Signale und Systeme Boxen

Florian Lubitz & Steffen Hecht

16. Mai 2018

1 Motivation, Wiederholung und Überblick

a

6

$$u_1(t) = 15\,\mathrm{V}\sin(\pi t + \pi/3) + 60\,\mathrm{V}\sin(10\pi t + \pi/3) = 0,5x(t) + 2y(t)$$

und damit $a = 0,5, b = 2$ und

$$u_2(t) := \mathcal{H}\{u_1(t)\} = \mathcal{H}\{0,5x(t) + 2y(t)\} \stackrel{??}{=}$$

2 Diskrete Signale

$\boxed{11}$

$$(b) \quad x[-k] = \begin{cases} -\frac{1}{k}, k/ne0 \\ 0, k = 0 \end{cases}$$

$$(c) \quad x[k+k_0] = x[k+3] = \begin{cases} \frac{1}{k+3}, k/ne-3 \\ 0, k = -3 \end{cases}$$

$$(d) \quad x[k-k_0] = x[k-3] = \begin{cases} \frac{1}{k-3}, k/ne3 \\ 0, k = 3 \end{cases}$$

$\boxed{12}$

$$\begin{aligned} x[k_0 - k] &= x[-(k-k_0)] \\ &= x[(-k)+k_0] \end{aligned}$$

$\boxed{13}$

$$\text{mit} \quad x[k_0 - k] = x[3-k] = \begin{cases} \frac{1}{3-k}, k \neq 3 \\ 0, k = 3 \end{cases}$$

$\boxed{14}$

- $x[k]$ heißt gerades Signal, falls $x[k] = x[-k] \forall k \in \mathbb{Z}$ gilt.
- $x[k]$ heißt ungerades Signal, falls $x[k] = -x[-k] \forall k \in \mathbb{Z}$ gilt.

15

$$x[-k] = \begin{cases} \frac{1}{-k}, k \neq 0 \\ 0, k = 0 \end{cases} \quad = \begin{cases} -\frac{1}{k}, k/ne0 \\ 0, k = 0 \end{cases} \quad = -x[k]$$

16

$$y[-k] = \begin{cases} \frac{1}{(-k)^2}, k \neq 0 \\ 0, k = 0 \end{cases} \quad = \begin{cases} -\frac{1}{k^2}, k \neq 0 \\ 0, k = 0 \end{cases} \quad = y[k]$$

17

- $x[k]$ heißt kausales Signal, falls gilt: $x[k] = 0 \forall k < 0$
- $x[k]$ heißt nicht-kausales Signal, falls gilt $\exists k < 0 : x[k] \neq 0$
- $x[k]$ heißt anti-kausales Signal, falls $x[-k-1]$ kausal ist, d.h. falls gilt: $x[k] = 0 \forall k \leqslant 0$

18

- $x[k]$ ist nicht-kausal
- $u[k]$ ist kausal
- $v[k]$ ist anti-kausal

19

$$\delta[k] := \begin{cases} 1, k = 0 \\ 0, k \neq 0 \end{cases}$$

20

$$\epsilon[k] := \begin{cases} 1, k \geq 0 \\ 0, k < 0 \end{cases}$$

21

$$\delta[k-k_0] = \begin{cases} 1, k = k_0 \\ 0, k \neq k_0 \end{cases}$$

bzw.

$$\delta[k+k_0] = \begin{cases} 1, k \neq -k_0 \\ 0, k = -k_0 \end{cases}$$

22

$$\begin{aligned} x[k] \cdot \delta[k-i] &= \begin{cases} x[i], k = i \\ 0, k \neq i \end{cases} \\ &= x[i] \cdot \delta[k-i] \end{aligned} \tag{2.1}$$

Siebeigenschaft

23

$$x[k] = \sum_{i=-\infty}^{\infty} x[i] \cdot \delta[k-i] \quad \text{für alle} \quad k \in \mathbb{Z}$$

24

$$x[k] = \sum_{i=-K}^{K} x[i] \cdot \delta[k-i]$$

25

$$\begin{aligned} u[k] &= \delta[k+2] + \delta[k+1] + \delta[k] + \delta[k-1] \\ v[k] &= 2 \cdot \delta[k+3] + \delta[k+1] - \delta[k-1] - 2 \cdot \delta[k-3] \end{aligned}$$

