Group A

Assignment No: 4

Title of the Assignment: Write a program to solve a 0-1 Knapsack problem using dynamic programming or branch and bound strategy.

Objective of the Assignment: Students should be able to understand and solve 0-1 Knapsack problem using dynamic programming

Prerequisite:

1. Basic of Python or Java Programming
2. Concept of Dynamic Programming
3. 0/1 Knapsack problem

Contents for Theory:

1. **Greedy Method**
2. **0/1 Knapsack problem**
3. **Example solved using 0/1 Knapsack problem**

What is Dynamic Programming?

- Dynamic Programming is also used in optimization problems. Like divide-and-conquer method, Dynamic Programming solves problems by combining the solutions of subproblems.

- Dynamic Programming algorithm solves each sub-problem just once and then saves its answer in a table, thereby avoiding the work of re-computing the answer every time.

- Two main properties of a problem suggest that the given problem can be solved using Dynamic Programming. These properties are **overlapping sub-problems and optimal substructure**.

- Dynamic Programming also combines solutions to sub-problems. It is mainly used where the solution of one sub-problem is needed repeatedly. The computed solutions are stored in a table, so that these don‘t have to be re-computed. Hence, this technique is needed where overlapping sub-problem exists.

- For example, Binary Search does not have overlapping sub-problem. Whereas recursive program of Fibonacci numbers have many overlapping sub-problems.

Steps of Dynamic Programming Approach

Dynamic Programming algorithm is designed using the following four steps −

- Characterize the structure of an optimal solution.
- Recursively define the value of an optimal solution.
- Compute the value of an optimal solution, typically in a bottom-up fashion.
- Construct an optimal solution from the computed information.

Applications of Dynamic Programming Approach

- Matrix Chain Multiplication
- Longest Common Subsequence
- Travelling Salesman Problem

Knapsack Problem

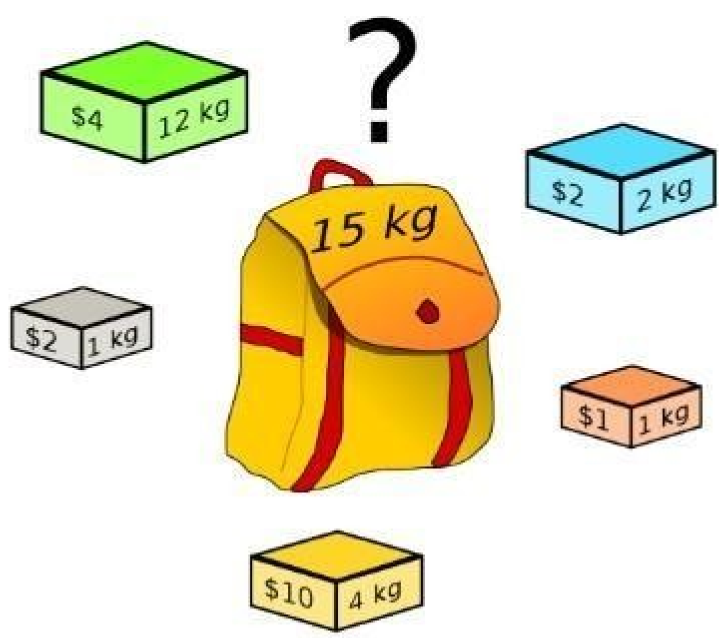
You are given the following-

- A knapsack (kind of shoulder bag) with limited weight capacity.
- Few items each having some weight and value.

The problem states-

Which items should be placed into the knapsack such that-

- The value or profit obtained by putting the items into the knapsack is maximum.
- And the weight limit of the knapsack does not exceed.



Knapsack Problem

Knapsack Problem Variants

Knapsack problem has the following two variants-

1. Fractional Knapsack Problem
2. 0/1 Knapsack Problem

0/1 Knapsack Problem-

In 0/1 Knapsack Problem,

- As the name suggests, items are indivisible here.
- We can not take a fraction of any item.
- We have to either take an item completely or leave it completely.
- It is solved using a dynamic programming approach.

