Discrete Quiz 5

Question 1. Do the prime factorization for this 6-digit positive integer: 510, 510.

Write your answer as a product of prime powers `p^a*q^b*r^c` or similar. Numbers p, q, r etc. should be in increasing order. All exponents (even those equal to 1) should be written explicitly.

Question 2. Are these statements true or false (do not forget that "all integers" include also negative numbers):

(A) For all integers a, b, c, if $a|c$ and $b|c$, then $(a+b)|c$.

(B) For all integers a, b, c, d, if $a|b$ and $c|d$, then $(ac)|(b+d)$.

(C) For all integers a, b, if $a|b$ and $b|a$, then $a = b$.

(D) For all integers a, b, c, if $a|(b+c)$, then $a|b$ and $a|c$.

(E) For all integers a, b, c, if $a|bc$, then $a|b$ or $a|c$.

(F) If p and q are primes (> 2), then $pq + 1$ is never prime.

Write your answer as a comma-separated string of T/F. For example, `T,T,T,T,T,T`.

Note. Even though you only write the answers, make sure that you are able to justify your answer. For true statements you should be able to find a reasoning; for false ones – a counterexample.

Question 3. Find $\mathrm{lcm}(24^{75}, 75^{24})$

Write your answer as a product of prime powers `p^a*q^b*r^c` or similar. Numbers p, q, r etc. should be in increasing order. All exponents (even those equal to 1) should be written explicitly.

Question 4. Convert $(100\,1100\,0011)_2$ to base 16, base 8 and base 7.

Write your answer as 3 comma-separated numbers. For the hexadecimal notation use capital letters A,B,C,D,E,F.

Question 5. Express the infinite periodic binary fraction $0.0001100110011\ldots_2 = 0.0(0011)$ as a rational number.

Note 1. In binary fractions a digit that is k places after the point is multiplied by 2^k. For example, 0.1_2 means $1/2$; 0.01_2 means $1/4$ and so on.

Note 2. You may need to use infinite geometric progression to find its value.

Write your answer as `P/Q`, where P, Q are both in decimal notation.

Question 6. Find the sum and the product of these two integers written in binary: $(101011)_2$, $(1101011)_2$.

Note. You may want to try the addition and multiplication algorithm directly in binary (without converting them into the decimal and back to the binary).

Write your answer as two comma-separated numbers (both written in binary).

Question 7. Write the first 10 powers of number 3 modulo 11: $3^1, 3^2, 3^3, \ldots, 3^{10}$.

Write your answer as a comma-separated list of ten remainders (mod 11), – numbers between 0 and 10.

Question 8. Alice has only 21-cent coins, Bob has 34-cent coins. Alice wants to pay Bob exactly 1 cent. Find two non-negative integers s, t that satisfy $21s - 34t = 1$.

Write your answer as two comma-separated integers.

Question 9. Consider the opposite situation: Alice has only 34-cent coins, Bob has only 21-cent coins. Find two non-negative integers s, t that satisfy $34s - 21t = 1$.

Write your answer as two comma-separated integers.

Question 10. Find 21^{-1} modulo 34. (This is a number z between 0 and 33 such that $21z \equiv 1 \pmod{34}$.)

Write your answer as a number modulo 34 (i.e. between 0 and 33).

Question 11. Solve the congruence equation $21x \equiv 11 \pmod{34}$.

Write your answer as a number modulo 34 (i.e. between 0 and 33).

Question 12. Solve the Bezout identity for the numbers $a = 390, b = 72$: Find any integers x, y satisfying the equation $390x + 72y = \gcd(390, 72)$.

Write your answer as two comma-separated integers.