1 Prerequisites: Installation of QUCS on host

Downloading the tarballs for QUCS and for the Verilog-AMS to SPICE-translator ADMS also cloning QUCS and ADMS from github in other folders and working on the installation from multiple folders resulted in many problems related to build process of `qucs-core`.

Giving up on the tarball installation folder, I went to the git-clone folder and worked until I gave up and went back again to the tar-ball folder then gave up and went so back and forth. Errors from not following installation instructions correctly, errors from uncomplete instructions which assumes a pre-knowledge in operating system internals (when super-user mode is supposed be used and when not, the necessity of the setting of enivroment varibles related to `LDPATH`).

Errouneously mixing installation commands for ADMS and for Qucs but also perhaps because of corruption of tar-balls due to limited and varying internet bandwith. Read on unix.stackexchange that a certain system administrator often encountered corrupted tarballs. Unfourtunately the reference to particular thread is lost but it does not seem unlikely to me given the fact that the `tar` command is old and might not have the same level of sofisticated error correction code as in modern git code, but I am speaking as layman I don't really know.

I also gave up on trying to get the Linux installation to work and tried running the Windows version "Qucs studio" on the Wine-platform for Linux but noticed differences in the display compared to what was shown in the document "Getting started with Qucs" and with Wine having the rumor of being buggy I did not dare to proceed. Besides that, I could not accept defeat from a "simple" installation. Done this before.

To be honest I don't know for sure why the installation suddenly worked. Did I finally follow the instructions or was it that the tarballs were corrupted or was it the enivroment variable or a combination of all these factors? Cloning the github repositories did not give the same errors which could be attributed to an possible existence of a check-sum verfification agorithm in git which possibly is not existing in the `tar` unpacking command or it could be the case that the build commands are different with respect to the git clone installation and therefore caught more attention and care.

Posted on Sourceforge[1] about the build related problems which was ignored because it contained a glaring mistake about the paths provided where I mixed up `/usr/local/bin` and `/usr/local/lib` and also posted another question on unix.stackexchange [2]. related to the Autotools build-tools suite.

According to the ADMS section of QUCS on Github [3] one is supposed to do the following when installing from a tar-ball (which is different from when installing from the github clone)

```
tar xvfz adms-x.x.x.tar.gz
cd adms-x.x.x
./configure --prefix=[/install/location/]
make install
```

The command `make install` does not work if one is not issuing it as a superuser, which is implied for those who know but is nowhere written in the instructions so under Linux Ubuntu one should issue `sudo make install`. It might be obvious to use super-user mode for someone with who is a complete novice but this is not always true. In some instances when installing in super-user mode the program is not availible in normal user-mode and one has to issue a `chmod` command to allow non superusers access.

The descision was for some reason to install ADMS in the location `/usr/local/lib` from reading on peoples problems who had

[1] https://sourceforge.net/p/qucs/discussion/311049/thread/88cc152f64/?limit=25
[2] https://unix.stackexchange.com/questions/715523/what-does-it-mean-when-aclocal-cannot-open-version
[3] https://github.com/Qucs/ADMS/blob/develop/README.md

succeeded with the installation on the net. `sudo make install` returned stating that the library admsXml could not be found which prompted me to run the configure portion of QUCS as `./configure --with-mkadms =/usr/local/lib/admsXml` but was still not found or not working.
It is also possible that I out of being carless have run `./configure --with-mkadms= /usr/local/bin/admsXml` which I stated on the Sourceforge site thus making a fool of myself. The tip using `--with-mkadms` was not from the official documentation but found on Stackexchange[4].

The ADMS build report which is dispalyed on the terminal during the installation also alludes to that it in some occations might be necessary to point to `LIBDIR` but not knowing what `LIBDIR` was meant to be, I decided to ignore until I realized that this might be crucial point.
Found this discussion on Stackexchange[5] relating to how to set enivroment varibles in Linux. I did not do it the official sanctioned Ubuntu way which according to a writer on Stackexchange is the adding of a `.conf` file to `/etc/ld.so.conf.d` and then in the `.conf` file write the path to the ADMS-library because it was not obvious where the `.conf` file then should be stored, so I resorted to editing `.bash_profile` of which I had a vague memory of having done some years ago and it worked then, but not the recommended way according to what someone said on Stackexchange... Using the rather difficult built in vi-editor `vi ~/.bash_profile` the addition was

```
LD_LIBRARY_PATH=/usr/local/lib
export LD_LIBRARY_PATH
```

The `.bash_profile` which I had edited before was empty when opened and information being lost from there after restart is discussed on mentioned thread but the work-around suggested results in an error if tried. After editing `.bash_profile` one is supposed to issue (at the terminal):

```
source ~/.bash_profile
sudo ldconfig
```

Finally, I still could not build the documentation because QUCS uses now obsolete libraries related to TexLive which are `texlive-math-extra`, and`pgf`. The `pgf` library is nowhere to be found because it is an instrinsic part of the standard TexLive distribution since years ago. Luckily QUCS provide the option to not build the documentation using the switch `--disable-doc` in the top-level configure script. QUCS also uses an obsololete, at least with respect to Ubuntu, Octave package for converting encapsulated postscript files `octave-epstk` which is nowhere to be found except for Debian Linux it seems.

Being still at the novice level in installing from source code and not having completed the full verfication of the installation (`make check`) before throwing out the code folder, the installation is not to be trusted. Dr. Dancila helped me with some tests in a Zoom-session but after the session ended disturbing warnings were detected which were not brought to Dr. Dancila's attention.

2 Verification of the installation with respect to RF-theory and with respect to performance as stated in the QUCS manual

Doing so many mistakes during the installation there can be no trust that something was not broken or corrupted working with both the tarball folder and the git cloned folder for both ADMS and QUCS. Due to the mistakes during the installation and the many problems encounterd also because of the fact that full built in checks of the installation were not performed before deleting the code folder, it feels necessary to evaluate the installation with respect to theory and record warnings and errors for evaluation before proceeding with the lab so to have some level of confidence that the

[4] https://stackoverflow.com/questions/36102809/qucs-core-configure-error-needs-admsxml
[5] https://stackoverflow.com/questions/13428910/how-to-set-the-environmental-variable-ld-library-path-in-linux

QUCS installation will actually perform as the authors designed it. Especially concerning was the following meassage after completing a zoom-session with Dr. Dancila and which was not shown to him.

```
Errors and Warnings:
--------------------

line 33: no trailing end-of-line found, continuing...
```

Does this impact the accuracy of the calculations? I certainly would not bet my life on that it does not!
Futher more we also want to verify that simple processes which will be used in the lab was not corrupted such that reading the touchstone file provided by Dr. Dancila and some of the built in functions which will be used in the lab before actually proceeding to the lab task. We also repeat the tests done together with the Dr. Dancila for full reference.

The following tests feels suitable to perform at minimum before proceeding with the actual lab-work.

(a) Reflection coefficient of load without transmission line should be transformed to the correct position in the Smith-chart.

(b) Calculation of Return Loss and Mismatch Loss should conform with Matlab/Octave.

(c) Reflection coefficient of load with 50 ohm transmission line should be transformed to correct position in the Smith chart

(d) Sweeping the length of the transmission line one half wave length shoul result in a plot of the reflection coefficient describing a full circle.

(e) Example 9-13 in David K. Cheng "'Field and wave Electromagnetics"' short circuited line is transformed to a certain impedance with a transmission line of 0.1λ.

(f) Plot of S-parameters verifying that the particular part of the touchstone file is read correctly.

(g) Plot of the built in function *Rolett*() to verify that it accurately performs the calculation of the Rolett stability condition.

(h) Plot of built-in functions *stabL*() and *stabS*() verifying input- and output-stability circles with respect to theory.

(i) Plot of the Noise-figure data verifying that the particular part of the Touchstone file is read correctly.

