Week 03 LAB: Objectives **1.5. Nested Quantifiers**  
1.5.a. Rewrite mathematical statements with nested quantifiers.  
1.5.b. Interpret quantifier expressions in human language.  
1.5.c. Negate multiple quantifiers.  
**1.8. Use Some More Proof Strategies**  
1.8.a. Explain proofs by analyzing cases.  
1.8.b. Explain proofs by counterexamples.

## Problem 1: Ulam Prime Spiral

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| Numbers from 1 to 49 placed in spiral order | Stanislaw Ulam (in addition to Ulam numbers) also suggested writing all numbers in a spiral. Then painted every prime number on the spiral. Prime numbers displayed a nice pattern to Ulam.  (1) Remember the arithmetic progression summation (2) Find the expression for vertical line: 1, 4, 15, 34, ...  (3) Find the expression for digonal line: 1, 9, 25, 49, ... (4) Find the expression for line going from “10” to the “South-West”: 10, 24, 46, ... |

## Problem 2: Rewrite with Predicates and Quantifiers

**(1)** Rewrite in predicate logic the following statement: **“f: R -> R is an odd function”**(such as f(x)=sin(x) or f(x)=x3. Switching the sign of its argument, the value switches sign). **(2)** Statement**: “f: R -> R is an even function”**(such as f(x)=cos(x) or f(x)=x2. Switching the sign of its argument, the value stays the same)  
**(3)** Statement: **“f: R -> R is a constant function”.**(All its values are the same).  
**(4)** Statement: **“f: R -> R is a periodic function”.**  
(After some period, all the values start repeating themselves.)  
**(5)** Statement: “F: Z+ -> Z+ is an eventually periodic   
(A sequence is eventually periodic if it can be made periodic by dropping some finite number of terms from the beginning. For example, the sequence of digits in the decimal expansion of 1/56 is eventually periodic:  
1 / 56 = 0 . 0 1 7 8 5 7 1 4 2 8 5 7 1 4 2 8 5 7 1 4 2 ... )  
**(6)** Statement: **“f is a bounded function”.**  
(All its values stay below some number – an upper bound, and also stay above some other number – a lower bound.)  
**(7)** Write in predicate logic: “f is a continuous function on R”.  
**(8)** Write in predicate logic: “lim f(x) = b, where x -> a.”.  
**(9)** Write in predicate logic: “lim f(x) = b, where x -> + infinity”.  
**(10)** Write in predicate logic: “f has a derivative in point x=a: f ’(a) = b.”  
**(11)** Write in predicate logic: “f is a strictly increasing function”  
**(12)** Write in predicate logic: “f is a (nonstrictly) decreasing function”; “f is a monotonous function”; “f has a local maximum when argument x=a”; “f is a convex function”