Rounding Error

Consider the function $y = x^3$

$$\frac{dy}{dx} = 3x^2$$

At $x = 1$, $\frac{dy}{dx} = 3(1)^2 = 3$

This is the exact answer.

Now, let's approximate!

$$\frac{dy}{dx} \approx \frac{f(x+h) - f(x)}{h} \quad (FD)$$

$$\approx \frac{f(x) - f(x-h)}{h} \quad (BD)$$

$$\frac{f(x+h) - f(x-h)}{} \quad (CD)$$

$$\approx \frac{f(x+h) - f(x-h)}{2h} \quad (1)$$

Forward Difference

Let $h = 0.1$

$$f(x+h) = (1.1)^3 = 1.3310$$

$$f(x) = (1.0)^3 = 1.0000$$

$$\frac{f(x+h) - f(x)}{h} = \frac{1.3310 - 1.000}{0.1}$$

$$= 3.3100$$

$$\neq 3 \; !!!$$

$$\Delta = |3.3100 - 3| = 0.31$$

Centered Difference :

$$\frac{dy}{dx} \approx \frac{f(x+h) - f(x-h)}{2h}$$

$$f(x+h) = (1.1)^3 = 1.3310$$

$$f(x-h) = (0.9)^3 = 0.7290$$

$$\frac{dy}{dx} \approx \frac{1.3310 - 0.7290}{2(0.1)}$$

$$= 3.0100$$

$$\Delta = |3.01 - 3| = 0.01$$

(Much better!)

What about at $h = 0.001$?

$$\Delta_{FD} = \left| \frac{(1.001)^3 - 1^3}{.001} - 3 \right|$$

$$= 0.003001$$

better, but not great.

$$\Delta \qquad \left| (1.001)^3 - (0.999)^3 - 3 \right|$$

$\Delta_{CD} =$ $\left| \frac{\quad}{2(.001)} \right|$ (

$= 1.0 \times 10^{-8}$ pretty good!

Let's keep going ☺, what about $h = 0.000001$ → try that on your calculator.

(Mine gives $\Delta_{CD} = 2.476 \times 10^{-11}$

For $h = 10^{-8}$, my calculator gives

$$\Delta_{CD} = 7.13 \times 10^{-9}$$

So, not as good as for 10^{-6} ????

This, is ROUNDING ERROR!