Lecture 7: Continuous distributions

BIOS 600 - Fall 2024

Supplemental Reading

These readings are optional. All material in homework and exams will be covered in lecture and lab, and books do not cover everything we do in lecture (even if there is a corresponding section).

However, for those with the very optional textbooks, the following sections correspond to today's lecture:

- Pagano and Gavreau: Section 7.4
- OpenIntro Statistics: 3.5, 4.1

Review: Discrete probability distributions

There are three rules for discrete probability distributions:

✅ Outcomes must be disjoint

✅ The probability of each outcome must be ≥ 0 and ≤ 1

✅ The sum of the outcome probabilities must add up to 1

Event	Probability
X = *pre*	0.10
X = *early*	0.27
X = *full*	0.57
X = *late/post*	0.06

Review: Expectation and variance

- The expectation is the average value (weighted by the probability of each value occurring)
- The variance describes the expected spread of values around the population expectation (thus, variance is in fact an expectation itself!)

> In small groups, complete today's participation exercise in Canvas.

Can we be more precise?

Letting X be the random variable that corresponds to how long a baby's gestation was, we could imagine subdividing further and further:

Event	Probability
X < 20 wk.	$P(X < 20)$
X = 20 to 21 wk.	etc.
X = 21 to 22 wk.	etc.
X = 22 to 23 wk.	etc.
$\vdots$	$\vdots$

Event	Probability
X < 20 wk.	$P(X < 20)$
X = 20 to 20.1 wk.	etc.
X = 20.1 to 20.2 wk.	etc.
X = 20.2 to 20.3 wk.	etc.
$\vdots$	$\vdots$

Can we be more precise?

- Now let gestational age X be a **continuous** random variable, which can take on *any* value, say from 0 to ∞.
- How might we define a continuous probability distribution that corresponds to X?

