SRI RAMACHANDRA

INSTITUTE OF HIGHER EDUCATION AND RESEARCH

(Category - I Deemed to be University) Porur, Chennai

SRI RAMACHANDRA ENGINEERING AND TECHNOLOGY

Day-2: 20-10-2020 MODULE -1 Assignment - 1

1. **Obtain the asymptotic bound for the recurrence relations given below using Master theorem.**

1. T (n) = 3T (n/2)+ n2 Solution: a=3 b=2 k=2 p=0 $\log_b a \simeq 1.5$ since $\log_b a<k$ (case 3)Answer: $\theta(n^2)$

2. T (n) = 4T (n/2)+ n2 Solution: a=4 b=2 k=2 p=0 $\log_b a \simeq 2$ since $\log_b a=k$ (case 1)Answer: $\theta(n^2 logn)$

3. T (n) = T (n/2) + 2n Solution: a=1 b=2 k=1 p=1 $\log_b a \simeq 0$ since $\log_b a<k$ (case 3)Answer: $\theta(2)$

4. T (n) = 2nT (n/2) + nn Masters theorem does not work a not a constant

5. T (n) = 16T (n/4)+ n Solution: a=16 b=4 k=1 p=0 $\log_b a \simeq 2$ since $\log_b a>k$ (case 1)Answer: $\theta(n^2)$

6. T (n) = 2T (n/2)+ n log n Solution: a=2 b=2 k=1 p=1 $\log_b a \simeq 1$ since $\log_b a=k$ (case 2)Answer:$\theta(n\log^2 n)$

7. T (n) = 2T (n/2)+ n/ log n Sol: a=2 b=2 k=1 p=-1 $\log_b a \simeq 1$ since $\log_b a=k$ (case 2)Answer:$\theta(nloglogn)$

8. T (n) = 2T (n/4)+ n0.51 Sol: a=2 b=4 k=0.51 p=0 $\log_b a \simeq 0.5$ since $\log_b a<k$ (case 3)Answer:$\theta(n^{0.51})$

9. T (n) = 0.5T (n/2)+ 1/n Sol: a=0.5 cannot be solved using masters theorem a<1

10. T (n) = 16T (n/4)+ n! Sol: a=16 b=4 k=c p=1 $\log_b a \simeq 2$ since $\log_b a<k$ (case 3)Answer:$\theta(n!)$

11. T (n) = p2T (n/2) + log n Sol: a=$\sqrt{2}$ b=2 k=0 p=1 $\log_b a \simeq 1/2$ since $\log_b a>k$ (case 1)Answer:$\theta(\sqrt{n})$

12. T (n) = 3T (n/2)+ n Sol: a=3 b=2 k=1 p=0 $\log_b a \simeq 1.58$ since $\log_b a>k$ (case 1)Answer:$\theta(n^{\log_2 3})$

13. T (n) = 3T (n/3)+ pn Sol: a=3 b=3 k=0.5 p=0 $\log_b a \simeq 1$ since $\log_b a>k$ (case 1)Answer:$\theta(n)$

14. T (n) = 4T (n/2)+ cn Sol: a=4 b=2 k=1 p=0 $\log_b a \simeq 2$ since $\log_b a>k$ (case 1)Answer:$\theta(n^2)$

15. T (n) = 3T (n/4)+ n log n Sol: a=3 b=4 k=1 p=1 $\log_b a \simeq 0.79$ since $\log_b a<k$ (case 3)Answer:$\theta(nlogn)$

16. T (n) = 3T (n/3)+ n/2 Sol: a=3 b=3 k=1 p=0 $\log_b a \simeq 1$ since $\log_b a=k$ (case 2)Answer:$\theta(nlogn)$

17. T (n) = 6T (n/3)+ n2 log n Sol: a=6 b=3 k=2 p=1 $\log_b a \simeq 1.63$ since $\log_b a<k$ (case3)Answer:$\theta(n^2 logn)$

18. T (n) = 4T (n/2)+ n/ log n Sol: a=4 b=2 k=1 p=-1 $\log_b a \simeq 2$ since $\log_b a>k$ (case1) Answer:$\theta(n^2)$

19. T (n) = 64T (n/8)− n2 log n Sol: a=64 b=8 k=2 p=1 $\log_b a \simeq 2$ since $\log_b a=k$ (case2) Answer:$\theta(n^2\log^2 n)$

