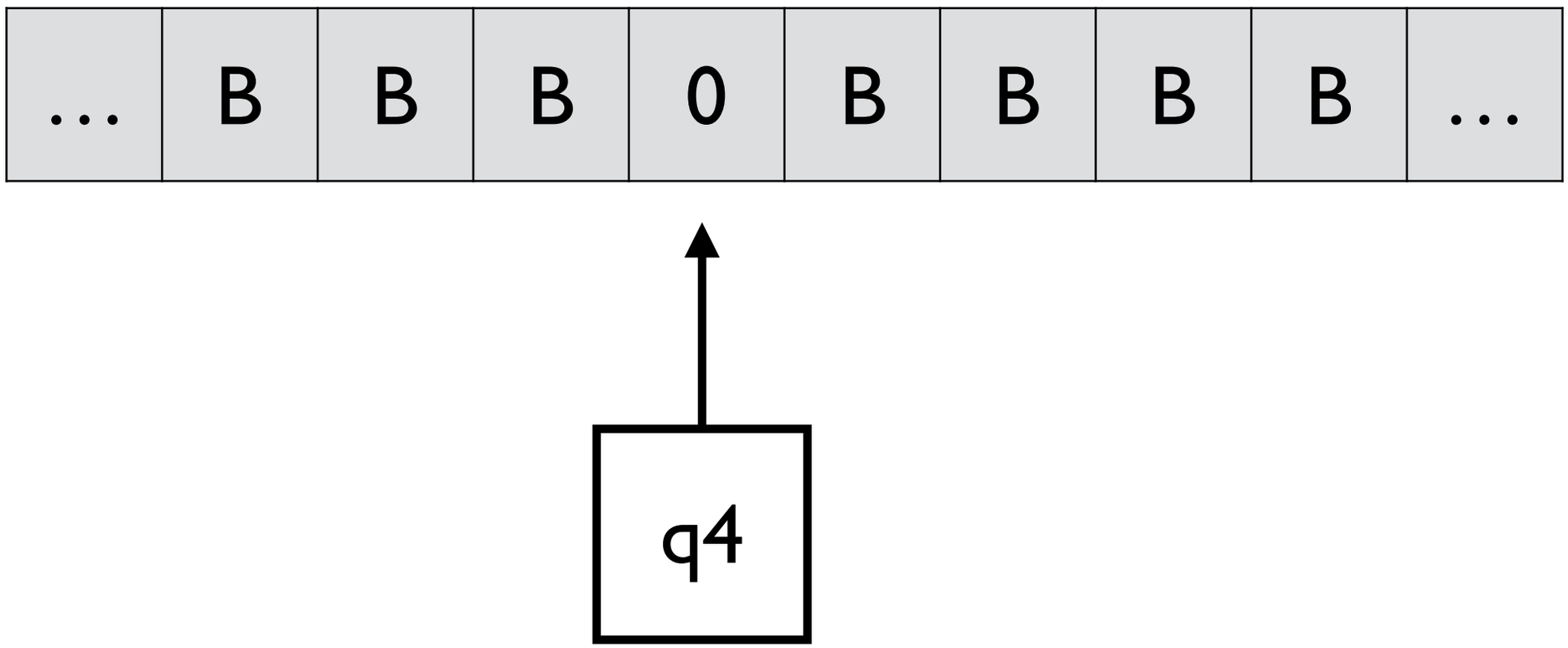


move left, changing I by B, until it finds B

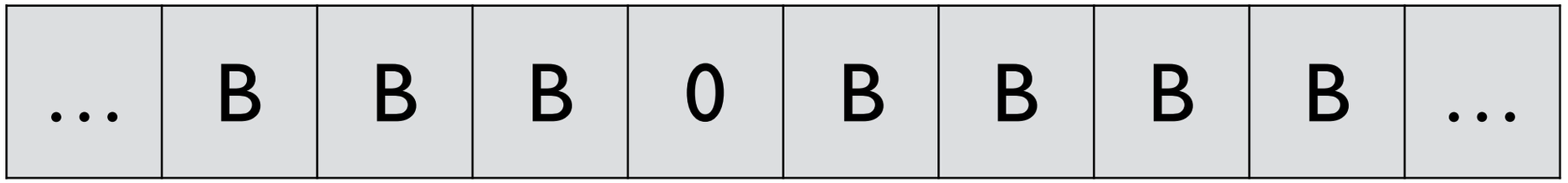
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

