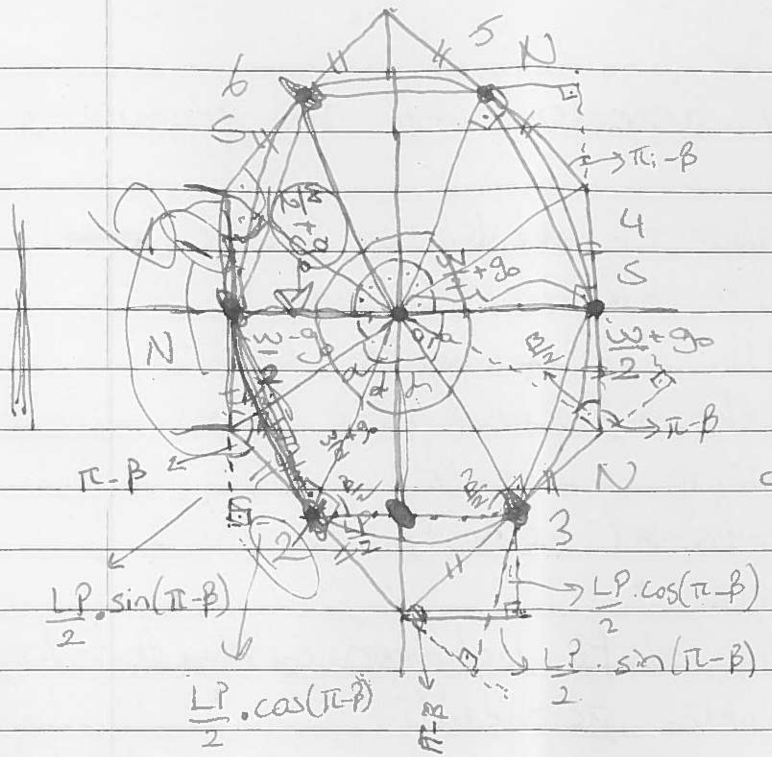
It's mostly dependent of the geometry.

NC : ~~Nukmber~~ Number of conductor loops (~~…~~)
NP : Number of poles (even)
w : width of conductor loops
l : legthe of conductor loops
i : passing current
ω : rotational velocity
t : time
ϕ : Phase shift between conductor loops
μ_0 : Magnetic permebility
g_0 : gap between poles and conductor loop where distance is lowest
O(0,0): Origin for specified coordinate system.
Br : Renenence field of a magnet
T : Thickness of a magnet

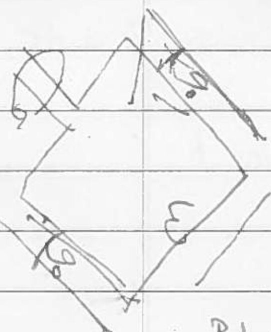


Angle between each ~~pole~~ pole placed

$$\beta = \frac{\pi(NP-2)}{NP}$$

due to geometry

Length of magnet poles $Lp = \left(\frac{w}{2}+g_0\right)\cdot\cot\frac{\beta}{2}\cdot 2$



$$Lp = (w+2g_0)\cdot\cot\frac{\beta}{2}$$

$$Lp = (w+2g_0)\cdot\cot\frac{\pi(NP-2)}{NP}$$

$P1 = (-w/2-g_0,\ 0)$

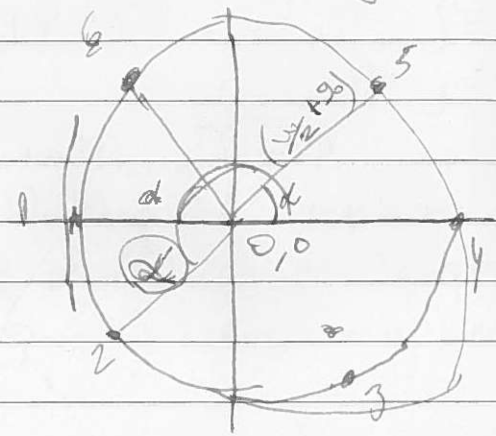
$P2 = \left(-w/2-g_0+LP/2\cdot\cos(\pi-\beta),\ \frac{LP}{2}\left(\sin(\pi-\beta)+1\right)\right)$

It's actually a circle.

Middle points of each pole edges on the tangent lines of the circle with radius → $\underline{w/2+g_0}$