The Toucstone file is as follows

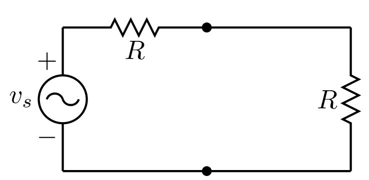
```
! Filename:       BFG520I.S2P       Version:   2.1
! NXP part #: BFG520                                    Date: Feb 1992
! Bias condition: Vce=6V, Ic=10mA
! IN LINE PINNING: same data as with cross emitter pinning.
#  MHz  S  MA  R  50
! Freq       S11            S21            S12            S22     !GUM [dB]
   40    .711  -14.1  23.473  170.8    .007   82.3    .974   -6.8 !    43.5
  100    .690  -34.5  22.218  158.5    .016   74.0    .931  -16.3 !    38.5
  200    .640  -64.3  19.183  141.5    .029   62.4    .816  -28.8 !    32.7
  300    .597  -88.3  16.207  128.8    .037   54.2    .703  -37.1 !    29.1
  400    .569 -106.1  13.627  119.5    .042   50.9    .613  -42.4 !    26.4
  500    .553 -119.9  11.673  112.5    .046   48.6    .544  -45.7 !    24.5
  600    .538 -130.7  10.152  107.0    .050   47.2    .493  -47.6 !    22.8
  700    .526 -139.2   8.947  102.5    .052   47.1    .453  -48.8 !    21.4
  800    .514 -146.7   7.968   98.6    .055   47.9    .422  -49.5 !    20.2
```

```
 900     .503 -154.1    7.148    95.1     .057    48.8     .396   -50.2 !     19.1
1000     .495 -160.2    6.488    92.4     .059    49.7     .374   -51.1 !     18.1
1200     .494 -171.7    5.468    87.0     .065    51.3     .344   -53.4 !     16.5
1400     .500 -131.8    4.748    82.0     .069    52.5     .331   -56.1 !     15.3
1600     .500  173.3    4.159    77.8     .075    55.2     .326   -57.3 !     14.1
1800     .494  166.0    3.722    74.3     .082    56.4     .317   -57.9 !     13.1
2000     .496  158.4    3.368    71.0     .087    58.2     .297   -59.5 !     12.2
2200     .515  151.9    3.080    67.9     .093    59.0     .279   -64.4 !     11.5
2400     .535  147.6    2.798    64.2     .097    60.7     .280   -71.4 !     10.8
2600     .540  144.2    2.579    61.6     .107    61.3     .298   -76.3 !     10.1
2800     .534  139.8    2.428    58.2     .114    60.9     .313   -77.7 !      9.6
3000     .541  133.9    2.259    55.3     .120    62.3     .309   -78.3 !      9.0
! Noise data:
! Freq.      Fmin        Gamma-opt          rn
  500        1.10     .330      27.0        .250
  900        1.25     .294      48.0        .260
 1000        1.30     .298      52.0        .270
 2000        1.90     .242     134.0        .160
```

2.1 Tests

(a) Reflection coefficient of load without transmission line should be transformed to the correct position in the Smith-chart.

When the dimensions of the circuit d are much less than the length of the wavelength λ the voltage depends only on their configuraion in the circuit but geometric distances from the source are of no importance, therefore sweeping the frequency when there is no difference in the voltage along the line should result in a constant reflection coefficient for all frequencies.



Figur 1: When $d << \lambda$ the dimensions does not impact any behaviour of the circuit

The formula for the reflection coefficient Γ is

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

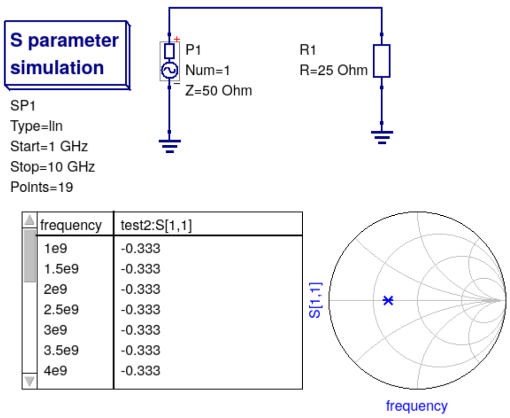
We are using normalized impedances so the formula is

$$\begin{aligned}\Gamma &= \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} \\ &= \frac{z_L - 1}{z_L + 1}\end{aligned}$$

We recieve

$$\begin{aligned}\Gamma &= \frac{z_L - 1}{z_L + 1} \\ &= \frac{25/50 - 1}{25/50 + 1} \\ &= -0.3333\end{aligned}$$

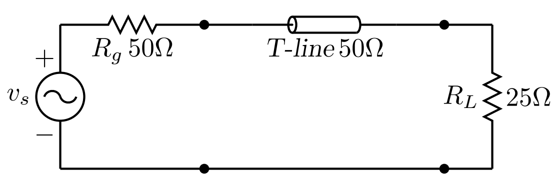
QUCS gives the following results



Figur 2: $R = 25$ Ohm

with no warnings or errors and thus we did not break anything during the installation here.

(b) Reflection coefficient of load with 50 ohm transmission line should be transformed to correct position in the Smith chart



Figur 3: When $d << \lambda$ the dimensions does not impact any behaviour of the circuit

If we have $Z_L = 25$ Ohm and 20 mm lossless transmissionline of 50 Ohm applying a voltage of 2 GHz, the theory states that the impedance seen at distance l from the load looking towards the load is

$$Z_i = R_0 \frac{Z_L + jR_0 tan\beta l}{R_0 + jZ_L tan\beta l}$$

What is βl ? βl is supposed to be $2\pi l/\lambda$ and we arrive to this because

$$\beta = \omega\sqrt{LC} \qquad u_p = \frac{1}{\sqrt{LC}}$$
$$\omega = 2\pi f \qquad u_p = f\lambda$$

So

$$\begin{aligned}\beta l &= \omega\sqrt{LC}\, l = 2\pi f\sqrt{LC}\, l \\ &= 2\pi\frac{u_p}{\lambda}\sqrt{LC}\, l = 2\pi\frac{1}{\cancel{\sqrt{LC}}\lambda}\cancel{\sqrt{LC}}\, l \\ &= 2\pi l/\lambda\end{aligned}$$

We apparently need a number for βl at 2 GHz where $l = 1$mm, the wavelength l is

$$\lambda = \frac{u_p}{f} = \frac{3 \cdot 10^8}{2 \cdot 10^9} = 0.15 \text{ m} = 150 \text{ mm}$$

then l/λ is

$$\frac{l}{\lambda} = \frac{20}{150} = 0.13333$$

so

$$\begin{aligned}Z_i &= R_0 \frac{Z_L + jR_0 tan\beta l}{R_0 + jZ_L tan\beta l} \\ &= 50\frac{25 + j50tan0.133333}{50 + j25tan0.13333} \\ &= 42.677 + 31.832i\end{aligned}$$

Normalized to the transmissionline impedance Z_i becomes z_i as

$$\begin{aligned}z_i &= Z_i/50 \\ &= \frac{42.677 + 31.832i}{50} \\ &= 0.8535 + 0.6366i\end{aligned}$$

To find out which reflection coefficient this represents we could either calculate Γ at the load and realize that $\Gamma = \Gamma(z' = 20mm)$ is Γ measured at the load multiplied with a phase factor e^{θ_γ} corresponding to 20mm or we can simply calculate

$$\Gamma_i = \frac{z_i - 1}{z_i + 1}$$

We will do both starting with the last stated equation

$$\begin{aligned}\Gamma_i &= \frac{z_i - 1}{z_i + 1} \\ &= \frac{0.8535 + 0.6366i - 1}{0.8535 + 0.6366i + 1} \\ &= 0.034843 + 0.331507i \\ &= 0.33\underline{/84}^\circ\end{aligned}$$

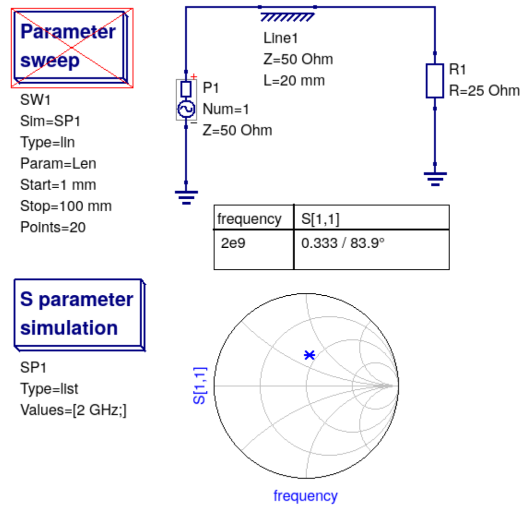
We also use We will do both starting with the last stated equation

$$
\begin{aligned}
\Gamma_i &= \Gamma_L e^{-2\gamma z'} \\
&= \Gamma_L e^{-j2\beta z'} \\
&= \Gamma_L e^{-j2\frac{2\pi z'}{\lambda}} \\
&= \Gamma_L e^{-j2\frac{2\pi 20}{150}} \\
&= 0.034843 + 0.331507i \qquad = 0.33\underline{/84}^\circ
\end{aligned}
$$

which is the same result as the first method of calculation. The last calculations were obtained using the followong Octave script