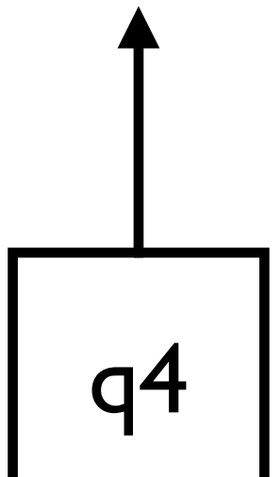


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



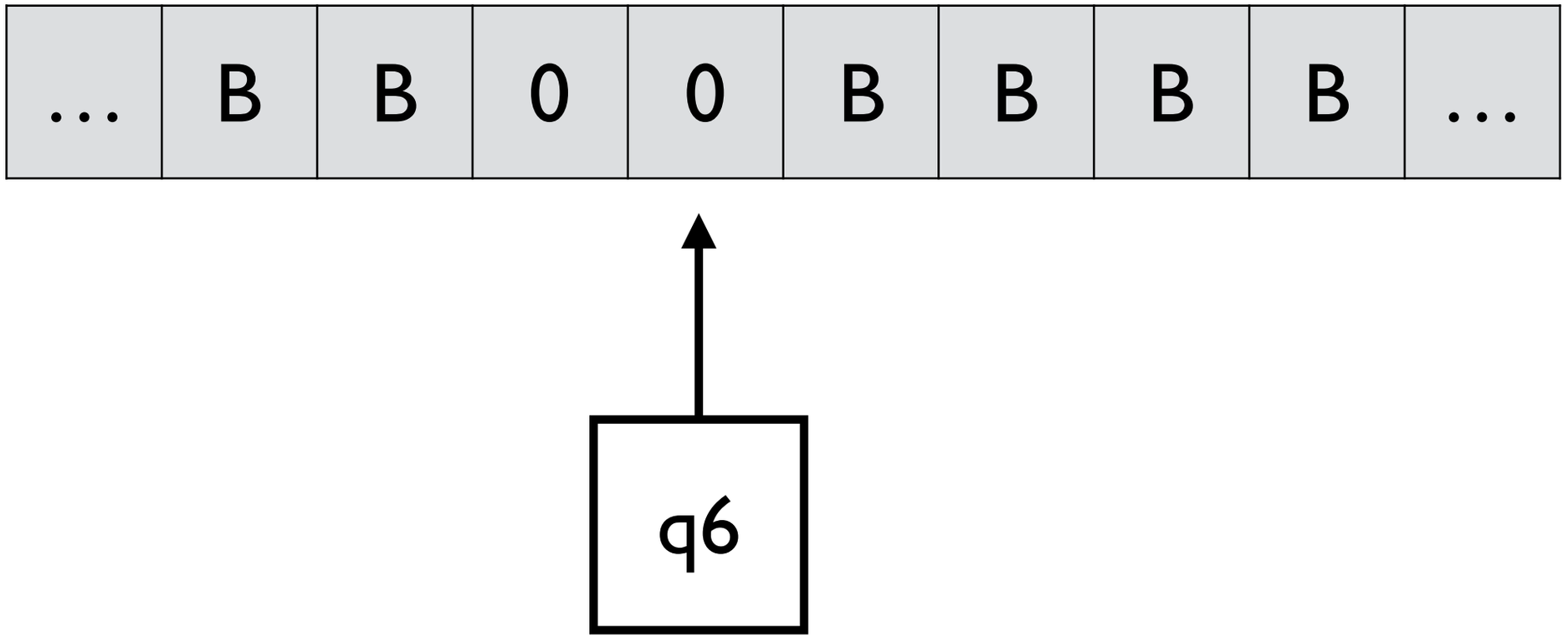
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			





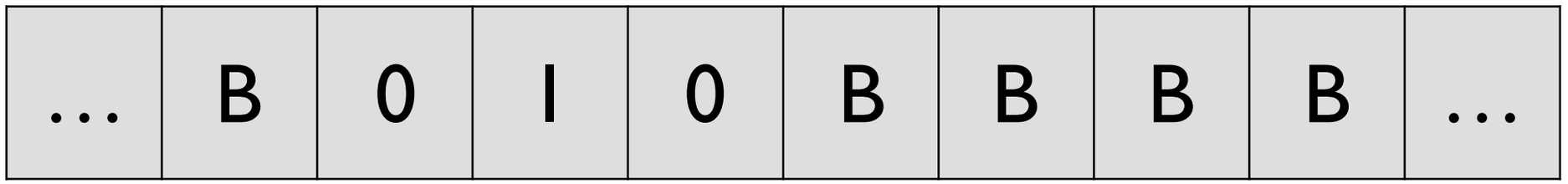
- change that B by 0
- enter the final state

