Assignment 14
Published on 2020-12-06,
Estimated Time: 30 minutes,
Max.grade 10‰

1 Bellman-Ford Algorithm

The Bellman-Ford algorithm solves the single source shortest paths problem in the case in which edge weights may be negative. It can work with directed graphs (and also undirected graphs; not discussed in this exercise). The algorithm initializes the distances to all the vertices u by $u.d = +\infty$. The only exception is the *source vertex*) which gets distance $s.d = 0$ (the distance to itself is 0).
After this initialization in a graph with n vertices it will perform $n-1$ identical iterations. In every iteration it considers all the edges in some order, and "relaxes" all the edges. See (Goodrich2011, p.640), where the edge relaxation procedure is described.
After that the Bellman-Ford algorithm performs one last iteration: If there are still relaxations that reduce distances even after n steps, this means that there is a negative loop in the original graph (and the shortest paths are not possible to compute as the distances can be reduced infinitely).

2 Problem

Consider the graph in Figure 1.

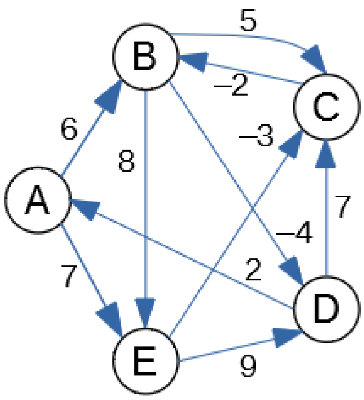

Figure 1: Graph diagram

(A) Find the following number:

$$X = (a + b + c) \bmod 5.$$

As a remainder it can take any value $0, 1, 2, 3, 4$. Depending on its value, select your source vertex from this table:

X	0	1	2	3	4
source	A	B	C	D	E

Redraw the graph and mark your source vertex by a double-circle (or shade it, or draw a short incoming arrow or apply some other highlighting).

(B) Create a table showing all the changes to all the distances to A, B, C, D, E as the relaxations are performed. In a single iteration the same distance can be relaxed (improved) multiple times. In this case the table cell shows how the distance is changed in multiple steps (e.g. $\infty \to 17 \to 11$). The table should display all n iterations (where $n = 5$ is the number of vertices). The last iteration is not supposed to change/relax any distance (otherwise we have a negative loop).

(C) Summarize the result: For each of the 5 vertices tell what is its minimum distance from the source. Also tell what is the shortest path how to get there. For example, if your source is E then you could claim that the shortest path $E \rightsquigarrow B$ is of length -5 and it consists of two edges (E, C), (C, B).