Discrete Structures (W3): Quiz

Question 1. We have the following truth table ("0" means False, "1" means "True"). Find the correct DNF (Disjunctive Normal Form). (Multiple answers may be true, since there may more than one way to write DNFs for the same Boolean function.)

3 Inputs

$2^3 = 8$ Combinations

X	Y	Z	F_3
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

(A)

$$(X \wedge \neg Y \wedge Z) \vee (\neg X \wedge Y \wedge Z)$$

(B)

$$(\neg Z \wedge Y \wedge Z) \vee (\neg Y \wedge Z)$$

(C)

$$(\neg X \wedge Z) \vee (X \wedge \neg Y \wedge Z)$$

(D)

$$(\neg X \wedge \neg Y \wedge Z) \vee (\neg X \wedge Y)$$

Question 2. Identify, which is the correct CNF (Conjunctive Normal Form) that computes the same thing as Peirce's arrow: $a \downarrow b = \neg(a \vee b)$. (Multiple answers may be true, since there may more than one way to write CNFs for the same Boolean function.)

(A)

$$(\neg a \vee \neg b) \wedge (\neg a \vee b) \wedge (a \vee \neg b)$$

(B)

$$(a \vee b) \wedge (\neg a \vee b) \wedge (a \vee \neg b)$$

(C)

$$\neg a \vee \neg b$$

(D)

$$\neg a \wedge (a \vee \neg b)$$

Question 3. We have predicates $A(p, i, r)$ (a Python program $p \in \mathcal{P}$ receives input $i \in \mathbb{Z}^+$ and answers with the result $r \in \mathbb{Z}^+$); $H(p, i)$ (program p halts on input i), and $C(i, r)$ (the correct/expected output for input i is r). Identify the predicate expression that expresses the following English sentence:
"For all inputs that are sufficiently large, the given program p halts and produces outputs that are correct; for some (finitely many) inputs p may loop forever, but it is never incorrect."

(A)

$$\forall p \in \mathcal{P}\ \forall i \in \mathbb{Z}^+\ \forall r \in \mathbb{Z}^+,$$
$$((i > M\ \wedge\ H(p, i)) \vee (A(p, i, r) \wedge C(i, r)))$$

(B)

$$\exists M \in \mathbb{Z}^+\ \forall i \in \mathbb{Z}^+\ \forall r \in \mathbb{Z}^+,$$
$$((i < M\ \wedge\ \neg H(p, i)) \vee (A(p, i, r) \wedge C(i, r)))$$

(C)

$$\forall p \in \mathcal{P}\ M \in \mathbb{Z}^+\ \forall i \in \mathbb{Z}^+\ \forall r \in \mathbb{Z}^+,$$
$$((i > M\ \wedge\ H(p, i)) \vee (A(p, i, r) \wedge C(i, r)))$$

(D)

$$\forall p \in \mathcal{P}\ \forall i \in \mathbb{Z}^+\ \forall r \in \mathbb{Z}^+,$$
$$((i > M\ \wedge\ H(p, i)) \vee (A(p, i, r) \wedge C(i, r)))$$

Question 4. For the quantifiers ($A(p, i, r)$, $H(p, i)$, $C(i, r)$) defined in the previous question, the following expression is given:

$$\forall p \in \mathcal{P} \exists i \in \mathbb{Z}^+\ \exists j \in \mathbb{Z}^+,$$
$$(j > i\ \wedge\ (H(p, i) \to H(p, j)).$$

Which sentence is expressed by this?
(A) Each Python program halts for at least 2 inputs.
(B) Each Python program halts for infinitely many inputs.
(C) There is no Python program that halts on exactly 1 input.
(D) For each Pyton program there is the largest input for which it halts.

Question 5. There is a set of 2 students $S = \{s_1, s_2\}$ and a set of 5 chairs $C = \{c_1, c_2, c_3, c_4, c_5\}$. Find, how many such functions $f : S \to C$ exist (a function by definition is a mapping that assigns a chair for each student; it is NOT always true that different students get different chairs or that all chairs are occupied).

Please find the total number of such functions, also the number of injective, surjective and bijective functions among them.

Your answer should be a comma-separated list of 4 numbers (e.g. 10,11,12,13 means that 10 is the total, 11 - injective, 12 - surjective, 13 - bijective).

Question 6. Given the following equation:

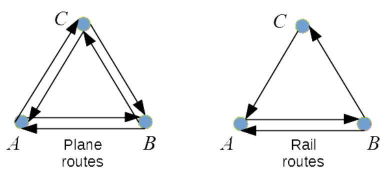
$$\forall u \in \mathbb{Z}^+\ \forall v \in \mathbb{Z}^+\ \forall p \in \mathbb{Z}^+\ \forall q \in \mathbb{Z}^+\ \exists k \in \mathbb{Z}^+,$$

$$S(x, u, y) \wedge S(x, v, z) \wedge S(d, p, y) \wedge S(d, q, z) \\ \rightarrow S(d, k, x).$$

Which statement is expressed by this expression?

(A) d is the greatest common divisor of y and z,
(B) x is the greatest common divisor of p and q,
(C) x is the greatest common divisor of y and z,
(D) d is the greatest common divisor of u and v.

Question 7.



Find the predicate expression that expresses this statement: "For the two given cities 'x' and 'y' there is a two-leg trip using planes, but there is no two-leg trip using rail." (We call a trip "two-leg", if it uses exactly two plane flights or exactly two rail links.)

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(A) forall x y z: City,
    Plane(x,z) /\ Plane(z,x) /\
~(Rail(x,z) /\ ~Rail(z,y)

(B) exists z: City,
    Plane(x,z) /\ Plane(z,x) /\
~(Rail(x,z) /\ ~Rail(z,y)

(C) forall x y: City, exists z: City,
    Plane(x,z) /\ Plane(z,x) /\
(~(Rail(x,z) \/ ~Rail(z,y))

(D) forall z: City,
    Plane(x,z) /\ Plane(z,x) /\
(~(Rail(x,z) \/ ~Rail(z,y))
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