CMPE 493
INTRODUCTION TO
INFORMATION RETRIEVAL

Term Weighting and the Vector Space Model

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Ranked retrieval

- Thus far, our queries have all been Boolean.
  - Documents either match or don't.
- Good for expert users with precise understanding of their needs and the collection.
  - Also good for applications: Applications can easily consume 1000s of results.
- Not good for the majority of users.
  - Most users incapable of writing Boolean queries (or they are, but they think it's too much work).
  - Most users don't want to wade through 1000s of results.
    - This is particularly true of web search.

Problem with Boolean search: feast or famine

- Boolean queries often result in either too few (=0) or too many (1000s) results.
- Query 1 [Boolean conjunction]:
  - "*justin bieber istanbul konseri*" → 283,000 hits - feast
- Query 2 [Boolean conjunction]:
  - "*justin bieber istanbul konseri yeri*" → 0 hits – famine

- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - In general: AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in ranked retrieval models, the system returns an ordering over the (top) documents in the collection with respect to a query
- Free text queries: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are two separate choices here, but in practice, ranked retrieval models have normally been associated with free text queries and vice versa

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
  - Indeed, the size of the result set is not an issue
  - We just show the top k ($\approx$ 10) results
  - We don't overwhelm the user

  - Premise: the ranking algorithm works

Importance of ranking:

- Viewing abstracts: Users are a lot more likely to read the abstracts of the top-ranked pages than the abstracts of the lower ranked pages.
- Clicking: Distribution is even more skewed for clicking
- In 1 out of 2 cases, users click on the top-ranked page.
- Even if the top-ranked page is not relevant, 30% of users will click on it.

- Getting the ranking right is very important.
- Getting the top-ranked page right is most important.

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query "match".

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be).

Jaccard coefficient

- Recall from Lecture 5: A commonly used measure of overlap of two sets A and B
- jaccard(A,B) = $|A \cap B| / |A \cup B|$
- jaccard(A,A) = 1
- jaccard(A,B) = 0 if $A \cap B = 0$
- A and B don't have to be the same size.
- Always assigns a number between 0 and 1.

Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: *ides of march*
- Document 1: *caesar died in march*
- Document 2: of *the long march*

Issues with Jaccard for scoring

- It doesn't consider *term frequency* (how many times a term occurs in a document)
- Rare terms in a collection are more informative than frequent terms. Jaccard doesn't consider this information.

Recall (Lecture 1): Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$

Term-document count matrices

- Consider the number of occurrences of a term in a document:
  - Each document is a count vector in $\mathbb{N}^{|V|}$: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- *John is quicker than Mary* and *Mary is quicker than John* have the same vectors
- This is called the bag of words model.

Term frequency tf

- The term frequency $tf_{t,d}$ of term t in document d is defined as the number of times that t occurs in d.
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term.
  - But not 10 times more relevant.
- Relevance does not increase proportionally with term frequency.

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- $0 \rightarrow 0$, $1 \rightarrow 1$, $2 \rightarrow 1.3$, $10 \rightarrow 2$, $1000 \rightarrow 4$, etc.
- Score for a document-query pair: sum over terms t in both q and d:

$$\text{score} = \sum (1 + \log \mathrm{tf}_{t,d})$$

- The score is 0 if none of the query terms is present in the document.

Document frequency

- Rare terms are more informative than frequent terms
  - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnophobic*)
- A document containing this term is very likely to be relevant to the query *arachnophobic*
- → We want a high weight for rare terms like *arachnophobic.*

Document frequency, continued

- Frequent terms are less informative than rare terms
- Consider a query term that is frequent in the collection (e.g., *high, increase, line*)
- A document containing such a term is more likely to be relevant than a document that doesn't
- But it's not a sure indicator of relevance.
- → For frequent terms, we want high positive weights for words like *high, increase, and line*
- But lower weights than for rare terms.
- We will use document frequency (df) to capture this.

idf weight

- df_t is the <u>document</u> frequency of t: the number of documents that contain t
  - df_t is an inverse measure of the informativeness of t
  - $df_t \leq N$ *(where N is the total number of documents in the collection)*
- We define the idf (inverse document frequency) of t by

$$\mathrm{idf}_t = \log_{10}(N/\mathrm{df}_t)$$

  - We use log (N/df$_t$) instead of N/df$_t$ to "dampen" the effect of idf.

idf example, suppose N = 1 million

term	df$_t$	idf$_t$
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$\mathrm{idf}_t = \log_{10}(N/\mathrm{df}_t)$$

There is one idf value for each term t in a collection.