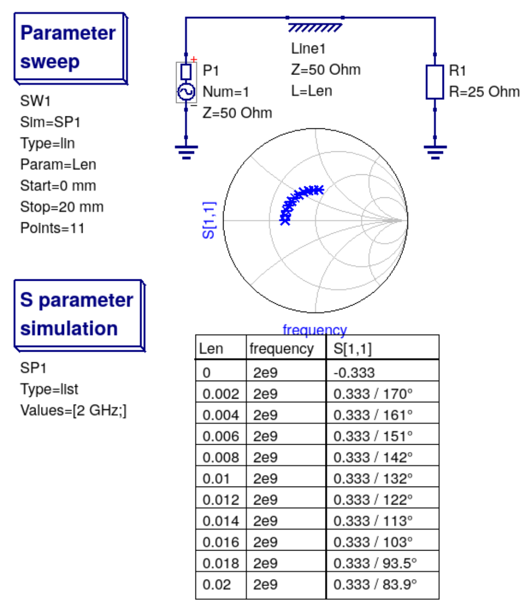
```
l=0.020
c = 3E+8
f=2E+9
lambda = c/f
beta=2*pi/lambda
R0=50;
ZL=25;

Zi=R0*((ZL+i*R0*tan(beta*l))...
   /(R0+i*ZL*tan(beta*l)))

zi=Zi/R0

Gamma_i = (zi-1)/(zi+1)

[THETA, R] = cart2pol...
  (real(Gamma_i), imag(Gamma_i))

THETA_GRAD=THETA*180/pi

zL=ZL/R0

Gamma_L = (zL-1)/(zL+1)
Gamma_i2 = Gamma_L*exp(-i*2*beta*l)
```

The QUCS installation also gives the same results. We also notice that the point seems to be correct located on the Smith Chart which is almost straight above the centerpoint of the Smithchart $(1, 0)$.



Figur 4: $R = 25$ Ohm, 20 mm 50 Ohm transmissionline

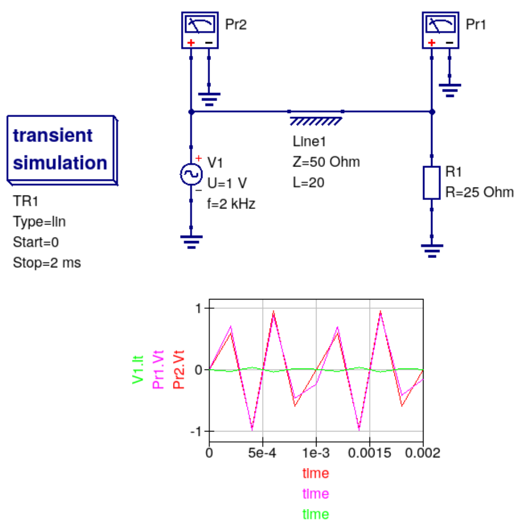
It is easier to verify the correctness of the point if the length of the line is swept from 0 to 20 mm so that ones sees on the Smithchart how the reflection coefficient changes when the length of the transmissionline varies. We expect to see a clockwise arc.



Len	frequency	S[1,1]
0	2e9	-0.333
0.002	2e9	0.333 / 170°
0.004	2e9	0.333 / 161°
0.006	2e9	0.333 / 151°
0.008	2e9	0.333 / 142°
0.01	2e9	0.333 / 132°
0.012	2e9	0.333 / 122°
0.014	2e9	0.333 / 113°
0.016	2e9	0.333 / 103°
0.018	2e9	0.333 / 93.5°
0.02	2e9	0.333 / 83.9°

Figur 5: Clockwise rotation of Γ is “Towards the generator”

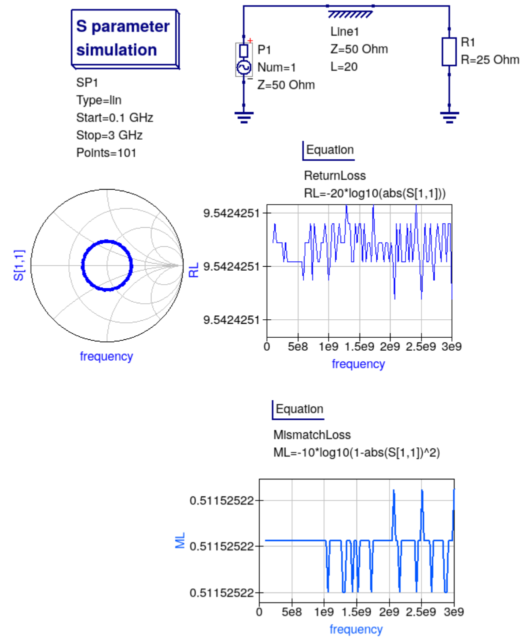
with no warnings or errors and thus we did not break anything during the installation here but we found the transient simulation to be broken.



Figur 6: Transient simulation is broken

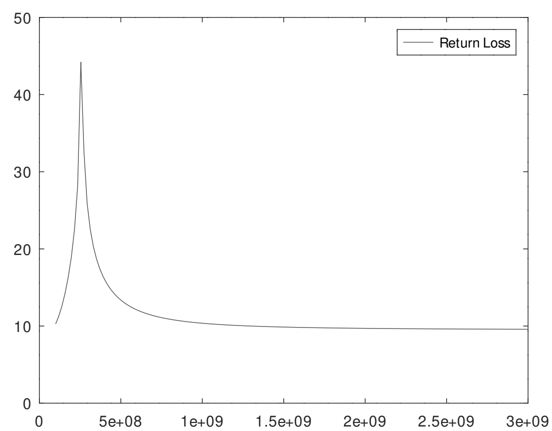
(e) Calculation of Return Loss and Mismatch Loss should conform with Matlab/Octave.

Here it looks like that QUCS is broken. The result does not conform with Matlab/Octave



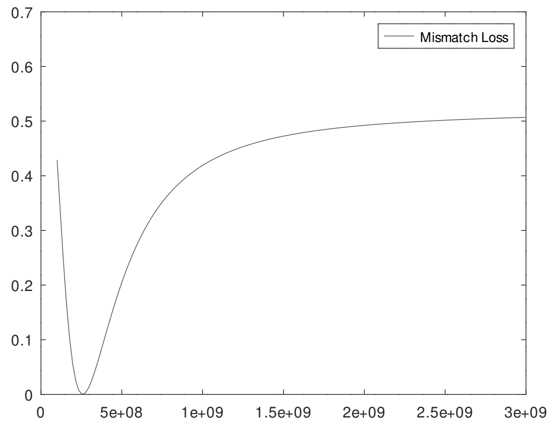
Figur 7: log-calculations are rubbish. Installation is broken

Octave gives the following Return Loss



Figur 8: Return Loss

and the following Mismatch Loss



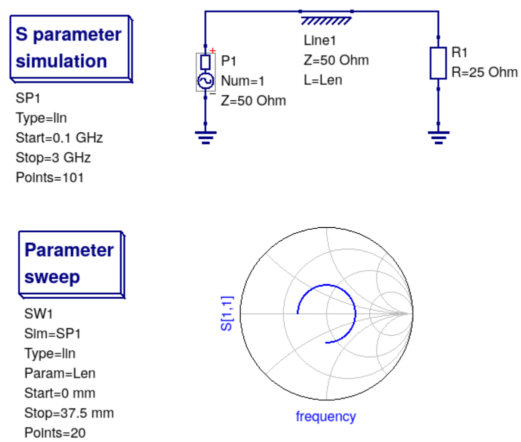
Figur 9: Mismatch Loss

From the following Octave code

```
l=0.020
c = 3E+8
f_start=0.1E+9;
f_stop=3.0E+9;
n_steps = 150;
step_size=(f_stop-f_start)/n_steps;
f=f_start:step_size:f_stop
GammaIn =zeros(size(f));

R0=50;
ZL=25;

for i=1:n_steps+1
      lambda = c/f(i)
      beta=2*pi/lambda

      Zi=R0*((ZL+i*R0*tan(beta*l))...
         /(R0+i*ZL*tan(beta*l)))

      zi=Zi/R0

      Gamma_i = (zi-1)/(zi+1);
      GammaIn(i)=Gamma_i;

 endfor

RL = -20*log10(abs(GammaIn));

ML = -10*log10(1-abs(GammaIn).^2);
figure(1)
plot(f,RL,";Return Loss;");

figure(2)
plot(f, ML, ";Mismatch Loss;")
```

(f) Sweeping the length of the transmission line one half wave length shoul result in a plot of the reflection coefficient describing a full circle.