0/1 Knapsack Problem Using Greedy Method-

Consider-

- Knapsack weight capacity = w
- Number of items each having some weight and value = n

0/1 knapsack problem is solved using dynamic programming in the following steps-

Step-01:

- Draw a table say ‗T' with (n+1) number of rows and (w+1) number of columns.
- Fill all the boxes of 0th row and 0th column with zeroes as shown-

	0	1	2	3		W
0	0	0	0	0	......	0
1	0					
2	0					
	......					
n	0					

T-Table

Step-02:

Start filling the table row wise top to bottom from left to right.

Use the following formula-

$$T(i, j) = \max \{ T(i-1, j), value_i + T(i-1, j - weight_i) \}$$

Here, T(i , j) = maximum value of the selected items if we can take items 1 to i and have weight restrictions of j.

- This step leads to completely filling the table.
- Then, value of the last box represents the maximum possible value that can be put into the knapsack.

Step-03:

- To identify the items that must be put into the knapsack to obtain that maximum profit,
- Consider the last column of the table.
- Start scanning the entries from bottom to top.
- On encountering an entry whose value is not same as the value stored in the entry immediately above it, mark the row label of that entry.
- After all the entries are scanned, the marked labels represent the items that must be put into the knapsack

Problem-.

For the given set of items and knapsack capacity = 5 kg, find the optimal solution for the 0/1 knapsack problem making use of a dynamic programming approach.

Item	Weight	Value
1	2	3
2	3	4
3	4	5
4	5	6

n = 4

w = 5 kg

(w1, w2, w3, w4) = (2, 3, 4, 5)

(b1, b2, b3, b4) = (3, 4, 5, 6)

Solution-

Given

- Knapsack capacity (w) = 5 kg
- Number of items (n) = 4

Step-01:

- Draw a table say ‗T' with (n+1) = 4 + 1 = 5 number of rows and (w+1) = 5 + 1 = 6 number of columns.
- Fill all the boxes of 0^{th} row and 0^{th} column with 0.

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0					
2	0					
3	0					
4	0					

T-Table

Step-02:

Start filling the table row wise top to bottom from left to right using the formula-

$$T(i, j) = \max \{ T(i-1, j), value_i + T(i-1, j - weight_i) \}$$

Finding T(1,1)-

We have,

- i = 1
- j = 1
- $(value)_i = (value)_1 = 3$
- $(weight)_i = (weight)_1 = 2$

Substituting the values, we get-

T(1,1) = max { T(1-1 , 1) , 3 + T(1-1 , 1-2) }

T(1,1) = max { T(0,1) , 3 + T(0,-1) }

T(1,1) = T(0,1) { Ignore T(0,-1) }

T(1,1) = 0

Finding T(1,2)-

We have,

- $i = 1$
- $j = 2$
- $(value)_i = (value)_1 = 3$
- $(weight)_i = (weight)_1 = 2$

Substituting the values, we get-

T(1,2) = max { T(1-1 , 2) , 3 + T(1-1 , 2-2) }

T(1,2) = max { T(0,2) , 3 + T(0,0) }

T(1,2) = max {0 , 3+0}

T(1,2) = 3

Finding T(1,3)-

We have,

- $i = 1$
- $j = 3$
- $(value)_i = (value)_1 = 3$
- $(weight)_i = (weight)_1 = 2$

Substituting the values, we get-

T(1,3) = max { T(1-1 , 3) , 3 + T(1-1 , 3-2) }

T(1,3) = max { T(0,3) , 3 + T(0,1) }

T(1,3) = max {0 , 3+0}

T(1,3) = 3

Finding T(1,4)-

We have,

- $i = 1$
- $j = 4$
- $(value)_i = (value)_1 = 3$
- $(weight)_i = (weight)_1 = 2$

Substituting the values, we get-

T(1,4) = max { T(1-1 , 4) , 3 + T(1-1 , 4-2) }

T(1,4) = max { T(0,4) , 3 + T(0,2) }

T(1,4) = max {0 , 3+0}

T(1,4) = 3

Finding T(1,5)-

We have,

- $i = 1$
- $j = 5$
- $(\text{value})_i = (\text{value})_1 = 3$
- $(\text{weight})_i = (\text{weight})_1 = 2$