Effect of idf on ranking

- Does idf have an effect on ranking for one-term queries, like
  - arachnophobic
- idf has no effect on ranking one term queries
  - idf affects the ranking of documents for queries with at least two terms
  - For the query arachnophobic person, idf weighting makes occurrences of arachnophobic count for more in the final document ranking than occurrences of person.

Collection vs. Document frequency

- The collection frequency of *t* is the number of occurrences of *t* in the collection, counting multiple occurrences.
- Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

- Which word is a better search term (and should get a higher weight)?

tf-idf weighting

- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$w_{t,d} = (1 + \log_{10} \mathrm{tf}_{t,d}) \times \log_{10}(N / \mathrm{df}_t)$$

- Best known weighting scheme in information retrieval
  - Note: the "-" in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, tf x idf
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

Final ranking of documents for a query

$$\mathrm{Score}(q,d) = \sum_{t \in q \cap d} \mathrm{tf.idf}_{t,d}$$

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

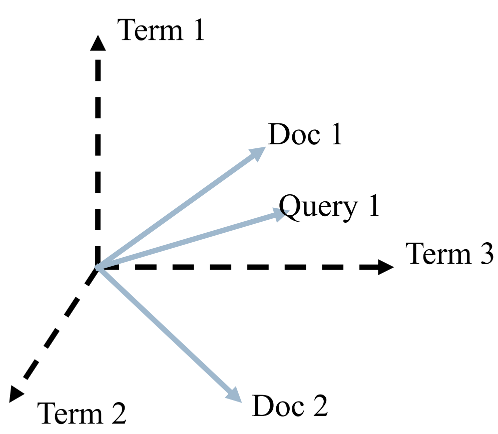
Documents as vectors

- So we have a |V|-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors - most entries are zero.

Queries as vectors

- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance
- Recall: We do this because we want to get away from the you're-either-in-or-out Boolean model.
- Instead: rank more relevant documents higher than less relevant documents

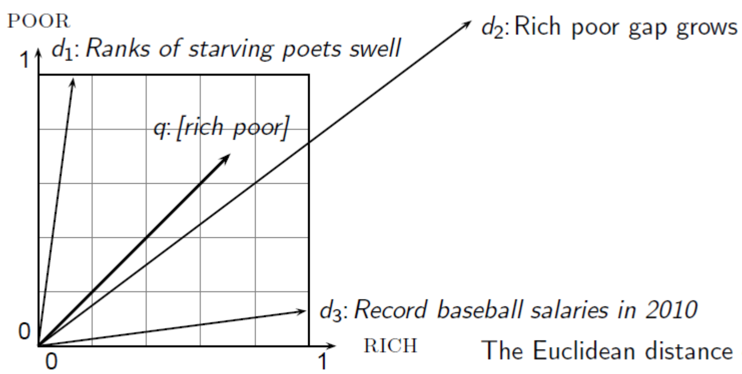
The Vector-space model



Formalizing vector space proximity

- First cut: distance between two points
  - (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea



The Euclidean distance of $\vec{q}$ and $\vec{d_2}$ is large although the distribution of terms in the query q and the distribution of terms in the document d_2 are very similar.

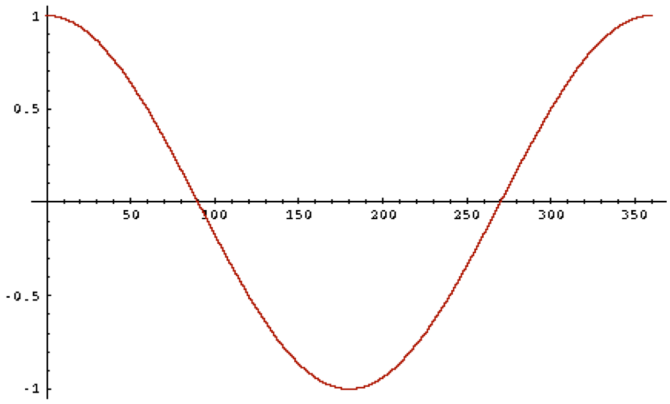
Use angle instead of distance

- Thought experiment: take a document *d* and append it to itself. Call this document *d′*.
- "Semantically" d and d′ have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.
- Key idea: Rank documents according to angle with query.