If we apply 3 GHz we will have a wave length of 100 mm, so sweeping the length of the transmission line from 0 mm to $\lambda/2 = 50$mm must result in a full circle drawn in clockwise direction. Here we sweep to 75% of $\lambda/2$ to verify that the circle grows clockwise "towards the generator".



Figur 10: Sweeping the lenght of the transmission line from 0 mm to 75% of $\lambda/2$

(g) Example 9-13 in David K. Cheng "'Field and wave Electromagnetics"' short circuited line is transformed to a certain impedance with a transmission line of 0.1λ.

The example states - use the Smith chart to find the imput impedance of a section of 50 Ω lossless transmission line that is 0.1 wavelength long and is terminated in a short circuit.

The formula for the input impedance is

$$Z_i = R_0 \frac{Z_L + jR_0 tan\beta l}{R_0 + jZ_L tan\beta l}$$

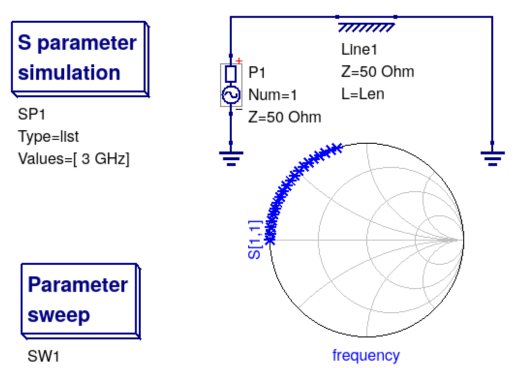
and setting $Z_L = 0$ and $l = 0.1\lambda$ gives

$$
\begin{aligned}
Z_i &= R_0 \frac{0 + jR_0 tan\beta l}{R_0 + j \cdot 0 \cdot tan\beta l} \\
&= jR_0 tan\beta l = jR_0 tan 2\pi \cdot 0.1 \\
&= 0 + 36.3271i
\end{aligned}
$$

which means that the normalized impedance $z_i = i0.726542528$ The corresponding reflection coefficient should read

$$
\begin{aligned}
\Gamma_i &= \frac{z_1 - 1}{z_1 + 1} \\
&= -0.3090 + 0.9511i
\end{aligned}
$$

Which also conforms to the QUCS calculation

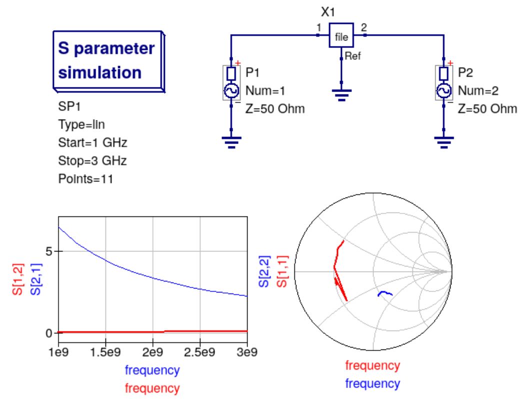


Len	frequency	S[1,1]
0.00474	3e9	-0.828+j0.561
0.00526	3e9	-0.789+j0.615
0.00579	3e9	-0.746+j0.665
0.00632	3e9	-0.701+j0.713
0.00684	3e9	-0.652+j0.758
0.00737	3e9	-0.601+j0.8
0.00789	3e9	-0.546+j0.838
0.00842	3e9	-0.49+j0.872
0.00895	3e9	-0.431+j0.902
0.00947	3e9	-0.37+j0.929
0.01	3e9	-0.308+j0.951

Figur 11: Sweeping the lenght of shortcircuited transmission line from 0 mm to 10mm at 3GHz

(h) Plot of S-parameters verifying that the particular part of the touchstone file is read correctly.

The reading is correct



Excerpt from Touhcstone for 1 GHz
Frequency in MHz/S11/S21/S12/S22
1000 .495 -160.2 6.488 92.4 .059 49.7 .374 -51.1 ! 18.1

frequency	S[1,1]	S[1,2]	S[2,1]	S[2,2]
1e9	0.495 / -160°	0.059 / 49.7°	6.49 / 92.4°	0.374 / -51.1°
1.2e9	0.494 / -172°	0.065 / 51.3°	5.47 / 87°	0.344 / -53.4°
1.4e9	0.5 / -132°	0.069 / 52.5°	4.75 / 82°	0.331 / -56.1°
1.6e9	0.5 / 173°	0.075 / 55.2°	4.16 / 77.8°	0.326 / -57.3°
1.8e9	0.494 / 166°	0.082 / 56.4°	3.72 / 74.3°	0.317 / -57.9°
2e9	0.496 / 158°	0.087 / 58.2°	3.37 / 71°	0.297 / -59.5°
2.2e9	0.515 / 152°	0.093 / 59°	3.08 / 67.9°	0.279 / -64.4°
2.4e9	0.535 / 148°	0.097 / 60.7°	2.8 / 64.2°	0.28 / -71.4°
2.6e9	0.54 / 144°	0.107 / 61.3°	2.58 / 61.6°	0.298 / -76.3°
2.8e9	0.534 / 140°	0.114 / 60.9°	2.43 / 58.2°	0.313 / -77.7°

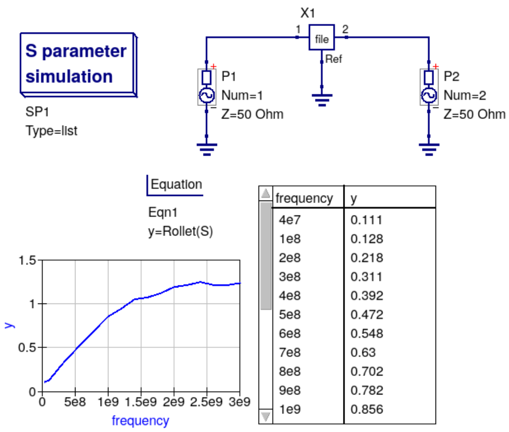
Figur 12: S-parameter sweep with excerpt of .s2p-file at 1GHz

though errors were detected by the simulator according to the log

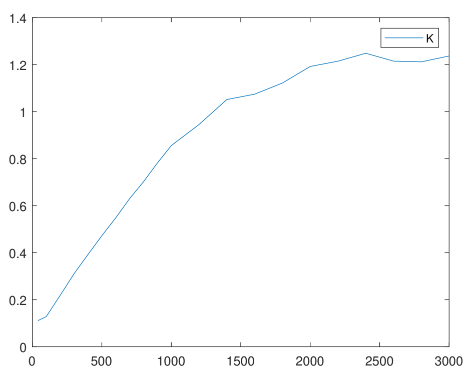
```
Errors occurred during simulation
on sön 04. sep 2022 at 22:19:18:426
Aborted.



Errors and Warnings:
--------------------

checker error, no actions defined: nothing to do
```

(i) Plot of the built in function $Rollet()$ to verify that it accurately performs the calculation of the Rollet stability condition.

