Signale und Systeme Boxen

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201

$$
\begin{aligned}
X(f) &= \sum_{n=0}^{\infty} r_n \cdot \delta(f - nf_0) \\
&= \hat{x}\frac{T_i}{T} \cdot \delta(f) + \sum_{n=1}^{\infty} 2\hat{x}\frac{T_i}{T} \cdot \left| si\left(n\pi\frac{T_i}{T}\right)\right| \cdot \delta(f - nf_0)
\end{aligned}
$$

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$$X_H(f) = 2\hat{x}\frac{T_i}{T} \cdot \left| si\left(n\pi\frac{T_i}{T}\right)\right| = 2\hat{x}\frac{T_i}{T} \cdot \left| si\left(\pi\frac{T_i}{T} \cdot \frac{f}{f_0}\right)\right|$$

203

$$f \in \{4f_0, 8f_0, 12f_0, \dots\}$$

204

$$c_0 := \frac{a_0}{2},\ c_n := \frac{1}{2}(a_n - jb_n),\ c_{-n} := \frac{1}{2}(a_n + jb_n) = c_n^*$$

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$$a_0 = 2c_0,\ a_n = c_n + c_{-n} = 2Re(c_n),\ b_n = j(c_n - c_{-n}) = -2Im(c_n)$$

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$$
\begin{aligned}
c_n &:= \frac{1}{2}(a_n - jb_n) = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\cos(n\omega_0 t)dt - j\frac{1}{T}\int_{-T/2}^{T/2} x(t)\sin(n\omega_0 t)dt \\
&= \frac{1}{T}\int_{-T/2}^{T/2} x(t)\underbrace{(\cos(n\omega_0 t) + j\sin(-n\omega_0 t))}_{e^{-jn\omega_0 t}}dt = \frac{1}{T}\int_{-T/2}^{T/2} x(t) \cdot e^{-jn\omega_0 t}dt
\end{aligned}
$$

$$c_{-n} = c_n^* = \frac{1}{T}\int_{-T/2}^{T/2} x(t) \cdot e^{jn\omega_0 t}dt$$

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$$\begin{aligned}
c_0 &= \frac{a_0}{2} = \hat{x}\frac{T_i}{T}\\
c_n &= \frac{1}{2}(a_n - jb_n) = \frac{a_n}{2} = \hat{x}\frac{T_i}{T}\cdot \mathrm{si}\left(n\pi\frac{T_i}{T}\right)\\
c_{-n} &= (c_n)^* = c_n
\end{aligned}$$

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$$c_k = \hat{x}\frac{T_i}{T}\cdot \mathrm{si}\left(k\pi\frac{T_i}{T}\right)$$

209

$$X_F(t) = \sum_{k=-\infty}^{\infty} c_k \cdot e^{jk\omega_0 t} \text{ für } c_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}dt$$

210

$$\begin{aligned}
X_F(t) &= \sum_{k=-\infty}^{\infty} \frac{\Delta\omega}{2\pi}\int_{-T/2}^{T/2} x(t')e^{-j\omega_k t'}dt' \cdot e^{j\omega_k t}\\
&= \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\left[\int_{-T/2}^{T/2} x(t')e^{-j\omega_k t'}dt'\right]\cdot e^{j\omega_k t}\cdot \Delta\omega
\end{aligned}$$

211

$$X_F(t) \overset{(T\to\infty)}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty}\underbrace{\left[\int_{-\infty}^{\infty} x(t)e^{-j\omega t'}\right]}_{=:X(\omega)} e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

212

$$x(t) = \delta(t) \circ\!\!-\!\!\bullet\, X(f) = \int_{-\infty}^{\infty}\delta(t)\cdot e^{-j2\pi f t}dt = e^0 = 1$$

213

$$\delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t} d\omega = \int_{-\infty}^{\infty} e^{j2\pi f t} df$$

214

$$\begin{aligned} x(t) \circ\!\!-\!\!\bullet\; X(\omega) &= \int_{-\infty}^{\infty} \hat{x}\cdot \mathrm{rect}(\frac{t}{T_i}) e^{-j\omega t} dt = \hat{x}\int_{-T_i/2}^{T_i/2} e^{-j\omega t} dt = \hat{x}\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T_i/2}^{T_i/2} \\ &= -\frac{\hat{x}}{j\omega}\left(e^{-j\omega\frac{T_i}{2}} - e^{j\omega\frac{T_i}{2}}\right) = -\frac{\hat{x}}{j\omega}\cdot 2j\cdot Im(e^{-j\omega\frac{T_i}{2}}) \\ &= -\frac{2\hat{x}}{\omega}\sin(-\omega\frac{T_i}{2}) = \hat{x}T_i\cdot \mathrm{si}(\omega\frac{T_i}{2}) \overset{(\omega=2\pi f)}{=} \hat{x}\cdot T_i\cdot \mathrm{si}(\pi f T_i) \end{aligned}$$

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$$X(f) = \delta(f - f_0) \bullet\!\!-\!\!\circ\; x(t) = \int_{-\infty}^{\infty} \delta(f-f_0)\cdot e^{j2\pi f t} df = e^{j2\pi f_0 t}$$

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$$c_1 x_1(t) + c_2 x_2(t) \circ\!\!-\!\!\bullet\; c_1 X_1(\omega) + c_2 X_2(\omega)$$

217

$$\mathrm{rect}(\frac{t}{2t}) \circ\!\!-\!\!\bullet\; 2T\cdot \mathrm{si}(T\omega) \text{ und } \mathrm{rect}(\frac{t}{4t}) \circ\!\!-\!\!\bullet\; 4T\cdot \mathrm{si}(2T\omega)$$

218

$$x(t) = 2\mathrm{rect}(\frac{t}{2t}) + 0.5\mathrm{rect}(\frac{t}{4t})$$

$$\circ\!\!-\!\!\bullet$$

$$X(\omega) = 2\cdot 2T\cdot \mathrm{si}(T\omega) + 0.54T\cdot \mathrm{si}(2T\omega) = 4T\mathrm{si}(T\omega) + 2T\mathrm{si}(2T\omega) = 4T\mathrm{si}(\pi 2Tf) + 2T\mathrm{si}(\pi 4Tf)$$

Todo list