20. T (n) = 7T (n/3)+ n2 Sol: a=7 b=3 k=2 p=0 $\log_b a \simeq 1.77$ since $\log_b a<k$ (case3) Answer:$\theta(n^2)$

21. T (n) = 4T (n/2)+ log n Sol: a=4 b=2 k=0 p=1 $\log_b a \simeq 2$ since $\log_b a>k$ (case1) Answer:$\theta(n^2)$

2. Solve the following recurrence relation using recursion tree method.

1. T(n) = T(n/5) + T(4n/5) + n

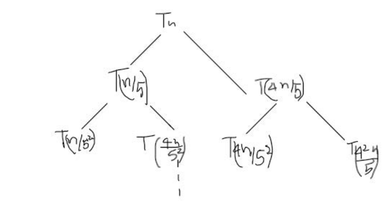
2. T(n) = 3T(n/4) + cn2

3. T(n) = cn+2T(n/2)

Module 2: Combinatorial Optimization

3. Design a greedy algorithmic technique using binary min heap to encode the word ***'mississippi'*** using variable length codeword. Calculate the number of bits may be required for encoding the message 'mississippi'?

2.1 $T(n) = T(n/5) + T(4n/5) + n$



height $= \left(\frac{4}{5}\right)^i n = 1$ | $i \log(4/5) = \log(1/n)$

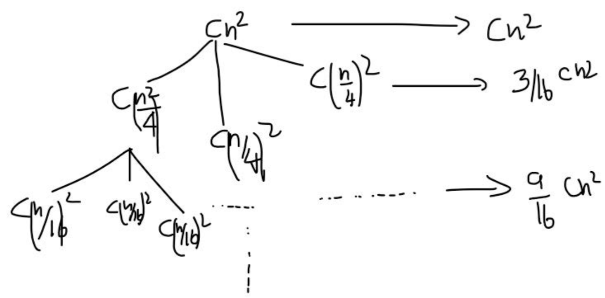
$\left(\frac{4}{5}\right)^i = 1/n$ | $i = \log_{5/4} n$

cost at last level $= 2^{\log_{5/4} n} \times T_1 = \Theta(n^{\log_{5/4} 2})$

$T(n) = n + n + n \cdots + \Theta(n^{\log_{5/4} 2})$

Ans: $\Theta(n \log_{5/4} n)$

2.2



Calculating the levels

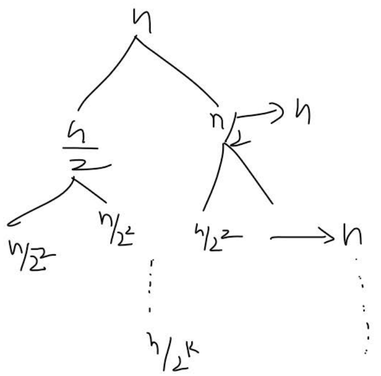
$n/4^x = 1 \Rightarrow x = \log_4 n$

$cn^2 + (1 + (3/16) + (3/16)^2 \cdots) + \Theta(n^{\log_4 3})$

most significant

$\boxed{\therefore T(n) = \Theta(n^2)}$

2.3



$\boxed{nK} \Rightarrow \boxed{\Theta(n\log n) + c}$

Assume that $n/2^k = 1$

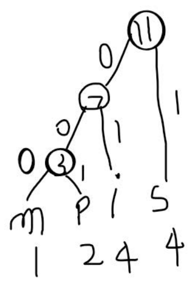
$n = 2^k \quad k = \log n$

3.

mississippi

letter	Freq	Bin
m	1	000
i	4	01
s	4	1
p	2	001

huffman tree



Chercter code $4 \times 8 = 32b$

Huffman code $(2 \times 3) + 2 + 1 = 9b$

Total size of the dictionary $41b$

total size of the message = (1x3)+4(2)+4(1)+2(3)= 21 bits

$\boxed{\therefore \text{ total compressed data size} = 62 \text{ bit}}$