We notice that the transistor is unstable for low frequencies ($K < 1$) Looks the same as the one calculated in Matlab



Figur 13: Rollet stability factor K



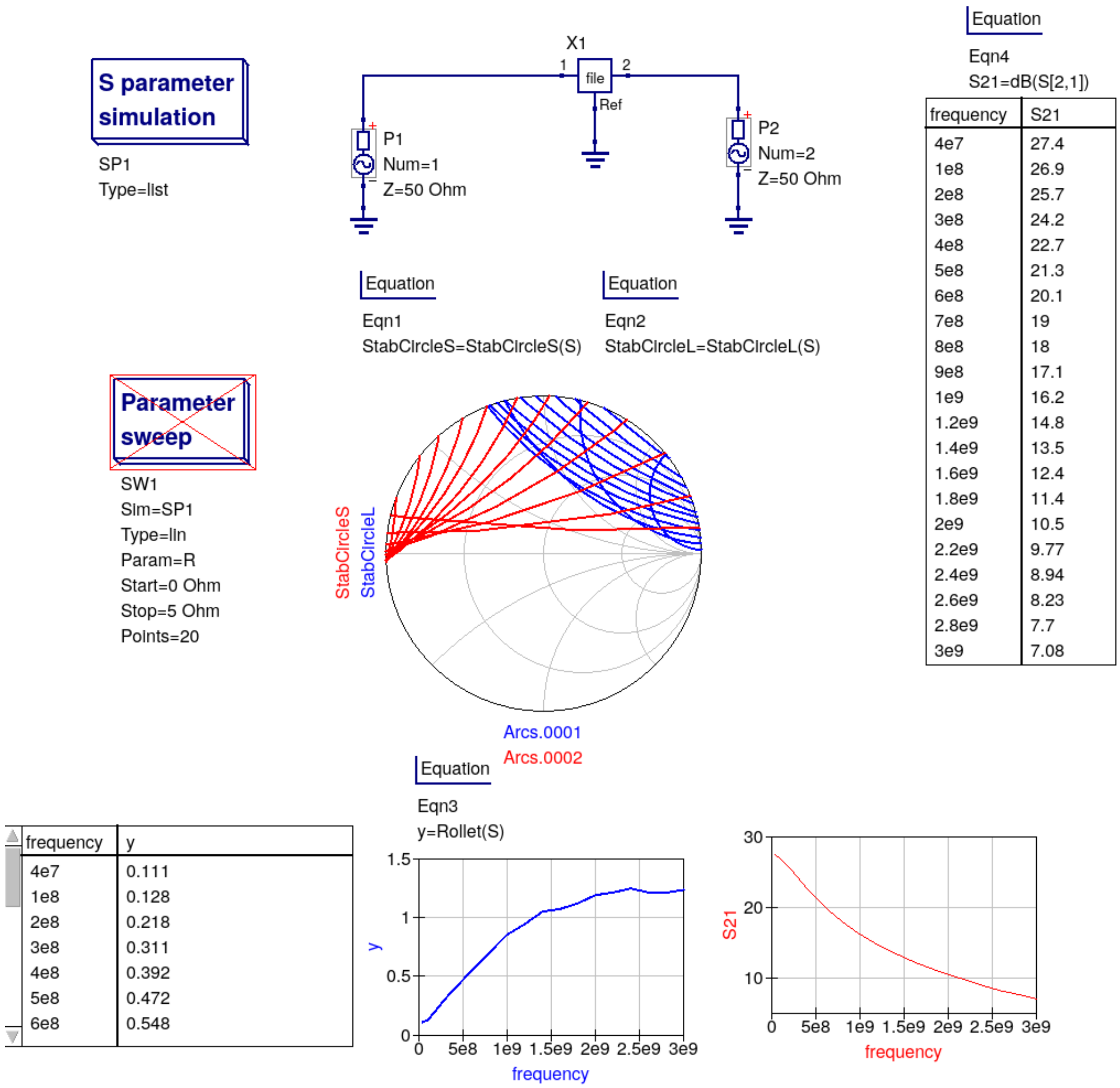
Figur 14: Rollet stability factor K calculated with Matlab

Cleand the .s2p file of the comments and the noisefigeure data and adapted code from Mathworks[6]

```
fid  = fopen('Spar.s2p', 'r') ;
 data = cell(1e6, 9) ;        % Prealloc.
 rCnt = 0 ;                   % Row counter.
 while ~feof(fid)
    rCnt = rCnt + 1 ;
    data{rCnt,1} = fscanf(fid, '%f', 1);
    data{rCnt,2} = fscanf(fid, '%f', 1);
    data{rCnt,3} = fscanf(fid, '%f', 1);
    data{rCnt,4} = fscanf(fid, '%f', 1);
    data{rCnt,5} = fscanf(fid, '%f', 1);
    data{rCnt,6} = fscanf(fid, '%f', 1);
    data{rCnt,7} = fscanf(fid, '%f', 1);
    data{rCnt,8} = fscanf(fid, '%f', 1);
    data{rCnt,9} = fscanf(fid, '%f', 1);
 end
 fclose(fid) ;
 data = data(1:rCnt,:) ;    % Truncate.
A=cell2mat(data);
f=A(:,1);
S11=zeros(size(f))
S21=zeros(size(f))
S12=zeros(size(f))
S22=zeros(size(f))
j=sqrt(-1);
for i=1:size(f)
 phi = deg2rad(A(i,3))
 phi1 = A(i,3)*pi/180
 S11(i)=A(i,2)*(cos(phi)+j*sin(phi))
 phi= deg2rad(A(i,5))
 S21(i)=A(i,4)*(cos(phi)+j*sin(phi))
 phi= deg2rad(A(i,7))
 S12(i)=A(i,6)*(cos(phi)+j*sin(phi))
 phi= deg2rad(A(i,9))
 S22(i)=A(i,8)*(cos(phi)+j*sin(phi))
end
Delta = abs(S11.*S22-S12.*S21)
K=(1-abs(S11).^2-abs(S22).^2+abs(Delta).^2)...
  ./(2*abs(S12.*S21))
plot(f,K)
legend('K')
```

(j) Plot of built-in functions *stabL*() and *stabS*() verifying input- and output-stability circles with respect to theory.

[6] https://se.mathworks.com/matlabcentral/answers/76197-how-to-read-strings-from-file-with-fscanf-or-sscanf-not-textscan



Figur 15: Input and output stability circles

3 Amplifier Theory, Chap. 12 in Pozar

In this section we will verify the equations presented in Pozar in the chapter about Microwave amplifiers. Pozar posts the following figure

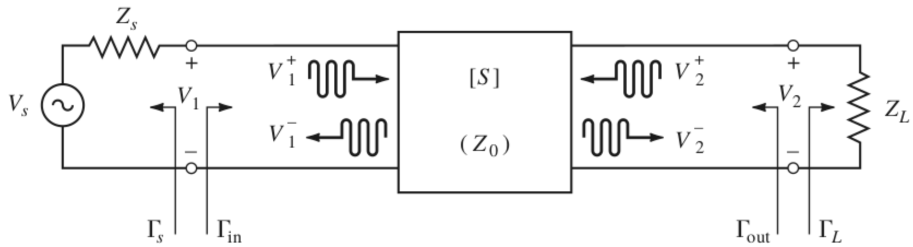


FIGURE 12.1 A two-port network with arbitrary source and load impedances.

Figur 16: Pozar figure Chap. 12

3.1 Γ_L and Γ_S

Pozar starts with the presentation of standard well known basic Electromagnetic Theory of the load reflection coefficient Γ_L

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

The source reflection coefficient looking into the load

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

Then he defines voltage scattering parameters which according to what I've seen differ between authors.

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$
$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$

This definition is apparently only valid when the source resistor and the load is equal to the characteristic impedance[7] and is in its general way defined as

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

where

$$a = \frac{1}{2}\frac{V + Z_oI}{\sqrt{|Re(Z_0)|}}$$
$$b = \frac{1}{2}\frac{V + Z_o^*I}{\sqrt{|Re(Z_0)|}}$$

See footnote[8]. It is unclear how one arrives at the Pozar expression of the scattering parameters from the general definition and for now will not try to show this. We think it is very unfourtunate that Pozar does not show this.

