# Discrete Sample Quiz 8

**Question 1** What is the probability that the sum of the numbers on two dice is even when they are rolled? (Express your answer as $P/Q$.)

**Question 2**
**(A)** What is the probability to get sequence "`Heads`, `Heads`, `Heads`, `Heads`, `Heads`, `Heads`" when tossing a coin?
**(B)** What is the probability to get sequence "`Heads, Tails, Heads, Tails, Heads, Tails, Heads, Tails`" when tossing a coin?
**(C)** What is the probability to get even number of "Heads" when tossing a coin 6 times?

**Question 3** Assume that there is the following gambling game involving dice (cube with six outcomes: $1, 2, 3, 4, 5, 6$ with equal probabilities). Player $A$ guesses a number $n$ between 1 and 6 (for example $n = 3$). Then he rolls a dice three times. We have the following four outcomes (events):
$E_1$: If one of the dice rolls equals Player's $A$ number $n$, then Player $A$ wins 1 euro.
$E_2$: If two of the dice rolls equal Player's $A$ number $n$, then Player $A$ wins 2 euros.
$E_3$: If all three dice rolls equal Player's $A$ number $n$, then Player $A$ wins 3 euros.
$E_4$: If none of the dice rolls equals $n$, then Player $A$ loses 1 euro.
**(A)** Find the probability for each event $E_1, E_2, E_3, E_4$.
**(B)** Find the expected value of the money that Player $A$ wins or loses during a single round of this game. (One round = three dice rolls as described above). Use this formula – a weighted sum of the euros multiplied by the respective probabilities:

$$P(E_1) \cdot 1 + P(E_2) \cdot 2 + P(E_3) \cdot 3 + P(E_4) \cdot (-1).$$

**Question 4** Find a probability that choosing a random month during a leap year, it would have exactly 5 Sundays.

**Question 5** Suppose you and a friend each choose at random an integer between 1 and 8, inclusive. For example, some possibilities are $(3, 7)$, $(7, 3)$, $(4, 4)$, $(8, 1)$, where your number is written first and your friends number second. Find the following probabilities:
**(A)** p(you pick 5 and your friend picks 8).
**(B)** p(sum of the two numbers picked is $< 4$).
**(C)** p(both numbers match).
**(D)** p(the sum of the two numbers is a prime).
**(E)** p(your number is greater than your friends number).

**Question 6** In a card-game "Zolīte" there are 26 cards: four Aces (worth 11 points each); four Kings (worth 4 points each); four Queens (worth 3 points each); four Jacks (worth 2 points each); four cards "10" (worth 10 points each); and also four "9", one "8" and one "7" (they are worth 0 points).
A single trick (*stiķis*) in this game consists of any three cards (you can assume that all the $C_{26}^3$ tricks have equal probabilities.)
**(A)** Let $X$ be the random variable that expresses the number of points of a single card out of the 26 cards. Find the expected value $E(X)$.
**(B)** Find the variance $V(X)$.
**(C)** Let $Y$ be the random variable that expresses the total number of points for all three cards $c_1, c_2, c_3$ in a random trick. Find the expected value $E(Y)$.
**(D)** Is $E(Y)$ exactly three times larger than $E(X)$?

**Question 7** You create a random bit string of length five (all 32 bit strings are equally probable). Consider these events:
$E_1$: the bit string chosen begins with 1;
$E_2$: the bit string chosen ends with 1;
$E_3$: the bit string chosen has exactly three 1s.
**(A)** Find $p(E_1 \mid E_3)$.
**(B)** Find $p(E_3 \mid E_2)$.
**(C)** Find $p(E_2 \mid E_3)$.

**Question 8** A tiny ant travels in 3D space. It starts in the point $(0; 0; 0)$ and makes 9 steps altogether: With equal probability each step can be parallel to either $x$, $y$ or $z$ axes (incrementing the corresponding coordinate).
**(A)** What is the probability that the ant has taken at least one step in the direction of each coordinate axis?
**(B)** What is the probability that the ant has reached the point $(3; 3; 3)$?