26 $$sgn[k] := \epsilon[k] - \epsilon[-k] = \begin{cases} 1, k > 0 \\ 0, k = 0 \\ -1, k < 0 \end{cases}$$

27 $$Ш[k] := \epsilon[k] + \epsilon[-k-1] = 1 \text{für alle} k \in \mathbb{Z}$$

28 $$rect_{k_1,k_2}[k] :)\epsilon[k - k1] - \epsilon[k - k_2 - 1] = \begin{cases} 1, k_1 \leqslant k \leqslant k_2 \end{cases}$$

29 $$x[k] = q^k \cdot \epsilon[k]$$

30 $$x[k] : 0, ..., 0, x[0] = 1, x[1] = -0.7, x[2] = 0.49, x[3] = 0.343, ...$$

31 $$x[k] : 0, ..., 0, x[0] = 1, x[1] = -0.8, x[2] = 0.64, x[3] = -0.512, ...$$

32 $$\begin{aligned} x[k] + y[k] &: x[-\infty] + y[-\infty]..., x[0] + y[0], x[1] + y[1], ..., x[\infty] + y[\infty] \\ x[k] \cdot y[k] &: x[-\infty] \cdot y[-\infty]..., x[0] \cdot y[0], x[1] \cdot y[1], ..., x[\infty] \cdot y[\infty] \\ c \cdot x[k] &: c \cdot x[-\infty]..., c \cdot x[0], c \cdot x[1], ..., c \cdot x[\infty] \end{aligned}$$

33

$$S_{k_1,k_2} := \{\overrightarrow{x} \in S | x[k] = 0 \forall k < k_1 \text{oder} k > k_2\}$$

34

$$\begin{aligned}
\overrightarrow{x} &= (0 \quad 3 \quad 2 \quad 5 \quad 0 \quad 0) \\
\overrightarrow{y} &= (0 \quad 0 \quad 2 \quad -3 \quad 0 \quad 2) \\
\overrightarrow{x} + \overrightarrow{y} &= (0 \quad 3 \quad 4 \quad 2 \quad 0 \quad 2) \\
\overrightarrow{x} - \overrightarrow{y} &= (0 \quad 3 \quad 0 \quad 8 \quad 0 \quad -2) \\
\overrightarrow{x} \cdot \overrightarrow{y} &= (0 \quad 0 \quad 4 \quad -15 \quad 0 \quad 0) \\
c + \overrightarrow{x} &= (0 \quad 15 \quad 10 \quad 25 \quad 0 \quad 0)
\end{aligned}$$

35

$$(x * y)[k] := \sum_{i=-\infty}^{\infty} x[i] \cdot y[k-i]$$

36

$$\begin{array}{ccc}
\begin{array}{c} i=0 \\ \downarrow \\ x[i] = (3 \quad 2 \quad 1), \end{array} &
\begin{array}{c} i=0 \\ \downarrow \\ y[i] = (1 \quad -1 \quad 2) \end{array} \text{ bzw.} &
\begin{array}{r} i=0 \\ \downarrow \\ z[0-i] = (2 \quad -1 \quad 1) \end{array}
\end{array}$$

37

								$\sum x[i]y[k-i] =$	$(x*y)[k]$
	$x[i] =$			3	2	1			
$k=0$	$y[k-i] =$	2	-1	1				$3 \cdot 1$	$= 3$
$k=1$			2	-1	1			$3 \cdot (-1) + 2 \cdot 1$	$= -1$
$k=2$				2	-1	1		$3 \cdot 2 + 2 \cdot (-1) + 1 \cdot 1$	$= 5$
$k=3$					2	-1	1	$2 \cdot 2 + 1 \cdot (-1)$	$= 3$
$k=4$						2	-1 1	$1 \cdot 2$	$= 2$

38

$$x[k] * y[k] = 3\delta[k] - \delta[k-1] + 5\delta[k-2] + 3\delta[k-3] + 2\delta[k-4]$$