From angles to cosines

- The following two notions are equivalent.
  - Rank documents in increasing order of the angle between query and document
  - Rank documents in decreasing order of cosine(query, document)
- Cosine is a monotonically decreasing function for the interval [0°, 180°]

From angles to cosines



Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm:

$$\|\vec{x}\|_2 = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d′ (d appended to itself) from earlier slide: they have identical vectors after length-normalization.
  - Long and short documents now have comparable weights

cosine(query,document)

Dot product

Unit vectors

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q} \bullet \vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|} \bullet \frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term i in the query
d_i is the tf-idf weight of term i in the document

$\cos(\vec{q},\vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$ … or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

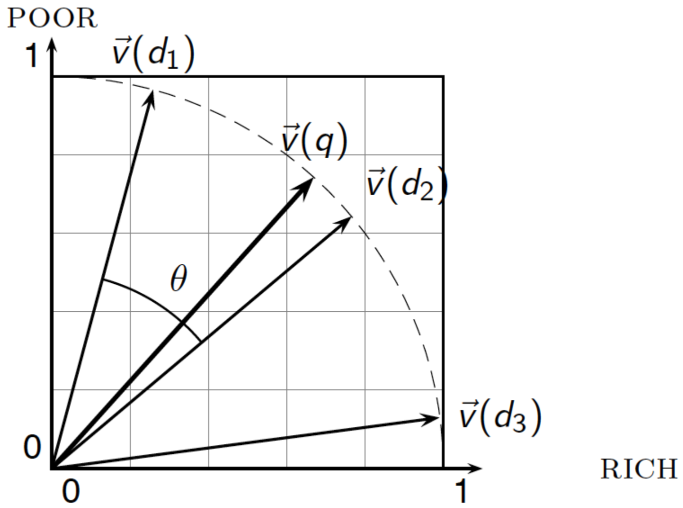
Cosine for length-normalized vectors

- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q},\vec{d}) = \vec{q} \cdot \vec{d} = \sum q_i d_i$$

for q, d length-normalized.

Cosine similarity illustrated



Cosine similarity amongst 3 documents

How similar are the novels

SaS: *Sense and Sensibility* (Jane Austen)

PaP: *Pride and Prejudice* (Jane Austen)

WH: *Wuthering Heights*? (Emily Bronte)

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈
0.789 × 0.832 + 0.515 × 0.555 + 0.335 × 0.0 + 0.0 × 0.0
≈ 0.94
cos(SaS,WH) ≈ 0.79
cos(PaP,WH) ≈ 0.69

Computing cosine scores

CosineScore(q)
float $Scores[N] = 0$
float $Length[N]$
for each query term t
do calculate $w_{t,q}$ and fetch postings list for t
    for each pair(d, $tf_{t,d}$) in postings list
    do $Scores[d] += w_{t,d} \times w_{t,q}$
Read the array $Length$
for each d
do $Scores[d] = Scores[d]/Length[d]$
return Top K components of $Scores[]$

Summary – vector space ranking

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top *K* (e.g., *K* = 10) to the user

Computing Scores in a Complete Search System

Outline

- Speeding up vector space ranking
- Putting together a complete search system
  - Will require learning about a number of miscellaneous topics and heuristics

Efficient cosine ranking

- Find the K docs in the collection "nearest" to the query $\Rightarrow$ K largest query-doc cosines.
- Efficient ranking:
  - Computing a single cosine efficiently.
  - Choosing the K largest cosine values efficiently.
    - Can we do this without computing all N cosines?

Special case – unweighted queries

- No weighting on query terms
  - Assume each query term occurs only once
- Then for ranking, don't need to normalize query vector

Faster cosine: unweighted query

FastCosineScore(q)

float $Scores[N] = 0$
for each d
do Initialize $Length[d]$ to the length of doc d
for each query term t
do calculate $w_{t,q}$ and fetch postings list for t
  for each pair(d, $\mathrm{tf}_{t,d}$) in postings list
  do add $\mathrm{wf}_{t,d}$ to $Scores[d]$
Read the array $Length[d]$
for each d
do Divide $Scores[d]$ by $Length[d]$
return Top K components of $Scores[]$

$w_{t,q}$ set to 1

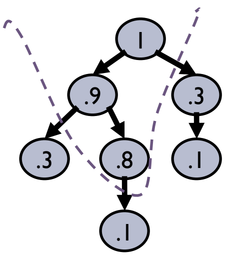
Figure 7.1 A faster algorithm for vector space scores.

