Final Exam

RBS, Discrete Structures

2020-04-23

## Problem 1

By we denote the set of all positive integers between and . This is our *universe* in which we define several subsets:

**(A)** Express using the sets (using set union , set intersection , set complement operations).  
**(B)** Express using the sets in a similar way.  
**(C)** Find - the size of the set .  
**(D)** Find - the size of the set .

## Problem 2

Let and be sets with sizes and and .

Calculate the largest and the smallest possible values for each of the following set sizes:

**(A)** .  
**(B)** .  
**(C)** - the powerset of a powerset of .  
**(D)** - the symmetric difference of the sets and .

## Problem 3

Consider the following recurrent sequence:

Assume that is another sequence satisfying the recurrence rule

(The first two members are not known.)

**(A)** Write the first members of this sequence ().  
**(B)** Write the characteristic equation for this sequence.  
**(C)** Write the general expression for an arbirary sequence satisfying the recurrent expression as a sum of two geometric progressions (you can leave unknown coefficients in your answer; just explain which ones they are).  
**(D)** Write the formula to compute (that would satisfy the initial conditions and ).

## Problem 4

Consider this code snippet in Python:

n = 1000  
sum = 0  
for i in range(1, n\*n+1):  
 for j in range(1,i+1):  
 sum += i % j

And a similar one in R:

n <- 1000  
sum <- 0  
for (i in 1:(n\*n)) {  
 for (j in 1:i) {  
 sum <- sum + i %% j  
 }  
}

**(A)** Explain in human language what this algorithm does.  
**(B)** Denote by the number of times the variable sum is incremented. Write the Big-O-Notation for . Find a function such that is in . (If there are multiple functions, pick the one with the slowest growth.)  
**(C)** Express the function precisely - how many times sum is incremented in terms of variable .

## Problem 5

Let be the set of all positive divisors of the number (including and itself).  
**(A)** What is the multiplication of all numbers in the set ?  
**(B)** Express this number as the product of prime powers.

## Problem 6

Define the following binary relationship on the set of integer numbers :  
We say that (numbers are in the relation ) iff

|  |  |  |
| --- | --- | --- |
| Item | Statement | True or False? |
| **(A)** | is reflexive |  |
| **(B)** | is symmetric |  |
| **(C)** | is antisymmetric |  |
| **(D)** | is transitive |  |
| **(E)** | iff |  |

For all items where you answered FALSE, specify a counterexample (values for some numbers that would make the condition true, but the conclusion false). If the statement was true, write “none”.

**(A)** counterexample \_\_\_

**(B)** counterexample \_\_\_

**(C)** counterexample \_\_\_

**(D)** counterexample \_\_\_

**(E)** counterexample \_\_\_

## Problem 7

Four people each has his own hat. After the meeting they leave their building in a hurry, everyone grabs some hat at random so that all permutations of the hats have equal probabilities.

Let the random variable denote the number of hats that were picked up correctly. (For example, if the hat assignment is this: , then , because two people got their own hats.)

**(A)** Find - the expected value of .  
**(B)** Find - the variance of .

## Problem 8

There was a crooked man who had a crooked 1 euro coin. On lucky days it would flip the *heads* with probability , and the *tails* with probability , but on unlucky days it was the opposite (, but ). There were equal probabilities of for lucky and unlucky days.

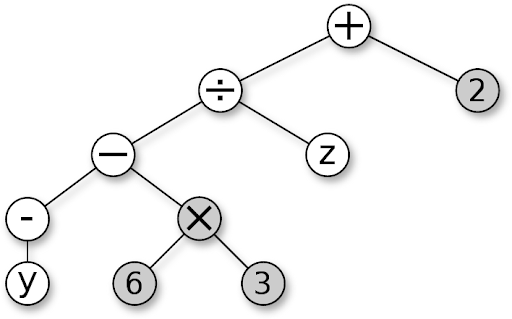
One morning he flipped the coin times and altogether got three *heads* and two *tails*.

Let us introduce the following events:

* (evidence): Five coin tosses result in three *heads* and two *tails*.
* (hypothesis): The current day is lucky.

**(A)** Find - the conditional probability of given that the day is lucky.  
**(B)** Find - the probability that the day is lucky and happens.  
**(C)** Find - the conditional probability of given that the day is not lucky.  
**(D)** Find - the probability that the day is unlucky and happens.  
**(E)** Find - as the sum of two probabilities ( happened on a lucky day and also happened on unlucky day).  
**(F)** Find the conditional probability - the likelyhood that the croocked man has a lucky day, given that the event has happened.

## Problem 9

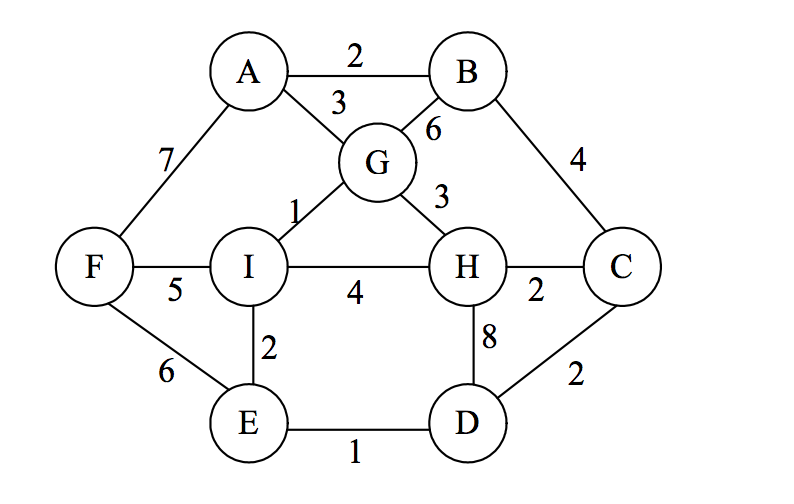
  
*Figure: A syntax tree for an expression*

The syntax tree describes an algebraic expression (please note the difference between the unary minus that flips the value of the variable and the binary minus that subtracts the two subexpressions: and ).

**(A)** Write the preorder DFS traversal of this tree.  
**(B)** Write the inorder DFS traversal of this tree.  
**(C)** Write the postorder DFS traversal of this tree.

*Note.* In all answers denote the unary minus with the tilde sign , but the regular/binary minus with .

## Problem 10

  
*Figure: Graph with 9 vertices*

Run the Prim’s algorithm on the following weighted graph, start growing the tree from the vertex .

|  |  |
| --- | --- |
| Step | Newly Added Edge |
| **Step 1** |  |
| **Step 2** |  |
| **Step 3** |  |
| **Step 4** |  |
| **Step 5** |  |
| **Step 6** |  |
| **Step 7** |  |
| **Step 8** |  |

What is the total weight of the obtained Minimum Spanning Tree? \_\_\_\_\_\_