Signale und Systeme Boxen

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1 Motivation, Wiederholung und Überblick

a

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$$u_1(t) = 15\,\mathrm{V}\sin(\pi t + \pi/3) + 60\,\mathrm{V}\sin(10\pi t + \pi/3) = 0,5x(t) + 2y(t)$$

und damit $a = 0,5, b = 2$ und

$$u_2(t) := \mathcal{H}\{u_1(t)\} = \mathcal{H}\{0,5x(t) + 2y(t)\} \overset{??}{=}$$

2 Diskrete Signale

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$$(b) \quad x[-k] = \begin{cases} -\frac{1}{k}, k/ne0 \\ 0, k = 0 \end{cases}$$

$$(c) \quad x[k + k_0] = x[k + 3] = \begin{cases} \frac{1}{k+3}, k/ne - 3 \\ 0, k = -3 \end{cases}$$

$$(d) \quad x[k - k_0] = x[k - 3] = \begin{cases} \frac{1}{k-3}, k/ne3 \\ 0, k = 3 \end{cases}$$

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$$\begin{aligned} x[k_0 - k] &= x[-(k - k_0)] \\ &= x[(-k) + k_0] \end{aligned}$$

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$$\text{mit} \quad x[k_0 - k] = x[3 - k] = \begin{cases} \frac{1}{3-k}, k \neq 3 \\ 0, k = 3 \end{cases}$$

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- $x[k]$ heißt gerades Signal, falls $x[k] = x[-k] \forall k \in \mathbb{Z}$ gilt.
- $x[k]$ heißt ungerades Signal, falls $x[k] = -x[-k] \forall k \in \mathbb{Z}$ gilt.

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$$x[-k] = \begin{cases} \frac{1}{-k}, k \neq 0 \\ 0, k = 0 \end{cases} = \begin{cases} -\frac{1}{k}, k/ne0 \\ 0, k = 0 \end{cases} = -x[k]$$

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$$y[-k] = \begin{cases} \frac{1}{(-k)^2}, k \neq 0 \\ 0, k = 0 \end{cases} = \begin{cases} -\frac{1}{k^2}, k \neq 0 \\ 0, k = 0 \end{cases} = y[k]$$

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- $x[k]$ heißt <u>kausales Signal</u>, falls gilt: $x[k] = 0 \forall k < 0$
- $x[k]$ heißt <u>nicht-kausales Signal</u>, falls gilt $\exists k < 0 : x[k] \neq 0$
- $x[k]$ heißt <u>anti-kausales Signal</u>, falls $x[-k-1]$ kausal ist, d.h. falls gilt: $x[k] = 0 \forall k \leqslant 0$

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- $x[k]$ ist nicht-kausal
- $u[k]$ ist kausal
- $v[k]$ ist anti-kausal

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$$\delta[k] := \begin{cases} 1, k = 0 \\ 0, k \neq 0 \end{cases}$$

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$$\epsilon[k] := \begin{cases} 1, k \geq 0 \\ 0, k < 0 \end{cases}$$

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$$\delta[k - k_0] = \begin{cases} 1, k = k_0 \\ 0, k \neq k_0 \end{cases}$$

bzw.

$$\delta[k + k_0] = \begin{cases} 1, k \neq -k_0 \\ 0, k = -k_0 \end{cases}$$

$\boxed{22}$

$$x[k] \cdot \delta[k-i] = \begin{cases} x[i], k = i \\ 0, k \neq i \end{cases}$$
$$= x[i] \cdot \delta[k-i] \tag{2.1}$$

Siebeigenschaft

$\boxed{23}$

$$x[k] = \sum_{i=-\infty}^{\infty} x[i] \cdot \delta[k-i] \quad \text{für alle} \quad k \in \mathbb{Z}$$

$\boxed{24}$

$$x[k] = \sum_{i=-K}^{K} x[i] \cdot \delta[k-i]$$

$\boxed{25}$

$$u[k] = \delta[k+2] + \delta[k+1] + \delta[k] + \delta[k-1]$$
$$v[k] = 2 \cdot \delta[k+3] + \delta[k+1] - \delta[k-1] - 2 \cdot \delta[k-3]$$

$\boxed{26}$

$$sgn[k] := \epsilon[k] - \epsilon[-k] = \begin{cases} 1, k > 0 \\ 0, k = 0 \\ -1, k < 0 \end{cases}$$

$\boxed{27}$

$$Ш[k] := \epsilon[k] + \epsilon[-k-1] = 1 \text{für alle} k \in \mathbb{Z}$$

$\boxed{28}$

$$rect_{k_1,k_2}[k] :)\epsilon[k-k1] - \epsilon[k-k_2-1] = \begin{cases} 1, k_1 \leqslant k \leqslant k_2 \end{cases}$$