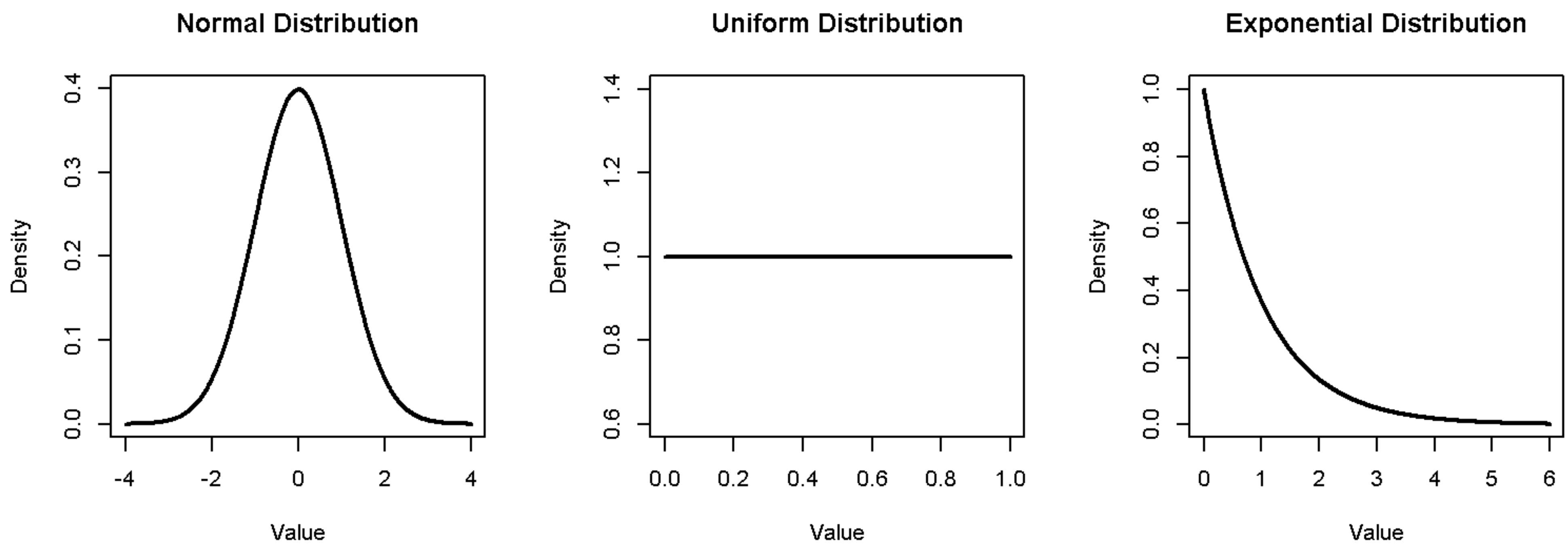
Continuous probability distributions

- The probability that a continuous variable equals any specific value is `0`
- No use tabulating - there is an *uncountably* infinite number of possible values they can be, all with $P(X = x) = 0$
- The distribution is given by a **probability density function**, helps us describe probabilities for *ranges* of values.

Density functions