Since the load voltage coefficient Γ_L, looking into the load is the quotient between the backward travelling wave V_2^+ and the forward travelling wave V_2^-, which is

$$\Gamma_L = \frac{V_2^+}{V_2^-}$$

and the source reflection coefficient Γ_S looking into the generator also is the backward wave (to the load) divided with the forward wave (into the generator)

$$\Gamma_S = \frac{V_1^+}{V_1^-}$$

Pozar rewrites equation (1) where he replaces V_2^+ with $\Gamma_L V_2^-$

$$V_1^- = S_{11}V_1^+ + S_{12}\Gamma_L V_2^- \quad (1)$$
$$V_2^- = S_{21}V_1^+ + S_{22}\Gamma_L V_2^- \quad (2)$$

[7] https://en.wikipedia.org/wiki/Scattering_parameters
[8] https://se.mathworks.com/discovery/s-parameter.html

3.2 Γ_{in} -Reflection coefficient looking into the network

Pozar arrives at Γ_{in} the reflection coefficient looking into the network, which is the backward wave divided with the forward wave.

$$\Gamma_{in} = \frac{V_1^-}{V_1^+}$$

and finds an expression for Γ_{in} as a function of the S-parameters by solving for V_2^- in equation (1) and inserting it into equation (2) and then solving for the quotient $\frac{V_1^-}{V_1^+}$ thus obtaining a new expression for Γ_{in}

$$\begin{aligned} V_1^- &= S_{11}V_1^+ + S_{12}\Gamma_L V_2^- \iff \\ V_2^- &= \frac{V_1^- - S_{11}V_1^+}{S_{12}\Gamma_L} \end{aligned}$$

Inserting the expression of V_2^- into (2) gives

$$\begin{aligned} V_2^- &= S_{21}V_1^+ + S_{22}\Gamma_L V_2^- \qquad (2) \\ \frac{V_1^- - S_{11}V_1^+}{S_{12}\Gamma_L} &= S_{21}V_1^+ + S_{22}\Gamma_L \frac{V_1^- - S_{11}V_1^+}{S_{12}\Gamma_L} \end{aligned}$$

We collect the terms with V_1^- to the left and the terms with V_1^+ to the right but first we clean the denominator on the left hand side

$$\begin{aligned} V_1^- - S_{11}V_1^+ = {} & S_{12}\Gamma_L S_{21}V_1^+ \\ & + S_{12}\Gamma_L S_{22}\Gamma_L \frac{V_1^- - S_{11}V_1^+}{S_{12}\Gamma_L} \end{aligned}$$

We see that $S_{12}\Gamma_L$ cancels up and down on the second term of the right hand side

$$\begin{aligned} V_1^- - S_{11}V_1^+ = {} & S_{12}\Gamma_L S_{21}V_1^+ \\ & + S_{22}\Gamma_L V_1^- - S_{22}\Gamma_L S_{11}V_1^+ \end{aligned}$$

Now we are ready to collect the terms with V_1^- and V_1^+ left and right

$$\begin{aligned} V_1^- - S_{22}\Gamma_L V_1^- = {} & S_{11}V_1^+ + S_{12}\Gamma_L S_{21}V_1^+ \\ & - S_{22}\Gamma_L S_{11}V_1^+ \end{aligned}$$

We break out

$$V_1^-(1 - S_{22}\Gamma_L) = V_1^+(S_{11} + S_{12}\Gamma_L S_{21} - S_{22}\Gamma_L S_{11})$$

and solve for V_1^-/V_1^+

$$\frac{V_1^-}{V_1^+} = \frac{S_{11} + S_{12}\Gamma_L S_{21} - S_{22}\Gamma_L S_{11}}{1 - S_{22}\Gamma_L}$$

This does not look like Pozar's expression but if we break out S_{11} from the numerator we get

$$\frac{V_1^-}{V_1^+} = \frac{S_{11}(1 - S_{22}\Gamma_L) + S_{12}\Gamma_L S_{21}}{1 - S_{22}\Gamma_L}$$

and we write the right hand side as two terms

$$\frac{V_1^-}{V_1^+} = \frac{S_{11}(1 - S_{22}\Gamma_L)}{1 - S_{22}\Gamma_L} + \frac{S_{12}\Gamma_L S_{21}}{1 - S_{22}\Gamma_L}$$

Now we see that a common term is cancelled up and down at the first term on the right hand side and we arrive at Pozar's expression

$$\frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

Thus Pozar arrived at an expression for Γ_{in} only involving S-parameters and Γ_L

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

3.3 Γ_{out} -Reflection coefficient looking into the network from the load side

Obviously Γ_{out} looking into the network from the load side must be (by symmetry)

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

which Pozar also states without evidence, but we can show this also for completeness.

The reflection coefficient looking into the source Γ_S having the network at the back is the backward moving wave V_1^+ going into the network divided by the forward going wave travelling into the source V_1^-

$$\Gamma_S = \frac{V_1^+}{V_1^-} \iff$$
$$V_1^+ = \Gamma_S V_1^-$$

which means that we can re-write Pozar's S-parameter definition

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$
$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$

as

$$V_1^- = S_{11}\Gamma_S V_1^- + S_{12}V_2^+$$
$$V_2^- = S_{21}\Gamma_S V_1^- + S_{22}V_2^+$$

We are hunting for the expression Γ_{out} looking into the network from the load side so we obviously must solve for V_1^- from the first equation above and insert it into the second

$$V_1^- = S_{11}\Gamma_S V_1^- + S_{12}V_2^+ \iff$$
$$V_1^-(1 - S_{11}\Gamma_S) = S_{12}V_2^+ \iff$$
$$V_1^- = \frac{S_{12}V_2^+}{1 - S_{11}\Gamma_S}$$

Inserting the expression of V_1^- into the second equations

$$V_2^- = S_{21}\Gamma_S V_1^- + S_{22}V_2^+$$
$$= S_{21}\Gamma_S \frac{S_{12}V_2^+}{1 - S_{11}\Gamma_S} + S_{22}V_2^+$$

Solving for V_2^-/V_2^+

$$\frac{V_2^-}{V_2^+} = S_{21}\Gamma_S \frac{S_{12}}{1 - S_{11}\Gamma_S} + S_{22}$$

Which is the wanted expression if we just write it in order

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

3.4 Availible power to the network P_{in}

Pozar want to derive an expression for the power gain $G = P_L/P_{in}$ further down the road. It is is the quotient of the power delivered to the load to the power availible to the network. Pozar is not clear, I thought that he wanted to establish a power-wave

$$P_{in} = \frac{1}{Z_0}\frac{V_1^+}{\sqrt{2}}\frac{V_1^{+*}}{\sqrt{2}}$$
$$= \frac{1}{2Z_0}|V_1^+|^2$$

but this is not the case. He defines P_{in} as

$$P_{in} = \frac{1}{2Z_0}V_1 \cdot V_1^*$$

where the factor an half comes from scaling to effective values.

He starts with establishing that the voltage $V_1 = V_1^+ + V_1^-$ is simply the voltage divder expression Because

$$V_1 = V_S\frac{Z_{in}}{Z_{in} + Z_S} = V_1^+ + V_1^- = V_1^+(1 + \Gamma_{in})$$

so he will be able to write

$$
\begin{aligned}
P_{in} &= \frac{1}{2Z_0} V_1 \cdot V_1^* \\
&= \frac{1}{2Z_0} (V_1^+(1+\Gamma_{in}))(V_1^+(1+\Gamma_{in})^* \\
&= \frac{1}{2Z_0} (V_1^+(1+\Gamma_{in}))(V_1^{+*}(1+\Gamma_{in}))^*
\end{aligned}
$$

because Γ_{in} is a complex number so

$$
\begin{aligned}
P_{in} &= \frac{1}{2Z_0} V_1^+ V_1^{+*}(1+\Gamma_{in})(1+\Gamma_{in}^*) \\
&= \frac{|V_1^+|^2}{2Z_0}(1+\Gamma_{in})((1+\Gamma_{in}^*) \\
&= \frac{|V_1^+|^2}{2Z_0}(1+|\Gamma_{in}|^2)
\end{aligned}
$$

because

$$
\begin{aligned}
(1+jb)(1-jb) &= 1^2 - jb + jb - j^2 b^2 \\
&= 1^2 + b^2 = 1 + |jb|^2
\end{aligned}
$$

but Pozar claims

$$
P_{in} = \frac{|V_1^+|^2}{2Z_0}(1-|\Gamma_{in}|^2)
$$

What is he using? Does he not take the complex conjugate? Does he perhaps take

$$
\begin{aligned}
P_{in} &= \frac{1}{2Z_0} V_1^2 \\
&= \frac{1}{2Z_0}(V_1^+(1+\Gamma_{in}))(V_1^+(1+\Gamma_{in}))
\end{aligned}
$$

but Γ_{in} is still complex

$$
\begin{aligned}
(1+jb)(1+jb) &= 1^2 + jb + jb + j^2 b^2 \\
&= 1 - b^2 + 2jb
\end{aligned}
$$

so

$$
\begin{aligned}
P_{in} &= \frac{1}{2Z_0} V_1^2 \\
&= \frac{(V_1^+)^2}{2Z_0}(1 - \Gamma_{in}^2 - 2j\Gamma_{in})
\end{aligned}
$$

No Pozar is not taking the square ———————————-
Then he is using

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

which is recasted to

$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$$

because

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \iff$$
$$\Gamma_{in}(Z_{in} + Z_0) = Z_{in} - Z_0$$

