Courant Number

Physics → we run into problems when the distance traveled in one time step, Δt, is greater than our distance step size, Δx

1. convection with constant velocity, c

$$c \Delta t \leq \Delta x$$

$$\Delta x = \frac{x_{range}}{(n_x - 1)}$$

$$\Delta t = \frac{t_{range}}{(n_t - 1)}$$

$$\rightarrow \quad c \cdot \frac{t_{range}}{(n_t - 1)} \leq \frac{x_{range}}{(n_x - 1)}$$

$$c\, t_{range} \leq (n_t - 1)\, x_{range}$$

$(n_x - 1)$

$$\boxed{n_t \geq \frac{c(n_x-1)\, t_{range}}{x_{range}} + 1}$$

2. Convection in 1D → inviscid flow.

$$u \Delta t \leq \Delta x$$

$$\frac{u \Delta t}{\Delta x} \leq 1$$

varying!

$$\frac{u_{max} \Delta t}{\Delta x} \leq 1$$

it turns out that sometimes we need this to be smaller than 1, depending on the type of discretization scheme that is used.

$$\sigma \equiv \frac{u_{max} \cdot \Delta t}{\Delta x} \leq \sigma_{max}$$

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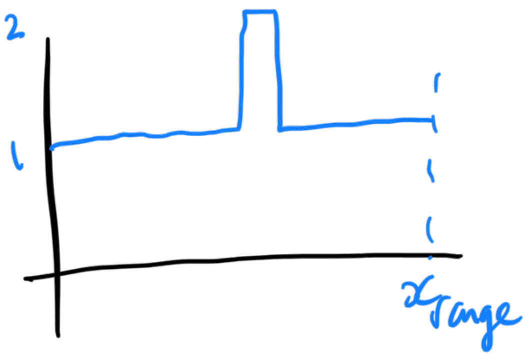
Example :

$$dx = \frac{x_{range}}{n_x - 1}$$



$$dt = \frac{\sigma_{max} \cdot dx}{u_{max}}$$

$$\sigma_{max} = 1$$

$$u_{max} = 2$$

$$dt = \frac{dx}{2}$$

$$dt = \frac{t_{range}}{n_t - 1}$$

$$n_t = \frac{t_{range}}{dt} + 1 = \frac{2 \cdot t_{range}}{x_{range}} (n_x - 1) + 1$$

For $n_x = 401$, $x_{range} = 2.0 - 0$, $t_{range} = 12$

$$n_t = \frac{2(12)}{20}(400) + 1$$

$$\boxed{n_t = 481}$$