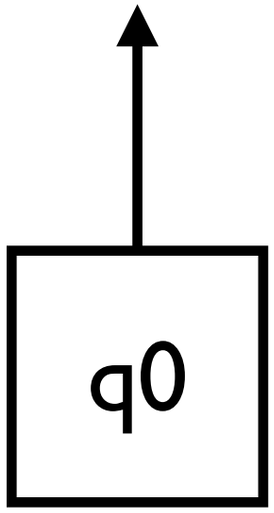
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

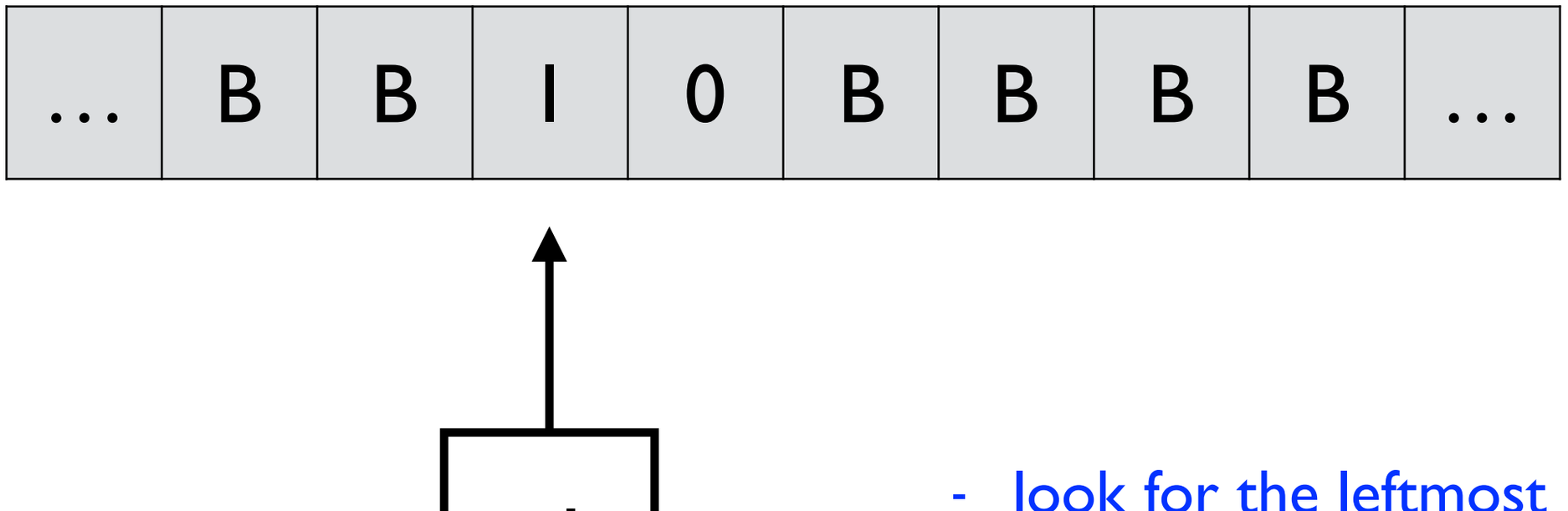
e.g., f(1,1)