coordinate on circle will follow

$$x^2+y^2=(w/2+g_0)^2$$



For every middle point of the magnets

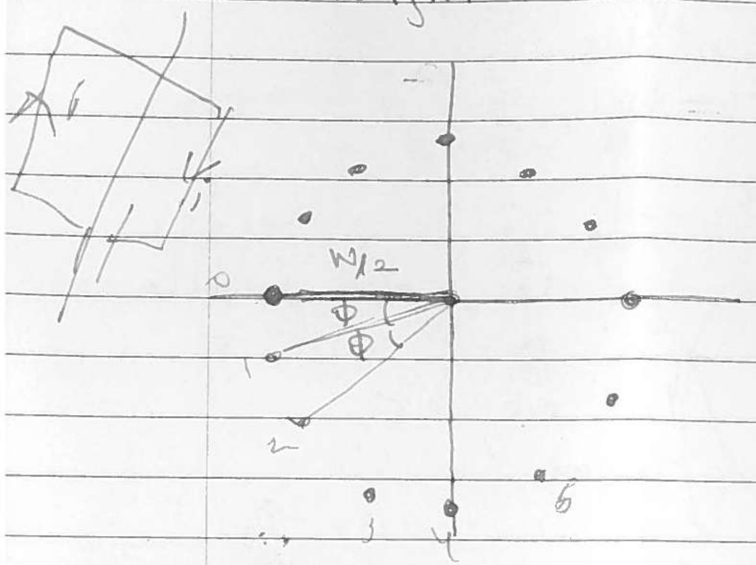
$$\alpha = \frac{2\pi}{NP}$$

for $j = 0$ to $NP-1$

$$x_j = (w/2+g_0)\cdot\cos\left(\pi + j\cdot\frac{2\pi}{NP}\right)$$

$$y_j = (w/2+g_0)\cdot\sin\left(\pi + j\cdot\frac{2\pi}{NP}\right)$$

Position of semi loops also important and can be maintained through similiar method, since they are rotating around of commutator which is placed in the origin



$\Phi = \frac{\pi}{NC}$ (Phase shift of each loop)

for $j = 0$ to $2 \times NC - 1$

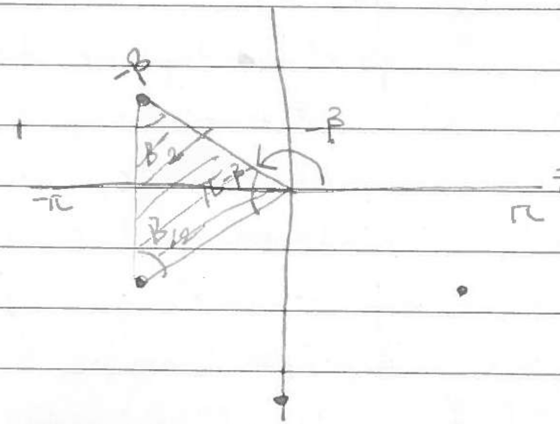
$$\mathrm{Pos}\,C(j) = \begin{cases} x_j = w/2 \cdot \cos\left(\pi + \omega . t + j\frac{\pi}{NC}\right) \\ y_j = w/2 \cdot \sin\left(\pi + \omega . t + j\frac{\pi}{NC}\right) \end{cases}$$

Depending on magnets poles, the reaction of wires will be changing as pull or push.

Polarity of Poles for each magnet Pole j

$$PP(j) = \begin{cases} \text{if } j \equiv 1 \bmod 2, \ 1 \quad (N) \\ \text{if } j \equiv 0 \bmod 2, \ -1 \quad (S) \end{cases}$$

In order to define a function for polarity of conductor loops or commutator, let us assume magnets define zones



for $j = 0$ to $NP-1$ each zone pair

$$ZP(j) = (\underbrace{-\beta + j(\pi - \beta)}_{1}, \underbrace{-\beta + (j+1)(\pi - \beta)}_{2})$$

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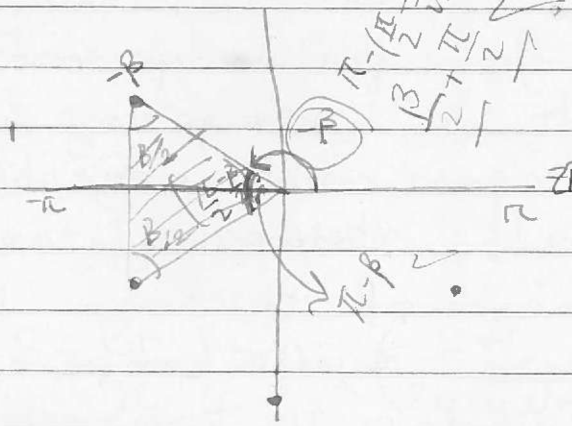


for $j=0$ to N^{p-1} each zone pair

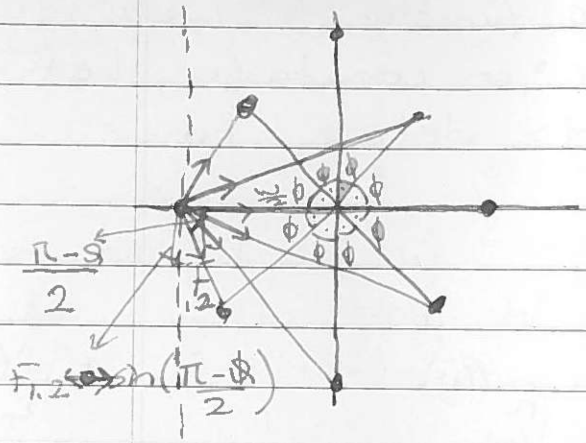
$$ZP(j) = \big(\underbrace{\cancel{-\beta}}_{1 \,\to\, \frac{\beta}{2}+\frac{\pi}{2}} + j(\pi-\beta),\ \underbrace{\cancel{-\beta}}_{2 \,\to\, \frac{\beta}{2}+\frac{\pi}{2}} + (j+1)(\pi-\beta)\big)$$

$$Zp(j) = \left(\frac{\beta}{2}+\frac{\pi}{2}+j(\pi-\beta),\ \frac{\beta}{2}+\frac{\pi}{2}+(j+1)(\pi-\beta)\right)$$

