Step 5: 2D Linear Convection

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + c\frac{\partial u}{\partial y} = 0$$

[<u>Note</u>: imagine $\vec{v} = c\hat{i} + c\hat{j}$

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y}$$

Then $\vec{v}\cdot\vec{\nabla} = c\frac{\partial}{\partial x} + c\frac{\partial}{\partial y}$

$$(\vec{v}\cdot\vec{\nabla})u = c\frac{\partial u}{\partial x} + c\frac{\partial u}{\partial y}$$]

Discretization scheme:

time → FORWARD DIFFERENCE

space → BACKWARD DIFFERENCE

(i → x-direction

j → y-direction)

$n \to$ time-direction.

$$\frac{u_{ij}^{n+1} - u_{ij}^{n}}{\Delta t} + c\,\frac{u_{ij}^{n} - u_{(i-1)j}^{n}}{\Delta x} + c\,\frac{u_{ij}^{n} - u_{i(j-1)}^{n}}{\Delta y} = 0$$

$$\boxed{\begin{aligned} u_{ij}^{n+1} = u_{ij}^{n} &- c\,\frac{\Delta t}{\Delta x}\left(u_{ij}^{n} - u_{(i-1)j}^{n}\right) \\ &- c\,\frac{\Delta t}{\Delta y}\left(u_{ij}^{n} - u_{i(j-1)}^{n}\right) \end{aligned}}$$