Probability density functions may be given graphically, satisfying the following two rules:

- The density must be non-negative everywhere ($f(x) \geq 0$ for all x from $-\infty$ to ∞)
- The total area under the density must be 1



Density functions

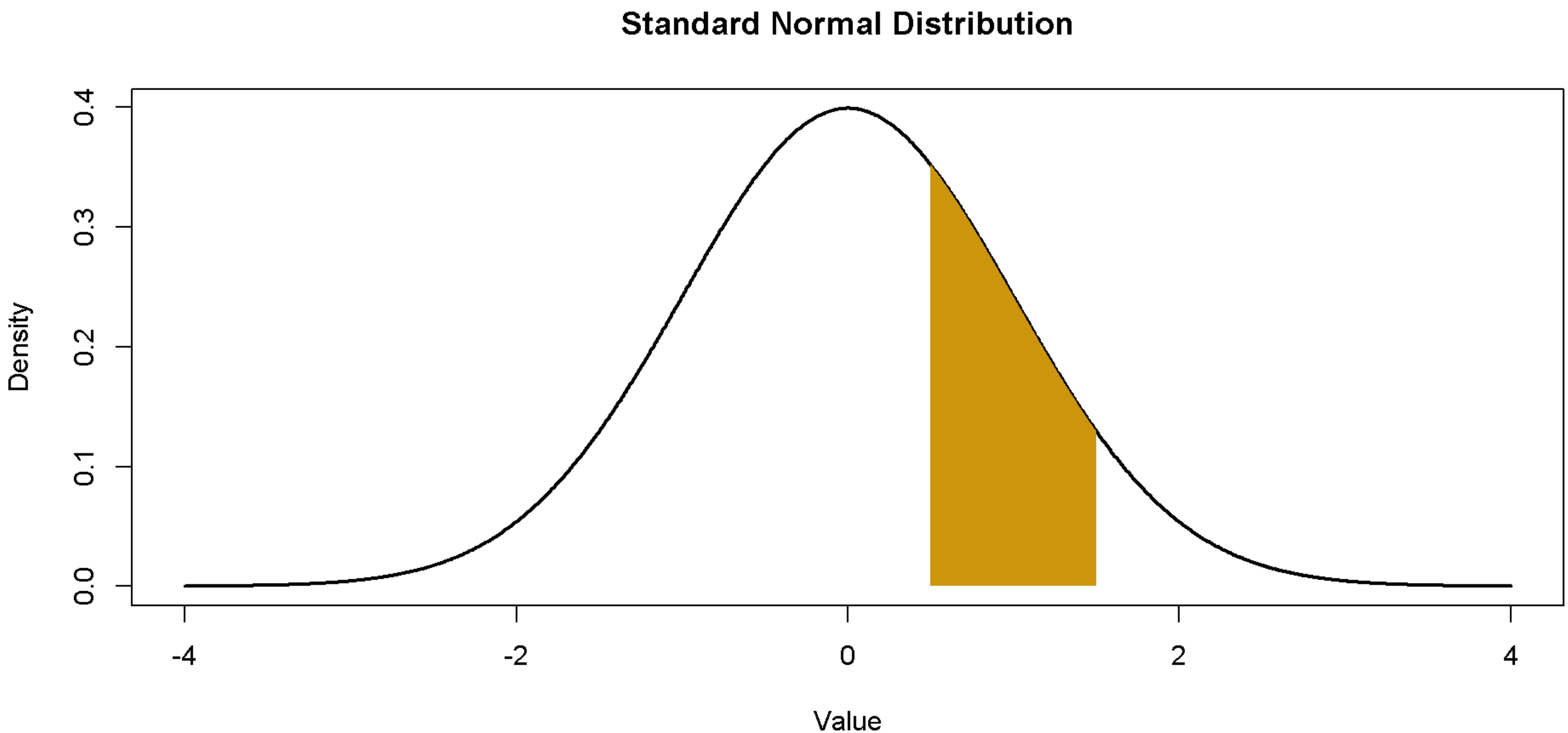
We can define events for continuous distributions and assign probabilities to them using density functions:

- Suppose X follows some density function $f(x)$
- We are interested in the event "X lies between a and b"
- We calculate the following probability:

$$P(a < X < b) = \int_a^b f(x)dx$$

> 🚧 Note
>
> Computers do this for us these days 🙌 ; no need to worry about the expression above)

What would this look like graphically?



The normal (Gaussian) distribution

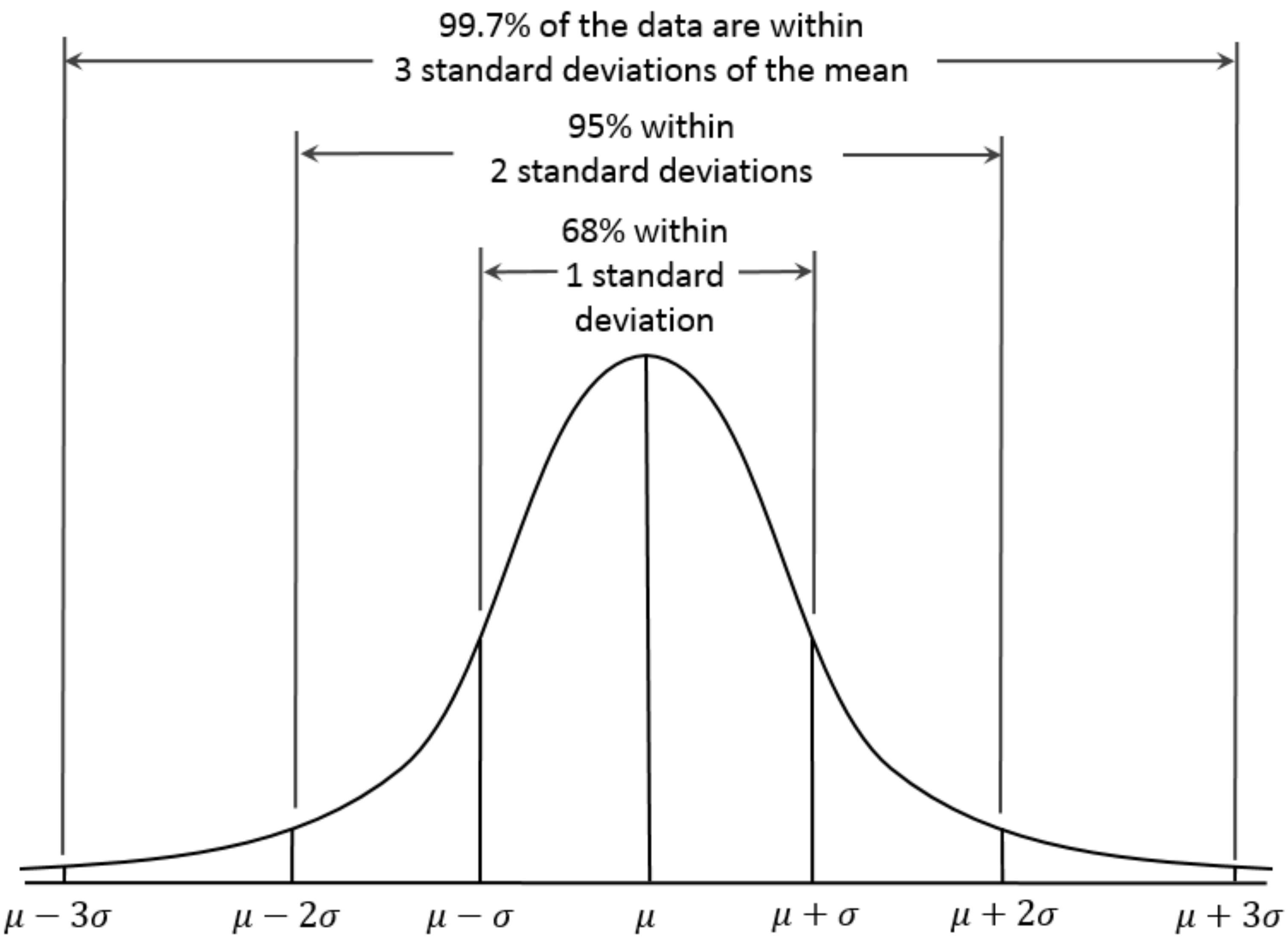
For the normal distribution,

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$$

where μ is the mean and σ^2 is the variance.

- We often write $N(\mu, \sigma^2)$.

68-95-99.7


99.7% of the data are within
3 standard deviations of the mean
95% within
2 standard deviations
68% within
1 standard
deviation
$\mu - 3\sigma$
$\mu - 2\sigma$
$\mu - \sigma$
μ
$\mu + \sigma$
$\mu + 2\sigma$
$\mu + 3\sigma$

Standardization

- The normal distribution is a family of distributions of a specific form. There are an infinite amount of possible distributions, since μ can be any real number and σ^2 can be any positive number.
- It would be very cumbersome to have to individually think about a $N(0,20)$ vs. $N(2.5,2)$ vs. $N(694,1549)$ vs. .... distribution, depending on the situation.
- In practice, we could calculate a **standard score** that gives the number of standard deviations away from the mean an observation from a particular population is.

Question

Why would we want to standardize?

z-scores

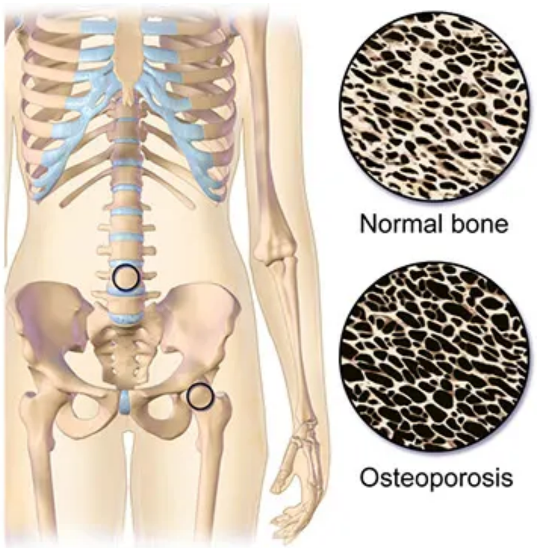
- A **z-score** tells us how many population standard deviations an observation is away from the population mean.
- They provide ways to compare results across many different measurement scales, since z-scores are *unitless*

$$z = \frac{x - \mu}{\sigma}$$

(note the use of population parameters μ and σ)

- So, a z-score of 1.2 is 1.2 standard deviations above a mean; a z-score of 0.8 is 0.8 standard deviations below the mean.

Osteoporosis



- According to NHANES, the mean bone mineral density for a 65 year old white woman is 809 mg/cm^2, with a standard deviation of 140 mg/cm^2.
- Suppose you are a 65 year old white woman whose bone density is 698 mg/cm^2.

Should you be very concerned about osteoporosis?