Discrete Signal Processing

EE698V - Machine Learning for Signal Processing

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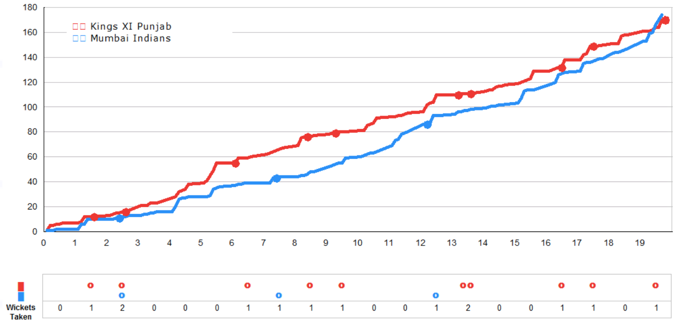
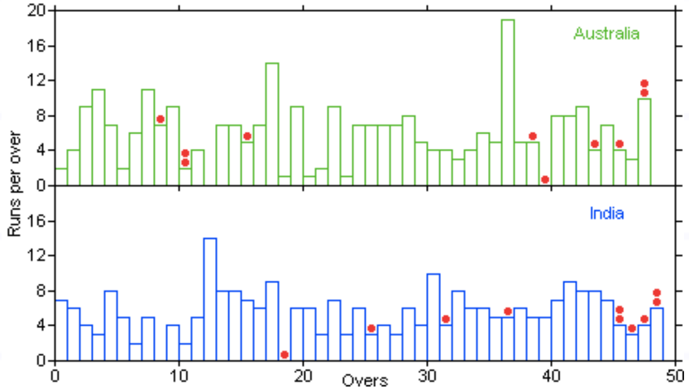
Logistics

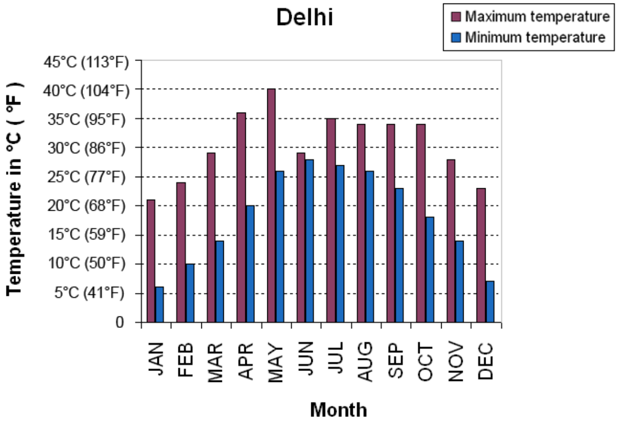
- Piazza classroom:
  - piazza.com/iitk.ac.in/firstsemester2019/ee698v

