Alternative Homework 3: Linear Optimization

Note. This is a parody of MIT OCW content.
See https://ocw.mit.edu/terms/. The original assignments and related materials can be retrieved from https://bit.ly/2YcyvII, https://bit.ly/30R4j7S and https://bit.ly/2YcgCtW.

Question 1. A warehouse has to ship 480 boxes of apples, 400 boxes of pears and 230 boxes of quinces every day. It can ship any product by mail, by courier or by drone. The couriers can ship up to 420 boxes of any product; drones can ship up to 250 boxes. Regular mail can ship any number of boxes. Each shipment method has different costs, different speed (and therefore – different earnings). The money earned by the warehouse per every box (depending on the product and its shipment method) is shown in the table:

Product	Mail	Courier	Drone
Apples	4 EUR	12 EUR	7 EUR
Pears	4 EUR	13 EUR	9 EUR
Quinces	8 EUR	14 EUR	11 EUR

(A) Write a Linear Program in a form that is an appropriate starting point for a simplex method.
(B) Write all the steps for the simplex method with Dantzig pivot rule, which is the most common one. (Pivot rules can be found e.g. here: https://bit.ly/3fDMzkn).
(C) Find the actual number of boxes to ship by each method to maximize the money earned. Are the answers obtained from the simplex method integer numbers?

Question 2. A team of n members would like to travel a distance d from A to B as quickly as possible. All of them can walk and they also have one scooter – initially the scooter is located in the point A and can be used to carry only one person at a time. For each person i ($1 \le i \le n$), we know his/her walking speed w_i and his/her speed s_i when travelling on the scooter. The goal is to find a way to bring all n people to destination so as to minimize the time at which the last person arrives. The scooter can be left by any member of the group on the side of the road, and picked up by anyone else of the group. Members of the group can also walk or use the scooter backwards (towards A) if that helps.

(A) Consider the case where $n = 3$, $w_1 = w_2 = 1$, $s_1 = s_2 = 6$, $w_3 = 2$, $s_3 = 8$ and $d = 100$. Find the fastest way for everyone to travel the distance d.
(B) For a general instance (general n and arbitrary speeds), write a linear program whose value is always a lower bound on the time needed for the n-person team to travel a distance d. This should be a small linear program; the number of variables and constraints should be $O(n)$ (and not dependent on d, or the number of 'legs' of the solution).
(C) Write a different variant of the linear program, if nobody is allowed to walk or to use the scooter in the backwards direction (towards A). Can this restriction increase the total travel time?
(This is a parody of Problem 4, see https://bit.ly/3ectpC2.)

Question 3. You are given the following points in the plane:

$$(1,3), (2,5), (3,6), (5,8), (8,13).$$

We denote them by $(x_1, y_1), \ldots, (x_5, y_5)$ respectively.
(A) Find a line $ax + by = c$ that approximately passes through these points (no line is a perfect fit). Write a linear program (you don't need to solve it) to find the line that minimizes the maximum absolute error:

$$\max_{i \in \{1,\ldots,5\}} |ax_i + by_i - c|.$$

(B) Is there a linear program that minimizes the total quadratic error? It is expressed like this:

$$\sum_{i \in \{1,\ldots,5\}} (ax_i + by_i - c)^2.$$

(This is a parody of Problem 7.8, see p.240 https://bit.ly/2YcgCtW.)