Computing the K largest cosines: selection vs. sorting

- Typically we want to retrieve the top K docs (in the cosine ranking for the query)
  - not to totally order all docs in the collection
- Can we pick off docs with K highest cosines?
- Let J = number of docs with nonzero cosines
  - We seek the K best of these J

Use heap for selecting top K

- Binary tree in which each node's value > the values of children (max-heap)
- Takes O(J) time to build the heap. Then, for each of K "winners": O(1) time to read the max element,
  and O(log J) time to maintain heap property.
- O(J) time to select top K using heap vs. O(J log J) time when sorting used.



Bottlenecks

- Primary computational bottleneck in scoring: cosine computation
- Can we avoid all this computation?
- Yes, but may sometimes get it wrong
  - a doc *not* in the top *K* may creep into the list of *K* output docs
  - Is this such a bad thing?

Cosine similarity is only a proxy

- User has a task and a query formulation
- Cosine matches docs to query
- Thus cosine is anyway a proxy for user happiness
- If we get a list of *K* docs "close" to the top *K* by cosine measure, should be ok

Generic approach

- Find a set A of *contenders*, with $K < |A| << N$
  - A does not necessarily contain the top K, but has many docs from among the top K
  - Return the top K docs in A
- Think of A as <u>pruning</u> non-contenders
- The same approach is also used for other (non-cosine) scoring functions
- Will look at several schemes following this approach

Index elimination

- Basic algorithm FastCosineScore only considers docs containing at least one query term
- Take this further:
  - Only consider high-idf query terms
  - Only consider docs containing many query terms

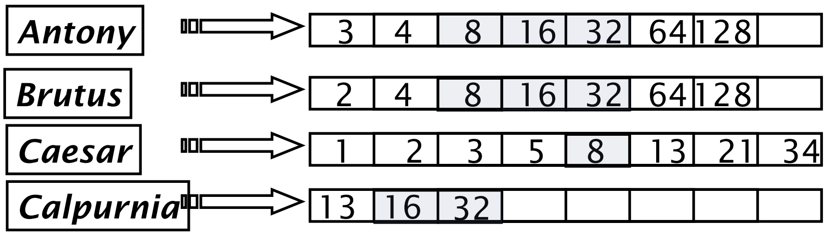
High-idf query terms only

- For a query such as *catcher in the rye*
- Only accumulate scores from *catcher* and *rye*
- Intuition: *in* and *the* contribute little to the scores and so don't alter rank-ordering much
- Benefit:
  - Postings of low-idf terms have many docs → these (many) docs get eliminated from set *A* of contenders

Docs containing many query terms

- Any doc with at least one query term is a candidate for the top *K* output list
- For multi-term queries, only compute scores for docs containing several of the query terms
  - Say, at least 3 out of 4
- Easy to implement in postings traversal

3 of 4 query terms



Scores only computed for docs 8, 16 and 32.

Champion lists

- Precompute for each dictionary term *t*, the *r* docs of highest weight in *t*'s postings
  - Call this the champion list for *t*
  - (aka fancy list or top docs for *t*)
- Note that *r* has to be chosen at index build time
  - Thus, it's possible that *r* < *K*
- At query time, only compute scores for docs in the champion list of some query term
  - Pick the *K* top-scoring docs from amongst these

Static quality scores

- We want top-ranking documents to be both *relevant* and *authoritative*
- *Relevance* is being modeled by cosine scores
- *Authority* is typically a query-independent property of a document
- Examples of authority signals
  - Wikipedia among websites
  - Articles in certain newspapers
  - A paper with many citations
  - (Pagerank)

Modeling authority

- Assign a *query-independent* quality score in [0,1] to each document *d*
  - Denote this by *g(d)*
- Thus, a quantity like the number of citations is scaled into [0,1]

Net score

- Consider a simple total score combining cosine relevance and authority
- net-score(q,d) = $g(d)$ + cosine(q,d)
  - Can use some other linear combination than an equal weighting
- Now we seek the top K docs by net score

Top K by net score – fast methods

- First idea: Order all postings by $g(d)$
- Key: this is a common ordering for all postings
- Thus, can concurrently traverse query terms' postings for
  - Postings intersection
  - Cosine score computation

Why order postings by *g(d)*?

