Section I

1. Random fields/processes

a. Field: space, process: time
b. All aleatory (natural randomness)
c. A collection of random variables that are correlated
d. Two quantities for describing random processes
    i. Autocorrelation: R(\tau) ~ exp(-|tau|^2) (Gaussian process)
    ii. Power spectrum
e. Stationary: relative location; non-stationary: absolute location
f. Simulation of correlated RV:
    i. T matrix: matrix of eigenvectors of cov(x)
    ii. Covariance function: R(tau)*sigma^2
g. KL expansion
    i. Used to simulate random field

$$\varpi_M(x,\theta) = \overline{\varpi}(x) + \sum_{i=1}^{M} \sqrt{\lambda_i}\,\xi_i(\theta) f_i(x)$$

h. Polynomial chaos
    i. Multiple forms (Hermite works best w/ normal variables)

2. Time-dependent Reliability Analysis

a. Equivalent Static Method
    i. Type I - Gaussian Process (Largest: Gumbel, Smallest)
    ii. Type II - Lognormal (Frechet)
    Type I and Type II: model Load, Type III: model Capacity
    iii. Type III - Bounded process (Weibull)
    Example: Weibull - strength, time to failure
b. Outcrossing approach
    Outcrossing event ~ Poisson Process

3. Surrogate models

a. Algebraic
    i. Polynomial chaos expansion
    ii. Regression (least squares, ML, linear polynomial)
    iii. Support vector machines
    iv. Gaussian Process (GP) / Kriging
    v. Radial Basis Function
    vi. Neural Network (regression in multiple layers)
b. Distribution (interested in joint dist. of input & output) -> prob. Space surrogates
    i. Copula, https://en.wikipedia.org/wiki/Copula_(probability_theory)

ii. Mixture models, weighted sum of individual distribution
iii. Bayesian networks
iv. Hidden Markov models (HMM)

3. Stochastic finite elements (SFEM) - handle spatial variability

a. Intrusive approach -> you go into the code, find analytical gradients
b. Non-intrusive “ -> treat as a black box
   Adjoint Gradient or Surrogate Model

4. Sensitivity analysis

a. Local (only good at that region)
   i. FORM (approx. the limit state func. With a first order (linear) approx., gives the most sensitive input locally)
   ii. Just the gradient of the limit state function
b. Global (Variance-based uncertainty)
   i. The contribution from the change of inputs to the change of outputs
   ii. Manov’s presentation pdf
   iii. https://www.sciencedirect.com/science/article/pii/S0951832016300266 (single loop sampling -- uniform probability spacing)
   iv. https://link.springer.com/article/10.1007/s00158-018-2077-1 (importance sampling using kernels)
   v. https://arxiv.org/pdf/1806.06285.pdf (derivative-based upper bound on total effects index)
   vi. https://arxiv.org/abs/1810.00955 (active subspace)

5. Bayesian methods (inference)

a. Data on X -> figure out the distribution of X
   i. Parametric: Histogram - Parametric distributions - Distribution type, Distribution parameters
   ii. Non-parametric: numerical fit
b. Parameter estimation
   i. MLE (if error is normal, its same with Least square)
   ii. LS ~ errors Normal
   iii. Bayesian: represent your lack of knowledge with a prob. Dist.
   Construction of posterior dist:
      1. Conjugate dist.
      2. Sampling-based Methods
         a. MCMC - generate samples based on a distribution (Converge samples to a target distribution)
            i. Metropolis

ii. Gibbs
iii. Slice sampling
iv. Adaptive improvements

b. Particle-filter methods
    i. Sequential importance re-sampling
    ii. Rao-Blackwelization, smoothing

Section II

1. Data uncertainty quantification

(x has both aleatory and epistemic uncertainties, how much does each side contribute)

a. D: distribution type, epistemic; theta_x: parameter(s) of the distribution, epistemic; U: auxiliary variable: aleatory
    i. Paper: Separating the contributions of variability and parameter uncertainty in probability distributions (https://www.sciencedirect.com/science/article/pii/S0951832012002542)
    ii. Paper: Distribution type uncertainty due to sparse and imprecise data (https://www.sciencedirect.com/science/article/pii/S0888327012002713)
    iii. Paper: Reliability analysis under epistemic uncertainty (https://www.sciencedirect.com/science/article/pii/S0951832016301119)
b. How to choose a distribution D
    i. Sum of squared (chi square - PDF)
    ii. Max error: K-S (CDF)
    iii. Likelihood ratio (Bayes factor)

2. Model uncertainties

3 sources:

a. Model Form uncertainties
    i. Validation
        1. Is the model representing the real world?
        2. Compare prediction vs. observation (MAE, RMSE)
    ii. Calibration
    iii. Fidelity (how good the physics is represented)
b. Model parameters
    i. Calibration (MLE, Bayesian, etc.)
c. Numerical approximations
    i. Verification (Numeric error quantification)
        1. Code verification
        2. Solution verification
    ii. Resolution (eg. number of mesh)
d. Model Discrepancy (Model Error)
    i. Model Form Error
    ii. Numerical solution error
        1. Deterministic error

            a. Discretization error (Richardson Extrapolation to quantify)
            b. Truncation error
        2. Stochastic error
            a. Surrogate model error
            b. Uncertainty propagation error
    iii.