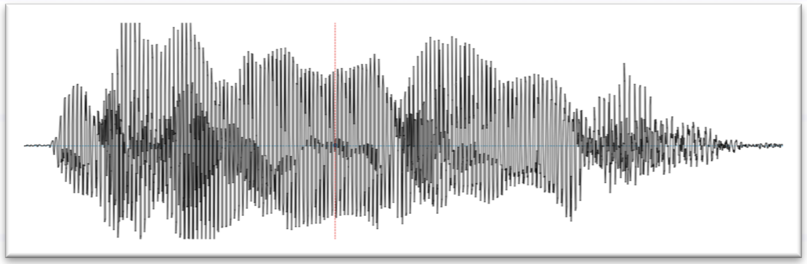
Time Series

- An ordered collection of numbers

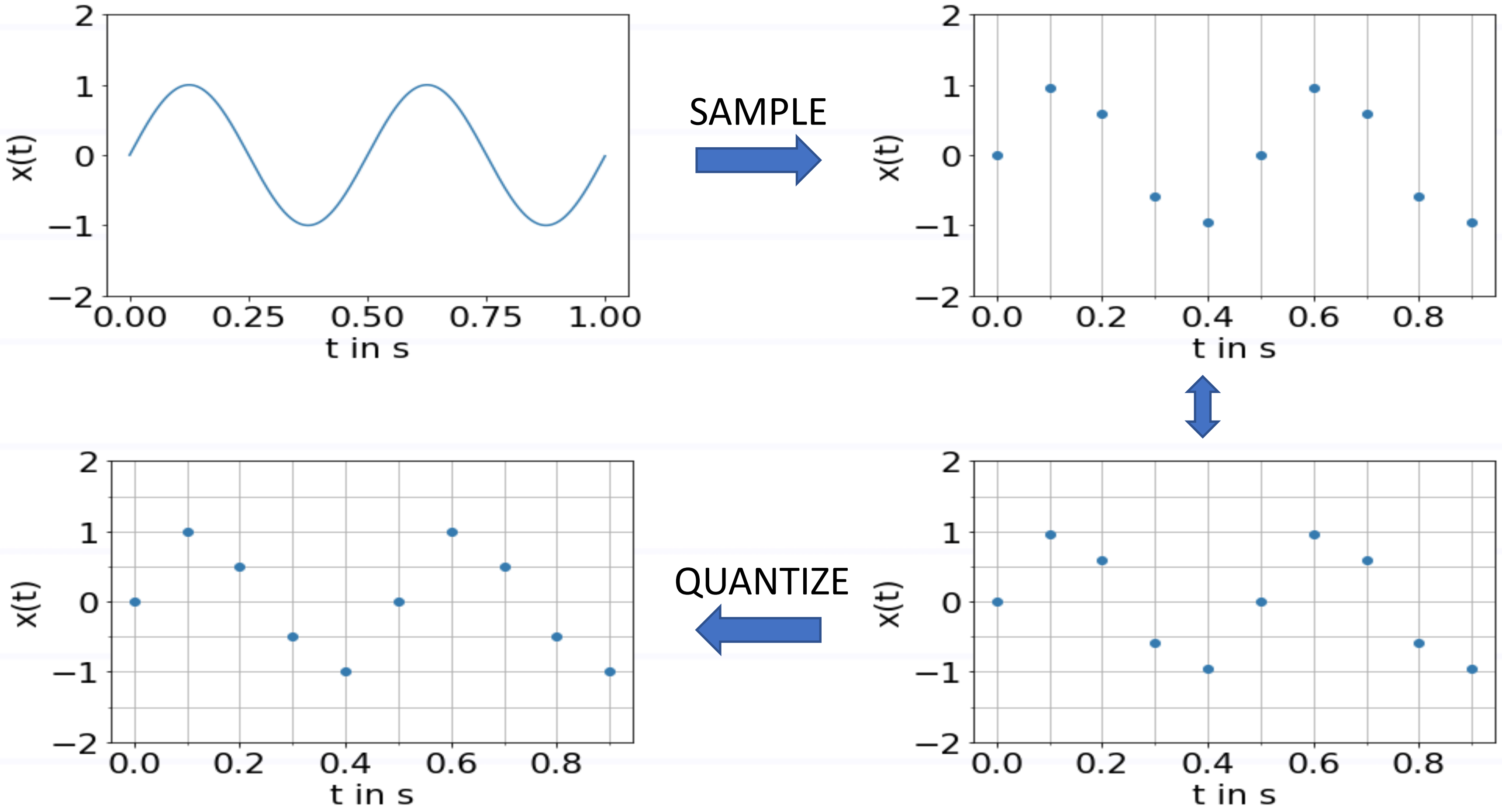








Sampling and Quantization



Time Series Analysis

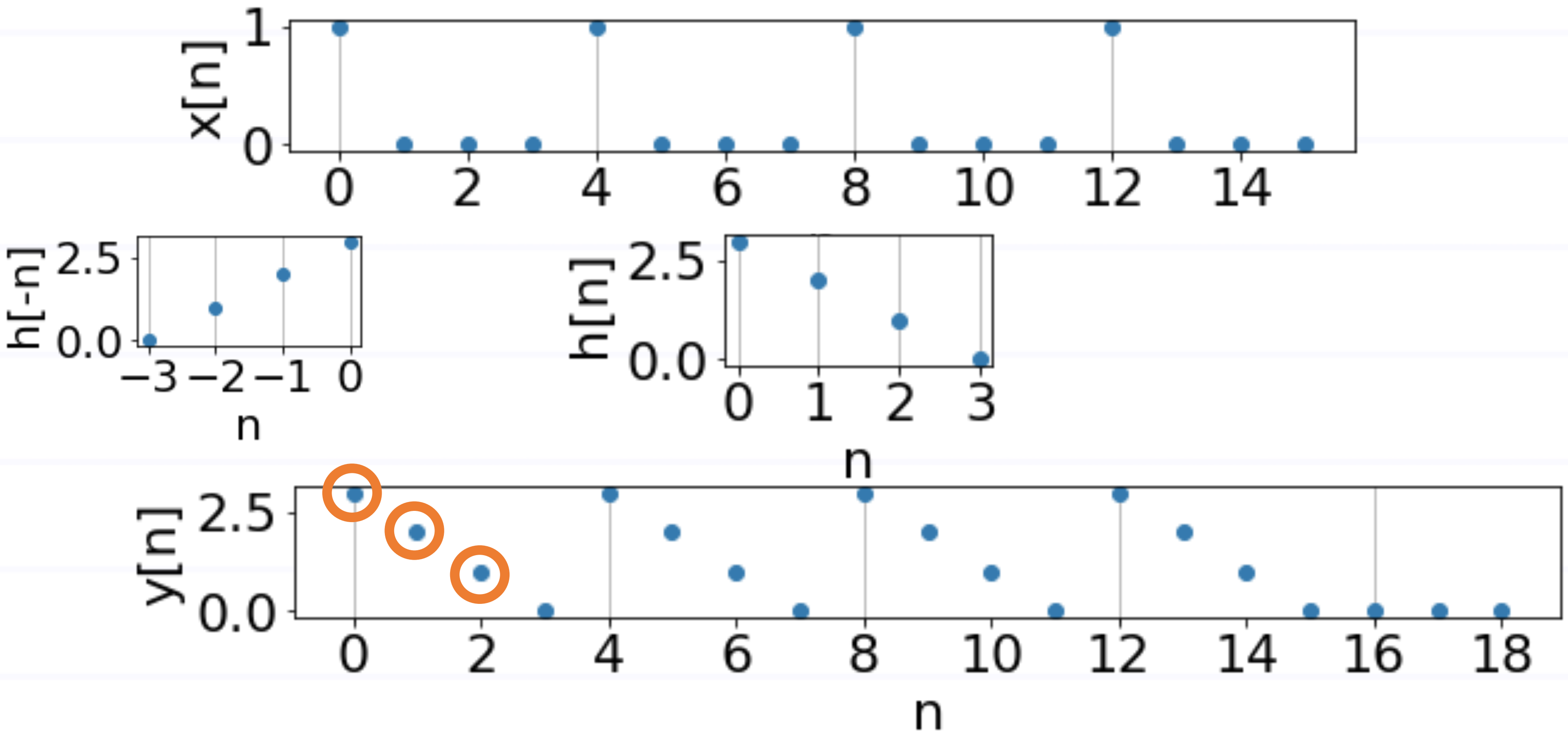
- Data Compression
  - compact description
- Data Explanation
  - Factors affecting the data, and their relationships
- Data Denoising
- Prediction of future values
- Classification

Convolution

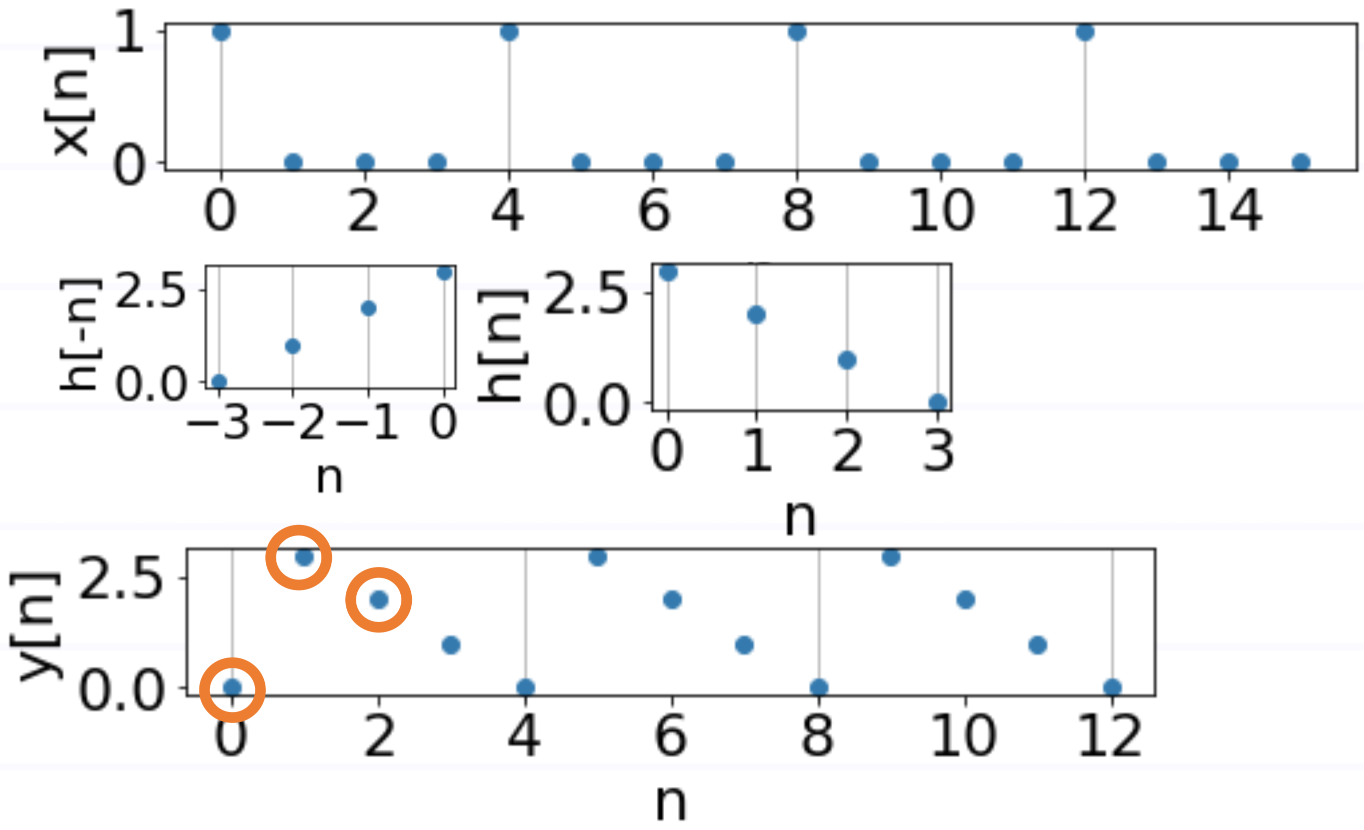
$$y[n] = x[n] * h[n] = \sum_{m} x[m]h[n-m]$$