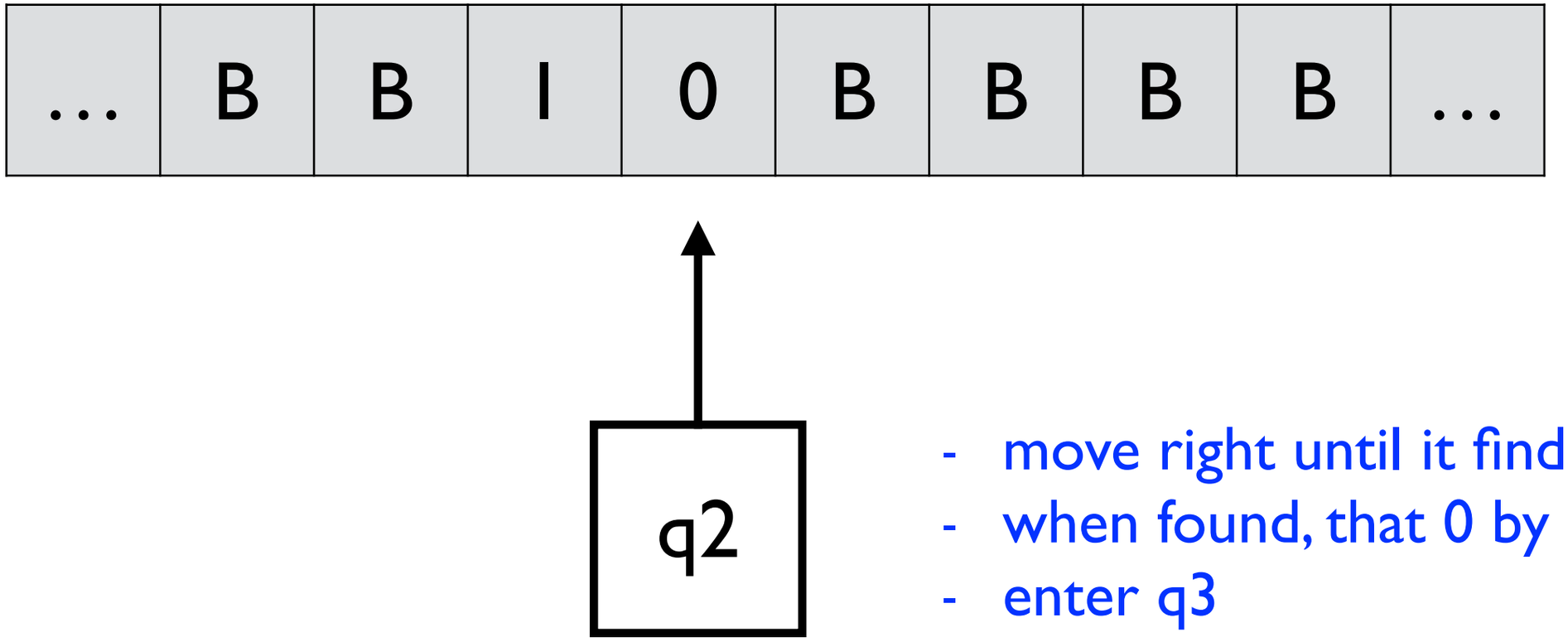
- change 0 by B
- enter q1

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



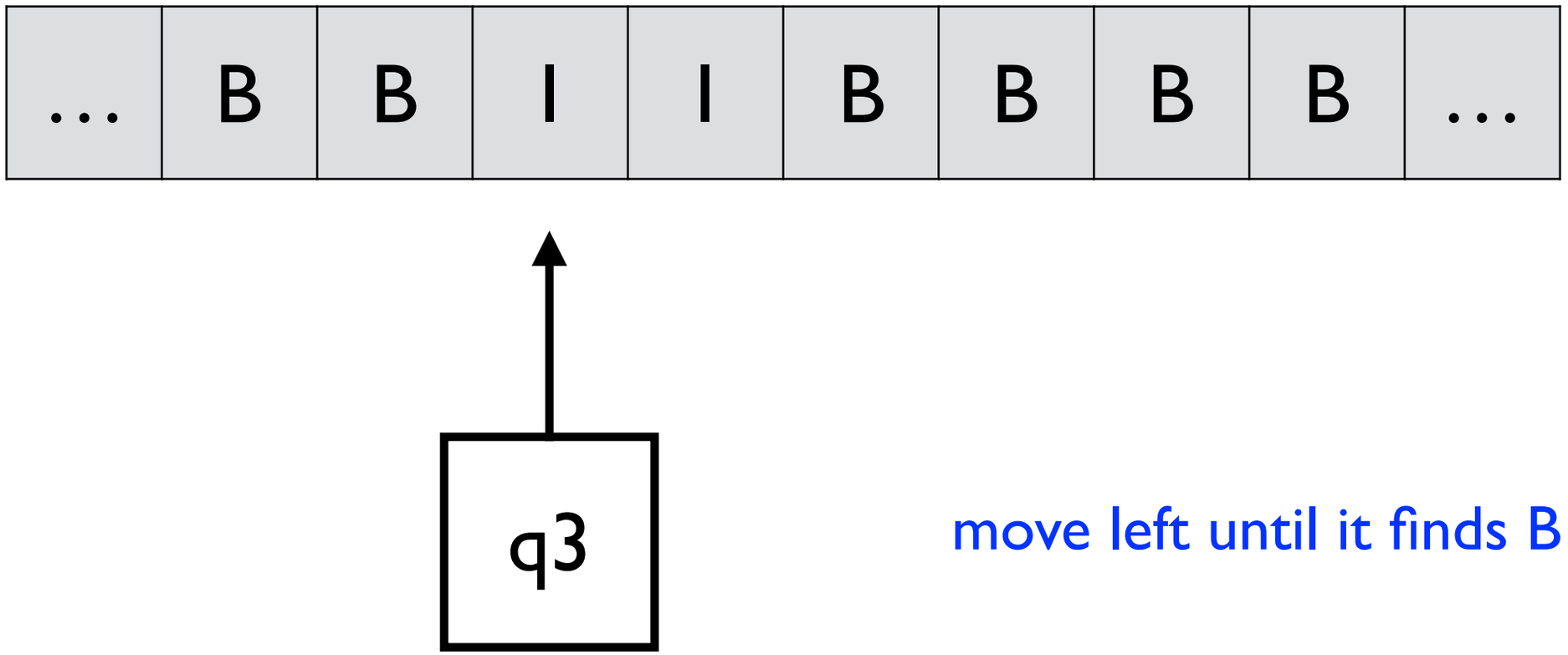
- look for the leftmost 1
- when found, enter q2