$\boxed{29}$

$$x[k] = q^k \cdot \epsilon[k]$$

$\boxed{30}$

$$x[k] : 0, ..., 0, x[0] = 1, x[1] = -0.7, x[2] = 0.49, x[3] = 0.343, ...$$

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$$x[k] : 0, ..., 0, x[0] = 1, x[1] = -0.8, x[2] = 0.64, x[3] = -0.512, ...$$

$\boxed{32}$

$$\begin{aligned} x[k] + y[k] &: x[-\infty] + y[-\infty]..., x[0] + y[0], x[1] + y[1], ..., x[\infty] + y[\infty] \\ x[k] \cdot y[k] &: x[-\infty] \cdot y[-\infty]..., x[0] \cdot y[0], x[1] \cdot y[1], ..., x[\infty] \cdot y[\infty] \\ c \cdot x[k] &: c \cdot x[-\infty]..., c \cdot x[0], c \cdot x[1], ..., c \cdot x[\infty] \end{aligned}$$

$\boxed{33}$

$$S_{k_1,k_2} := \{\overrightarrow{x} \in S | x[k] = 0 \forall k < k_1 \text{oder} k > k_2\}$$

$\boxed{34}$

$$\begin{aligned} \overrightarrow{x} &= (0 \quad 3 \quad 2 \quad 5 \quad 0 \quad 0) \\ \overrightarrow{y} &= (0 \quad 0 \quad 2 \quad -3 \quad 0 \quad 2) \\ \overrightarrow{x} + \overrightarrow{y} &= (0 \quad 3 \quad 4 \quad 2 \quad 0 \quad 2) \\ \overrightarrow{x} - \overrightarrow{y} &= (0 \quad 3 \quad 0 \quad 8 \quad 0 \quad -2) \\ \overrightarrow{x} \cdot \overrightarrow{y} &= (0 \quad 0 \quad 4 \quad -15 \quad 0 \quad 0) \\ c + \overrightarrow{x} &= (0 \quad 15 \quad 10 \quad 25 \quad 0 \quad 0) \end{aligned}$$

$\boxed{35}$

$$(x * y)[k] := \sum_{i=-\infty}^{\infty} x[i] \cdot y[k-i]$$

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$$x[i] = (\underset{\substack{\downarrow\\ i=0}}{3} \quad 2 \quad 1), \quad y[i] = (\underset{\substack{\downarrow\\ i=0}}{1} \quad -1 \quad 2) \text{ bzw. } \quad z[0-i] = (2 \quad -1 \quad \underset{\substack{\downarrow\\ i=0}}{1})$$

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	$x[i] =$			3	2	1			$\sum x[i]y[k-i] =$	$(x*y)[k]$
$k=0$	$y[k-i] =$	2	-1	1					$3 \cdot 1$	$= 3$
$k=1$			2	-1	1				$3 \cdot (-1) + 2 \cdot 1$	$= -1$
$k=2$				2	-1	1			$3 \cdot 2 + 2 \cdot (-1) + 1 \cdot 1$	$= 5$
$k=3$					2	-1	1		$2 \cdot 2 + 1 \cdot (-1)$	$= 3$
$k=4$						2	-1	1	$1 \cdot 2$	$= 2$

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$$x[k] * y[k] = 3\delta[k] - \delta[k-1] + 5\delta[k-2] + 3\delta[k-3] + 2\delta[k-4]$$

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$$x[i] = (\underset{\substack{\downarrow\\ i=-43}}{-1} \quad 3 \quad -2) \text{ und}$$

$$y[i] = (\underset{\substack{\downarrow\\ i=19}}{1} \quad -2 \quad 4 \quad -1) \text{ bzw. } y[-i] = (-1 \quad 4 \quad -2 \quad 1)$$

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$i = -43$ (pfeil)

k	$x[i]$				-1	3	-2			$(x*y)[k]$
-24	$y[k-i] =$	-1	4	-2	1					-1
-23			-1	4	-2	1				$2+3=5$
-22				-1	4	-2	1			$-4-6-2=-12$
-21					-1	4	-2	1		$1+12+4=17$
-20						-1	4	-2	1	$-3-8=-11$
-19							-1	4	-2 1	2

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$$
\begin{aligned}
(x * y)[k] = &- \delta[k+24] + 5\delta[k+23] - 12\delta[k+22] + 17\delta[k+21] \\
&- 11\delta[k+20] + 2\delta[k+19]
\end{aligned}
$$

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$$x[k] * y[k] \in \mathbf{S}_{a+c,b+d} \quad \text{und hat Länge} \quad n+m-1.$$

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I) Kommutativität: $x * y = y * x$

II) Assoziativität: $w * (x * y) = (w * x) * y$ und $c \cdot (x * y) = (c \cdot) * y$

III)