- soak a seed, it sprouts in 10 hours

Convolution: with zero padding



Convolution: without zero padding

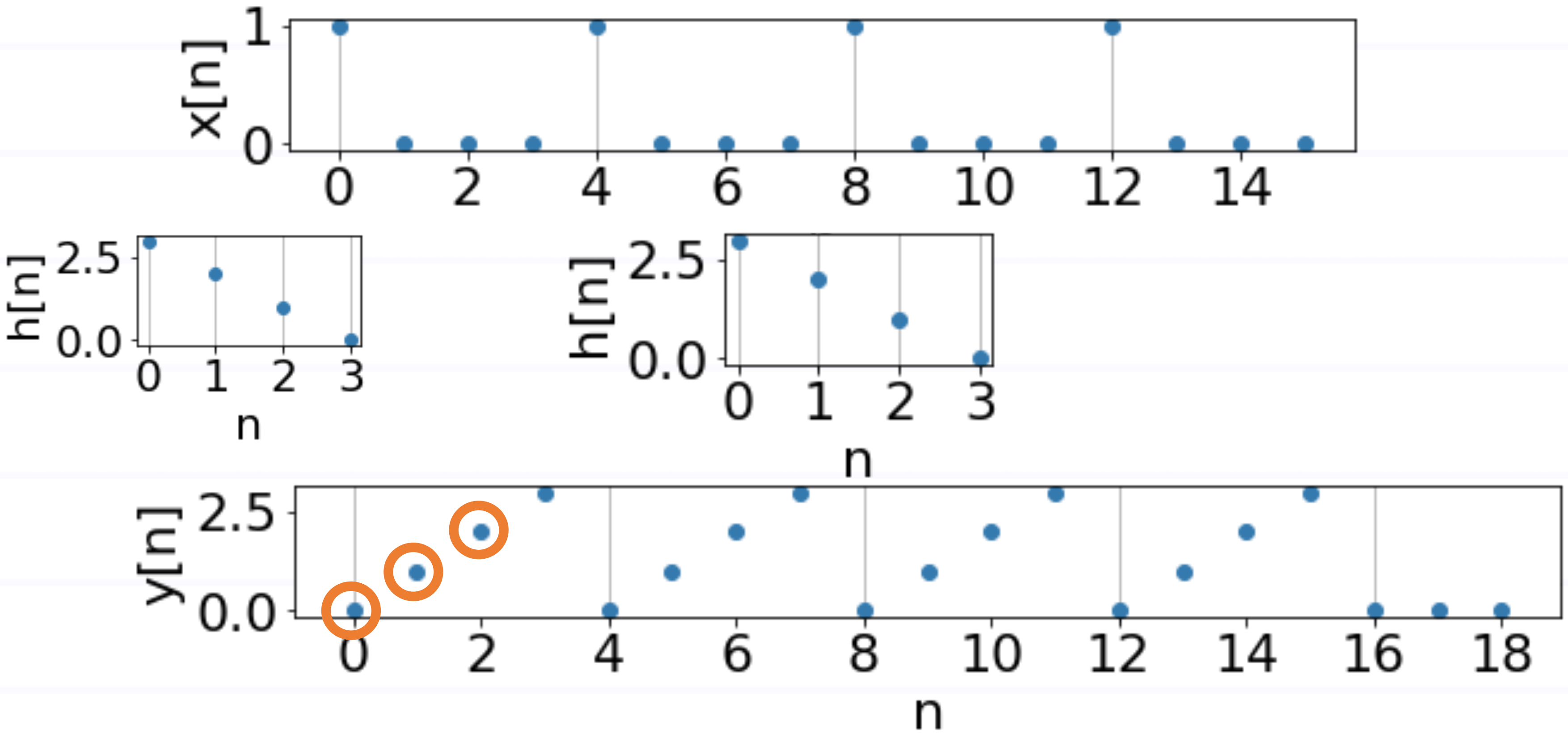


Cross-correlation

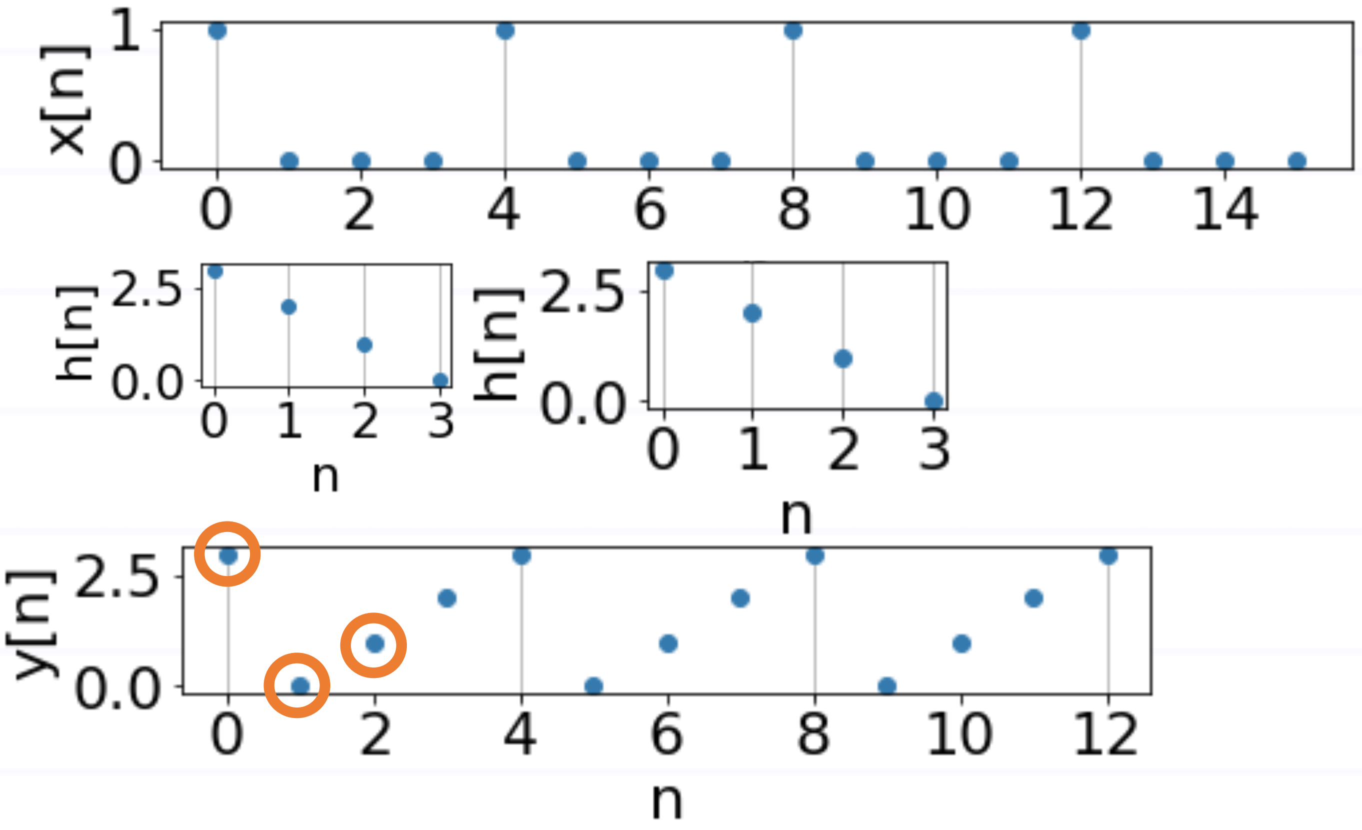
$$y[n] = x[n] \otimes h[n] = \sum_{m} x[m]h[m+n]$$