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



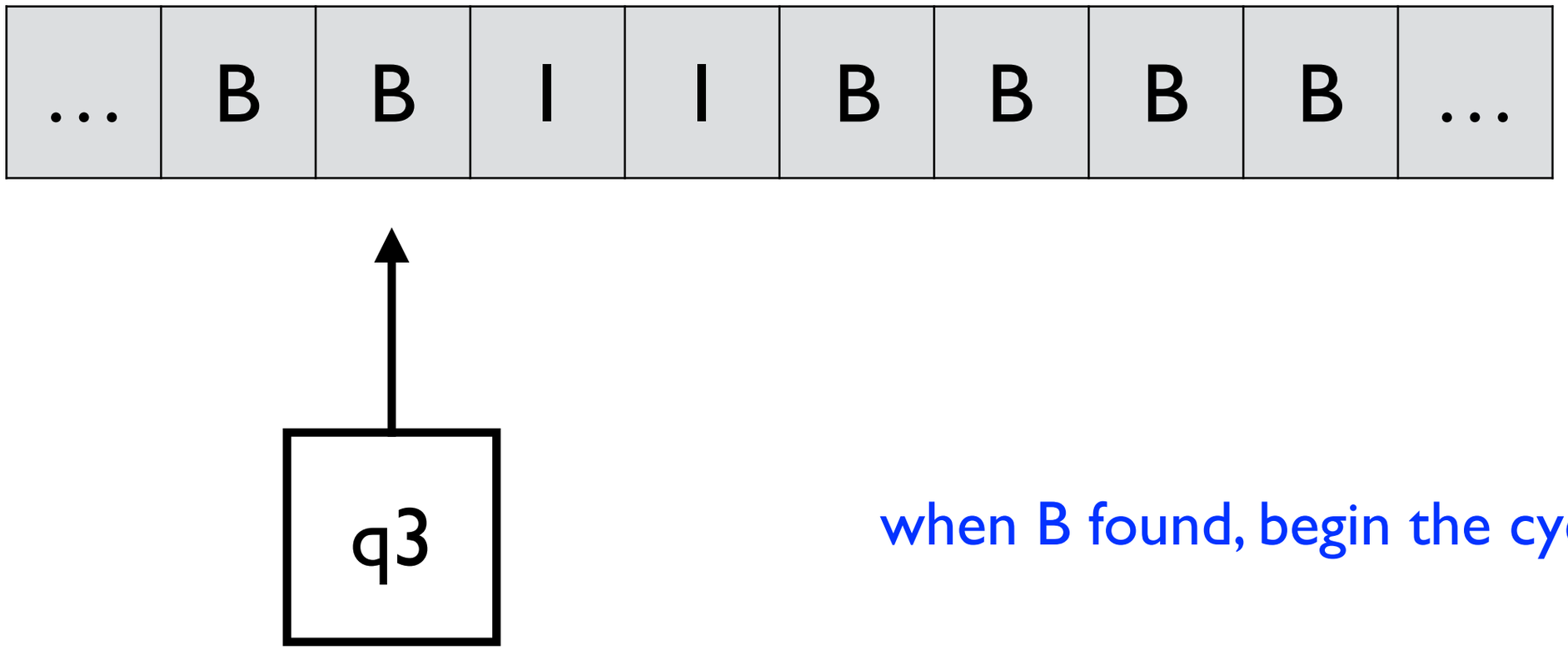
- move right until it finds 0
- when found, that 0 by 1
- enter q3