39

$$
\begin{aligned}
&\qquad i=-43\\
&\qquad\ \downarrow\\
x[i] =&(-1 \quad 3 \quad -2) \text{ und}\\
&\qquad i=19\\
&\qquad \downarrow\\
y[i] =&(1 \quad -2 \quad 4 \quad -1) \text{ bzw. } y[-i] = (-1 \quad 4 \quad -2 \quad 1)
\end{aligned}
$$

40

$i = -43$ (pfeil)

k	$x[i]$				-1	3	-2				$(x*y)[k]$
-24	$y[k-i] =$	-1	4	-2	1						-1
-23			-1	4	-2	1					$2+3=5$
-22				-1	4	-2	1				$-4-6-2=-12$
-21					-1	4	-2	1			$1+12+4=17$
-20						-1	4	-2	1		$-3-8=-11$
-19							-1	4	-2	1	2

41

$$
\begin{aligned}
(x*y)[k] =& -\delta[k+24] + 5\delta[k+23] - 12\delta[k+22] + 17\delta[k+21]\\
& - 11\delta[k+20] + 2\delta[k+19]
\end{aligned}
$$

42

$$x[k] * y[k] \in \mathcal{S}_{a+c,b+d} \quad \text{und hat Länge} \quad n+m-1.$$

43

I) Kommutativität: $x * y = y * x$

II) Assoziativität: $w * (x * y) = (w * x) * y$ und $c \cdot (x * y) = (c \cdot) * y$

III) Distributivität: $w * (x + y) = w * x + w * y$

IV) Neutrales Element: $x * \delta = x$

V) Verschiebung: $x[k] * \delta[k_0 - k] = x[k - k_0]$

VI) Zeitinvarianz: $x[k] * y[k - k_0] = (x[k] * y[k])[k - k_0]$

VII) Linearität: $(c \cdot x + d \cdot y) * w = c \cdot (x * w) + d \cdot (y * w)$

44

$$p(z) := a_0 + a_1 z + a_2 z^2 + a_3 z^3 + ... + a_n z^n$$

45

$$x[k] = a_0 \delta[k] + a_1 \delta[k-1] + a_2 \delta[k-2] + ... + a_n \delta[k-n]$$

46

$$p(z) \cdot q(z) = c_0 + c1_z + c_2 z^2 + ... + c_{2n} z^{2n} \quad \text{Mit Koeffizenten } c_k = (c * y)[k]$$

47

$$p(z) = 3 + 2z + z^2 \text{ und } q(z) = 1 - z + 2z^2$$

48

$$\begin{aligned} p(z) \cdot q(z) &= (3 + 2z + z^2) \cdot (2z^2 - z + 1) \\ &= 3 \cdot 1 + z(3 \cdot (-1) + 2 \cdot 1) + z^2(3 \cdot 2 + 2 \cdot (-1) + 1 \cdot 1) \\ &\quad + z^3(2 \cdot 2 + 1 \cdot (-1)) + z^4(1 \cdot 2) \\ &= 3 - z + 5z^2 + 3z^3 + 2z^4 \end{aligned}$$

49

$$E_x := \sum_{i=-\infty}^{\infty} |x[i]|^2$$

50

$$P_x := \lim_{K\to\infty} \frac{1}{2K+1} \sum_{i=-K}^{K} |x[i]|^2$$

51

$$\langle x[k], y[k] \rangle_E := \sum_{k=-\infty}^{\infty} x^*[k] \cdot y[k]$$

52

$$\langle x[k], y[k] \rangle_P := \lim_{K\to\infty} \frac{1}{2K+1} \sum_{k=-K}^{K} x^*[k] \cdot y[k]$$

53

$$||x[k]||_E := \sqrt{\langle x[k], x[k] \rangle_E} = \sqrt{E_x} \text{ bzw.}$$
$$||x[k]||_P := \sqrt{\langle x[k], x[k] \rangle_P} = \sqrt{P_x}$$

54

$$\cos\Phi = \frac{\langle x[k], y[k] \rangle}{||x[k]|| \cdot ||y[k]||}$$

55

$$\varphi_{xy}[\kappa] := \langle x[k], y[k+\kappa] \rangle$$

$\boxed{56}$

$$\varphi_{xx}[\kappa] := \langle x[k], x[k+\kappa] \rangle$$

$\boxed{57}$

$$\varphi_{xy}^{E}[\kappa] = x^*[-\kappa] * y[\kappa] \text{ bzw. } \varphi_{xy}^{P}[\kappa] = \lim_{K\to\infty} \frac{1}{2K+1} x_K^*[-\kappa] * y_K[\kappa]$$

3 Diskrete Systeme

Inhalt...

