Cellular Automata Based Model for the Prediction of Oil Slicks Behavior

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Abstract: *The paper describes the results of a study that examined the relevance of cellular automata for the prediction of oil slicks behavior. We propose a three dimensional Cellular automaton model encompassing the effects of oil vertical transportation, evaporation, water and wind currents, dissolution, deposition and emulsification. The proposed model is used to simulate hypothetical oil slick behavior.*

Keywords: Cellular Automata, Oil slicks

1. Introduction

1.1. Definition of cellular automata

Cellular automata, firstly introduced by Ulam and Von Neumann [7], are a special class of finite automata that can be described by the 3-tuple of Eq. (1) and that contain large numbers of simple identical components with only local interconnections.

$$A = (S, N, \delta) \tag{1}$$

In the above equation S is a nonempty set, called the state set, $N \subseteq \mathbb{Z}^2$ is the neighborhood, and $\delta : S^N \rightarrow S$ is the local transition rule.
CAs have enough expressive power to represent phenomena of arbitrary complexity and, at the same time, they can be accurately simulated by digital computers because of their intrinsic discreteness [10]. CAs have been successfully used in many complex systems and processes where local interactions are involved, such as photolithography, traffic simulation, lava flow and immune system modeling [4].

1.2. Components of cellular automata

A lattice of N identical finite-state machines (i.e. cells) each with an identical pattern of local connections to other cells for input and output is called a *cellular space*. Each cell is denoted by an index i and its state at time t is denoted s_i^t (where $s_i^t \in S$). Cell i together with the cells to which the cell i is connected is called the neighborhood of the cell i and is denoted η_i^t.
Local transition rule $\delta : S^N \rightarrow S$ gives the update state s_i^{t+1} for each cell i as a function of η_i^t.
Typically CA works in a discrete manner. That is to say time goes step by step and a global clock provides an update signal for all cells.

A two-dimensional CA is illustrated in Figure 1. The neighborhood consisting of the cell itself and the four bordering cells (to the north, south, east, and west) is called "von Neumann neighborhood" (Figure 1a). The two dimensional neighborhood consisting of the cell itself and the eight bordering cells is called the "Moore neighborhood" (Figure 1b).

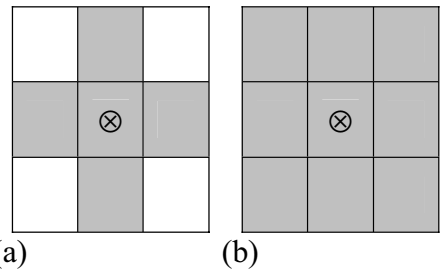

Figure 1: (a) The von Neumann neighborhood. (b) The Moore neighborhood. In both cases the cell to be updated is marked with ⊗.

The application of cellular automata for the prediction of oil slick movement and spreading was proposed in [4], where a simple two-dimensional cellular automaton was described. However, the problem was constrained to horizontal spreading and evaporation. Despite the important general laws established in that paper, the model described there was

oversimplified and did not take into account many important physical processes affecting the movement and spreading of oils. Moreover, the assumptions underlying the model are not made explicit in that paper. It is important to remember that any model simplifies reality and that a cellular automaton model is just one of a number of model types that reduce the complexity of the original problem into a more manageable form.

2. Enhanced CA model

2.1. Three dimensional cellular automaton

Our model is based on the non-uniform three dimensional cellular automaton with two sets of rules. Like one- and two-dimensional, three-dimensional automata are specified by an initial state and a set of transition rules used to transform the state of each cell in each time step. Our three-dimensional automaton consists of an n*n*2 cube of cells. The goal is to have one set of cells describing surface spreading and another set of cells describing sub-surface spreading. In each time step the new value of a cell is computed based on the current value of the cell and the current value of its 17 direct neighbors. This configuration is shown in Figure 2.

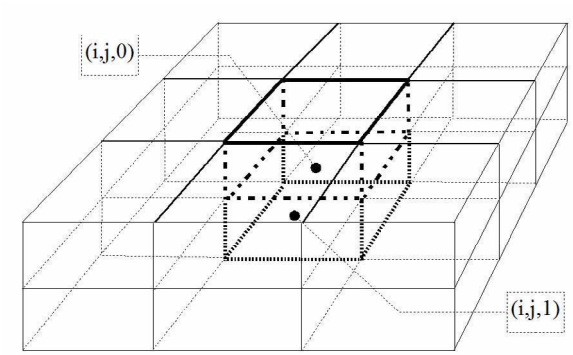

Figure 2: Three dimensional cellular automaton. It consists of n*n*2 cube of cells. Here, a cubical cell's neighbors are those cubes which share an edge or a vertex with the central cell.

