IE 714: Oil Seeds - Supply Chain Planning
Assignment 2

Akul Bansal/17i190011, Kapsikar Suyog Pramod/183190008, Gulfan Khan/17i190003

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1 Introduction

There are eight growing areas G1 to G8 which are used for growing oil seeds for bio-diesel production. Each of them have a potential yield of 1500 tonnes per month. To extract oil processing facilities have to be set up at some of these growing areas and yield of other areas can be sent to these facilities. This will incur some transportation cost.

For setting up processing facility there are three alternative choices of T1, T2 and T3 technology. Each technology has different fixed and processing costs. They also have different capacity of operation. Based on technology used different proportions of two outputs are produced. One is the crude oil which will be sent to the refinery and other is seed cake which has to be sent to an extraction plant for further extraction of crude oil. Sending material between places will incur some transportation cost.

Next, an extraction plant can have 3 levels of operation which have different capacities and so does different fixed and processing costs for oil extraction. The decision regarding opening of the extraction plants has to be taken. The crude oil extracted at an extraction plant will be sent to refinery and it will incur some transportation cost.

Our objective is to minimize the total cost - fixed cost, processing cost and transportation cost while deciding:

- Which technologies to set and where to set them ?
- How the yield is transported between growing areas ?
- Whether to open extraction plant or not ?
- Which levels will extraction plant operate at ?
- How the seed cake is transported between processing facility and extraction plants?

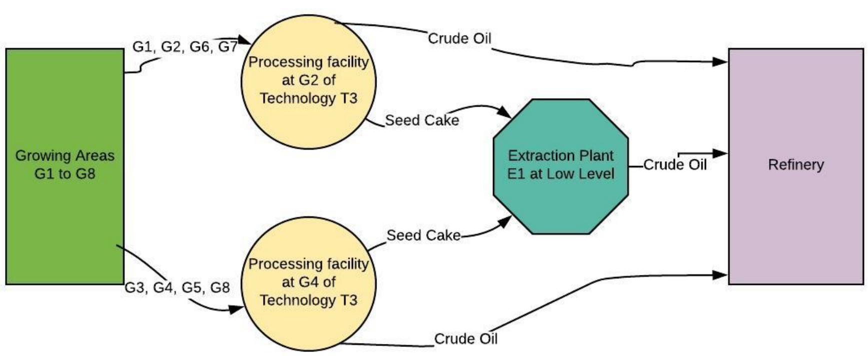

Figure 1: Schematic view of final supply chain

2 Mixed Integer Programming Model

We propose a Mixed-Integer Linear Programming Model to take above decisions. We first list the assumptions that we have made while formulating.

2.1 Assumptions

- All the growing areas give yield of exact 1500 tonne per month.
- There is no lower bound on production of crude oil, but all the oil seeds must be processed and all the seed cake must also be processed.
- All the oil seeds grown at a growing area must only go to exactly one of the processing facility (if multiple processing facilities happen to exist).
- A processing facility can have exactly one technology alternative installed in it.
- A extraction plant can have exactly one level of operation.
- Extraction plant if it is required to set up then it is assumed opening cost is also incorporated in the fixed cost of the level that has been given in data.
- Model neglects the cost for production of oil seeds and also neglects the scrapping cost of the output of extraction plant other than the crude oil.

2.2 Decision Variables

We introduce a mix of integer and real valued decision variables for formulating the model. We first begin with binary variables and state the condition when they take value 1 and otherwise they will take value 0.

p_{el} if extraction plant e is operating at level l
g_{jt} if processing facility j has technology t
z_j^1 if processing facility j uses any technology
z_e^2 if extraction plant e is operating
x_{ij} if anything is sent from growing area i to processing facility j

Continuous variables used in formulation are:
q_{ijt} quantity sent from growing area i to processing facility j with technology t
s_{jr} quantity of crude oil sent from processing facility j to the refinery
s_{jtel} quantity of seed cake sent from a processing facility j operating with technology t to the extraction plant e operating at level l