- Find similar segment, shifted in time

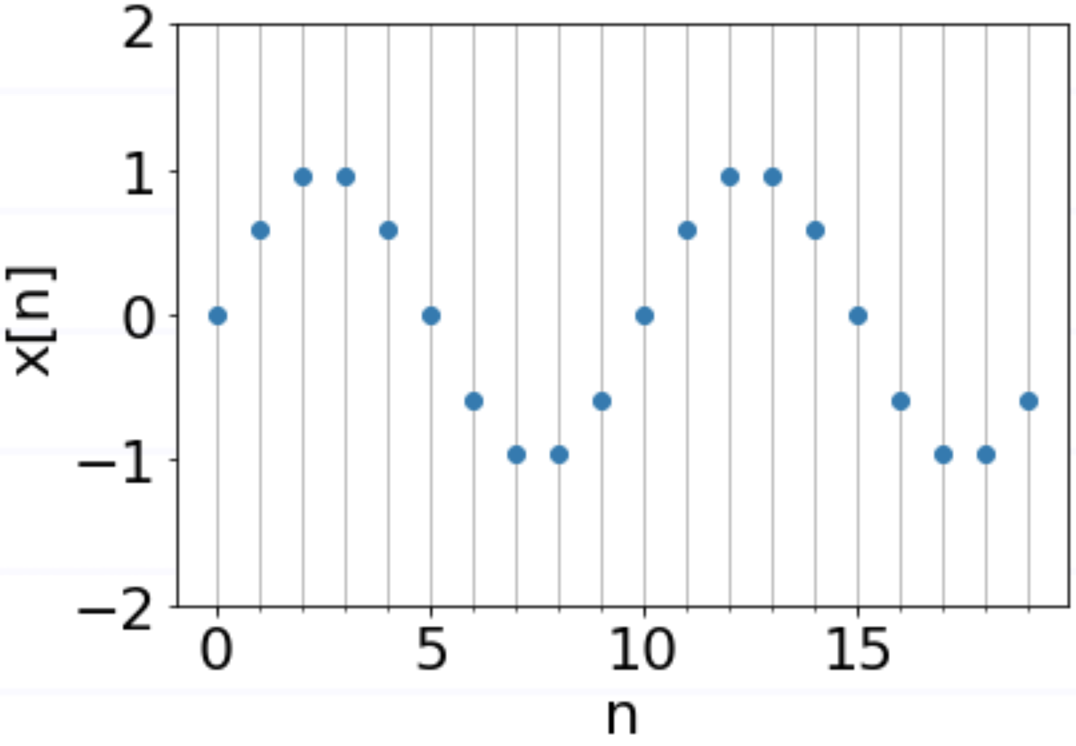
Cross Correlation: with zero padding

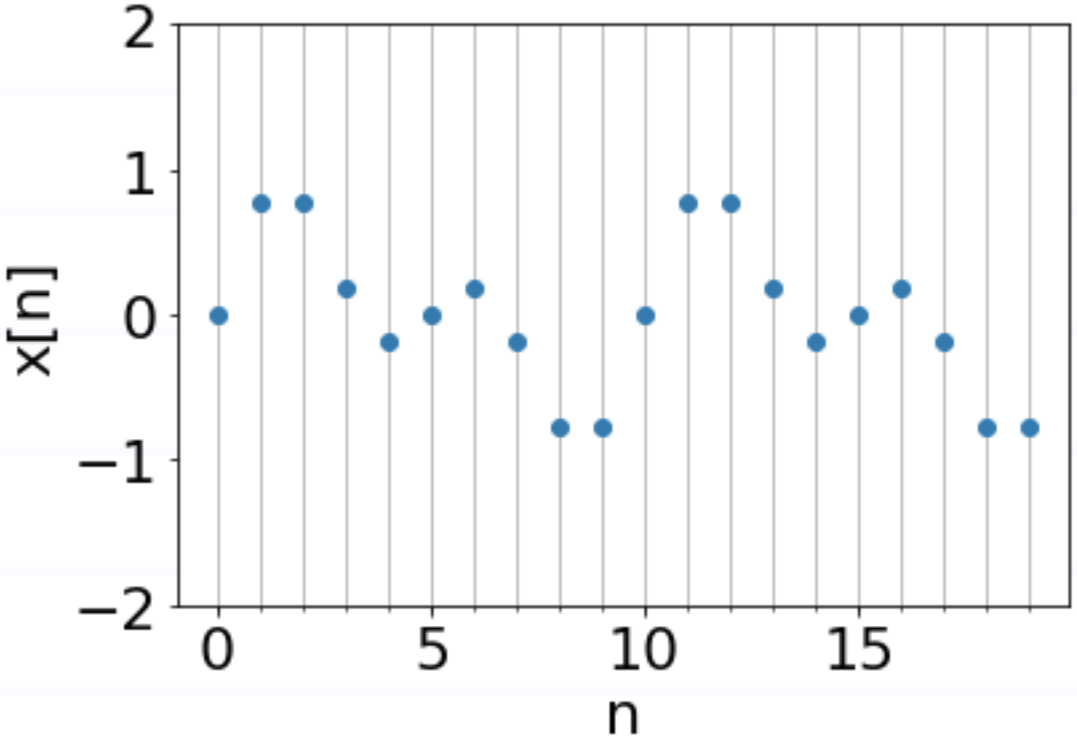


Cross Correlation: without zero padding



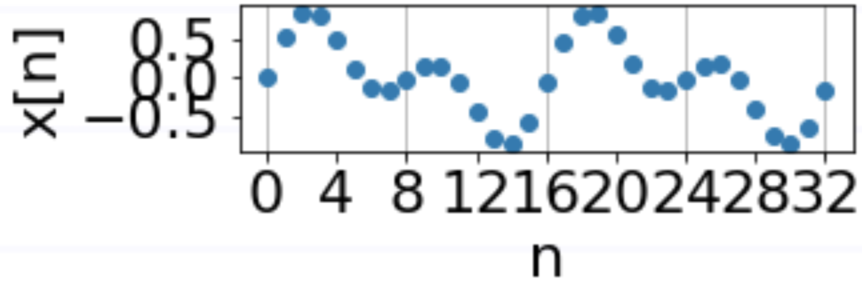
Periodic Signals

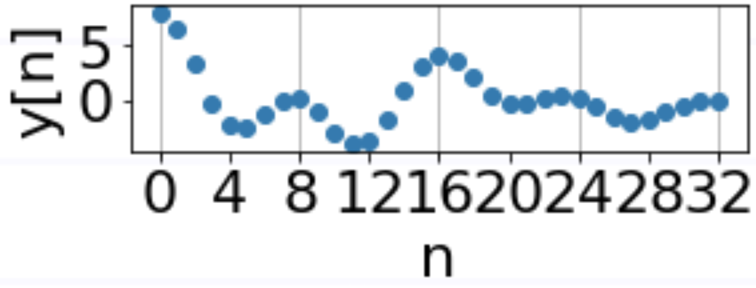




Periodic Signals

- What is the fundamental frequency, F0 (in samples)?



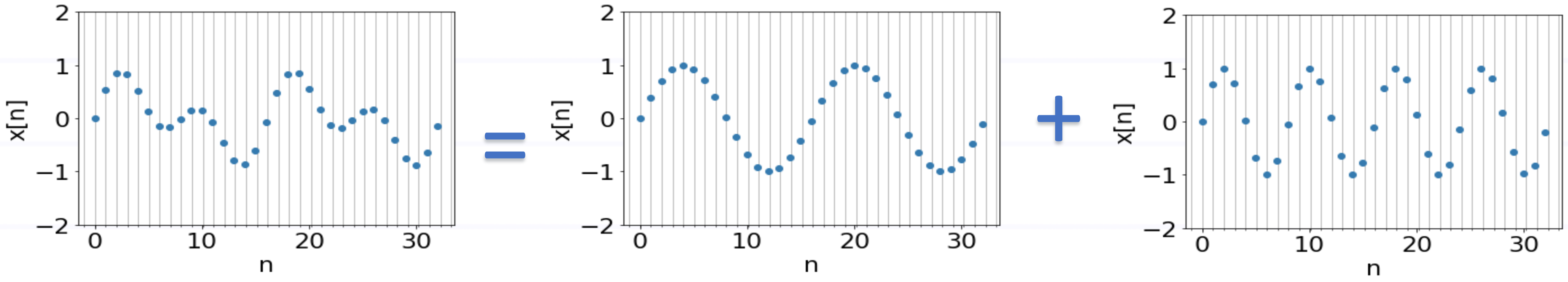


Auto Correlation,
i.e., cross-correlation with itself

Works for any shape of a unit

Periodic Signals

- Decompose them into standard periodic basis



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Periodic Signals – a sinusoid

$$x[n] = X sin(2\pi f n + \phi)$$

A sinusoid is:

- very well-behaved (continuous, differentiable)
- characterized by 3 parameters:
  - Amplitude, X
  - Frequency, f
  - Phase, ϕ
- Orthogonal basis

Periodic Signals

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] sin(2\pi \frac{k}{N} n + \phi[k])$$

- f = k/N
- For N samples, we can go upto N sinusoidal components
- How to estimate X[k] and ϕ[k]?

Fourier Transforms

$$x(t) = \sum_{k} X[k] e^{jk\omega_0 t} X$$

$$X_k = \frac{1}{T} \int x(t) e^{-jk\omega_0 t} dt$$

- *x(t)* is continuous and periodic
- X_k is discrete and non-periodic

Continuous time Fourier Series
Ref: Oppenheim, Ch-3

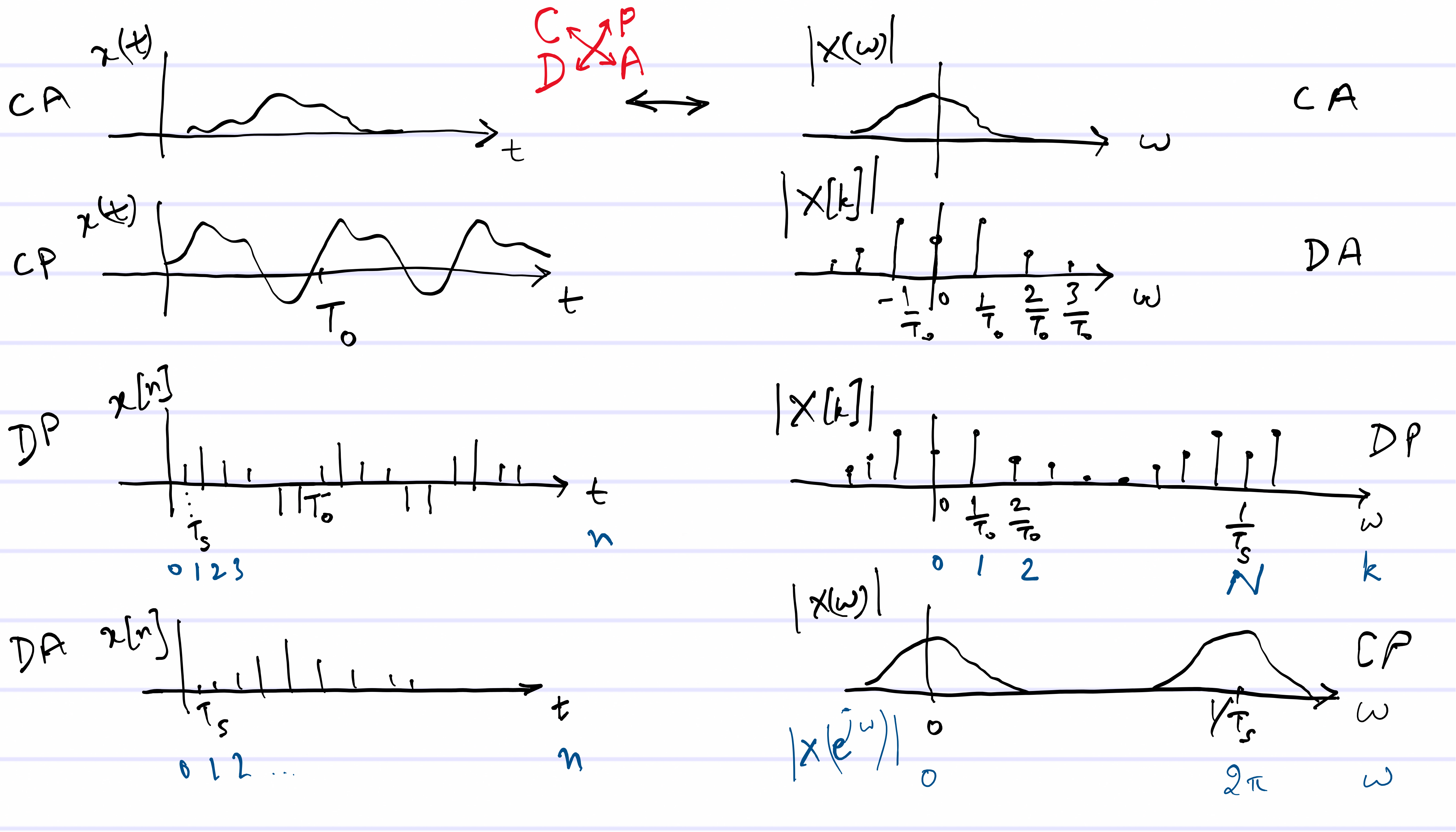
Fourier Transforms

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

- *x(t)* is continuous, non-periodic
- *X(ω)* is continuous, non-periodic

Continuous time Fourier Transform
Ref: Oppenheim, Ch-4


C ↔ P
D ↔ A
CA
x(t)
t
|X(ω)|
ω
CA
CP
x(t)
T0
t
|X[k]|
-1/T0 0 1/T0 2/T0 3/T0
ω
DA
DP
x[n]
Ts
T0
t
n
0 1 2 3
|X[k]|
0 1/T0 2/T0 1/Ts
ω
0 1 2 N k
DP
DA
x[n]
Ts
t
n
0 1 2 ...
|X(ω)|
0 1/Ts
ω
CP
|X(e^jω)|
0 2π ω