Our CA model is capable of simulating oil processes of horizontal and vertical spreading, evaporation, dissolution, shoreline deposition and emulsification. These are the processes that affect the fate of spilled oil in the first few days to weeks of a spill and which may dramatically change the nature of the oil. Contribution of long term processes such as biodegradation, photo-oxidation and sedimentation, are not included in our model, as they are less important for the initial prediction of the behavior of spilled oil.

The state of the (i, j, k) cell at the time t, $C^t_{i,j,k}$ is:

$$C^t_{i,j,k} = \begin{Bmatrix} LF, M^t_{i,j,k}, CD^t_{i,j,k}, CV^t_{i,j,k}, WD^t_{i,j,k}, \\ WV^t_{i,j,k}, EM^t_{i,j,k}, MD^t_{i,j,k} \end{Bmatrix} \quad (2)$$

LF is a one bit flag which indicates land (LF=1) or water area (LF=0). $M^t_{i,j,k}$ is the oil mass at time t in the are corresponding to cell (i, j, k) $CV^t_{i,j,k}$ and $WV^t_{i,j,k}$ are the water and wind velocity at time t in the area of the (i, j, k) cell. $CD^t_{i,j,k}$ and $WD^t_{i,j,k}$ are the flags that indicate the directions of the current and wind directions at time t in the area of the (i, j, k) cell. The number of bits in these flags is three; hence eight directions can be indicated (N, S, E, W, NE, NW, SE and SW). $EM^t_{i,j,k}$ is the mass of water content in emulsion and $MD^t_{i,j,k}$ is the mass of the dissolute oil at time t in the area of the (i, j, k) cell.
The initial distribution of oil mass in the cells $\left(M^0_{i,j,k}\right)$ is an input to the model. Direction and speed of the water and wind are assumed to be known at all times as they are also input to the model.

In the paper already referred to, in order to model oil slick movement and spreading, two-dimensional automaton with Moore Neighborhood is used. The driving force for oil transport between two cells is the difference of oil mass. As a first order approximation, the transported oil mass is proportional to the oil mass difference between two cells.

We find this approach to be practical and consistent with the general mechanism of oil-slick spreading and with Gravity-Viscous formulation of Fay and Hoult, [1]. However, it fails to include the effect of oil viscosity on the spreading of the slick. This is a major flaw, as field studies have proven that it has an important effect. Our model overcomes this limitation by addressing the spreading of oil using modified diffusion coefficient which is derived from Johansen's equation [3]. The modified coefficient is given by:

$$D = m^2 \cdot \alpha^{1/2} \cdot v_o$$
$$\alpha = g \cdot \frac{\rho_o}{\rho_w \cdot v_w} \quad (3)$$

In (3) m is a constant of proportionality to be found experimentally and ρ_o, ρ_w, v_o, v_w are densities and viscosities of oil and water respectively. This spreading occurs during the early stages of the oil spill. This is simply due to the fact that oil, being a liquid, will not remain in a pile. However, the viscosity of oil retards the rate of oil motion. That is, light oil spreads more rapidly than heavy one.
Oil spilled directly on water spreads radially until the slick reaches some terminal thickness [5]. For heavy crudes this terminal thickness equals to 0.01 cm; while for less viscous substances like gasoline, kerosene and light diesel fuel it equals to 0.001 cm.

2.2. Vertical transportation

Our model adds another degree of freedom by allowing free vertical movement. Dispersion of a surface slick results in the formation of droplets that are entrained into the water column and transported with the subsurface currents. Vertical diffusivity generally ranges between 1 and $200\,\mathrm{cm}^2/\mathrm{s}$ depending on a number of environmental factors and is a strong function of wave height. We use vertical diffusion coefficient defined by (4) which was, in a slightly different form, suggested in [9]:

$$E_z = 0.028 \cdot \left(\frac{h_w^2}{T} \right) \cdot e^{\frac{-4\pi}{L}} \quad (4)$$

where L is wave length, T is wave period, and h_w is taken as the significant wave height. It is well documented that there is a strong cut-off of vertical diffusivity with depth, thus our model limitation to only two sets of n*n cells restricts the processes to those that are significant since they occur on or near the water surface.