3. Model verification/calibration/validation

a. Steps
    i. Develop a conceptual model, construct a mathematical equation
    ii. Develop numerical solution procedure
    iii. Verification
        1. Code verification
        2. Solution verification
    iv. Calibration (parameter estimation)
    v. Model Validation
        1. Definition: The process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended use of the model
        2. Paper: Quantitative model validation techniques: New insights (https://www.sciencedirect.com/science/article/pii/S0951832012002414)
b. Model validation metrics
    i. Statistical confidence intervals
        1. Area metric
        2. Normalizing residuals
    ii. Hypothesis testing based methods (Statistical)
        1. Classical hypothesis testing
            a. Z test
            b. T test
        2. Bayesian hypothesis testing
            a. Bayes Factor B (>1 supports H_0)
            b. Confidence measure Pr(H_0|y_D) = B/(B+1)
            c. Four scenarios: total risk (total probability rule)
            Paper: Bayesian risk-based decision method for model validation under uncertainty (https://www.sciencedirect.com/science/article/pii/S0951832006000718)
        3. Reliability analysis based techniques (Probabilistic)
            d. Distance metric (y_D - y_m)
            e. Area metric
c. Model selection
    i. Multi-fidelity model (tradeoff between accuracy and cost)

1. Use low fidelity model to run multiple times until error is unacceptable, and use high fidelity model to correct.

4. Uncertainty aggregation and reduction

a. Roll up Sankararaman, S., and Mahadevan, S., "Integration of calibration, verification and validation for uncertainty quantification in engineering systems," *Reliability Engineering & System Safety*, Vol. 138, pp. 194-209, 2015.
b. Uncertainty aggregation flow details: Nannapaneni, S., and Mahadevan, S., "Reliability Analysis under Epistemic Uncertainty," Reliability Engineering & System Safety, Vol. 155, pp. 9-20, 2016.
c. http://www.gdr-mascotnum.fr/media/etics2018_mahadevan.pdf
d. Paper: Relative contributions of aleatory and epistemic uncertainty sources in time series prediction (https://www.sciencedirect.com/science/article/pii/S0142112315002832)

5. Bayesian networks for information fusion

a. Composite distribution (use Bayes factor to calculate the weights of distribution type)
b. Auxiliary variable: u is the same as adding model error, e_m

6. UQ in multi-physics and multi-scale

7. Decision-making under uncertainty

a. Only Aleatory: RBDO or RDO
    i. Coupled
    ii. Decoupled (OUU formulations)
        1. Decoupled: (similar to previous, reliability measure) opt. ↔ reliab.
        2. Single-loop (RBDO): Kuhn-Tucker conditions, FORM: instead of having 2 routines, built-in 1 routine for only FORM with Single-Loop (Thus, no need to calculate beta, reliability analysis) → reduce comp. Effort, inc. d.v., x_p is not a d.v. but each x_d, it changes
b. Aleatory + Epistemic

i. RBDO
   1. Mean value of P[g $\leq 0]$ or P[P[g $\leq 0] \geq Pf1] \geq Pf2 \rightarrow$ $Product\ certification$, $Pf1 = 0.95$ & $Pf2$ =0.95 -> having a conf.in reliability.
   2. Dual problem: max. P[P[g $\leq 0] \geq Pf1]$, s.t. cost $\leq$ target

Test Resource Allocation
Testing & Simulation Optimization
Calibration & Validation Tests (2 types)
1. Test design -> max. Info. gain -> K-L div.
2. Test selection -> meet target system-level prediction unc. -> # of tests at diff. Levels of complexity (physics, component), diff. Kinds of tests.Has to be done with synthetic data since test is not done yet!

7.1. System design

7.2. Model development (Resource alloc. usually w/ synthetic data, for unc. reduction)

7.2.1. Testing

7.2.1.1. Calibration (# of tests, test inputs, sensor placement)
7.2.1.2. Validation

7.2.2 Modeling

7.2.2.1 Fidelity
7.2.2.2 Scale
7.2.2.3 Resolution

Goal: Accuracy
Obj.: min. Error (bias,i.e. Cross-valid., + var), s.t. cost<budget

To est. predic. (for other tests)
1. *Roll-up post. (post. After valid.) -> non-intrusive*
2. *Embedded dicrepency*
3. *Abhinav method (go back to govn. eq.) -> intrusive (source code is available)*

7.3 Manufacturing (product quality) - > min. var(quality) -> bias+var. : dynamic process not like model predict.

7.4 Performance, Operations & Health Management:

7.4.1 Intervals/Schedule (general): how often? Where to (hotspot)? What to measure?

7.4.2 Active\passive measurement\sensing (sensor passively collects, active: you send a signal and detect)

7.4.3 Mission profile -> Resilience (to make sure system can effectively respond to an adverse event): all done under unc:

7.4.3.1 response -> immediate
7.4.3.2 rerouting/reconfig.
7.4.3.3 remediation
7.4.3.4 recovery

8. Non-probabilistic methods

Both 8.1. And 8.2. Do not update knowledge (unlike Bayesian framework)

8.1. Interval Analysis

- Only useful for forward problems.
- Inverse is done by expert.
- Correlation is an issue (w/out corr. Interval is wider)
- Non-linear func. (use opt.)
- Binary combinations of variable (2^n)

8.2. Fuzzy Sets/Possibility Theory

- Extension of classical set theory
- Allows ambiguity (unlike classical set) -> partial membership (in your mind)
  - Vauge\not crisp info (fuzzy)
- possibility/degree of membership
- Correlation is limiation
- New info. Not updated is an issue

8.3. Evidence Th.

- Extension of Bayesian prob. Th.
- Basic belief (prob.) assignment (BBA)
- Allows a little more flexibility than prob. Th. to capture expert's opinion/
- <u>Evidence Combination RUle (Dempster Rule)</u>
  - <u>Nothing but Bayes' Thm. in which you can only do x1 & x2 not (x1,x2)</u>
  - <u>Evidence Th. expends the discernment</u> (the quality of being able to grasp and comprehend what is obscure : skill in discerning: an act of perceiving or discerning something)
- Belief (lb) vs. plausibility (ub), prob. in between

Info gap Th. (Caltech)

concentration -of-measure inequalities (Michael Ortiz)