

Time Series Final Assessment

Exercise 3: PPI AR(3) and State-Space Model

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1. Fitted AR(3) Model

The estimated AR(3) model is

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2),$$

where

$$\phi_1 = 0.2873, \quad \phi_2 = 0.1226, \quad \phi_3 = 0.0903, \quad \sigma_\varepsilon = 0.0078.$$

The estimated AR(3) parameters ($\phi_1 \approx 0.2873$, $\phi_2 \approx 0.1226$, $\phi_3 \approx 0.0903$) indicate a decaying but persistent memory in the PPI growth rates. The innovation standard deviation ($\sigma_\varepsilon \approx 0.0078$) is small, reflecting that most month-to-month fluctuations are modest after demeaning.

2. State-Space Model with Measurement Error

The state-space representation is given by

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \phi_3 x_{t-3} + w_t, \quad w_t \sim N(0, \sigma_w^2),$$

$$y_t = x_t + e_t, \quad e_t \sim N(0, \sigma_e^2).$$

The estimated parameters are

$$\phi_1 = 0.2873, \quad \phi_2 = 0.1226, \quad \phi_3 = 0.0903,$$

$$\sigma_w = 0.0065, \quad \sigma_e = 0.0035.$$

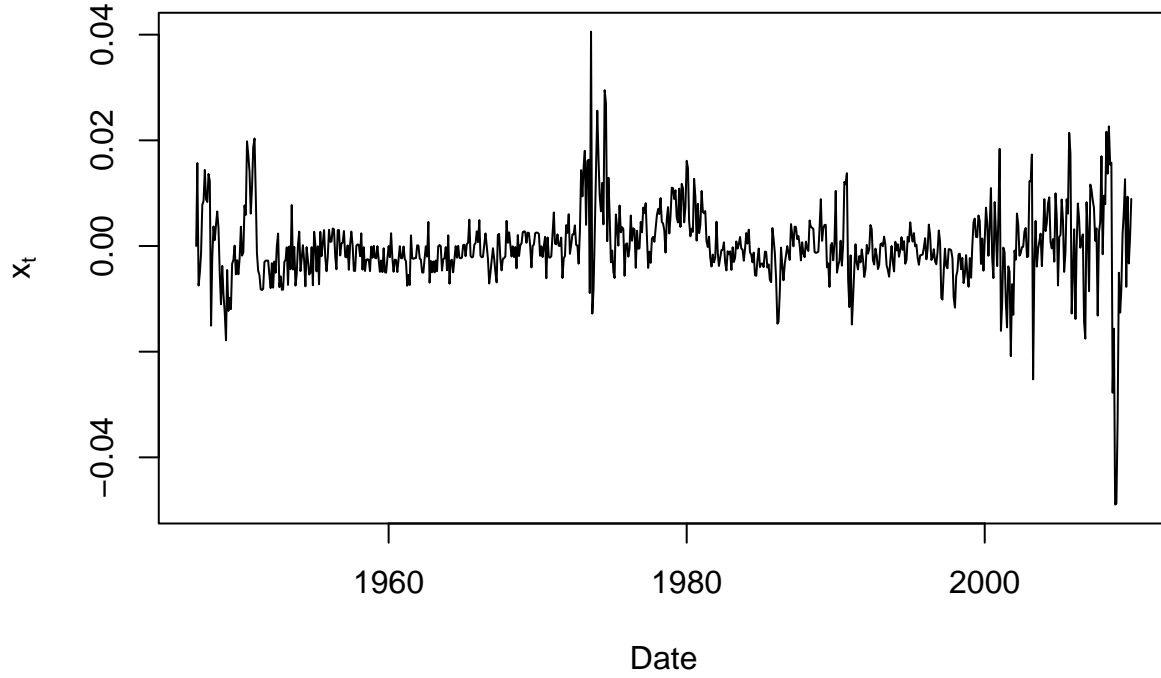


Figure 1: Smoothed estimate of x_t over time

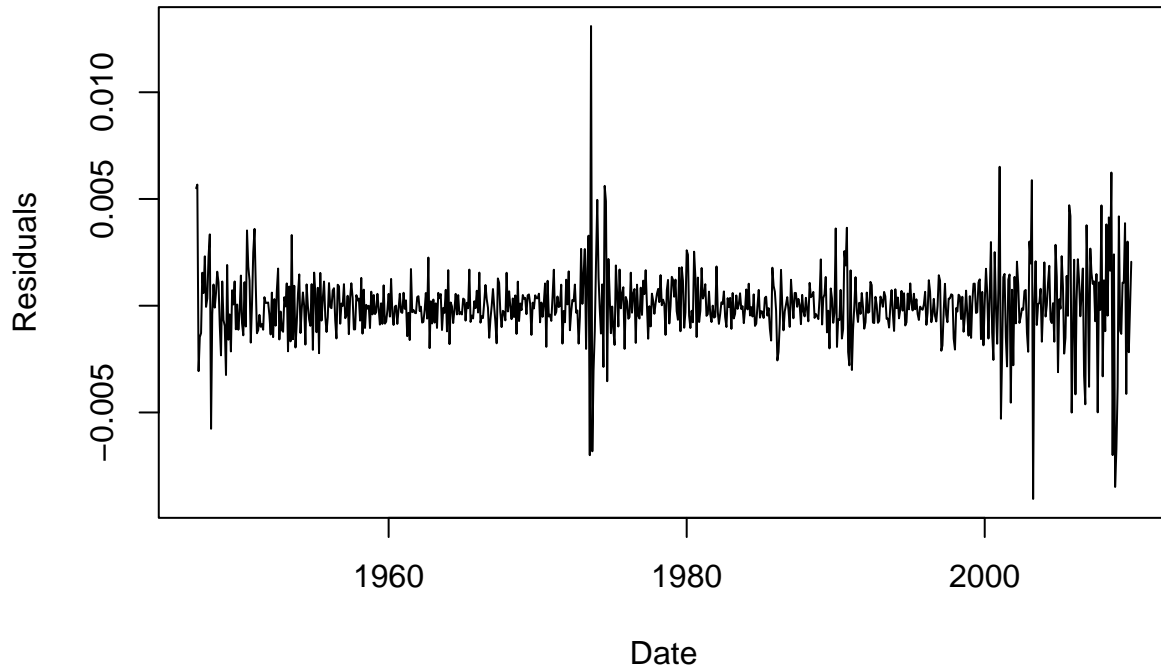


Figure 2: Filtered response residuals over time

In the measurement-error state-space formulation, we recover similar AR coefficients alongside estimated process noise ($\sigma_w \approx 0.0065$) and measurement noise ($\sigma_e \approx 0.0035$). The fact that σ_w exceeds σ_e suggests that genuine latent dynamics dominate over observational error in the logged PPI changes. The smoothed latent path captures major economic shocks, and the filtered residuals appear statistically well-behaved.

3. Comments

Both the AR(3) and the state-space models demonstrate strong performance in capturing the dynamics of monthly PPI log-differences over the 1947–2009 period. The AR(3) fit isolates the core autoregressive structure, revealing moderate persistence and low innovation variance. Enhancing this with the state-space representation yields a decomposition of total variability into latent process noise and measurement error, which informs on the underlying signal-to-noise ratio and highlights that most volatility arises from actual economic shifts rather than data irregularities.

The smoothed latent trajectory aligns well with known historical events—such as the energy shocks of the 1970s and the early 2000s downturn—suggesting that the latent state effectively tracks genuine inflationary pressures. Filtered residuals exhibit no pronounced autocorrelation or heteroskedasticity, further validating the model assumptions.

For future analysis, one may extend this framework by incorporating seasonal components, exogenous covariates (e.g., commodity prices or interest rates), or by comparing forecasting accuracy against simpler benchmarks. Overall, these results provide a robust statistical foundation for both descriptive study and predictive modeling of producer price inflation.