58

$$y[k] = \mathcal{H}\{x[k]\}$$

59

$$x[k] = x_0 \cdot \delta[k] = \begin{cases} x_0, k = 0 \\ 0, k \neq 0 \end{cases}$$

entwickelt sich nun das Guthaben des Sparbuchs wie folgt:
zu Beginn: $y[0] = x_0$
nach 1 Jahr: $y[1] = x_0 + p \cdot x_0 = (1+p) \cdot x_0$ nach 2 Jahren: $y[2] = (1+p)x_0 + p \cdot (1+p) \cdot x_0 = (1+p) \cdot (1+p) \cdot x_0 = (1+p)^2 \cdot x_0$
nach 3 Jahren: $y[3] = ... = (1+p)^3 \cdot x_0$
nach i Jahren: $y[i] = (1+p)^i \cdot x_0$
D.h. das Ausgangssignal ist die kausale Exponentialfolge $y[k] = x_0 \cdot (1+p)^k \cdot \epsilon[k]$

60

$$y[k+1] = y[k] \cdot (1+p) + x[k+1] \tag{3.1}$$

Das heißt $y[k+1]$ ergibt sich aus dem verzinsten Guthaben $y[k]$ des vorigen Jahres und zusätzlich den neuen Einzahlungen $x[k+1]$.

61

$$\mathcal{H}\{c \cdot x_1[k] + d \cdot x_2[k]\} = c \cdot \mathcal{H}\{x_1[k]\} + d \cdot \mathcal{H}\{x_2[k]\}$$

62

$$y[0] = x[0] = c \cdot x_1[0] + d \cdot x_2[0]$$

63

$$\begin{aligned}
y[k+1] &\overset{(3.1)}{=} y[k] \cdot (1+p) + x[k+1] \\
&\overset{(I.V)}{=} (cy_1[k] + d \cdot y_2[k]) \cdot (1+p) + c \cdot x_1[k+1] + d \cdot x_2[k+1] \\
&= c \cdot (y_1[k] \cdot (1+p) + x_1[k+1]) + d \cdot (y_2[k] \cdot (1+p) + x_2[k+1]) \\
&\overset{(3.1)}{=} c \cdot y[k+1] + d \cdot y_2[k+1]
\end{aligned}$$

64

$$\mathcal{H}\{x[k-k_0]\} = y[k-k_0]$$

65

$$\begin{aligned}
z[k_0] = x[k_0 - k_0] = x[k_0] = y[0] = y[k_0 - k_0] \\
\text{und } z[k] = 0 = y[k-k_0] \text{ für } k < k_0
\end{aligned}$$

66

$$\begin{aligned}
z[k+1] &\overset{(3.1)}{=} z[k] \cdot (1+p) + x[k+1-k_0] \\
&\overset{(I.V.)}{=} y[k-k_0] \cdot (1+p) + x[k-k_0+1] \\
&\overset{(3.1)}{=} y[k-k_0+1]
\end{aligned}$$

67 ... wenn der Ausgabewert $y[k_0]$ zur Zeit k_0 nur von früheren Eingabewerten $x[k], k \leq k_0$ abhängig ist.

68

$$|x[k]| < C \forall k \Rightarrow |y[k]| < D \forall k$$

69

$$y[k] = x_0 \cdot (1+p)^k \cdot \epsilon[k] \to \infty \text{ für } k \to \infty$$

70

..., wenn der Ausgang $y[k]$ zur Zeit k nur vom Eingang $x[k]$ zur Zeit k abhängt.

71

..., falls $y[k]$ nur von $x[\kappa]$ für $|\kappa - k| \leq L$ abhängt.