Polarity of each semi loops k in rotation at zone j

$$PC(k) = \begin{cases} \text{if } Z_P(j)_1 \leq \pi + \omega t + \frac{k\pi}{NC} \leq Z_P(j)_2 \text{ in mod } 2\pi \\ PC(k) \times PP(j) = 1 \end{cases}$$

Due to passing current each wire of conductor loop would be attracting others.



The force on semi loop, due to magnetical atraction will purely be dependent on current passing on each wires, distance between. The necessary component of the Force in our case is, its component perpendicular to loop's plane.

For each conductor j, total of force applied by other conductors k, according to ampere's law, as perpendicular to loop j's plane

$$\overline{F_{cj}} = \sum_{k=1}^{2NC} \sin\left(\frac{\pi - (k-j)\phi}{2}\right) \frac{PC(j).PC(k).l.i^2.M_0}{2\pi.\sqrt{(x_j - x_k)^2 + (y_j - y_k)^2}}$$

where $k \neq j$.

Each magnet due to their magnetical nature will be affecting, moving conductor loops.

For a magnet of length L, width w, thickness D, the magnetic flux density the magnet created on a point way in the air as follows

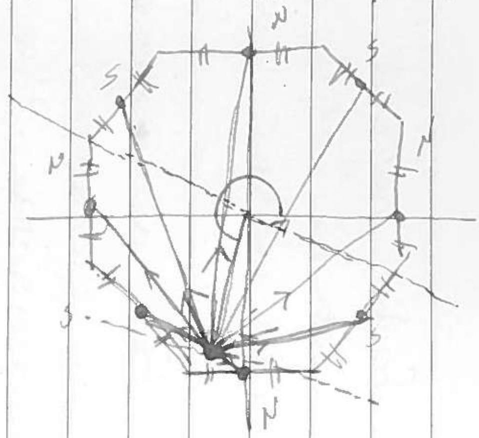
$$B = \frac{Br}{\pi}\left[\arctan\left(\frac{LW}{2z\sqrt{4z^2+L^2+W^2}}\right) - \arctan\left(\frac{LW}{2(D+z)\sqrt{4(D+z)^2+L^2+W^2}}\right)\right]$$

On a magnet of a block shape

$$B = \frac{B_r}{\pi}\left[\arctan\left(\frac{LW}{2z\sqrt{4z^2+L^2+W^2}}\right) - \arctan\left(\frac{LW}{2(D+z)\sqrt{4(D+z)^2+L^2+W^2}}\right)\right]$$

Length of each magnet is uniformal and calculated as L_p. Let's assume width of our magnets is as much as length of our conductor loop ℓ, Thickness of our magnets is T.

$$B = \frac{B_r}{\pi}\left[\arctan\left(\frac{\ell L_p}{2z\sqrt{4z^2+L_p^2+\ell^2}}\right) - \arctan\left(\frac{\ell L_p}{2(T+z)\sqrt{4(D+z)^2+L_p^2+\ell^2}}\right)\right]$$



For calculation of z we could use our position functions.

$j = 0$ to $2N_C - 1$ angular position of each conductor loop is

$$\pi + \omega t + j\frac{\pi}{N_C}$$

Then angular position of perpendicula plane of each loop is

$$\pi - \frac{\pi}{2} + \omega t + j\frac{\pi}{N_C} = \frac{\pi}{2} + \omega t + j\frac{\pi}{N_C}$$

and slope of perpendiculer plane of each loop is

$$m_1 = \tan\left(\frac{\pi}{2} + \omega t + j\frac{\pi}{N_C}\right)$$

Slope of force vectors created by magnetical field of each magnet ~~k~~ on each loop j

$$m_2 = \frac{y_k - y_j}{x_k - x_j}$$

Angle of forces to perpendicular plane is

$$\Theta = \arctan\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$$

For each semi conductor loop j, total force applied by magnets k perpendicular to conductor plate is

$$F_{mj} = \sum_{k=1}^{NP} i \cdot \ell \cdot PP(k) \cdot PC(j) \cdot \cos\theta \cdot \frac{Br}{\pi} \cdot \left[\arctan\left(\frac{\ell \cdot Lp}{2 \cdot \underbrace{\sqrt{(x_k - x_j)^2 + (y_k - y_j)^2}}_{z} \cdot \sqrt{4z^2 + Lp^2 + \ell^2}} \right) - \arctan\left(\frac{\ell \cdot Lp}{2\,(t+z)\sqrt{4(z+T)^2 + Lp^2 + \ell^2}} \right) \right]$$

Total Force generated by DC motor is

$$\sum_{j=1}^{2NC-1} F_{cj} + F_{mj}.$$