2.3 Parameters

Parameters defined in the model are as follows:
T_{ij} is the transportation cost per tonne from growing area i to processing facility j
F_t is the fixed cost for setting up a processing facility with technology t
P_t is the processing cost per tonne at processing facility with technology t

T_{jr} transportation cost per tonne from processing facility j to the refinery

T_{je} transportation cost per tonne from processing facility j to the extraction plant e

F^1_{el} fixed cost for setting up an extraction plat e at level l

P^1_{el} processing cost per tonne at an extraction plant e at level l
s_{er} quantity of crude oil sent from extraction plant e to the refinery
T_{er} Transportation cost per tonne from extraction plant e to the refinery
cap_l maximum quantity level l of an extraction plant can process per month
cap_t maximum quantity technology t of a processing facility can process per month
α_t fraction of oil seed converted to crude oil at technology t

2.4 Optimization Model

Objective Function *Minimize*

$$\sum_{ijt} q_{ijt} * T_{ij} + \sum_{jt} g_{jt} * f_t + \sum_{ijt} q_{ijt} * P_t + \sum_{j} s_{jr} * T_{jr} + \sum_{el}\sum_{j}(\sum_{t} s_{jtel}) * T_{je} +$$
$$\sum_{el} p_{ij} * F^1_{el} + \sum_{el}\sum_{jt} s_{jtel} * P^1_{el} + \sum_{j} s_{er} * T_{er}$$

Objective function captures cost of transporting between growing areas, fixed cost of setting up a technology, processing cost at growing areas, transportation cost of sending the crude oil directly to refinery, transportation cost of sending the seed cake to extraction plant, fixed cost of operating an extraction plant at some level, processing cost at extraction plant and final transportation cost to the refinery.

Constraints put in our model are described below. They include business constraints, assumptions mentioned above and some inequalities to make the model linear.

This constraint ensures no yield is sent to a growing area where technology has not been set up.

$$\sum_{t} q_{ijt} \leq 1500 * z^1_j \quad \forall j \tag{1}$$

Next constraint captures the relationship between binary variable x_{ij} and continuous variable q_{ijt}. It says two growing areas are connected in the network then across all technologies 1500 units is sent from i to j.

$$\sum_{t} q_{ijt} \geq 1500 * x_{ij} \quad \forall i \tag{2}$$

Next, it is enforced that each growing area must send its produce to exactly one growing area. Together with constraint (1) and (4) this means that yield of each growing area is processed by some technology.

$$\sum_{j} x_{ij} = 1 \quad \forall i \tag{3}$$

$$x_{ij} \leq z^1_j \quad \forall i, j \tag{4}$$

This constraint ensures that processing facility can have atmost 1 technology.

$$\sum_t g_{jt} = z_j^1 \forall j \tag{5}$$

Following constraint captures the capacity constraint at the processing facility based on the technology used there.

$$\sum_i q_{ijt} \leq \sum_t g_{jt} * cap_t \quad \forall j, t \tag{6}$$

Next is a flow balance constraint at each processing facility j capturing the quantity sent directly to the refinery.

$$s_{jr} \geq \sum_i \sum_t q_{ijt} * \alpha_t \quad \forall j \tag{7}$$

Here, capacity at each extraction plant based on the level it is operated at is enforced.

$$\sum_t \sum_e s_{jtel} \leq p_{el} * cap_l \quad \forall l \tag{8}$$

Like processing facility extraction plant can have at most one level.

$$\sum_l p_{el} = z_e^2 \quad \forall e \tag{9}$$

The next three constraints are put to linearize the model. They capture the relationship between continuous and binary variables.

$$s_{jtel} \leq M * p_{el} \quad \forall e, l \tag{10}$$

s_{jtel} takes value 0 if there is no technology at processing facility j

$$s_{jtel} \leq M * g_{ijt} \quad \forall j, t \tag{11}$$

q_{ijt} takes value 0 if there is no technology at processing facility j

$$q_{ijt} \leq M * g_{ijt} \quad \forall i \tag{12}$$

Note that all continuous variables are greater than or equal to 0 and big-M is taken to be 1500*8. Main constraints in our model above are capacity constraint, ensuring all yield is processed , setting up only one technology, flow balance constraints at processing plants and extraction plants. Other constraints have been put to capture relationship between several variables.