2.3. Evaporation

Evaporation is the most important weathering process that accounts for the largest loss in oil mass during the first day immediately following a spill. Due to evaporation oil mass in the slick is continuously reduced. Evaporation is especially important in the modeling of lighter hydrocarbon spills, since it is well documented that light oils can lose as much as 75% of the original volume within the first few days of the spill. Since the amount of evaporation and the speed at which it occurs depend mainly upon volatility of the oil and upon the surface temperature, we have adapted the single component approximation from [9] which is given in the CA form by (5):

$$\left[0.165(\%D) + 0.45(T^t - 15) \right] \cdot \ln(t_m) \cdot m_{i,j,k}^t \quad (5)$$

In (5) %D is the percentage (by weight) distilled at 180 °C, t_m is the time corresponding to a model time step and T^t is the temperature of the environment at time step t.

We use the expression (5) to describe the influence of evaporation on a viscosity as well. This effect is given by the equation (6) which was suggested in [5]:

$$\mu = \mu_0 \cdot e^{(c_E \cdot F_{em})} \quad (6)$$

F_{em} is the mass fraction of evaporated oil, and the non dimensional empirical constant C_E varies with oil type between 1 and 10, with higher values for more viscous products

2.4. Water and wind-driven currents

The oil spilled in to the water is transported by a combination of winds and currents. The combined effect of water and wind-driven currents greatly affect oil transport from one cell to another and causes the advection of surface and subsurface oil.
Most of the spill trajectory simulations lump all of the wind effects together into a wind drift factor, usually taken to be three percent of the wind speed. This is an extremely usefully approach which has been used in [4] as well, but it does simplify a great deal of what is going on. We propose a more sophisticated model which is able to simulate time-varying wind driven factor which, for north direction, is given by (7):

$$\left|N^t_{w_{i,j,1}}\right| = R^t_1 \cdot \frac{WV^t_{i,j,1}}{WV_{\max}}$$

$$\left|N^t_{c_{i,j,1}}\right| = \frac{CV^t_{i,j,1}}{CV_{\max}}$$

$$\left|N^t_{w_{i,j,2}}\right| = R^t_2 \cdot \frac{WV^t_{i,j,1}}{WV_{\max}}$$

$$\left|N^t_{c_{i,j,2}}\right| = \frac{CV^t_{i,j,1}}{CV_{\max}} \quad (7)$$

$$R^t_1 = \begin{cases} 0,045 \ if & M^t_{i,j,1} <= 0,01 \cdot M^t_{i,j,0} \\ 0,03 \ \ if \ 0,01 \cdot M^t_{i,j,0} < M^t_{i,j,1} <= 0,1 \cdot M^t_{i,j,0} \\ 0,015 \ if & M^t_{i,j,1} >= 0,1 \cdot M^t_{i,j,0} \end{cases}$$

$$R^t_2 = \frac{R^t_1}{10}$$

$N^t_{w_{i,j,k}}$ models the wind-driven current, whereas $N^t_{c_{i,j,k}}$ models the water current. Analog equations are used for other seven directions. The explanation behind (7) is that as the oil weathers to tar balls, the oil may spend a significant part of its time below the surface and away from the effects of the wind, thus reducing the drift factor. Sum of both water and wind constants is called a directional constant. The north direction constant, for example, is given by:

$$N^t_{i,j,k} = N^t_{w_{i,j,k}} + N^t_{c_{i,j,k}} \quad (8)$$

2.5. Dissolution

In our model the total dissolution rate is calculated by (9):

$$N^t_{i,j,k} = K \cdot A_s \cdot S \quad (9)$$

where N is the total dissolution of the slick in grams, K is the dissolution mass transfer coefficient (set to $3.0 \cdot 10^{-6}\ m/s$, in accordance with [5]), A_s is the slick area in square meters corresponding to cell (i, j, k) and S is the oil solubility in the water. The solubility of the oil will change with the time as the oil weathers. Huang and Monastero [2] suggested a simple relationship for oil solubility that was adopted for our model:

$$S = S_0 \cdot e^{-0,1 \cdot t} \quad (10)$$

where S_0 is the initial solubility at the time of the spill.

