Homework 8

Discrete Structures
Due Tuesday, March 2, 2021

Submit each question separately in .pdf format only

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1. P is the set of all quadratic polynomials $ax^2 + bx + c$ (having real coefficients, $a \neq 0$). There is a relation R between two polynomials $p_1, p_2 \in P$ (written $p_1 R p_2$) iff p_2 can be obtained by multiplying p_1 with a real number.

   (a) Is R reflexive? Is it irreflexive? Neither reflexive nor irreflexive?

   (b) Is R symmetric? Is it asymmetric? Neither symmetric nor asymmetric?

   (c) Is R antisymmetric?

   (d) Is R transitive?

   (e) Is R an equivalence relation? (If it is, build a subset $P^* \subseteq P$ such that P^* contains exactly one representative from each equivalence class).

2. Write all the partitions of a three-element set $S = \{A, B, C\}$. Introduce a relation ρ between any two partitions P_1, P_2 (write $P_1 \rho P_2$) iff P_2 can be obtained from P_1 subdividing some class in P_1 into two non-empty subclasses (and leaving any other classes unchanged). Represent the relation ρ as a directed graph (label the vertices as circles with their partitions written inside).

   Relation R is the transitive closure of ρ. How many pairs does R contain? Is it a partial or total order relation?