Discrete Fourier Transform

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j \frac{2\pi}{N} kn}; \quad n = 0, 1, \ldots, N-1$$

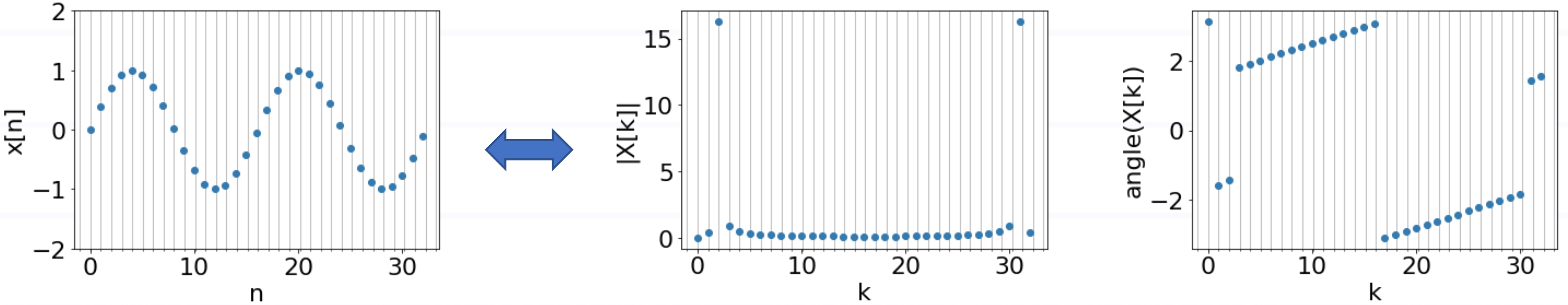
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn}; \quad k = 0, 1, \ldots, N-1$$

Ref: Quatieri, Chapter-2

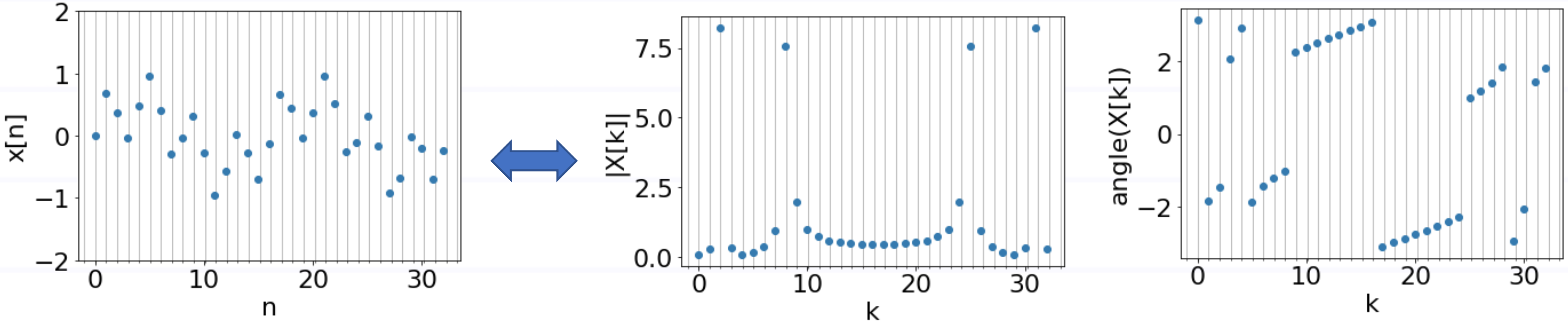
Discrete Fourier Transform

- If $x[n]$ is real,
  - $X[-k] = X^*[k]$, i.e.,
    - $|X[k]|$ is an even function
    - angle($X[k]$) is an odd function