- Under *g(d)*-ordering, top-scoring docs likely to appear early in postings traversal
- In time-bound applications (say, we have to return whatever search results we can in 50 ms), this allows us to stop postings traversal early
  - Short of computing scores for all docs in postings

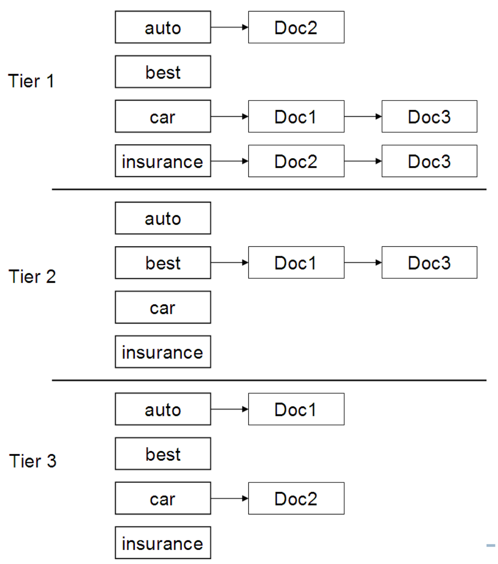
High and low lists

- For each term, we maintain two postings lists called *high* and *low*
  - Think of *high* as the champion list
- When traversing postings on a query, only traverse *high* lists first
  - If we get more than *K* docs, select the top *K* and stop
  - Else proceed to get docs from the *low* lists
- Can be used even for simple cosine scores, without global quality *g(d)*
- A means for segmenting index into two <u>tiers</u>

Tiered indexes

- Break postings up into a hierarchy of lists
  - Most important
  - …
  - Least important
- Can be done by *g(d)* or another measure
- Inverted index thus broken up into <u>tiers</u> of decreasing importance
- At query time use top tier unless it fails to yield *K* docs
  - If so drop to lower tiers

Example tiered index



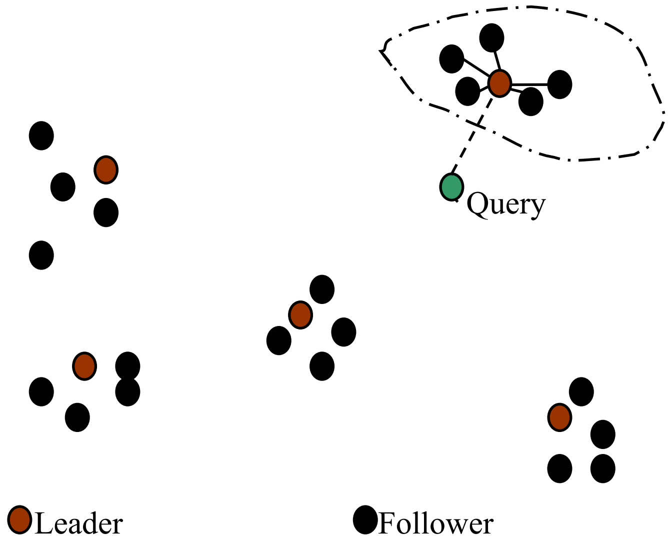
Cluster pruning: preprocessing

- Pick √N *docs* at random: call these *leaders*
- For every other doc, pre-compute nearest leader
  - Docs attached to a leader: its *followers;*
  - Likely: each leader has ~ √*N* followers.

Cluster pruning: query processing

- Process a query as follows:
  - Given query *Q*, find its nearest *leader L.*
  - Seek *K* nearest docs from among *L*'s followers.

Visualization



Why use random sampling

- Fast
- Leaders reflect data distribution

General variants

- Have each follower attached to b_1 (e.g., 3) nearest leaders.
- From query, find b_2 (e.g., 4) nearest leaders and their followers.