Collecting terms with Z_{in} to the right and terms with Z_0 on the left

$$Z_0 + \Gamma_{in} Z_0 = Z_{in} - \Gamma_{in} Z_{in}$$

We break out Z_0 on the left and we break out Z_{in} on the right

$$Z_0(1 + \Gamma_{in}) = Z_{in}(1 - \Gamma_{in})$$

We solve for Z_{in}

$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$$

He replaces Z_{in} with this expression in

$$V_S \frac{Z_{in}}{Z_{in} + Z_S} = V_1^+(1 + \Gamma_{in})$$
$$V_S \frac{Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}}}{Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}} + Z_S} = V_1^+(1 + \Gamma_{in})$$

which becomes

$$V_S \frac{Z_0(1 + \Gamma_{in})}{Z_0 \frac{(1-\Gamma_{in})1+\Gamma_{in}}{1-\Gamma_{in}} + (1 - \Gamma_{in}) Z_S} = V_1^+(1 + \Gamma_{in})$$

$1 + \Gamma_{in}$ cancels right and left and $1 - \Gamma_{in}$ cancels in the left most term of the denominator on the left hand side.

$$V_S \frac{Z_0}{Z_0(1 + \Gamma_{in}) + (1 - \Gamma_{in}) Z_S} = V_1^+$$

He also replaces Z_S with the recasted version of Γ_S which is the reflection coefficient looking into the source

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \iff$$
$$Z_S = Z_0 \frac{1+\Gamma_S}{1-\Gamma_S}$$

Inserting the expression of Z_S

$$V_1^+ = V_S \frac{Z_0}{Z_0(1+\Gamma_{in}) + (1-\Gamma_{in})Z_S}$$
$$V_1^+ = V_S \frac{Z_0}{Z_0(1+\Gamma_{in}) + (1-\Gamma_{in})Z_0\frac{1+\Gamma_S}{1-\Gamma_S}}$$

We multiply and divide the left term in the denominator with $1-\Gamma_S$ and invert this bottom factor to the numerator.

$$V_1^+ = V_S \frac{Z_0(1-\Gamma_S)}{Z_0(1+\Gamma_{in})(1-\Gamma_S) + (1-\Gamma_{in})Z_0(1+\Gamma_S)}$$

Z_0 cancels up and down. We exapand the denominator

$$V_1^+ = V_S \frac{Z_0(1-\Gamma_S)}{Z_0(1+\Gamma_{in})(1-\Gamma_S) + (1-\Gamma_{in})Z_0(1+\Gamma_S)}$$
$$= V_S \frac{(1-\Gamma_S)}{1 - \cancel{\Gamma_S} + \cancel{\Gamma_{in}} - \Gamma_{in}\Gamma_S + 1 + \cancel{\Gamma_S} - \cancel{\Gamma_{in}} - \Gamma_{in}\Gamma_S}$$

remains

$$V_1^+ = V_S \frac{(1-\Gamma_S)}{1 - \Gamma_{in}\Gamma_S + 1 - \Gamma_{in}\Gamma_S}$$
$$= V_S \frac{(1-\Gamma_S)}{2 - 2\Gamma_{in}\Gamma_S}$$

Breaking out 1/2

$$V_1^+ = \frac{V_S}{2} \frac{1-\Gamma_S}{1-\Gamma_{in}\Gamma_S}$$

so P_{in} becomes if we use Pozar's definition

$$P_{in} = \frac{|V_1^+|^2}{2Z_0}(1-|\Gamma_{in}|^2)$$
$$= \frac{|\frac{V_S}{2}\frac{1-\Gamma_S}{1-\Gamma_{in}\Gamma_S}|^2}{2Z_0}(1-|\Gamma_{in}|^2)$$
$$= \frac{|V_S|^2|1-\Gamma_S|^2}{8|1-\Gamma_{in}\Gamma_S|^2 Z_0}(1-|\Gamma_{in}|^2)$$

Pozar also says that P_{in} becomes this with his definition of P_{in}

$$P_{in} = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2(1-|\Gamma_{in}|^2)}{|1-\Gamma_{in}\Gamma_S|^2}$$

3.5 Power delivered to the load P_L

Pozar states that P_L is

$$P_L = \frac{|V_2^-|^2}{2Z_0}(1-|\Gamma_L|^2)$$

possibly by a symmetry argument because he is saying that

$$P_{in} = \frac{|V_1^+|^2}{2Z_0}(1-|\Gamma_{in}|^2)$$

For a more involved expression of P_L He solves for V_2^- of the scattering matrix definition to elimate V_2^- in expression of P_L

$$\begin{aligned}
V_2^- &= S_{21}V_1^+ + S_{22}V_2^+ \\
&= S_{21}V_1^+ + S_{22}\Gamma_L V_2^- \\
V_2^-(1-S_{22}\Gamma_L) &= S_{21}V_1^+ \\
V_2^- &= \frac{S_{21}V_1^+}{1-S_{22}\Gamma_L}
\end{aligned}$$

Inserting V_2^- in the expression of P_L becomes

$$\begin{aligned}
P_L &= \frac{|V_2^-|^2}{2Z_0}(1-|\Gamma_L|^2) \\
&= \frac{\left|\frac{S_{21}V_1^+}{1-S_{22}\Gamma_L}\right|^2}{2Z_0}(1-|\Gamma_L|^2) \\
&= \frac{\left|S_{21}V_1^+\right|^2}{|1-S_{22}\Gamma_L|^2 2Z_0}(1-|\Gamma_L|^2)
\end{aligned}$$

Then he is inserting

$$V_1^+ = \frac{V_S}{2}\frac{1-\Gamma_S}{1-\Gamma_{in}\Gamma_S}$$

$$
\begin{aligned}
P_L &= \frac{\left|S_{21}V_1^+\right|^2}{|1-S_{22}\Gamma_L|^2 2Z_0}(1-|\Gamma_L|^2) \\
&= \frac{\left|S_{21}\frac{V_S}{2}\frac{1-\Gamma_S}{1-\Gamma_{in}\Gamma_S}\right|^2}{|1-S_{22}\Gamma_L|^2 2Z_0}(1-|\Gamma_L|^2) \\
&= \frac{|S_{21}V_S(1-\Gamma_S)|^2}{4|1-\Gamma_{in}\Gamma_S|^2|1-S_{22}\Gamma_L|^2 2Z_0}(1-|\Gamma_L|^2) \\
&= \frac{|S_{21}|^2|V_S|^2|1-\Gamma_S|^2}{8|1-\Gamma_{in}\Gamma_S|^2|1-S_{22}\Gamma_L|^2 Z_0}(1-|\Gamma_L|^2)
\end{aligned}
$$

asserting Pozar's expression of P_L Equation 12.7

$$
P_L = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_L|^2)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2}
$$

3.6 Power gain P_L/P_{in}

If we use Pozar's expression of P_{in} and divide P_L/P_{in}

$$
\begin{aligned}
G = \frac{P_L}{P_{in}} &= \frac{\frac{|S_{21}|^2\cancel{|V_S|^2}\cancel{|1-\Gamma_S|^2}}{\cancel{8}\cancel{|1-\Gamma_{in}\Gamma_S|^2}|1-S_{22}\Gamma_L|^2\cancel{Z_0}}(1-|\Gamma_L|^2)}{\frac{\cancel{|V_S|^2}}{\cancel{8Z_0}}\frac{\cancel{|1-\Gamma_S|^2}(1-|\Gamma_{in}|^2)}{\cancel{|1-\Gamma_{in}\Gamma_S|^2}}} \\
&= \frac{|S_2 1|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2(1-|\Gamma_{in}|^2)}
\end{aligned}
$$

3.7 P_{avs} - the maximum power availible from the source

Pozar continues defining P_{avs} as the maximum power availible from the source

$$
\begin{aligned}
P_{avs} &= P_{in}\Big|_{\Gamma_{in}^*=\Gamma_S} \\
&= \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2(1-|\Gamma_{in}|^2)}{|1-\Gamma_{in}\Gamma_S|^2}\Bigg|_{\Gamma_{in}^*=\Gamma_S} \\
&= \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2(1-|\Gamma_S^*|^2)}{|1-\Gamma_S^*\Gamma_S|^2}
\end{aligned}
$$

We have that $\Gamma_S^*\Gamma_S = |\Gamma_S|^2$

$$
\begin{aligned}
P_{avs} &= \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2(1-|\Gamma_S^*|^2)}{|1-|\Gamma_S|^2|^2} \\
&= \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{|1-|\Gamma_S|^2|}
\end{aligned}
$$

which Pozar writes as

$$
P_{avs} = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{(1-|\Gamma_S|^2)}
$$

Apparently $|1-|\Gamma_S|^2| = (1-|\Gamma_S|^2)$ which is true if $|\Gamma_S|^2 \leq 1$.

