

1D Burger's Equation.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} - \nu \frac{\partial^2 u}{\partial x^2} = 0$$

Solution : $u(t) = -2\nu \frac{\frac{\partial \phi}{\partial x}}{\phi} + 4$
 with $\phi = e^{-x^2/4\nu} + e^{-\frac{(x-2\nu)^2}{4\nu}}$

$$\frac{\partial \phi}{\partial x} = e^{-x^2/4\nu} \left(-\frac{2x}{4\nu} \right) + e^{-\frac{(x-2\nu)^2}{4\nu}} \left(-\frac{2(x-2\nu)}{4\nu} \right)$$

$$\frac{-2\nu \frac{\partial \phi}{\partial x}}{\phi} = \frac{e^{-x^2/4\nu} (x) + e^{-\frac{(x-2\nu)^2}{4\nu}} (x-2\nu)}{e^{-x^2/4\nu} + e^{-\frac{(x-2\nu)^2}{4\nu}}}$$

$$u(t) = \frac{x e^{-x^2/4\nu} + (x-2\nu) e^{-\frac{(x-2\nu)^2}{4\nu}}}{e^{-x^2/4\nu} + e^{-\frac{(x-2\nu)^2}{4\nu}}}$$

+ 4