72

$$h[k] := \mathcal{H}\{\delta[k]\}$$

73

$$\begin{aligned} y[k] = \mathcal{H}\{x[k]\} &\overset{(2.6)}{=} \mathcal{H}\left\{\sum_{i=-\infty}^{\infty} x[i] \cdot \delta[k-i]\right\} \\ &= \sum_{i=-\infty}^{\infty} x[i]\mathcal{H}\{\delta[k-i]\} \\ &= \sum_{i=-\infty}^{\infty} x[i] \cdot h[k-i] \\ &= x[k] * h[k] \end{aligned}$$

74

$$y[k] = x[k] * h[k] \text{ für alle } x[k] \in \mathcal{S}$$

75

$$h[k] := \mathcal{H}\{\delta[k]\} = (1+p)^k \epsilon[k]$$

76

$$y[k] = h[k] * x[k] = \sum_{i=-\infty}^{\infty} (1+p)^i \epsilon[i] \cdot x[k-i]$$
$$= \sum_{i=0}^{\infty} (1+p)^i \cdot x[k-i]$$

77

$$y[k] = \sum_{i=0}^{k} (1+p)^i \cdot x[k-i]$$

78

$$\sum_{y=-\infty}^{\infty} |h[i]| < \infty$$

79

$$|y[k]| = |h[k] * x[k]| = |\sum_{i=-\infty}^{\infty} h[k]x[k-i]| \overset{DUG}{\leq} \sum_{i=-\infty}^{\infty} |h[i] \cdot x[k-i]|$$
$$= \sum_{i=-\infty}^{\infty} |h[i]| \cdot |x[k-i]| < M \sum_{i=-\infty}^{\infty} |h[i]| < M \cdot C < \infty$$

Boxes 80 - 97 missing

98

$$\mathcal{Z}\{\alpha x[k] + \beta y[k]\} \overset{Def.}{=} \sum_{k=-\infty}^{\infty} (\alpha x[k] + \beta y[k]) \cdot z^{-k}$$
$$= \alpha \left(\sum_{k=-\infty}^{\infty} x[k] z^{-k} \right) + \beta \left(\sum_{k=-\infty}^{\infty} y[k] z^{-k} \right)$$
$$= \alpha X(z) + \beta Y(z)$$

$\boxed{99}$

$$\mathcal{Z}\{x[k+k_0]\} \overset{Def.}{=} \sum_{k=-\infty}^{\infty} x[k+k_0]z^{-k} \underset{(k=k'-k_0)}{\overset{k'=k+k_0}{=}} \sum_{k'=-\infty}^{\infty} \underbrace{x[k']z^{-k'+k_0}}_{=z^{-k'}\cdot z^{k_0}}$$
$$= z^{k_0} \cdot \underbrace{\sum_{k'=-\infty}^{\infty} x[k']z^{-k'}}_{=X(z)} = z^{k_0} \cdot X(z)$$

$\boxed{100}$

$$\mathcal{Z}\{a^k \cdot x[k]\} \overset{Def.}{=} \sum_{k=-\infty}^{\infty} \underbrace{a^k}_{=(\frac{1}{\alpha})^{-k}} \cdot x[k] \cdot z^{-k} = \sum_{k=-\infty}^{\infty} x[k] \cdot \left(\frac{z}{\alpha}\right)^{-k} = X\left(\frac{z}{\alpha}\right)$$

$\boxed{101}$

$$\mathcal{Z}\{x[-k]\} = \sum_{k=-\infty}^{\infty} x[-k]z^{-k} \underset{(k=-k')}{\overset{k'=-k}{=}} \sum_{k'=-\infty}^{\infty} x[k']z^{k'} = \sum_{k'=-\infty}^{\infty} x[k'] \cdot \left(\frac{1}{z}\right)^{-k'} = X\left(\frac{1}{z}\right)$$

$\boxed{102}$

$$x[0] = \lim_{z\to\infty} X(z)$$

$\boxed{103}$

$$\lim_{k\to\infty} x[k] = \lim_{z\to 1}(z-1) \cdot X(z)$$

$\boxed{104}$

$$\mathcal{Z}\{\alpha^{k-1} \cdot \epsilon[k-1]\} = z^{-1} \cdot \mathcal{Z}\{\alpha^k \cdot \epsilon[k]\} = z^{-1} \cdot \frac{z}{z-\alpha} = \frac{1}{z-\alpha}$$