Parametric and Zone Indexes

Parametric and zone indexes

- Thus far, a doc has been a sequence of terms
- In fact documents have multiple parts, some with special semantics:
  - Author
  - Title
  - Date of publication
  - Language
  - Format
  - etc.
- These constitute the metadata about a document

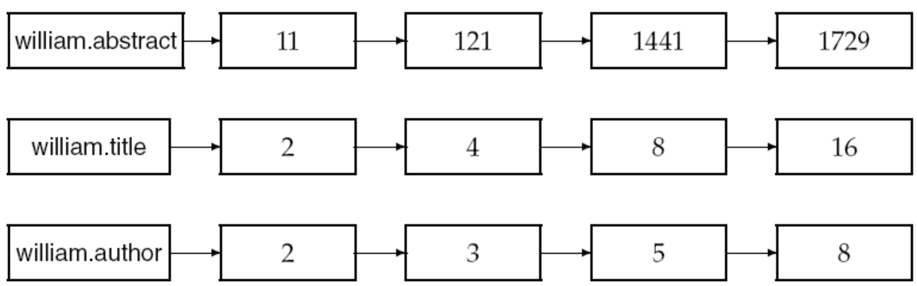
Fields

- We sometimes wish to search by these metadata
  - E.g., find docs authored by William Shakespeare in the year 1601, containing *alas poor Yorick*
- Year = 1601 is an example of a field
- Also, author last name = shakespeare, etc
- Field or parametric index: postings for each field value
- Field query typically treated as conjunction
  - (doc *must* be authored by shakespeare)

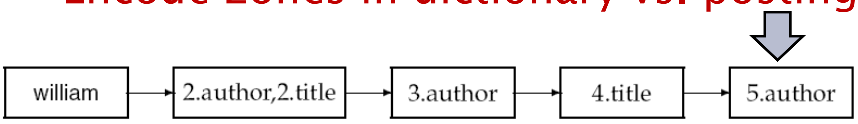
Zone

- A <u>zone</u> is a region of the doc that can contain an arbitrary amount of text e.g.,
  - Title
  - Abstract
  - References …
- Build inverted indexes on zones as well to permit querying
- E.g., "find docs with *merchant* in the title zone and matching the query *gentle rain*"

Example field/zone indexes



Encode zones in dictionary vs. postings.



Query term proximity

- Free text queries: just a set of terms typed into the query box – common on the web
- Users prefer docs in which query terms occur within close proximity of each other
- Let *w* be the smallest window in a doc containing all query terms, e.g.,
- For the query *strained mercy* the smallest window in the doc *The quality of mercy is not strained* is 4 (words)
- Would like scoring function to take this into account.

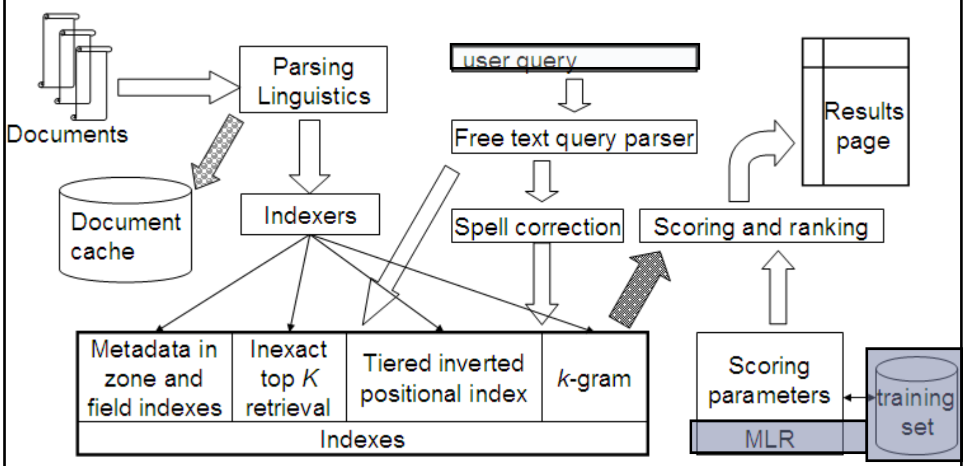
Aggregate scores

- We've seen that scoring functions can combine cosine, static quality, proximity, etc.
- How do we know the best combination?
- Some applications – expert-tuned
- Increasingly common: machine-learned

Query parsers

- Free text query from user may in fact spawn one or more queries to the indexes, e.g. query *rising interest rates*
  - Run the query as a phrase query
  - If <K docs contain the phrase *rising interest rates*, run the two phrase queries *rising interest* and *interest rates*
  - If we still have <K docs, run the vector space query *rising interest rates*
  - Rank matching docs by vector space scoring
- This sequence is issued by a query parser

Putting it all together



References

- *Introduction to Information Retrieval*, chapters 6 & 7.
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