Discrete Quiz 8

Question 1 Two chess queens are placed on two different places in a 4×4 chess-board. Assume that all the $\binom{16}{2}$ possibilities how they are placed have equal probabilities. Find a probability that one queen *attacks* the other. (Two queens attack each other, if they are located on the same horizontal, vertical or diagonal). Write your answer as a rational fraction: P/Q

Question 2 Assume that you are generating 10-bit sequences (a string of 0's and 1's). All the 2^{10} sequences have equal probabilities. Find the probability of an event that the 10-bit sequence does NOT contain two consecutive 0's anywhere.
Write your answer as a rational fraction: P/Q

Question 3 Two players have 4 cubic dices. Instead of the usual numbers, their faces have the following numbers on their faces:
Dice A: 4, 4, 4, 4, 0, 0;
Dice B: 3, 3, 3, 3, 3, 3;
Dice C: 6, 6, 2, 2, 2, 2;
Dice D: 5, 5, 5, 1, 1, 1.
In a single round Players Alice and Bob randomly select two of the dices (with equal probabilities they can select any of the six pairs - (A, B), or (A, C), or (A, D), or (B, C), or (B, D), or (C, D)). There can be three outcomes:
Outcome 1: They have selected "opposite dices" - pairs (A, C) or (B, D). In this case the payoff is zero (nobody pays anything to the other).
Outcome 2: The dices win in the "clockwise manner" ($A > B$ or $B > C$ or $C > D$ or $D > A$) - then Alice wins 1 euro.
Outcome 3: The dices win in the "counter-clockwise manner" ($B > A$ or $C > B$ or $D > C$ or $A > D$) - then Bob wins 1 euro.
(*Note.* The expression $A > B$ means that the number that rolled out on the the dice A was larger than the number on dice B; but $B > A$ denotes the opposite

event.)
Find the expected value - how much money Alice is expected to win in a single round of such a game.
Write your answer as a rational fraction: P/Q
For example, if the expected win for Alice is 0.10 EUR, then write 1/10. If Alice is expected on average to lose 0.10 EUR per one round of this game, then write -1/10.

Question 4 For every year we count the number of Friday's that fall on the 13th date of some month (such as Friday, March 13, 2020). Denote this count by X – it is your random variable. Find the expected value and the variance of X. Round them to the nearest thousandth.
Write your answer as two comma-separated numbers:

D.DDD,D.DDD.

Question 5 What is the probability that a randomly chosen positive integer between 1 and 600 is not divisible by either 6 or 10?
Write your answer as a rational fraction: P/Q

Question 6 A chip factory Intel adds one toy animal to every bag of the chips they produce. There are three sorts of animals - Aligators, Bears or Cats (and they have equal probability of 1/3). Find the expected number of the chip bags one needs to purchase to collect all three animals.
Write your answer as a rational fraction: P/Q

Question 7 You create a random bit string of length five (all 32 bit strings are equally probable). Consider these events:
E_1: the bit string chosen begins with 1;
E_3: the bit string chosen has exactly three 1s.
(A) Find $p(E_1 \mid E_3)$.
(B) Find $p(E_3 \mid E_1)$.
Write your answer as a comma-separated rational fractions P1/Q1,P2/Q2