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Answer 1

1.a) To determine if the statement is a tautology or contradiction, we'll create a truth table for $(p \to q) \oplus (p \wedge \neg q)$:

p	q	$\neg q$	$p \wedge \neg q$	$p \to q$	$(p \to q) \oplus (p \wedge \neg q)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T

The truth table shows that $(p \to q) \oplus (p \wedge \neg q)$ is always true (T) ; so it's a **tautology**.

1.b) To prove the propositional equivalence, we can use the given tables and laws:

$$
\begin{array}{rcl|l}
p \to ((q \vee \neg p) \to r) & \equiv & p \to (\neg(q \vee \neg p) \vee r) & \text{By using (If elimination)}\textbf{Table 7} \\
 & \equiv & p \to ((\neg q \wedge p) \vee r) & \text{By using (De Morgan's laws)} \ \textbf{Table 6} \\
 & \equiv & \neg p \vee ((\neg q \wedge p) \vee r) & \text{By using (If elimination)} \ \textbf{Table 7} \\
 & \equiv & (\neg p \vee (\neg q \wedge p)) \vee r & \text{By using (Associatative Law)} \ \textbf{Table 6} \\
 & \equiv & ((\neg p \vee \neg q) \wedge (\neg p \vee p)) \vee r & \text{By using (Distributive Law)} \ \textbf{Table 6} \\
 & \equiv & ((\neg p \vee \neg q) \wedge T) \vee r & \text{By using (Negation Law)} \ \textbf{Table 6} \\
 & \equiv & (\neg p \vee \neg q) \vee r & \text{By using (Identity Law)}\textbf{Table 6} \\
 & \equiv & \neg(\neg p \vee \neg q) \to r & \text{By using (If introduction)}\textbf{Table 7} \\
 & \equiv & (p \wedge q) \to r & \text{By using (De Morgan's Law)} \ \textbf{Table 6}
\end{array}
$$

Thus, $p \to ((q \vee \neg p) \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

1.c) Logical Equivalences:

- $(p \wedge q) \to r \equiv (\neg p \wedge \neg q) \vee r$: **False**
- $(p \wedge q) \vee (\neg p \wedge \neg q) \equiv p \oplus q$: **False**
- $(p \vee q) \wedge (p \wedge \neg q) \equiv p$: **False**
- $(p \vee q) \wedge (p \vee \neg q) \equiv p$: **True**
- $p \oplus q \equiv \neg(p \leftrightarrow q)$: **True**

Answer 2

a) Player Can plays in a team that plays in league L.
$\exists x(P(\text{Can}, x) \wedge T(x, L))$

b) Every team in league S has at least one Turkish player.
$\forall x(T(x, \text{S}) \to \exists y(P(y, x) \wedge N(y, \text{Turkish})))$

c) Every team in league S has exactly one rival, which is also in league S.
$\forall x(T(x, S) \to \exists y(R(x, y) \wedge T(y, S) \wedge \forall z((R(x, z) \wedge T(z, S)) \to (z = y))))$

d) Team M has never won against a team with at least one English player.
$\forall x(W(\text{M}, x) \to \neg\exists y(N(y, \text{English}) \wedge P(y, x)))$

e) Exactly two Turkish players play on team G.
$\exists x \exists y((P(x, \text{G}) \wedge N(x, \text{Türk})) \wedge (P(y, \text{G}) \wedge N(y, \text{Turkish})) \wedge x \neq y) \wedge \forall z((N(z, \text{Turkish}) \to \neg P(z, \text{G})) \wedge (z \neq x \wedge z \neq y))$

f) There are some teams that play in more than one league.
$\exists x \exists y \exists z(T(x, y) \wedge T(x, z) \wedge (z \neq y))$

Answer 3

Table 1: Proof of $p \to q, (r \wedge s) \to p, (r \wedge \neg q) \vdash \neg s$

1.	$p \to q$	*premise*
2.	$(r \wedge s) \to p$	*premise*
3.	$(r \wedge \neg q)$	*premise*
4.	s	*assumption*
5.	r	$\wedge$e 3
6.	$\neg q$	$\wedge$e 3
7.	$(r \wedge s)$	$\wedge$i 4, 5
8.	$(r \wedge s) \to p$	*copy 2*
9.	p	$\to$e 7, 8
10.	$p \to q$	*copy 1*
11.	q	$\to$e 9, 10
12.	$\perp$	$\neg$e 6, 11
13.	$\neg s$	$\neg$i 4 − 12

Answer 4

a) Converting sentences to logic:

Some students need to study for the exam in order to pass.	$\equiv \exists x(P(x) \to S(x))$
Every student passed the exam.	$\equiv \forall x P(x)$

b)

Table 2: Proof of $\forall x P(x), \exists x(P(x) \to S(x)) \vdash \exists x S(x)$

1.	$\forall x P(x)$	*premise*
2.	$\exists x(P(x) \to S(x))$	*premise*
3.	$P(c) \to S(c)$	*assumption*
4.	$P(c)$	$\forall$e 1
5.	$S(c)$	$\to$e $3, 4$
6.	$\exists x S(x)$	$\exists$i 5
7.	$\exists x S(x)$	$\exists$e $2, 3-6$