3.8 P_{avn} - the maximum power availible from the network

Pozar defines P_{avn} ss the maximum power availible from the network which is when $\Gamma_{out}^* = \Gamma_L$

$$\begin{aligned}
P_{avn} &= P_L\Big|_{\Gamma_L=\Gamma_{out}^*} \\
&= \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_L|^2)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2}\Bigg|_{\Gamma_L=\Gamma_{out}^*} \\
&= \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_{out}^*|^2)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_{out}|^2|1-\Gamma_S\Gamma_{in}|^2}
\end{aligned}$$

Pozar the states that Γ_{in} must be evaluated for $\Gamma_L = \Gamma_{out}^*$ and that it can be shown that

$$|1-\Gamma_S\Gamma_{in}|^2\Bigg|_{\Gamma_L=\Gamma_{out}^*} = \frac{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)^2}{|1-S_{22}\Gamma_{out}^*|^2}$$

I've tried to show this but cannot. But we insert this and see what happens

$$\begin{aligned}
P_{avn} &= \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_{out}^*|^2)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_{out}|^2|1-\Gamma_S\Gamma_{in}|^2} \\
&= \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_{out}^*|^2)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_{out}|^2\frac{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)^2}{|1-S_{22}\Gamma_{out}^*|^2}}
\end{aligned}$$

The expression $(1-|\Gamma_{out}^*|^2)$ in the numerator seems to cancel and we can invert the expression $|1-S_{22}\Gamma_{out}^*|^2$ from the denominator to the numerator but that's it

$$P_{avn} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2\cancel{|1-S_{22}\Gamma_{out}^*|^2}|1-\Gamma_S|^2}{\cancel{|1-S_{22}\Gamma_{out}|^2}|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)}$$

Cancellation up and down

$$P_{avn} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2|1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)}$$

3.9 G_A - Availible power gain

Pozar defines G_A as the maximum power possible delivered from the network P_{avn} to the maximum power availibe from the source which he derived from conjugately matched reflection coefficients

$$\begin{aligned}
G_A = \frac{P_{avn}}{P_{avs}} &= \frac{\frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2|1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)}}{\frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{(1-|\Gamma_S|^2)}} \\
&= \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)}
\end{aligned}$$

which confirms Pozar's equation 12.12

3.10 G_{TU} Tranceducer power gain

Pozar defines the tranceducer power gain G_{TU} as the power delivered to the load P_L to the maximum power avialible from the source P_{avs}

$$
\begin{aligned}
G_{TU} = \frac{P_L}{P_{avs}} &= \frac{\frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_L|^2)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2}}{\frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{(1-|\Gamma_S|^2)}} \\
&= \frac{|S_{21}|^2(1-|\Gamma_L|^2)(1-|\Gamma_S|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2}
\end{aligned}
$$

A special case, writes Pozar, arises when $\Gamma_L = 0$ and $\Gamma_S = 0$ which would correspond to a non-resonant network which reduces $G_{TU} = S_{21}^2$ in contrast to a resonant network. He separates G_{TU} as

$$
\begin{aligned}
G_{TU} &= \frac{|S_{21}|^2(1-|\Gamma_L|^2)(1-|\Gamma_S|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2} \\
&= \underbrace{\frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}}_{G_S} \cdot \underbrace{|S_{21}|^2}_{G_0} \cdot \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}_{G_L}
\end{aligned}
$$

4 Stability

Stability requires $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ We verified Pozar's expression of the above. We have the backward voltage wave to the forward volatage wave looking into the network from the source side.

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

and similarly the backwardmoving voltage wave looking into the network from the load side (moving into the load) to the forward moving voltage wave (reflected from the load).

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$

so stability requires

$$
\begin{aligned}
|\Gamma_{in}| &= \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} \right| < 1 \\
|\Gamma_{out}| &= \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} \right| < 1
\end{aligned}
$$

Of the requirement for Γ_{in} Pozar derives the so called "output stability circle" which poses restrictions on Γ_L for stability. We've checked the input and output stability circles already and they don't look correct comparing with the calculated Rollet stabilty graf.

5 Design for maximum gain

We know that for maximum power transfer which also means resonance in passive networks $\Gamma_{in}^* = \Gamma_S$ and $\Gamma_{out}^* = \Gamma_L$

$$
\begin{aligned}
G_{TU} &= \underbrace{\frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}}_{G_S} \cdot \underbrace{|S_{21}|^2}_{G_0} \cdot \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}_{G_L} \\
&= \underbrace{\frac{1-|\Gamma_S|^2}{|1-|\Gamma_S|^2|^2}}_{G_S} \cdot \underbrace{|S_{21}|^2}_{G_0} \cdot \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}_{G_L}
\end{aligned}
$$

Up and down factors cancels and we get

$$G_{TU} = \underbrace{\frac{1}{|1-|\Gamma_S|^2|}}_{G_S} \cdot \underbrace{|S_{21}|^2}_{G_0} \cdot \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}_{G_L}$$

To achive this we solve

$$\Gamma_S^* = \Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$
$$\Gamma_L^* = \Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$

which gives the solutions

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|}}{2C_1}$$

where

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - \Delta$$
$$C_1 = S_{11} - \Delta \cdot S_{22}^*$$

and Γ_L is given by

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|}}{2C_2}$$

where

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - \Delta$$
$$C_2 = S_{22} - \Delta \cdot S_{11}^*$$

We are requested to provide a design for 2.45GHz but we don't have S-parameters for that frequecy so we decided on 2.4 GHz and hoping that the design would have enough bandwidth. For the desing we used Octave and verified each step with a Matlab library[9]

```
clear all
% addpath("/home/lasse/ewa")
%Add above statement to console input
i=sqrt(-1);
disp("S-Params at 1.8GHz BFG520 ...
       Common Emitter 6V10mA")
S11_r = 0.494
Theta_S11_grad=166
S21_r=3.722
Theta_S21_grad=74.3
S12_r=0.082
Theta_S12_grad= 56.4
S22_r=0.317
Theta_S22_grad= -57.9
%Call to the EMW-library
disp("")
S=smat([0.494 166.0 3.722 74.3 0.082 ...
     56.4 0.317 -57.9 ])
disp("")
```

We convert to cartesian coordinates, not shown here, but the full program will in the appendix We did the Rollet-stability test

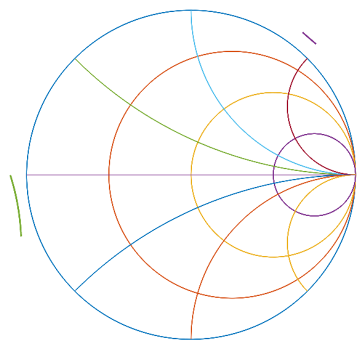
```
%Stability check
disp('Stability check')
Delta = S11*S22-S12*S21
K_Stab=(1-abs(S11)^2-abs(S22)^2+abs(Delta)^2)...
      /(2*abs(S12*S21))
Abs_Delta = abs(Delta)
if(K_Stab>1 & Abs_Delta<1)
 disp("Stability OK")
else
 disp("Stability not OK")
end
```

Which resulted in $K = 1.1220$ and $|\Delta| = 0.175$ which means that it stable at the frequecy though unstable at lower frequencies as previously shown. Stability circles using the toolbox are using commands

```
[cL,rL] = sgcirc(S,'l');
[cG,rG] = sgcirc(S,'s');
smith;
smithcir(cL, rL, 1.1, 1.5);
smithcir(cG, rG, 1.1, 1.5);
```

Unfourtunately the toolbox doesn't seem to give the option to insert a legend but the upper is the load stability circle and the lower is the source stability cirlce. They are both outside the unit circle and this means that any source and load impedances will not result in instability.

[9] http://eceweb1.rutgers.edu/~orfanidi/ewa/



Figur 17: Load stabilty circle (upper). Source stability circle (lower)

We calculated to source and load reflection coefficients Γ_s and Γ_L for a conjugate match which means $\Gamma_s^* = \Gamma_{in}$ and $\Gamma_L^* = \Gamma_{out}$