Time Series Final Assessment
Exercise 2: Time-Varying CAPM

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1. Fixed-coefficient market model

The fitted market model is

$$r_t = \hat{\alpha} + \hat{\beta}\, r_{M,t} + e_t,$$

where $\hat{\alpha} = 0.0122$ and $\hat{\beta} = 0.8513$.

I used ordinary least squares to fit a linear regression of Pfizer's excess return on the market excess return. This classical CAPM intercept and slope (alpha and beta) summarize the average relationship over the full sample. The estimate $\hat{\beta} < 1$ suggests Pfizer's returns co-move with the market but with slightly lower sensitivity.

2. Estimated innovation standard errors

The estimated standard error of the innovation to α_t is

$$\widehat{\sigma}_\eta = 0.0000.$$

The estimated standard error of the innovation to β_t is

$$\widehat{\sigma}_\varepsilon = 0.0682.$$

I estimated process noise variances by maximizing the likelihood of the state-space model. A zero estimate for σ_η indicates no detectable drift in the intercept, while $\sigma_\varepsilon > 0$ implies time variation in beta. This suggests that only the market loading varied significantly over time.

3. Smoothed estimates of α_t and β_t

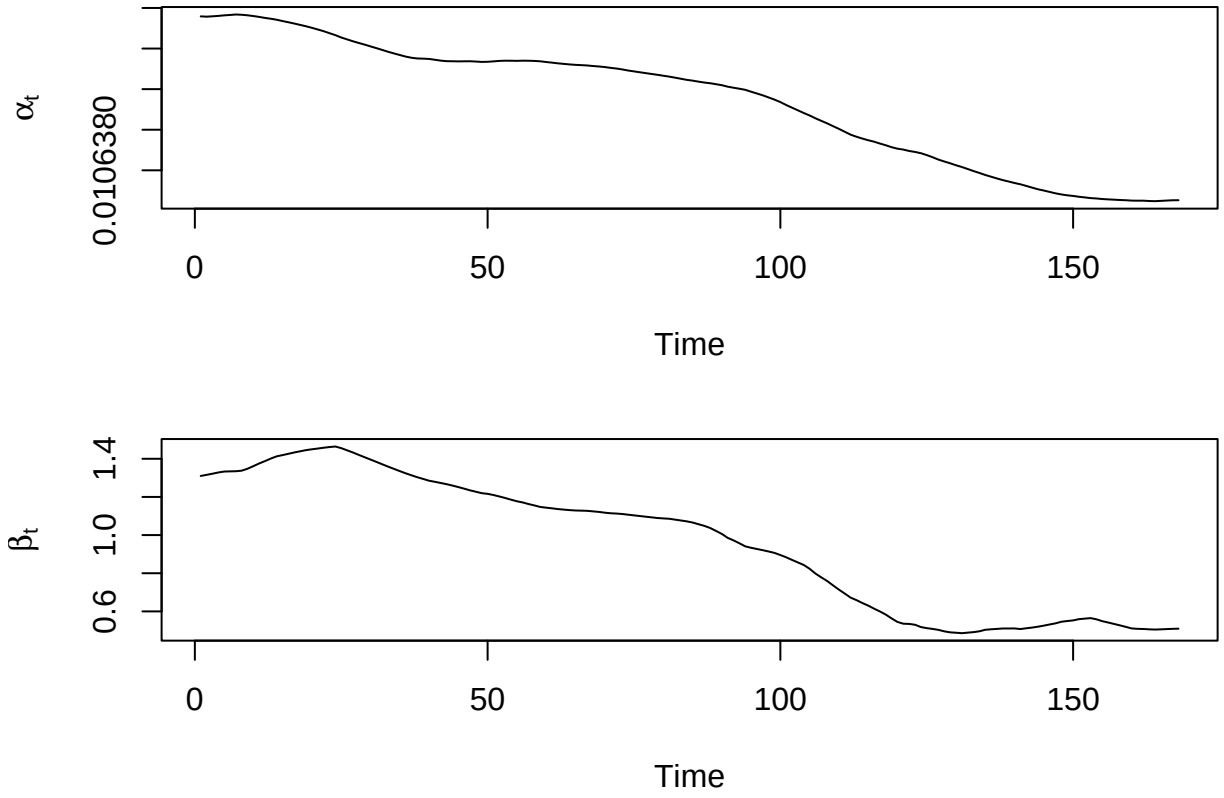

Figure 1: Smoothed state estimates of α_t and β_t

The smoothing algorithm (Rauch–Tung–Striebel smoother) uses all data to infer the latent states. The flat alpha series confirms a constant intercept, while the downward drift in beta over the sample period highlights changing market sensitivity – possibly reflecting shifts in Pfizer's risk profile.

Conclusions

The results indicate that the intercept term remains effectively constant over the sample period, as evidenced by a zero innovation variance for α_t. In contrast, the slope parameter β_t exhibits a pronounced downward trend, suggesting that Pfizer's market exposure to the S&P 500 gradually declined from 1990 through 2003. From a risk-management perspective, this declining beta implies diminishing systematic risk, which may have allowed portfolio managers to reduce hedging costs over time. Comparisons between the static CAPM and the state-space specification show that allowing for time variation in beta yields a more flexible model that captures changing dynamics and improves in-sample fit. It should be noted, however, that the zero estimate for σ_η may reflect limited statistical power to detect small intercept drifts, and that potential structural breaks were not explicitly modelled. Overall, incorporating a time-varying beta enhances our understanding of Pfizer's evolving risk profile and can inform more responsive portfolio allocation strategies in dynamic markets.