Discrete Fourier Transform



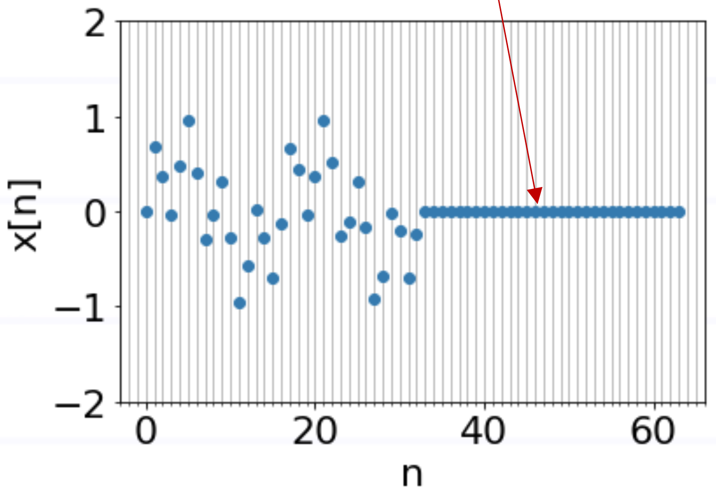
$x[n] = \sin(2\pi * 2/32 * n)$

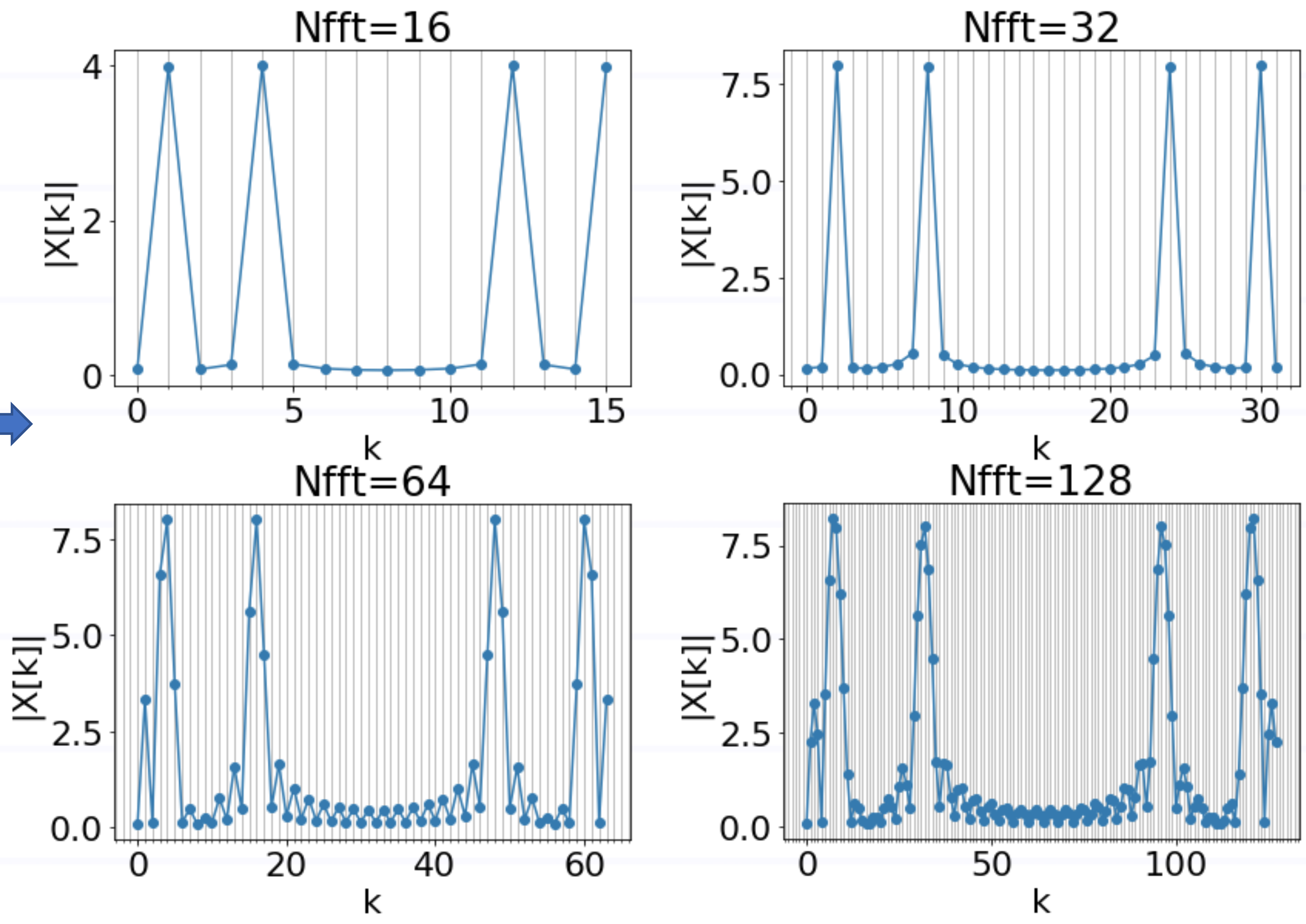


$x[n] = 0.5 * \sin(2\pi * 2/32 * n) + 0.5 * \sin(2\pi * 8/32 * n)$

Discrete Fourier Transform

- Zero padding



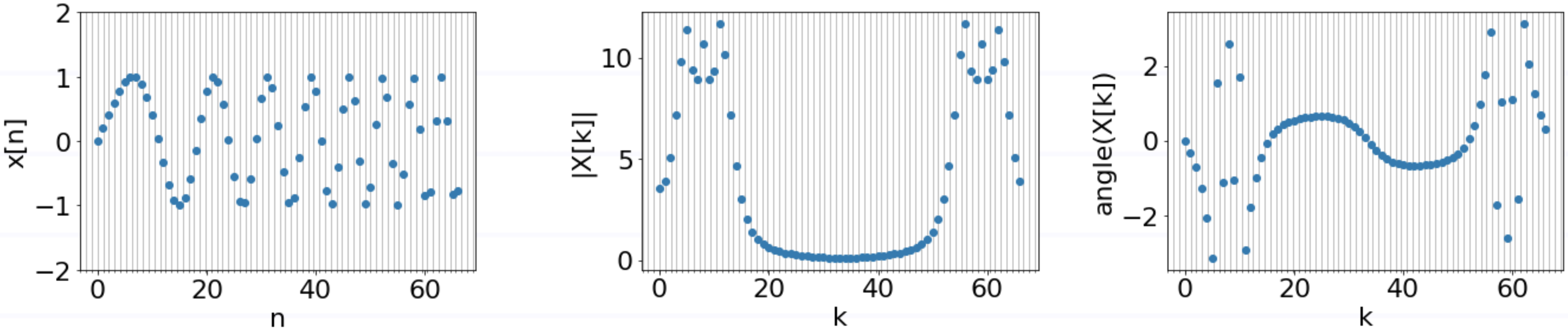


Why Spectral Representation

- Analysis of periodicity
- Compact
- Disentangles

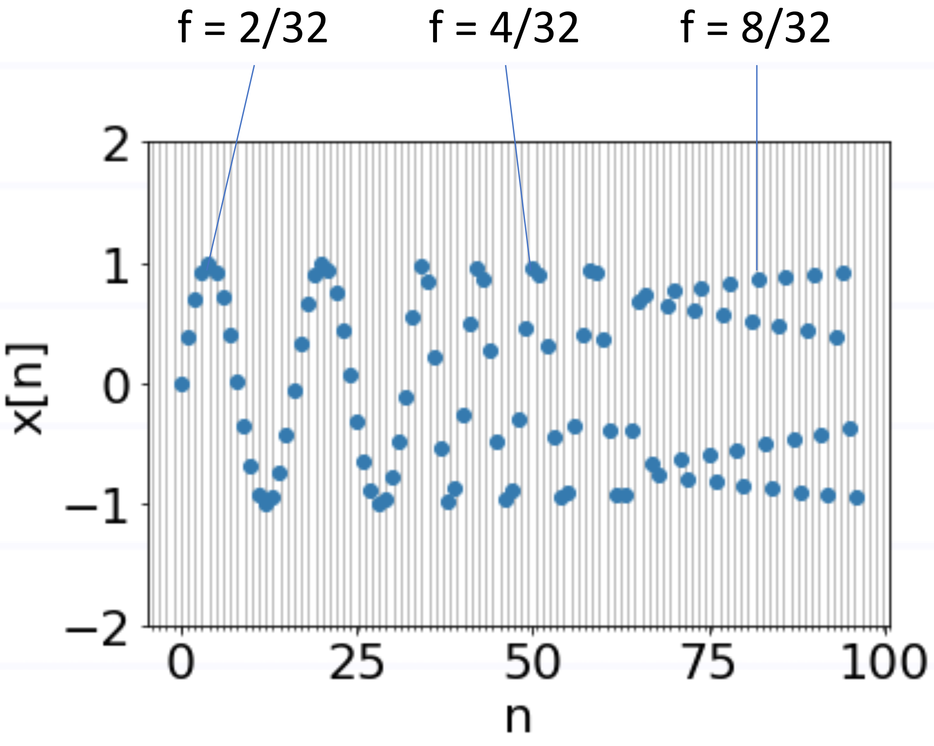
Discrete Fourier Transform

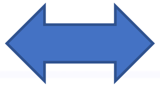
- This representation is valid is the signal is stationary, i.e., its characteristics do not change with time
- How about a non-stationary signal? a mess?

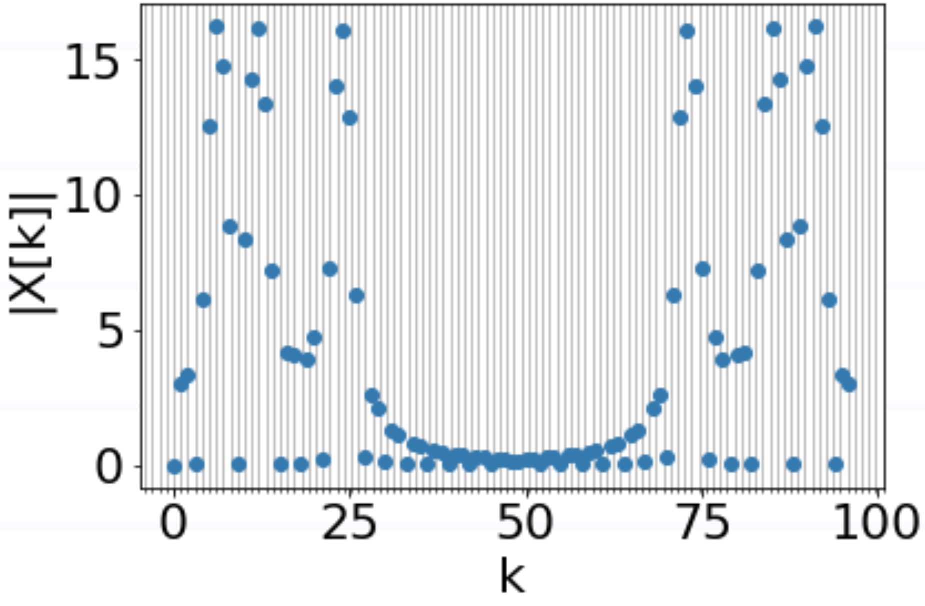


x[n] = sin(2π * f * n); where, f varies from 2/32 to 8/32 linearly with n

Discrete Fourier Transform







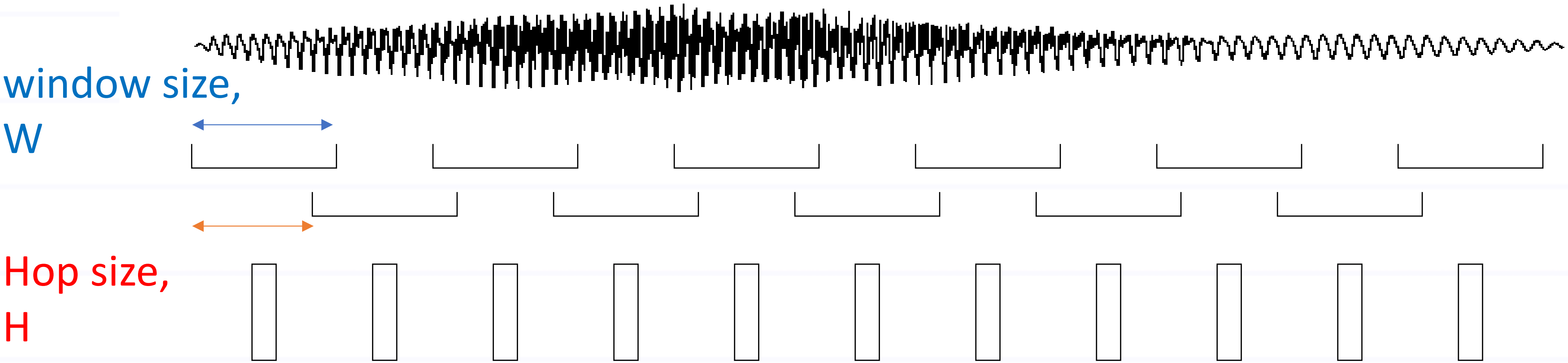
x[n] = sin(2π * f * n)

