Discrete Structures (W2): Quiz

Question 1. Let $p \in \mathbb{Z}^+$ be a positive integer. Translate into predicate logic: "p is a prime number." (Prime numbers have exactly two positive divisors: 1 and the number itself).
Note. Use "infix" notation in your expressions: write $a \mid b$ whenever a divides b; write $x < y$, if x is less than y.

(Write predicate expression; specify domains for quantifiers.)

Question 2. We define $P(n)$ to be true iff n is a prime. For example, $P(2)$, $P(3)$, $P(5)$ etc. are true, but $P(1)$, $P(4)$ etc. are all false.
Translate into predicate logic: "There are arbitrarily large primes", i.e. there is no such thing as the largest prime. (Use just the $P(n)$ and inequaly symbols as predicates.)

(Write predicate expression; specify domains for quantifiers.)

Question 3. You can express the exclusive OR as a *disjunction of conjunctions*:

$$a \oplus b \equiv (a \wedge \neg b) \vee (\neg a \wedge b).$$

Indeed, for $a \oplus b$ to be true, you should either have a true and b false: $(a \wedge \neg b)$ or a false and b true: $(\neg a \wedge b)$.

Express this truth table as a *disjunction of conjunctions* as well – list all cases when it takes value T:

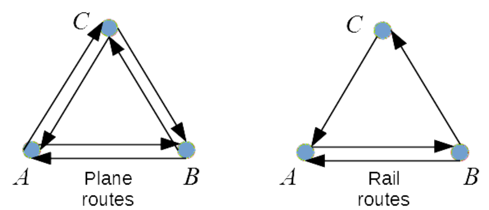
p	q	r	$E(p,q,r)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

(Write Boolean expression as disjunction of conjunctions)

Question 4. There are altogether 10 children: $\{c_1, \ldots, c_{10}\}$ and 10 hats: $\{h_1, \ldots, h_{10}\}$. Initially, every child c_i has his own hat h_i. When they were about to leave a party, there was an electricity blackout, and they grabbed hats at random (not necessarily their own). Predicate $G(i,j)$ is true iff child c_i grabbed hat h_j.
(a) Write the domain set and the range set of the predicate function G.
(b) Translate this statement into predicate logic: "Nobody grabbed his/her own hat."

Question 5. Translate these two sequences into predicate logic:
(a) In the interval of real numbers $(0;1)$ there is no smallest number.
(b) For the function $f(x) = x^2 - x$ defined on $(0;1)$ there exists the smallest value.

Question 6. Predicates Plane(x,y), Rail(x,y) show how to travel between cities A, B, C.



They have these following truth tables:

Plane	A	B	C
A	F	T	T
B	T	F	T
C	T	T	F

Rail	A	B	C
A	F	T	F
B	T	F	T
C	T	F	F

Find the truth values of these statements:
(a) $\forall x \exists y, \neg P(x,y)$
(b) $\forall x \forall y \exists z, P(x,z) \wedge P(z,y)$
(c) $\exists x \exists y \exists z, Q(x,y) \wedge Q(y,z) \wedge Q(z,x)$
(d) $\forall x \forall y \exists z, Q(x,z) \wedge Q(z,y)$
Note. In the truth tables the first argument is represented by row, the second is represented by column. For example, Rail(`C`, `B`) = `F` (3rd row, 2nd column).

Question 7. Two positive real numbers $x, y \in \mathbb{R}^+$ are given. Translate into predicate logic this statement: "Values x and y are the same, if we round them to two decimal places." In Python this predicate can be written like this:

```
def P(x,y):
    return(round(x,ndigits=2) ==
      round(y,ndigits=2))
```

Note. Rounding x to two decimal places finds the number $\frac{p}{100}$ closest to x. If two are equally close, then round up. (For example, 3.14159 rounds to 3.14; 3.144999 rounds to 3.14, but 3.145 rounds to 3.15.)