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



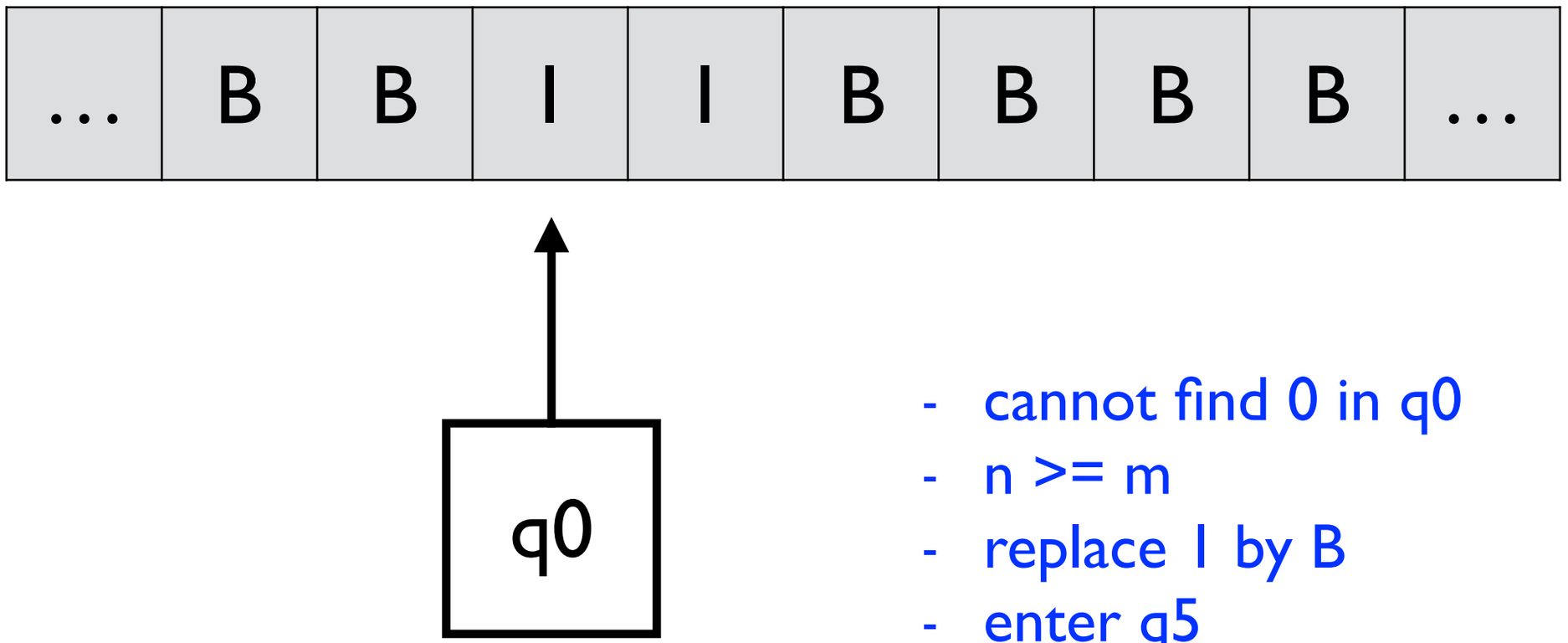
move left until it finds B

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



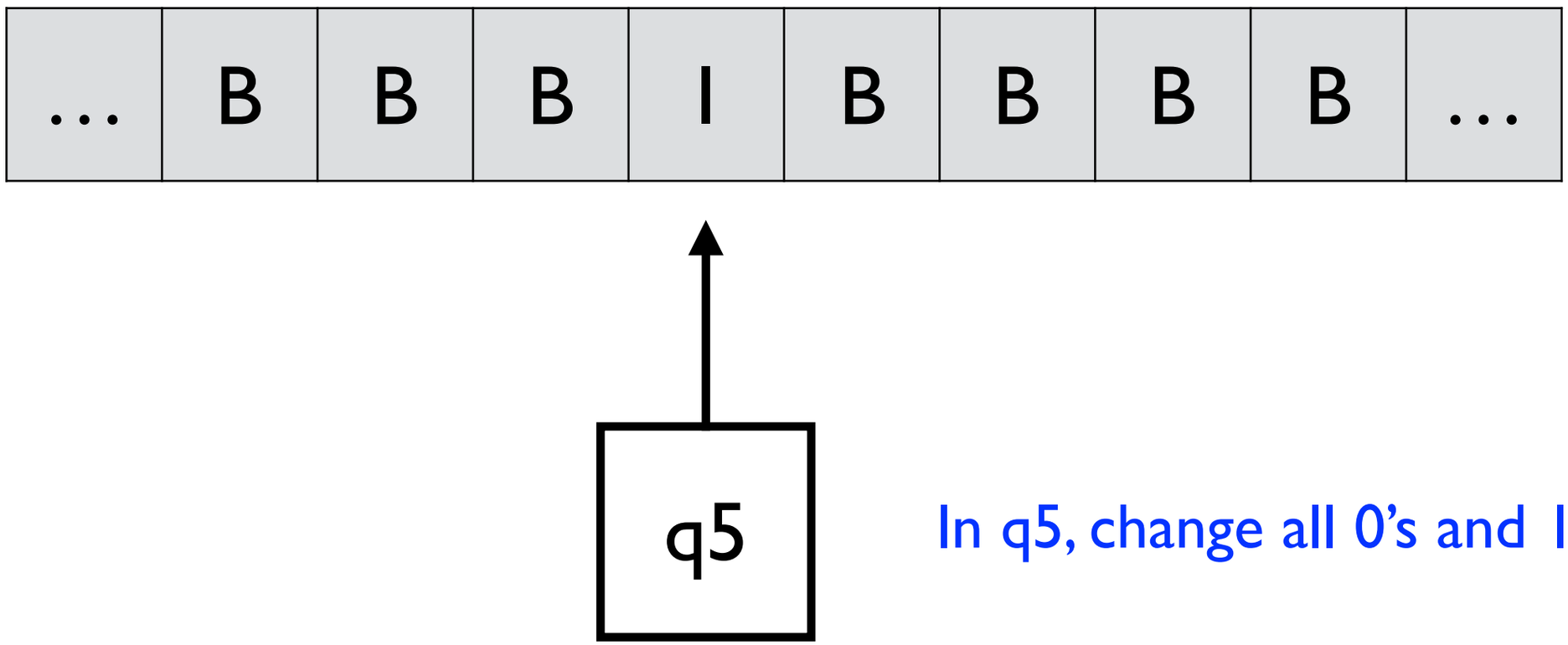
