Discrete Quiz 4

Question 1. Define the universe U to be all possible remainders when we divide by 360: $\{0, 1, 2, \ldots, 359\}$. Also define 3 subsets in this universe:

$$\begin{cases} K_2 &= \{x \in U \mid x \text{ divisible by } 2\}, \\ K_3 &= \{x \in U \mid x \text{ divisible by } 3\}, \\ K_5 &= \{x \in U \mid x \text{ divisible by } 5\}, \end{cases}$$

Denote by Φ the subset of U containing all numbers that are mutually prime with 360 (no common divisors greater than 1): $\Phi = \{1, 7, 11, 13, \ldots, 359\}$. Which set equality is valid regarding the subset Φ:

(A) $\Phi = (K_2 \cup K_3 \cup K_5)$
(B) $\Phi = (K_2 \cap K_3 \cap K_5)$
(C) $\Phi = \left(\overline{K_2} \cup \overline{K_3} \cup \overline{K_5}\right)$
(D) $\Phi = \left(\overline{K_2} \cap \overline{K_3} \cap \overline{K_5}\right)$
(E) $\Phi = \left(\overline{K_2 \cap K_3} \cup \overline{K_2 \cap K_5} \cup \overline{K_3 \cap K_5}\right)$

Pick your answer as a single letter like this: `G`

Question 2. Find the size of the set you constructed in the previous example.

Write your answer as a single non-negative integer like this: `17`

Question 3. We have the following sets:
A is the set of all finite sequences of even positive positive numbers (such as $(6, 22, 10, 14, 2, 6)$, and so on)
B is the set of all infinite nondecreasing lists of even positive numbers (such as $(40 \leq 40 \leq 42 \leq 46 \leq \ldots)$, and so on)
C is the set of all infinite nonincreasing lists of even positive numbers (such as $(64 \geq 58 \geq 58 \leq 54 \geq \ldots)$, and so on).
Clearly, all three sets are infinite. Determine their cardinalities - which list of cardinalities is equal to the list $(|A|, |B|, |C|)$?
(A) $(|\mathbb{N}|, |\mathbb{N}|, |\mathbb{N}|)$. **(B)** $(|\mathbb{N}|, |\mathbb{R}|, |\mathbb{N}|)$. **(C)** $(|\mathbb{N}|, |\mathbb{N}|, |\mathbb{R}|)$. **(D)** $(|\mathbb{N}|, |\mathbb{R}|, |\mathbb{R}|)$. **(E)** $(|\mathbb{R}|, |\mathbb{R}|, |\mathbb{R}|)$.

Pick your answer as a single letter like this: `G`

Question 4. Let $f(x) = (x^2) \ \mathbf{mod}\ 11$. Find the set $f(S)$ if $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Write the list of elements of $f(S)$ as a sorted list like this: `1,2,3`

Question 5. How many 2-element sets are there in the powerset $\mathcal{P}(\{\{\mathtt{A}, \mathtt{B}\}, \mathtt{C}, \mathtt{D}, \mathtt{E}\})$?

Write your answer as a non-negative integer like this: `17`

Question 6. Given two sets $A = \{x, y\}$ and $B = \{x, \{x\}\}$, check, if statements are true or false:
(A) $x \subseteq B$.
(B) $\emptyset \in \mathcal{P}(B)$.
(C) $\{x\} \subseteq A - B$.
(D) $|\mathcal{P}(A)| = 4$.

Write your answer as a sorted list of letters (which are true) like this: `A,B,C,D`

Question 7. We define functions $g : A \to A$ and $f : A \to A$, where $A\{1, 2, 3, 4\}$ by listing all argument-value pairs: $f = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$, $g = \{(1, 3), (2, 1), (3, 4), (4, 2)\}$. Find the value pairs for the function $(f \circ g)^{-1}$.

Write your answer as a comma-separated list like this: `(1,1),(2,2),(3,3),(4,4)`

Question 8. Find the value of this infinite sum: $1 - 1/3 + 1/9 - 1/27 + 1/81 - \ldots$.

Write your answer as a simple fraction: `P/Q`

Question 9. It is known that the function $f(n) = n^3 + 88n^2 + 3$ is in $O(n^3)$ – its asymptotic growth is as fast as the growth of the function $g(n) = n^3$. $\exists C \in \mathbb{Z}^+\ \exists n_0 \in \mathbb{Z}^+\ \forall n \in \mathbb{Z}^+$,
$(n > n_0 \to |f(n)| \leq C \cdot |g(n)|)$ Find the smallest positive integer C that would satisfy the above definition, and for your C find the smallest possible n_0.

Write your answer (C, n_0) as a pair of two numbers like this: `17,17`

Question 10. "Big O notation" allows to arrange functions according to the their growth rate for large n. Identify, which list of functions is such that the first element of this list is in the big-O of the next element of that list and so on. (Intuitively, the first element in the list is the slowest growing function, the last element is the fastest growing one.)

(1) $\log(n^{10})$, **(2)** $(\log n)^2$, **(3)** $\log \log n$, **(4)** $n \log n$, **(5)** $\log(n!)$, **(6)** $\log 2^n$.

Write your answer as a comma-separated list like this: `1,2,3,4,5,6`

Question 11. Digits of all rational numbers P/Q in $(0; 1)$ are eventually periodic: they infinitely repeat some group of digits (the period) starting from some place. For example, the fraction $11/205 = 0.05(36585)$ has period of 5 digits and a pre-period "05" of just two digits. Find the predicate logic expression that tells that sequence of digits $d(1), d(2), d(3), \ldots$ is eventually periodic (it may have pre-period of any length, including length zero).
(A) $\exists N \in \mathbb{Z}^+\ \exists T \in \mathbb{Z}^+\ \forall n \in \mathbb{Z}^+$,
$(n \geq N - 1 \to d(n) = d(n + T))$.
(B) $\exists N \in \mathbb{Z}^+\ \forall n \in \mathbb{Z}^+\ \exists T \in \mathbb{Z}^+$,
$(n \geq N - 1 \to d(n) = d(n + T))$.
(C) $\forall n \in \mathbb{Z}^+\ \exists N \in \mathbb{Z}^+\ \exists T \in \mathbb{Z}^+$,
$(n \geq N - 1 \to d(n) = d(n + T))$.
(D) $\forall n \in \mathbb{Z}^+\ \forall N \in \mathbb{Z}^+\ \exists T \in \mathbb{Z}^+$,
$(n \geq N - 1 \to d(n) = d(n + T))$.

Pick your answer as a single letter like this: `G`