2.6. Deposition

An oil slick will most likely reach a shoreline sometime after a spill occurs. The model described in [4] assumed that in that case no oil transport will occur between a cell representing the oil-slick parcel and the cell representing a land area. However, based on the exponential decay function similar to that of the half-life formulation, the mass of oil remaining on the shoreline can be related to its original mass by (11):

$$Mdep^t_{i,j,k} = M^t_{i,j,k} \cdot e^{-d \cdot t_m}$$

$$d = \left(\frac{-\ln \frac{1}{2}}{\lambda} \right) \quad (11)$$

where λ is half-life of oil. The half life is the time needed for 50% of the oil to disappear from the sea surface. $Mdep^t_{i,j,k}$ is the mass of oil accumulated at the cell (i, j, k) at the time t assuming that this cell represents land (LF=1), while $M^t_{i,j,k}$ is the mass of oil that would have been accumulated had this cell been a water area.

2.7. Emulsification

Emulsification is felt by many to be the second most important behavioral characteristic after evaporation. Emulsification is the incorporation of water into oil, and is the opposite of dispersion. It occurs when two liquids (e.g. oil and water) that cannot be naturally mixed combine. The formation of emulsion can drastically change the properties and characteristics of oil spills. In addition, the density of the resulting emulsion can be as great as 1.03 g/ml compared to a starting density as low as 0.80 g/ml. Due to its high viscosity the physical spreading of the spill decreases and such emulsified oil is difficult to remove from water surface.

In general, heavier oils emulsify more rapidly than lighter and emulsification almost never occurs during spills of gasoline, kerosene and diesel fuels.

It has been acknowledged that the theory developed by Mackay et al. [6] is the best simulation of emulsification. We adopt an equation for the water content in cellular automata as depicted by (12):

$$EM_{i,j,k}^{t+1} = EM_{i,j,k}^{t} + K_A \cdot (U+1)^2 \cdot (1-K_{bW}) \cdot t_m \tag{12}$$

where $EM_{i,j,k}^{t+1}$ is the fractional water content at time t in the area of the (i, j, k) cell, K_a is an empirical constant, U is the wind speed, K_b is a constant with the value of approximately 1.33 and t_m is the time corresponding to a model time step.

The influence of emulsification on density (and hence also on viscosity) is given by (13)

$$\delta_E = W \cdot \delta_W + (1-W) \cdot \delta_O \tag{13}$$

In which δ_E, δ_W and δ_O are density of emulsion, water and oil respectively, whereas W represents the water content fraction of the emulsion.

2.8. CA Formulation

In order to include all of the aforementioned effects we give the CA formulation by (14):

$$Mm_{i,j,k}^{t+1} = \begin{cases} M_{i,j,k}^{t} + m \cdot D \cdot \begin{bmatrix} \left(1+N_{i,j,k}^{t}\right)\cdot\left(M_{i-1,j,k}^{t} - M_{i,j,k}^{t}\right) + \left(1+S_{i,j,k}^{t}\right)\left(M_{i+1,j,k}^{t} - M_{i,j,k}^{t}\right) + \\ \left(1+W_{i,j,k}^{t}\right)\left(M_{i,j-1,k}^{t} - M_{i,j,k}^{t}\right) + \left(1+E_{i,j,k}^{t}\right)\left(M_{i,j+1,k}^{t} - M_{i,j,k}^{t}\right) \end{bmatrix} + \\ m \cdot r \cdot D \cdot \begin{bmatrix} \left(1+NW_{i,j,k}^{t}\right)\left(M_{i-1,j-1,k}^{t} - M_{i,j,k}^{t}\right) + \left(1+SW_{i,j,k}^{t}\right)\left(M_{i+1,j-1,k}^{t} - M_{i,j,k}^{t}\right) + \\ \left(1+NE_{i,j,k}^{t}\right)\left(M_{i-1,j-1,k}^{t} - M_{i,j,k}^{t}\right) + \left(1+SE_{i,j,k}^{t}\right)\left(M_{i+1,j+1,k}^{t} - M_{i,j,k}^{t}\right) \end{bmatrix} + \\ m \cdot r \cdot E_z \cdot \begin{bmatrix} \left(M_{i-1,j,(k+1)\%2}^{t} - M_{i,j,k}^{t}\right) + \left(M_{i+1,j,(k+1)\%2}^{t} - M_{i,j,k}^{t}\right) + \\ \left(M_{i,j-1,(k+1)\%2}^{t} - M_{i,j,k}^{t}\right) + \left(M_{i,j+1,(k+1)\%2}^{t} - M_{i,j,k}^{t}\right) + \\ \left(M_{i-1,j-1,(k+1)\%2}^{t} - M_{i,j,k}^{t}\right) + \left(M_{i+1,j-1,(k+1)\%2}^{t} - M_{i,j,k}^{t}\right) + \\ \left(M_{i-1,j-1,(k+1)\%2}^{t} - M_{i,j,k}^{t}\right) + \left(M_{i+1,j+1,(k+1)\%2}^{t} - M_{i,j,k}^{t}\right) \end{bmatrix} + \\ m \cdot E_z \cdot \left(M_{i,j,(k+1)\%2}^{t} - M_{i,j,k}^{t}\right) - \left[0.165(\%D) + 0.45(T^t - 15)\right] \cdot \ln(t_m) \cdot m_{i,j,k}^{t} \\ \qquad - Mdep_{a,b,c}^{t} - N_{i,j,k}^{t}, \\ Mdep_{a,b,c}^{t} \in (\eta_{i,j,k}^{t} \setminus M_{i,j,k}^{t}), if \ \frac{M_{i,j,k}^{t} + EM_{i,j,k}^{t}}{\delta_E \cdot a \cdot b} \le TT \\ M_{i,j,k}^{t+1} - p \cdot t_m \cdot T^t \qquad otherwise \end{cases} \tag{14}$$