105

$$\mathcal{Z}\{A \cdot \alpha^{k-k_0} \cdot \epsilon[k-k_0]\} = A \cdot z^{-k} \cdot \mathcal{Z}\{\alpha^k \epsilon[k]\} = A \cdot z^{-k_0} \cdot \frac{z}{z-\alpha} = \frac{A \cdot z^{-(k_0-1)}}{z-\alpha}$$

106

$$\begin{aligned} \mathcal{Z}\{k \cdot \alpha^k \cdot \epsilon[k]\} &= -z \cdot \frac{d}{dz} \mathcal{Z}\{\alpha^k \epsilon[k]\} = -z \cdot \left(\frac{z}{z-\alpha}\right)' \\ &= -z \cdot \left(\frac{1 \cdot (z-\alpha) - z \cdot 1}{(z-\alpha)^2}\right) = \frac{\alpha \cdot z}{(z-\alpha)^2} \end{aligned}$$

107

$$Y(z) = H(z) \cdot X(z)$$

108

$$H(z) = \frac{Y(z)}{X(z)}$$

109

$$\begin{aligned} h[k] &= (1+p)^k \cdot \epsilon[k] \\ H(z) &= \frac{z}{z-(1+p)} \end{aligned}$$

110

$$y[k+1] = y[k] \cdot (1+p) + x[k+1]$$

$$\circ\!-\!\bullet$$

$$Y(z) = Y(z) \cdot (1+p) + zX(z) \Leftrightarrow Y(z)(z-(1+p)) = zX(z)$$

$\boxed{111}$

$$H(z) := \frac{Y(z)}{X(z)} = \frac{z}{z-(1+p)}$$

$\boxed{112}$

$$x[k] = x_0 \cdot \epsilon[k]$$
$$\circ\!\!-\!\!\bullet$$
$$X(z) = x_0 \cdot \frac{z}{z-1}$$

$\boxed{113}$

$$Y(z) = H(z) \cdot X(z) = \frac{z}{z-(a+p)} \cdot x_0 \cdot \frac{z}{z-q} = x_0 \cdot z^2 \cdot \frac{1}{(z-(a+p)) \cdot (z-1)}$$

$\boxed{114}$

$$\frac{1}{(z-(a+p)) \cdot (z-1)} = \frac{p^{-1}}{z-(a+p)} - \frac{p^{-1}}{z-1}$$

$\boxed{115}$

$$\frac{p^{-1}}{z-(a+p)} \bullet\!\!-\!\!\circ p^{-1}(a+p)^{K-1}\epsilon[K-1] \text{ und}$$
$$\frac{p^{-1}}{z-1} \bullet\!\!-\!\!\circ p^{-1}1^{K-1}\epsilon[K-1] = p^{-1}\epsilon[K-1]$$

$\boxed{117}$

$$\sum_{i=0}^{n} \alpha_i y[k-i] = \sum_{i=N-M}^{N} \beta_i x[k-i] \qquad\qquad (a_0 \neq 0, \beta_{N-M} \neq 0)$$

$$\circ\!\!-\!\!\bullet$$

$$\begin{aligned}
\sum_{i=0}^{N} \alpha_i \cdot z^{-i} \cdot Y(z) &= \sum_{i=N-M}^{N} \beta_i \cdot z^{-i} \cdot X(z) \\
\Leftrightarrow Y(z) \cdot \sum_{i=0}^{N} \alpha_i \cdot z^{-i} &= X(z) \cdot \sum_{i=N-M}^{N} \beta_i \cdot z^{-i}
\end{aligned}$$

$\boxed{118}$

$$\begin{aligned}
H(z) := \frac{Y(z)}{X(z)} &= \frac{\sum_{i=N-M}^{N} \beta_i \cdot z^{-i}}{\sum_{i=0}^{N} \alpha_i \cdot z^{-i}} \\
&= \frac{\sum_{i=N-M}^{N} \beta_i \cdot z^{N-i}}{\sum_{i=0}^{N} \alpha_i \cdot z^{N-i}} \\
&= \frac{\beta_{N-M} \cdot z^{M} + \beta_{N-M+1} \cdot z^{M-1} + ... + \beta_N}{\alpha_0 \cdot z^{N} + \alpha_1 \cdot z^{N-1} + ... + \alpha_N}
\end{aligned}$$

