Uzdevumi 2020.g. 21. februāra nodarbībai

Uzdevums 2.1: Determine all positive integers n such that n has a multiple with all non-zero digits in its decimal notation.

Uzdevums 2.2: A *wobbly number* is a positive integer whose digits are alternately nonzero and zero with the units digit being nonzero. Determine all positive integers that do not divide any wobbly numbers.

Uzdevums 2.3: Let n be a given integer with $n \geq 4$. For a positive integer m let S_m denote the set $\{m, m+1, \ldots, m+n-1\}$. Determine the minimum value of $f(n)$ such that every $f(n)$-element subset of S_m (for every m) contains at least three pairwise relatively prime elements.

Uzdevums 2.4: By $\sigma(k)$ we denote the sum of all positive divisors of k (including 1 and k itself). For every positive integer n, prove that

$$\frac{\sigma(1)}{1} + \frac{\sigma(2)}{2} + \ldots \frac{\sigma(n)}{n} \leq 2n.$$

Note. In the last two problems let gpf(n) denote the greatest prime factor of an integer n. We also define gpf$(1) =$ gpf$(-1) = 1$, and gpf(0) is undefined.

Uzdevums 2.5: Show that there exist infinitely many positive integers n such that gpf$(n^4 + 1)$ is greater than $2n$.

Uzdevums 2.6: Find all polynomials $P(n)$ with integer coefficients satisfying both properties:

- $P(n^2) \neq 0$ for all integers $n = 0, 1, 2, \ldots$ and
- There exists $M > 0$ such that gpf$(P(n^2)) - 2n \leq M$ for all integers $n = 0, 1, 2, \ldots$.