A Quasi-stationary Signal

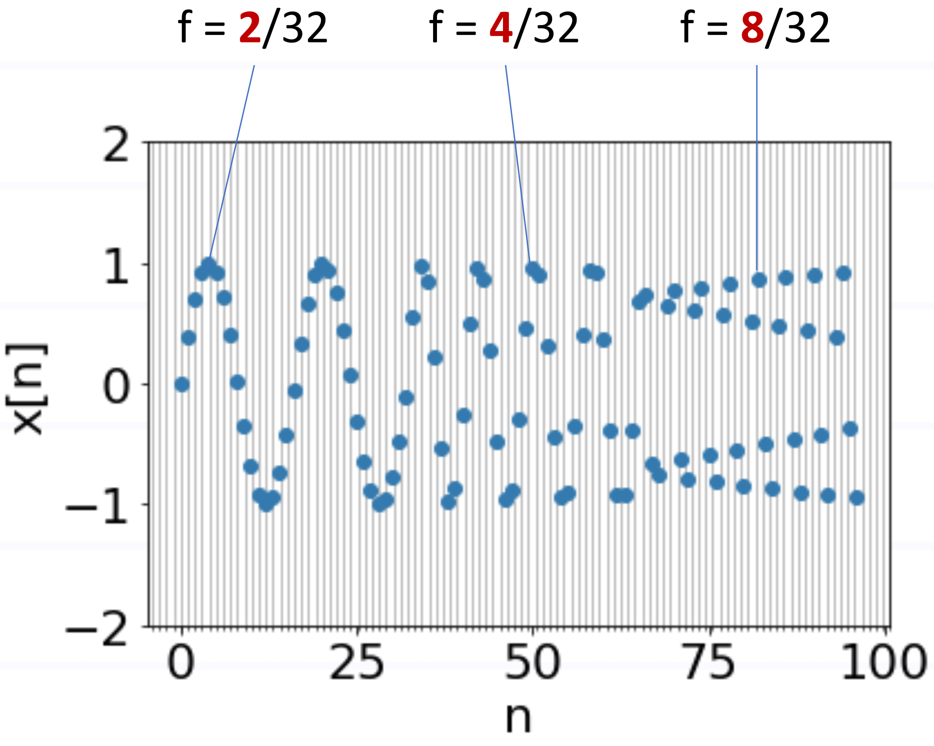
almost stationary in small intervals

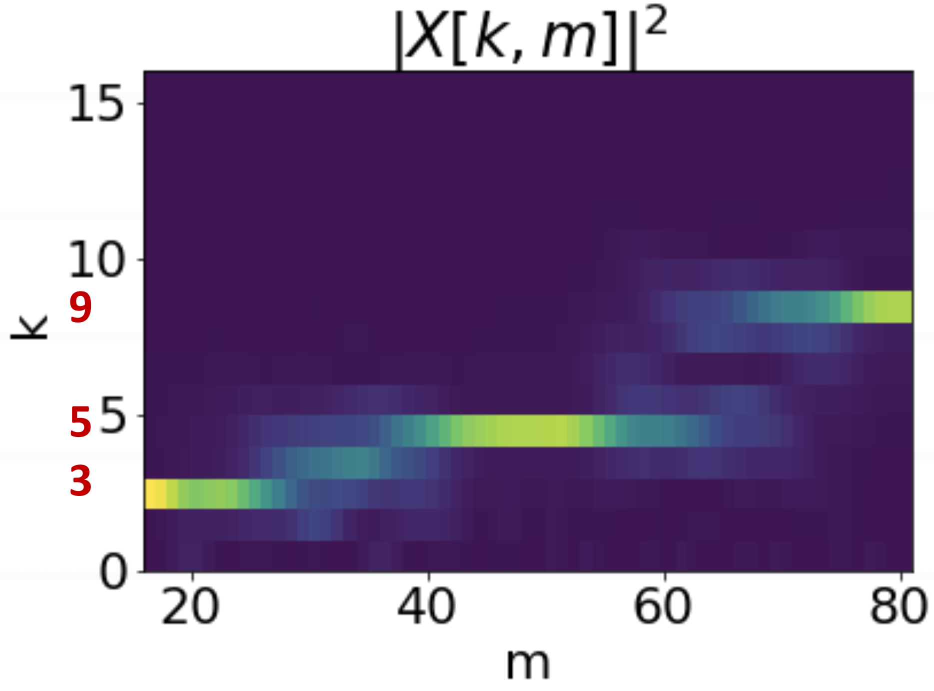
Short-time Fourier Transform (STFT)

- Intuitively,



Short-time Fourier Transform (STFT)





W = 32
H = 1

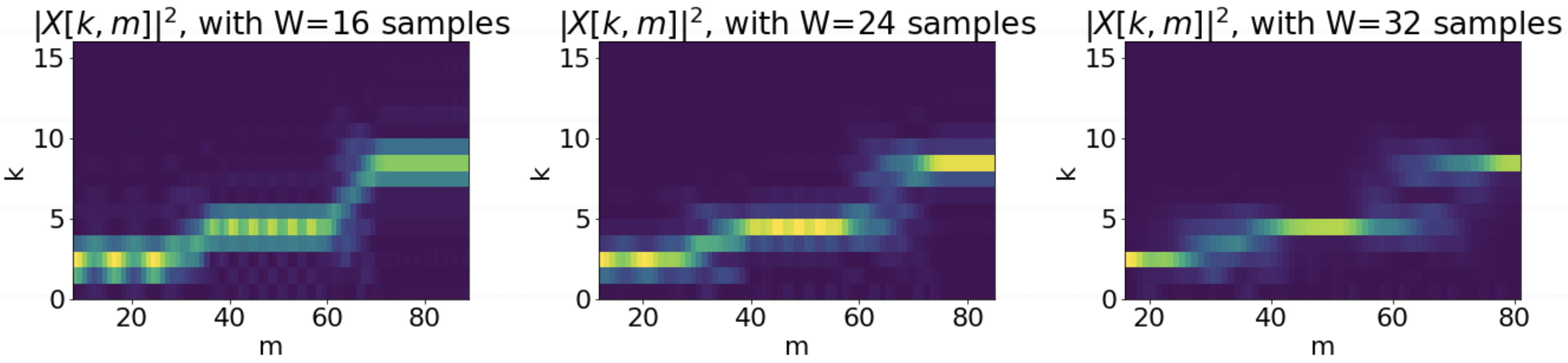
x[n] = sin(2π * f * n)

A Quasi-stationary Signal

STFT preserves the signal characteristics, both in time and frequency

Short-time Fourier Transform (STFT)

- The effect of changing window size, W

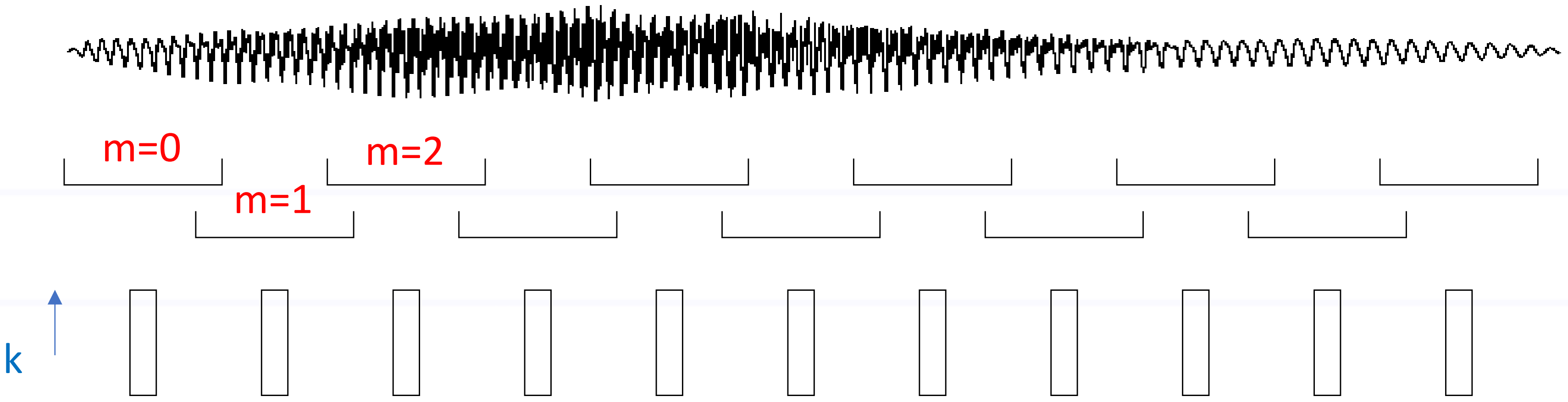


- Notice the trade off between **time** resolution vs **frequency** resolution
- Heisenberg uncertainty principal