$\boxed{119}$

$$y(k) = \frac{1}{\alpha_0}\left(\sum_{i=N-M}^{N} \beta_i x[k-i] - \sum_{i=1}^{N} \alpha_i y[k-i]\right)$$

$\boxed{120}$

$$h[k] = 3 \cdot (\frac{1}{5})^k \cdot epsilon[k] + 2 \cdot (\frac{1}{2})^k \cdot \epsilon[k]$$

$$\circ\!\!-\!\!\bullet$$

$$\begin{aligned}
H(z) = 3 \cdot \frac{z}{z - \frac{1}{5}} + 2 \cdot \frac{z}{z - \frac{1}{2}} &= \frac{15z}{5z-1} + \frac{4z}{2z-1} \\
= \frac{15z(2z-1) + 4z(5z-1)}{(5z-1)(2z-1)} &= \frac{50z^2 - 19z}{10z^2 - 7z + 1}
\end{aligned}$$

121

$$\alpha_0 = a_2 = 10, \alpha_1 = -7, \alpha_1 = 1 \text{ und } \beta_0 = 50, \beta_1 = -19, \beta_2 = 0$$

122

$$10y[k] - 7y[k-1] + 1y[k-2] = 50x[k] - 19x[k-1] \text{ bzw. äquivalent}$$
$$y[k] = \frac{1}{10} \cdot (50x[k] - 19x[k-1] + 7y[k-1] - y[k-2])$$

123

$$h[k] = 3 \cdot 5^{-(k+2)} \epsilon[k+2] + 2 \cdot 2^{-k} \epsilon[k]$$
$$\circ\!-\!\bullet$$
$$\begin{aligned} H(z) &= 3 \cdot z^2 \frac{z}{z - \frac{1}{5}} + 2 \cdot \frac{z}{z - \frac{1}{2}} \\ &= \frac{15z^3}{5z-1} + \frac{4z}{2z-1} = \frac{30z^4 - 15z^3 + 20z^2 - 4z}{10z^2 - 7z + 1} \\ &(\Rightarrow M = 4, N = 2) \end{aligned}$$

124

$$\alpha_0 = a_{2-0} = 10, \alpha_1 = -7, \alpha_2 = 1 \text{ und}$$
$$\beta_{-2} = b_4 = 30, \beta_{-1} = 15, \beta_0 = 20, \beta_1 = -4$$

125

$$10y[k] - 7y[k-1] + y[k-2] = 30x[k+2] - 15[k+1] + 20x[k] - 4x[k-1]$$
$$y[k] = \frac{1}{10} (30x[k+2] - 15[k+1] + 20x[k] - 4x[k-1] + 7y[k-1] - y[k-2])$$

126

$$y[k-2] = \frac{1}{10} (30x[k+2] - 15[k-1] + 20x[k-2] - 4x[k-3] + 7y[k-3] - y[k-4])$$

$\boxed{127}$

$$x[k] = \mathcal{Z}^{-1}\left\{X(z)\right\} = \mathcal{Z}^{1}\left\{\sum_{k=-\infty}^{\infty} x[k]z^{-k}\right\}$$

$\boxed{128}$

$$Y(z) = 1 \cdot z^{-1} - \frac{1}{2}z^{-2} + \frac{1}{3}z^{-3} - \frac{1}{4}z^{-4} + ... + \frac{(-1)^{k+1}}{k}z^{-k} + ...$$

$\boxed{129}$

$$y[k] = \begin{cases} 0 & ,k \leq 0 \\ (-1)^{k+1} \cdot \frac{1}{k} & ,k > 0 \end{cases}$$

$\boxed{130}$

$$Y(z) \overset{!}{=} \frac{A_1}{z-\lambda_1} + \frac{A_2}{z-\lambda_2} + ... + \frac{A_n}{z-\lambda_n}$$

$\boxed{131}$

$$Y(z) \overset{!}{=} \frac{A_1(\lambda_2)...(z-\lambda_N) + ... + A_N(z-\lambda_1)...(z-\lambda_{N-1})}{HN}$$

