# Discrete Structures Lab01: Hints

**Lemma 1:** For all propositions $a$, $\neg\neg a \to a$.
**Proof:**

- Assume that $\neg\neg a$ is true.
- We sort two cases by "classic" axiom (Excluded Middle): either $a$ or $\neg a$ must be true.
- If $a$ is true, we are happy.
- Otherwise $\neg a$ is true (along with $\neg\neg a$ obtained before). This is a contradiction.
- Therefore $a$ must be true in all cases (it is either trivial, or a contradiction).

**Lemma 2:** For all propositions $a$ and $b$, $\neg(a \to b) \to a$.
**Proof:**

- Assume that $\neg(a \to b)$ is true.
- We have to prove that $a$ is true. By Lemma 1, we will prove instead that $\neg nega$ is true, then it will also imply $a$.
- We will assume that $\neg a$ is true, and attempt to get a contradiction (this means that $\neg\neg a$ must be true).
- Let's prove now that $a \to b$ is true - this would be an immediate contradiction with $\neg(a \to b)$.
- To prove $a \to b$, assume that $a$ is true and let's prove $b$. But earlier we assumed that $\neg a$.
- $a$ and $\neg a$ cannot be simultaneously true. This is a contradiction.

**Peirce Lemma:** For all propositions $a$ and $b$, $((a \to b) \to a) \to a$.

**Hint.** Just use Lemma 1 and 2 for this. And also the "classic" axiom: Sort 2 cases when $(a \to b)$ or $\neg(a \to b)$ are true.

**Lemma 4:** For all propositions $a$ and $b$, $(\neg b \to \neg a) \to (a \to b)$.
This is the opposite direction from a well-known contrapositive ($\neg b \to a$ and $a \to b$ mean the same thing.)

**Hint.** Use "classic" axiom (Excluded middle) on $b$.

**Lemma 5:** For all propositions $a, b, c, d, e$,

$$((((a \to b) \to (\neg c \to \neg d)) \to c) \to e) \to \neg a \to (d- > e).$$

**Hint.** Indeed, assume that $(((a \to b) \to \neg c \to \neg d) \to c) \to e$; also assume $\neg a$ and $d$. Then you can prove $(((a \to b) \to \neg c \to \neg d) \to c)$ which is similar to what you need.

After all this, you can do Sample20 from the Coq lab.