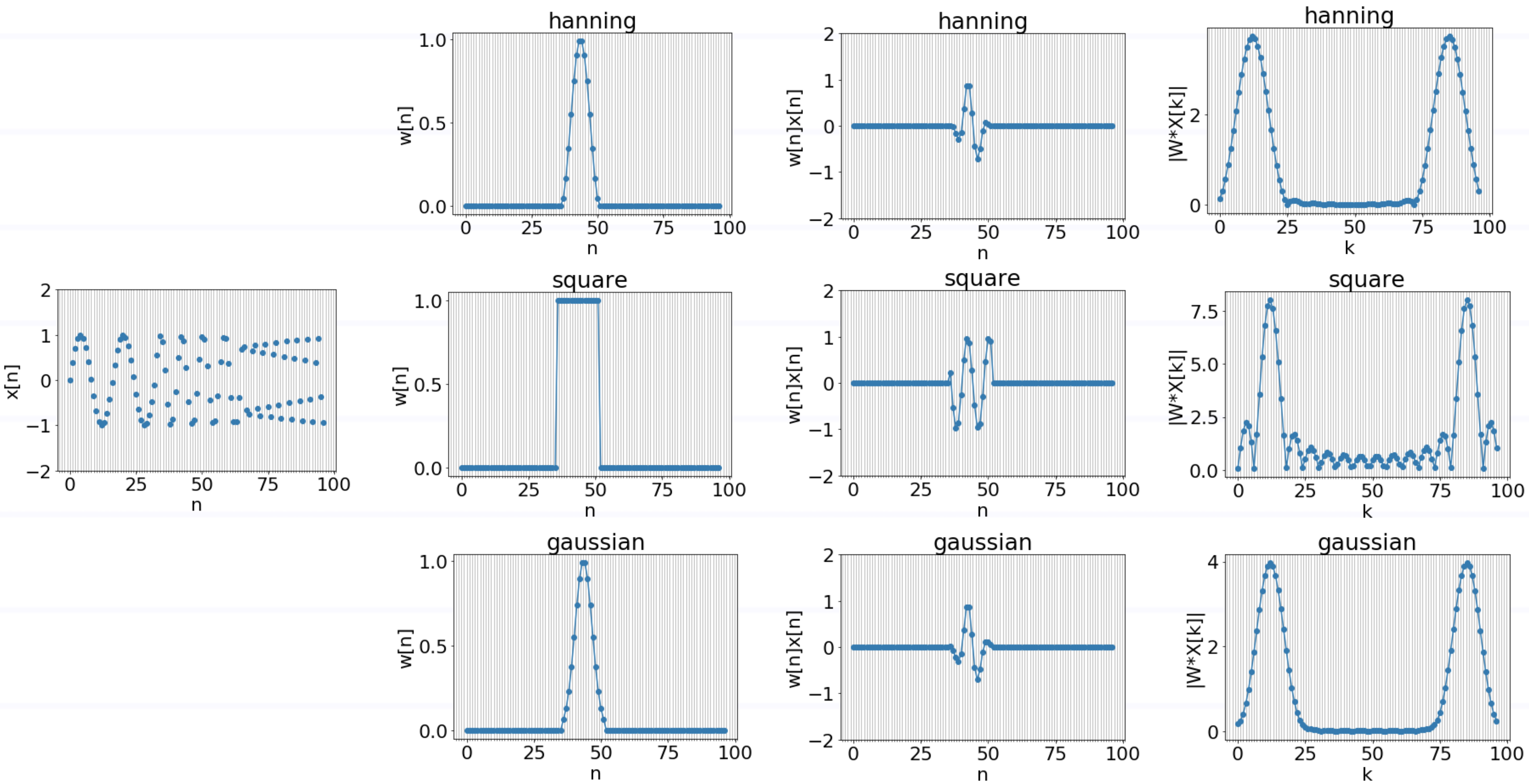
Short-time Fourier Transform (STFT)

- Mathematically,

$$X[k,m] = \sum_{n=0}^{N-1} x[n]w[n-mH]e^{-j\frac{2\pi}{N}kn}; \quad k = 0, 1, ..., N-1$$

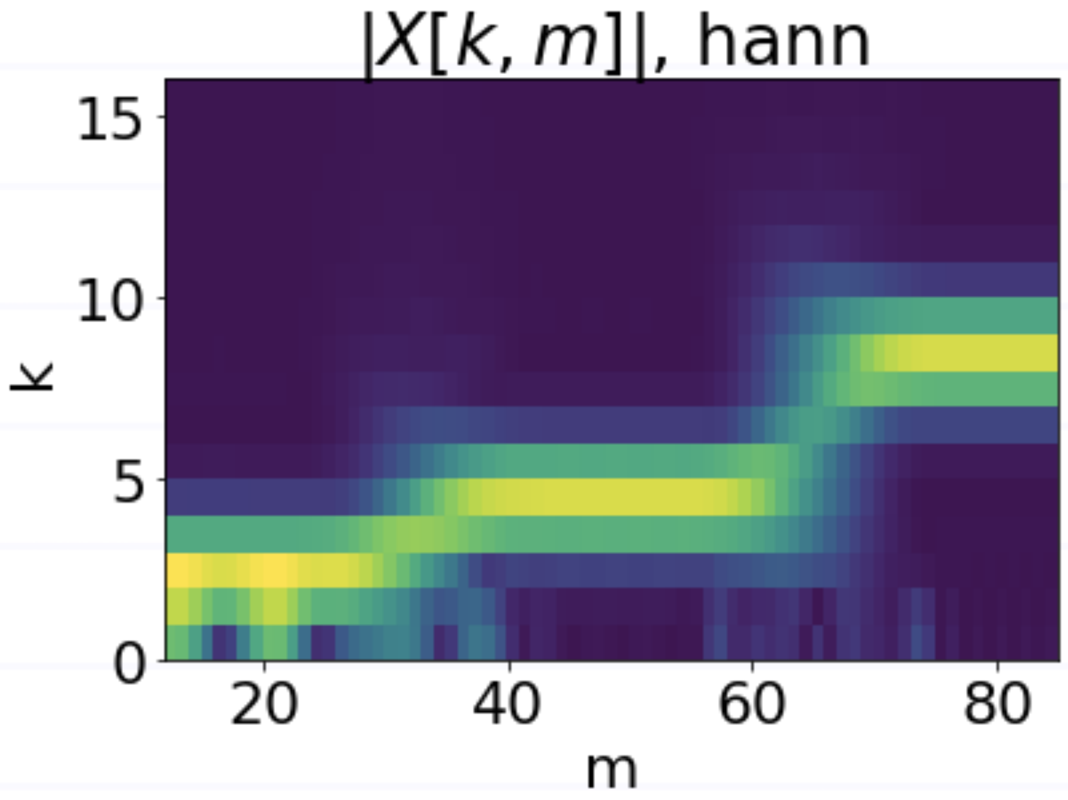


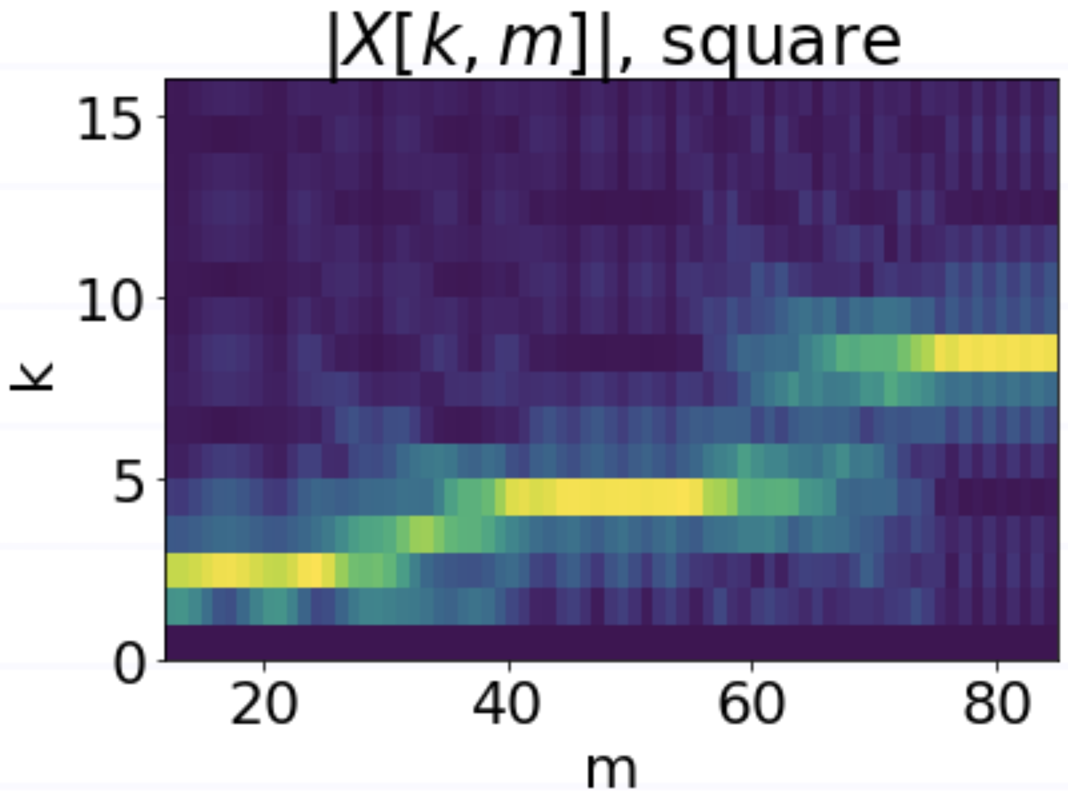
Window



Sharp discontinuities in signal cause more side lobes in |X[k]|

Spectrogram with windowing





Fast Fourier Transform

- FFT, $O(N\log N)$, is a faster implementation of DFT, $O(N^2)$
- Using FFTs in your computations results in computational gains

Topics studied

- Convolution: transformation of a signal
- Correlation: similarity check
- Periodicity
- Fourier Transform: represent periodic signals
- Short-time Fourier Transform: signals changing over time

References

- https://www.tutorialspoint.com/digital_signal_processing/index.htm
- Quatieri, T. F. (2006). *Discrete-time speech signal processing: principles and practice*. Pearson Education India (Chapter 2)
- Oppenheim, A.V., Willsky, A.S., & Nawab, H.S. (1996). Signals and systems. *Pearson press, USA*.