

## Discrete Sample Quiz 11

### Question 1 (Rosen7e, Ch.10, Q11-Q14).

- (A)  $K_n$  (the complete graph on  $n$  vertices) has ..... edges and ..... vertices.  
 (B)  $K_{m,n}$  (the complete bipartite graph on sets of sizes  $m, n$ ) has ..... edges and ..... vertices.  
 (C)  $W_n$  (wheel graph on  $n$  vertices –  $n$ -gonal pyramid viewed from above) has ..... edges and ..... vertices.  
 (D)  $Q_n$  ( $n$ -dimensional cube) has ..... edges and ..... vertices.

### Question 2 (Rosen7e, Ch.10, Q15-Q17).

- (A) The length of the longest simple circuit in  $K_5$  is .....  
 (B) The length of the longest simple circuit in  $W_{10}$  is .....  
 (C) The length of the longest simple circuit in  $K_{4,10}$  is .....

*Note.* A simple circuit in (Rosen2019) is defined as a circular sequence of vertices  $v_0, v_1, \dots, v_n = v_0$ , where each two vertices are connected by an edge (and it does not contain any edge more than once).

**Question 3 (Rosen7e, Ch.10, Q19-Q24).** In each example find the dimensions of a matrix; and number of 0s and 1s in it: Find  $X, Y, Z, T$ .

- (A) The adjacency matrix for  $K_{m,n}$  has size (rows times columns)  $X \times Y$ ; it has  $Z$  0's and  $T$  1's.  
 (B) The adjacency matrix for  $K_n$  has size  $X \times Y$ ; it has  $Z$  0's and  $T$  1's.  
 (C) The adjacency matrix for  $C_n$  has size  $X \times Y$ ; it has  $Z$  0's and  $T$  1's.  
 (D) The adjacency matrix for  $Q_4$  has size  $X \times Y$ ; it has  $Z$  0's and  $T$  1's.  
 (E) The incidence matrix for  $W_n$  has size  $X \times Y$ ; it has  $Z$  0's and  $T$  1's.  
 (F) The incidence matrix for  $Q_5$  has size  $X \times Y$ ; it has  $Z$  0's and  $T$  1's.

*Note.* Adjacency matrix is a square matrix of size  $|V| \times |V|$ , but incidence matrix is a rectangular matrix of size  $|V| \times |E|$ .

### Question 4 (Rosen7e, Ch.10, Q28-Q31).

- (A) List all positive integers  $n$  such that  $K_n$  has an Euler circuit; what is its length in terms of  $n$ ?  
 (B) List all positive integers  $n$  such that  $Q_n$  has an Euler circuit.

### Question 5 (Rosen7e, Ch.10, Q43).

If  $G$  is a planar connected graph with 12 regions and 20 edges, then  $G$  has ..... vertices.

### Question 6 (Rosen7e, Ch.10, Q44).

If  $G$  is a planar connected graph with 20 vertices, each of degree 3, then  $G$  has ..... regions.

### Question 7 (Rosen7e, Ch.10, Q45).

If a regular graph  $G$  has 10 vertices and 45 edges, then each vertex of  $G$  has degree .....

*Note.* A regular graph is a graph where all vertices have the same degree.

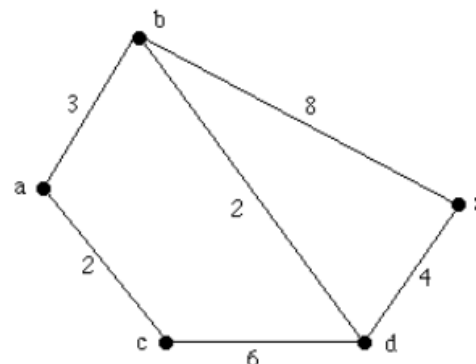
### Question 8 (Rosen7e, Ch.10, Q59-Q82).