In the equation (14) *(k+1)%2* has the meaning of a modulo 2 operation, *TT* denotes terminal thickness and *a* and *b* represent the length of two sides of the base of each cell in the cellular automaton. We use a constant *d* to absorb for the difference of the amount of oil transported between diagonal cells with regard to the amount of oil transported between adjacent cells.

Most of the oil components are to some degree water-soluble; hence some of them can dissolve into the water. Since the evaporation is a much faster process, the amount of dissolved oil will be small. Even though it only accounts for a negligible fraction of the oil mass balance, dissolution is an important process due to the possible biological harm of the dissolved oil.

3. Experiments and Results

An algorithm based on the described model has been tested in the framework of this research work. To determine the values of *m* and *d*, a hypothetical circular oil slick with diameter of 50 meters was introduced. The total oil mass was assumed to be 30000 kg, and followed a Gaussian distribution (i.e. oil mass decrease with the circle radius). In the conditions where no wind blow, where no water current exists and

where is no evaporation the algorithm should produce circular oil slicks. We use this trivial experiment just to calibrate the model. It was found that the model behaves this way when the values of m and d are set to 0.0034 and 0.22 respectively.

To test the model we have run a series of simulation with the same parameters (for all the applicable inputs) that were used in the numerical calculation [8].

Table 1. Input parameters for simulations

Parameter	Value
Oil type	sp. gr. =84
Terminal thickness (cm)	TT =0,001
Vertical diffusion coef. (cm^2/s)	E_z =100
Percentage distilled at 180 °C	%D =7,87
Half life (days)	λ =2
Spill volume (m^3)	=50
Air temperature °C	T =10
Water viscosity ($m^2\ s^{-1}$)	$v_w = 1{,}311 \cdot 10^{-6}$
Dyn. oil viscosity ($g\ cm^{-1}\ s^{-1}$)	μ_o =980.0
Maximum wind speed (km/h)	WV_{max} =200
Wind speed (km/h)	WV =5
Empirical constant C_E	C_E =5

Table 2. Results for sample simulations

Time (hrs)	Distance moved (m)	Oil Vol. (m^3)	Area (km^2)
0	0	50	0.17
25	34	43	0,32
50	52	39	0,41
75	72	32	0,52
100	97	25	0,63
125	108	19	0,77

Results from the Table 2 are obtained by summing the results across all cells in the simulation. As it can be seen by comparing the obtained results with [8] the results are very similar in terms of the quantity and precision.

It is worth noticing here that unfortunately there is a major lack of field data collected during actual or experimental spills that could be used for assessing and comparing accuracy of different models. Therefore the evaluation of models is usually limited to assessing the realism of the underlying assumptions made in deriving model formulas. Yet, by comparing the results with numerical calculations based on semi-empirical models we can conclude that a particular model performs reasonably well.

4. Conclusion

An enhanced model for the prediction of oil slick behavior using CA was presented in this paper. Contrary to the models described in [2] as well as to the most of the other models, our work is not based on a system of Partial Differential Equations. Describing all the parameters from our model using PDEs would lead to the system that would be difficult to handle and most probably impossible to solve for the general case. Cellular automata based model is an alternative that makes the model more universal. The introduction of the second set of cells in CA enhances the quality of the obtained results that may be considered satisfactory. By comparison to the published numerical methods we observed that the described model is very efficient and that the results are very promising.

5. References

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