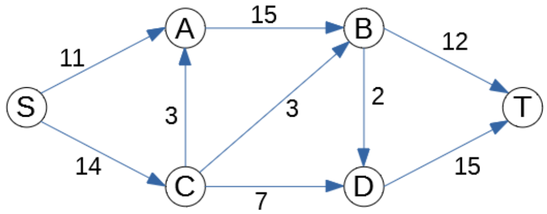
WRITTEN ASSIGNMENT 10



(A) Run Edmonds-Karp algorithm on the graph shown above. Every edge in the picture is labeled with a number showing the *capacity* of that edge.

For every phase highlight the the augmenting path (or simply list its vertices), find the *residual flow* of this augmenting graph. Draw a copy of the original graph where the residual flow is added. Namely, every age is labeled by two numbers `f/c` – the actual flow `f` (after adding the residual flow obtained in this step) and also the capacity `c` of the edge (it never changes).

During the next phase, show the next residual graph, highlight the augmenting path, find the residual flow. And next to that residual graph show a new copy of the original graph with updated flow numbers. Thus, every phase shows two oriented graphs:

- The current residual graph (initially – it is simply the given graph with all flows equal to 0). It only displays edge capacities and **not** flows (but it may include *reversed edges*). In this graph you can search (in BFS order) and highlight the augmenting path.
- The original graph with all the flows added. In this graph you must also show the flows using the notation with two numbers `f/c`.

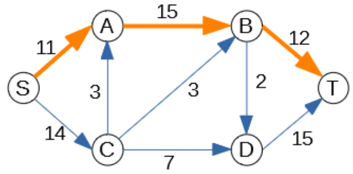
Note: In Edmonds-Karp algorithm visiting the successors of the source vertex s in the BFS order needs to know the ordering. Assume that all the vertices are arranged in alphabetical order.

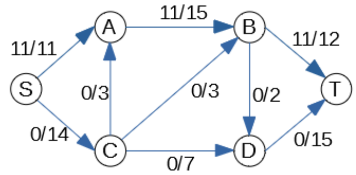
(B) Redraw the original graph with all the maximum flows (use the same two-number labels for edges `f/c`). Show the min-cut which prevents any further augmenting paths (either highlight with another color, or simply list the partition of graphs vertices into two disjoint sets that describe the cut).

Answer:

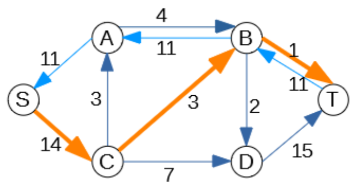
(A) Following Edmonds-Karp algorithm, we successfully select augmenting paths starting from the shortest ones (and lexicographically first – if there are multiple paths of the same length).

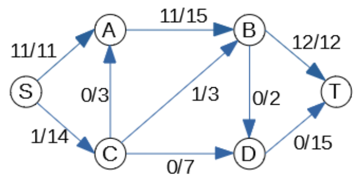
Phase 1: Push 11 units of flow over the augmenting path $S \to A \to B \to T$ highlighted in orange.



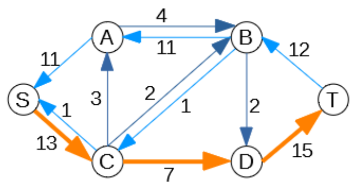


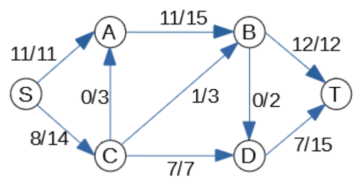
Phase 2: Push 1 unit of flow over the augmenting path $S \to C \to B \to T$.



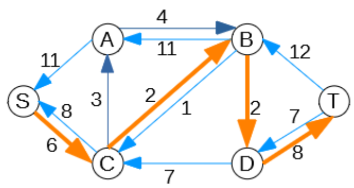


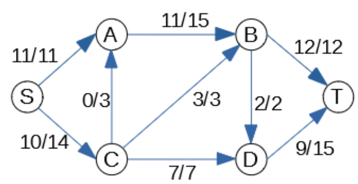
Phase 3: Push 7 units of flow over the augmenting path $S \to C \to D \to T$.



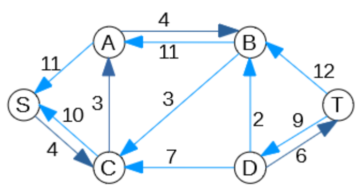


Phase 4: Push 2 units of flow over the augmenting path $S \to C \to B \to D \to T$.





The last residual graph does not contain any augmenting path from S to T, so the algorithm stops here. Overall, we have pushed $11 + 1 + 7 + 2 = 21$ units of flow.



(B) We redraw the flow graph (showing actual flows and capacities for each edge). The minimum cut is shown as red dashed line. It splits vertices into two disjoint groups: S, A, B, C and D, T; all the edges between them are saturated – the flow reaches capacity. As we know the capacity of a minimum cut must equal the maximum flow. This maximum flow (equalling the min cut capacity) is $7 + 2 + 12 = 21$.