- (A) A simple graph with 6 vertices, whose degrees are 2, 2, 2, 3, 4, 4.  
 (B) A simple graph with 8 vertices, whose degrees are 0, 1, 2, 3, 4, 5, 6, 7.  
 (C) A simple graph with degrees 1, 2, 2, 3.  
 (D) A simple graph with degrees 2, 3, 4, 4, 4.  
 (E) A simple graph with degrees 1, 1, 2, 4.  
 (F) A simple digraph with indegrees 0, 1, 2 and outdegrees 0, 1, 2.  
 (G) A simple digraph with indegrees 1, 1, 1 and outdegrees 1, 1, 1.  
 (H) A simple digraph with indegrees 0, 1, 2, 2 and outdegrees 0, 1, 1, 3.  
 (I) A simple digraph with indegrees 0, 1, 2, 4, 5 and outdegrees 0, 3, 3, 3, 3.  
 (J) A simple digraph with indegrees 0, 1, 1, 2 and outdegrees 0, 1, 1, 1.  
 (K) A simple digraph with indegrees: 0, 1, 2, 2, 3, 4 and outdegrees: 1, 1, 2, 2, 3, 4.  
 (L) A simple graph with 6 vertices and 16 edges.  
 (M) A connected simple planar graph with 5 regions and 8 vertices, each of degree 3.  
 (N) A graph with 4 vertices that is not planar.  
 (O) A planar graph with 10 vertices.  
 (P) A planar graph with 8 vertices, 12 edges, and 6 regions.  
 (Q) A planar graph with 7 vertices, 9 edges, and 5 regions.

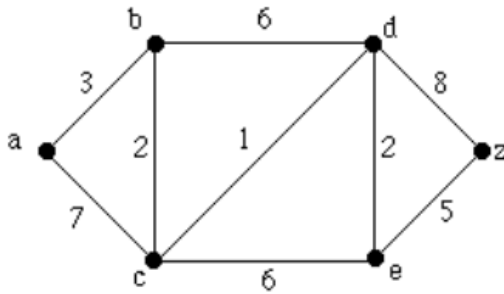
### Question 9 (Rosen7e, Ch.10, Q108).

Use Dijkstras Algorithm to find the shortest path length between the vertices a and z in these weighted graphs.

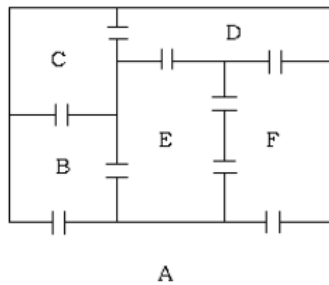
(A)



(B)

**Question 10 (Rosen7e, Ch.10, Q113).**

The picture at the right shows the floor plan of an office. Show that it is impossible to plan a walk that passes through each doorway exactly once, starting and ending at A.

**Hall's Marriage Theorem (Rosen2019, p.772)**

The bipartite graph  $G = (V, E)$  with partition of vertices into 2 disjoint sets  $V = X \cup Y$  has a *maximum matching* that saturates  $X$  iff for all  $A \subseteq X$  we have  $|X| \leq |N(X)|$ .

*Note.* A *matching* in a graph is a set of edges such that no two edges share a common endpoint. A *maximum matching* is matching containing the greatest number of edges. And a matching *saturates* a set  $X$ , if each vertex  $v \in X$  belongs to some matching edge.

**Question 11 (Königs Marriage Theorem)**

Prove that if all the vertices of a bipartite graph have the same degree, then it has a perfect matching.

(Quines2017, p.11); <https://cjquines.com/files/halls.pdf>

*Note.* A matching is *perfect*, if it saturates all vertices (every vertex has a pair).

**Question 12**

We have a regular deck of 52 playing cards, with exactly 4 cards of each of the 13 ranks. The cards have been randomly dealt into 13 piles, each with 4 cards in it. Prove that there is a way to take a card from each pile so that after we take a card from every pile, we have exactly a card of every rank.

(Quines2017, p.11).