3 Results

3.1 Single Supply Chain

On solving the model with PuLP framework on Python we find cost at optimality to be Rs.16026008 when the extraction plant E2 is not forced to be open and at Rs. 16302040 when extraction plant E2 is forced to open. This is achieved by opening processing facility at two locations G2, G4 with technology T3. We list all the results in the table below.

The network flow of material in the supply chain is depicted in tables below:

Growing Area	Sending To	Operating at	Quantity sent
G1	G2	T3	1500
G2	G2	T3	1500
G3	G4	T3	1500
G4	G4	T3	1500
G5	G4	T3	1500
G6	G2	T3	1500
G7	G2	T3	1500
G8	G4	T3	1500

Figure 2: Flow-Quantity-Technology network between growing areas

When E2 is forced to Open			
Processing Facility	Sending to extraction Plant	Operating at	Quantity sent
G2 (T3)	E1	Low	5004
G4 (T3)	E1	Low	3996
G4 (T3)	E2	Low	1008

When E2 is not forced to Open			
Processing Facility	Sending to extraction Plant	Operating at	Quantity sent
G2 (T3)	E1	Medium	5004
G4 (T3)	E1	Medium	5004

Figure 3: Flow-Quantity-Level network between growing Areas and extraction plant

We bifurcate the **objective function** to understand which aspect of cost contributes in what proportion in the final cost. We depict this in the table below

3.1.1 Whether to open EP1 or not ?

We note that when we keep the decision of operating E2 optional we find it optimal to only operate extraction plant E1. This is counter-intuitive because E2 is at the refinery and should be ideally better than going to (E1 then refinery). Looking at the costs above we notice that by operating only E1 the transportation cost of going from extraction plant to refinery has increased. This was expected. However, we see a substantial reduction in the transportation cost from processing facility to extraction plant. This happened because earlier we were sending the entire seed cake production to E2 which was resulting in high

Cost Components	E2 closed	E2 open
Transportation Cost between growing Areas	1575000	1575000
Fixed Cost of setting up technology	1250000	1250000
Transportation Cost of crude oil to refinery	1095600	1095600
Transportation cost to Extraction Plant	1651320	1973880
Fixed Cost at Extraction Plant	1500000	1250000
Processing Cost at Extraction Plant	4153320	4453560
Transportation cost from EP to Refinery	960768	864000
Processing Cost at Technology	3840000	3840000
TOTAL COST	16026008	16302040

Figure 4: Objective Value with two different assumptions

transportation cost per tonne. When E1 is opened we are sending seed cake production closer to the growing area and then only the proportion of oil extracted there is sent to refinery. While this increases the transportation cost of going from the extraction plant to refinery marginally, it drastically reduces the transportation cost of reaching the extraction plant. This conveys that operating E1 is a good idea. [1]

3.2 Multiple Supply Chains

We also wanted to understand the difference it would make if different components of the supply chain problem were solved independently and there was some communication between these segments. To do so, we solved the entire problem using two simple MILPs and two manual decisions.

- First, we addressed the problem of which technology to install manually.
- Second, we solved a MILP to decide where the technology was setup and how the flow happened for processing oil seeds.
- Using answer from above we next decided whether to open extraction plant E1 and what levels they should operate at. This decision was made manually.
- Based on answer of last question we solved a MILP to decide flow of seed cakes to extraction plants. This was the final decision taken.

On solving the problem in these multiple parts we find minimal cost possible to be 17559000. Original optimal solution is hence an improvement of 9.56% over current solution. This is a big improvement because based on Sensex data companies grow annually on average at 13%.

[1] Credits: Analysis regarding opening E1 was understood in discussions with Aakash Banik and Richa Dhingra(16i190005)