Discrete Sample Quiz 10

Question 1 (A reminiscence about variance)
Assume that you want to encode six-letter alphabet $\mathcal{A} = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}, \texttt{e}, \texttt{f}\}$ and transmit it over a computer network. You assign 2 or 3 bit codes to these letters:

a	00
b	01
c	100
d	101
e	110
f	111

For example, the 11-bit sequence `"10100100110"` means `"dace"`. Denote by X the random variable – the number of bits used to encode a single letter. (All 6 letters have equal probabilities.)
Find $E(X)$ and $V(X)$.
Write them as two fractions: `P1/Q1,P2/Q2`
(Separate the fractions by comma, do not leave any spaces.)

Question 2 (Rosen7e, Ch.9, Q10-Q23).
Determine whether the binary relation is: (1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive. Express your answer as as a 4-letter string of T/F (true/false values that are answer to these 4 questions). For example, `TFTT` etc.

(A) The relation R on $\{1, 2, 3, \ldots\}$ where aRb means $a \mid b$.

(B) The relation R on $\{w, x, y, z\}$ where $R = \{(w, w), (w, x), (x, w), (x, x), (x, z), (y, y), (z, y), (z, z)\}$.

(C) The relation R on $\mathbb{Z}$ where aRb means $|a - b| \leq 1$.

(D) The relation R on $\mathbb{Z}$ where aRb means $a \neq b$.

(E) The relation R on $\mathbb{Z}$ where aRb means that the units digit of a is equal to the units digit of b.

(F) The relation R on the set of all subsets of $\{1, 2, 3, 4\}$ where SRT means $S \subseteq T$.

(G) The relation R on the set of all people where aRb means that a is younger than b.

(H) The relation R on the set $\{(a, b) \mid a, b \in \mathbb{Z}\}$ where $(a, b)R(c, d)$ means $a = c$ or $b = d$.

Question 3 (Rosen7e, Ch.9, Q35-Q38).
Construct a matrix of the relations defined below. Output the matrix as a list of lists:
`[[a11,a12,...],[a21,a22,...],...]`

(A) R on $\{1, 2, 3, 4, 6, 12\}$ where aRb means $a \mid b$.

(B) R on $\{1, 2, 3, 4, 6, 12\}$ where aRb means $a \leq b$.

(C) R^2, where R is the relation on $\{1, 2, 3, 4\}$ such that aRb means $|a - b| \leq 1$.

Question 4 (Rosen7e, Ch.9, Q42).
Define

$$M_R = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

determine if R is: (1) reflexive (2) symmetric (3) antisymmetric (4) transitive. Express your answer as as a 4-letter string of T/F (true/false values that are answer to these 4 questions). For example, `TFTT` etc.

Question 5 (Rosen7e, Ch.9, Q47).
Let A be the set of all positive divisors of 60 (including 1 and 60 itself). Draw the Hasse diagram for the relation R on A where aRb means $a \mid b$.

Question 6 (Rosen7e, Ch.9, Q51).
Find the transitive closure of R if

$$M_R = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

Question 7 (Rosen7e, Ch.9, Q59).
Find the join of the 3-ary relation:

```
{ (Wages,MS410,N507),
  (Rosen,CS540,N525),
  (Michaels,CS518,N504),
  (Michaels,MS410,N510) }
```

and the 4-ary relation:

```
{ (MS410,N507,Monday,6:00),
  (MS410,N507,Wednesday,6:00),
  (CS540,N525,Monday,7:30),
  (CS518,N504,Tuesday,6:00),
  (CS518,N504,Thursday,6:00) }
```

with respect to the last two fields of the first relation and the first two fields of the second relation.

Question 8 (Rosen7e, Ch.9, Q69-Q71). Give an example of a relation or state that there are none.
(A) A relation on $\{a, b, c\}$ that is reflexive and transitive, but not antisymmetric.
(B) A relation on $\{1, 2\}$ that is symmetric and transitive, but not reflexive.
(C) A relation on $\{1, 2, 3\}$ that is reflexive and transitive, but not symmetric.

Question 9 (Rosen7e, Ch.9, Q73).
Suppose $|A| = 7$. Find the number of reflexive, symmetric binary relations on A.