Alternative Homework 1: Information Compression

Note. This is a parody of MIT OCW content. See https://ocw.mit.edu/terms/. The original assignments and related materials can be retrieved from TBD and TBD

Question 1. Run-length encoding is a popular variable-length lossless compressor used in fax machines, image compression, etc. Consider compression of S^n – an i.i.d. Bern(δ) source with very small $\delta = \frac{1}{128}$ using run-length encoding: A chunk of consecutive $r \leq 255$ zeros (resp. ones) is encoded into a zero (resp. one) followed by an 8-bit binary encoding of r (If there are > 255 consecutive zeros then two or more 9-bit blocks will be output). Compute the average achieved compression rate

$$\lim_{n\to\infty} \frac{1}{n} E(\ell(f(S^n))).$$

How does it compare with the optimal lossless compressor? *Hint:* Compute the expected number of 9-bit blocks output per chunk of consecutive zeros/ones; normalize by the expected length of the chunk.