when B found, begin the cycle

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



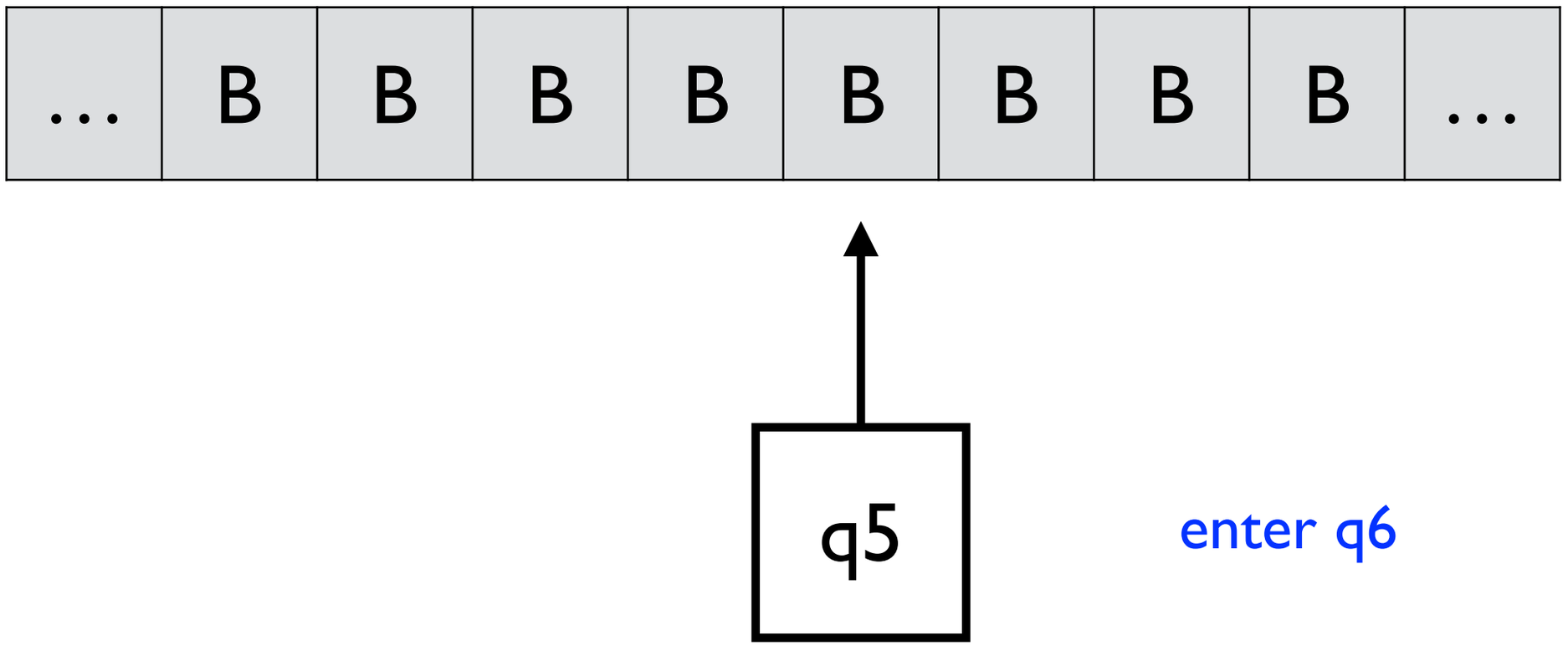
- cannot find 0 in q0
- n >= m
- replace 1 by B
- enter q5