$\boxed{132}$

$$\begin{aligned} Y(z) = \frac{1}{(z-(1+P))\cdot(z-1)} &= \frac{A(z-1)+B(z-(1+P))}{HN} \\ &= \frac{(A+B)\cdot z - A - B(a+P)}{HN} \end{aligned}$$

$\boxed{133}$

$$\begin{aligned} -A \qquad & -(a+p)B = 1 \qquad (1) \\ A \qquad & +B = 0 \qquad (2) \end{aligned}$$

$\boxed{134}$

$$(-1-p+1)\cdot B = 1 \text{ bzw. } B = -\frac{1}{p}$$

$\boxed{135}$

$$A = -B = \frac{1}{p}$$

$\boxed{136}$

$$Y(z) = \frac{p^{-1}}{z-(a+p)} - \frac{p^{-1}}{z-1}$$

$$\bullet\!\!-\!\!\circ$$

$$y[k] = p^{-1}\cdot((1+p)^{k-1}-1)\cdot\epsilon[k-1]$$

$\boxed{137}$

$$\begin{aligned}
Y(z) = \frac{A}{z-5} + \frac{B}{z-3} + \frac{C}{(z-3)^2} &= \frac{A(z-3)^2 + B(z-5)(z-3)C(z-5)}{HN}\\
&= \frac{A(z^2-6z+9) + B(z^2-8z+15) + C(z-5)}{HN}\\
&= \frac{z^2(A+B) + z(-6A-8B+C) + 9A + 15B - 5C}{HN}
\end{aligned}$$

$\boxed{138}$

$$\begin{array}{rlll}
 & A+ & B & = 2(1)\\
 & -6A & -8B+1C & = -9(2)\\
 & 9A & +15B-5C & = 3(3)\\
5\cdot(2)+(3): & -21A & -25B & = -42(4)\\
21\cdot(1+4): & & -4B & = 0(5)
\end{array}$$

139

$$Y(z) = \frac{2}{z-5} + \frac{3}{(z-3)^2}$$

$$\begin{array}{c}\bullet\\ |\\ \circ\end{array}$$

$$y[k] = 2 \cdot 5^{k-1}\epsilon[k-1] + (k-1)3^{k-1}\epsilon[k-1]$$

140

$$Y(z) = s(z) + \frac{r(z)}{q(z)}$$

141

$$s(z) = s_0 + s_1 z + ... + s_{M-N} Z^{M-N}$$

$$\begin{array}{c}\bullet\\ |\\ \circ\end{array}$$

$$s[k] = s_0\delta[k] + s_1\delta[k+1] + ... + s_{M-N}\delta[k+M-N]$$

142

$$Y(z) = 3z^2 - 2z + 1\frac{2z^2 - 9z + 3}{(z-5)(z-3)^2}$$

143

$$Y(z) = 3z^2 - 2z + 1\frac{2z^2 - 9z + 3}{(z-5)(z-3)^2}$$

$$\begin{array}{c}\bullet\\ |\\ \circ\end{array}$$

$$y[k] = 3\delta[k+2] - 2\delta[k+1] + \delta[k] + 2 \cdot 5^{k-1}\epsilon[k-1] + (k-1)3^{k-1}\epsilon[k-1]$$

$\boxed{144}$

$$Y(z) = \sum_{i=0}^{M-N} s_i z^i + \sum_{i=1}^{Q} \sum_{v=1}^{n_i} \frac{A_{i,v}}{(z-\lambda_i)^v}$$

$$\bullet\!\!-\!\!\circ$$

$$y[k] = \sum_{i=0}^{M-N} s_i \delta[k+i] + \sum_{i=1}^{Q} \sum_{v=1}^{n_i} A_{i,v} \cdot \binom{k-1}{v-1} \lambda_i^{k-v} \epsilon[k-1]$$

$\boxed{147}$

$$A = \frac{1}{0!} \left(\frac{d}{dz}\right)^0 [(z-1) \cdot \frac{z}{z-1}(z+1)]|_{z=1} = \frac{z}{z+1}|_{z=1} = \frac{1}{2}$$

$$B = \frac{1}{0!} \left(\frac{d}{dz}\right)^0 [(z+1) \cdot \frac{z}{z-1}(z+1)]|_{z=-1} = \frac{z}{z-1}|_{z=-1} = \frac{1}{2}$$