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

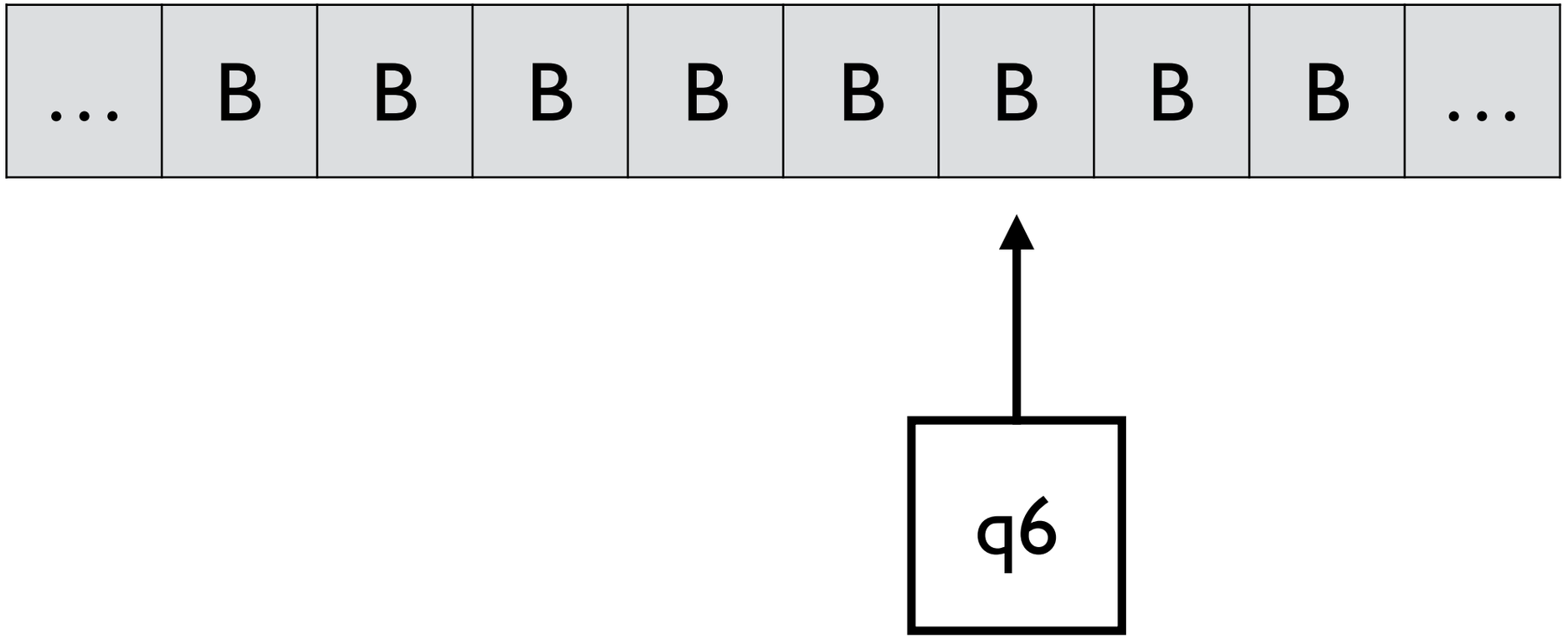


In q5, change all 0's and 1's to B

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			