Substituting the values, we get-

T(1,5) = max { T(1-1 , 5) , 3 + T(1-1 , 5-2) }

T(1,5) = max { T(0,5) , 3 + T(0,3) }

T(1,5) = max {0 , 3+0}

T(1,5) = 3

Finding T(2,1)-

We have,

- $i = 2$
- $j = 1$
- $(\text{value})_i = (\text{value})_2 = 4$
- (weight)i = (weight)2 = 3

Substituting the values, we get-

T(2,1) = max { T(2-1 , 1) , 4 + T(2-1 , 1-3) }

T(2,1) = max { T(1,1) , 4 + T(1,-2) }

T(2,1) = T(1,1) { Ignore T(1,-2) }

T(2,1) = 0

Finding T(2,2)-

We have,

- $i = 2$
- $j = 2$
- $(\text{value})_i = (\text{value})_2 = 4$
- $(\text{weight})_i = (\text{weight})_2 = 3$

Substituting the values, we get-

T(2,2) = max { T(2-1 , 2) , 4 + T(2-1 , 2-3) }

T(2,2) = max { T(1,2) , 4 + T(1,-1) }

T(2,2) = T(1,2) { Ignore T(1,-1) }

T(2,2) = 3

Finding T(2,3)-

We have,

- $i = 2$
- $j = 3$
- $(\text{value})_i = (\text{value})_2 = 4$
- $(\text{weight})_i = (\text{weight})_2 = 3$

Substituting the values, we get-

T(2,3) = max { T(2-1 , 3) , 4 + T(2-1 , 3-3) }

T(2,3) = max { T(1,3) , 4 + T(1,0) }

T(2,3) = max { 3 , 4+0 }

T(2,3) = 4

Similarly, compute all the entries.

After all the entries are computed and filled in the table, we get the following table-

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

T-Table

- The last entry represents the maximum possible value that can be put into the knapsack.
- So, maximum possible value that can be put into the knapsack = 7.

Identifying Items To Be Put Into Knapsack

Following Step-04,

- We mark the rows labelled ―1‖ and ―2‖.
- Thus, items that must be put into the knapsack to obtain the maximum value 7 are-

Item-1 and Item-2

Time Complexity-

- Each entry of the table requires constant time θ(1) for its computation.
- It takes θ(nw) time to fill (n+1)(w+1) table entries.
- It takes θ(n) time for tracing the solution since tracing process traces the n rows.
- Thus, overall θ(nw) time is taken to solve 0/1 knapsack problem using dynamic programming

Code :-

```
# code

# A Dynamic Programming based Python# Program

for 0-1 Knapsack problem # Returns the maximum

value that can# be put in a knapsack of capacity W




defknapSack(W, wt, val, n):

    dp =[0fori inrange(W+1)]   # Making the dp array



    fori inrange(1, n+1):   # taking first i elements

        forw inrange(W, 0, -1): # starting from back,so that we alsohave data of

                                    # previous computation when taking i-1
items

            ifwt[i-1] <=w:

                # finding the maximum value

                dp[w]  =max(dp[w],  dp[w-wt[i-1]]+val[i-1])



    returndp[W]   # returning the maximum value of knapsack




# Driver code

val =[60, 100, 120]

wt =[10, 20, 30]

W =50

n =len(val)

print(knapSack(W, wt, val, n))
```

Output
220

Conclusion-In this way we have explored Concept of 0/1 Knapsack using Dynamic approch

Assignment Question

1. **What is Dynamic Approach?**
2. **Explain concept of 0/1 knapsack**
3. **Difference between Dynamic and Branch and Bound Approach.Which is best?**
4. **Solve one example based on 0/1 knapsack(Other than Manual)**

Reference link

- https://www.gatevidyalay.com/0-1-knapsack-problem-using-dynamic-programming-appr oach/
- https://www.youtube.com/watch?v=mMhC9vuA-70
- https://www.tutorialspoint.com/design_and_analysis_of_algorithms/design_and_analysi s_of_algorithms_fractional_knapsack.htm

Group A

Assignment No: 5

Title of the Assignment: Design n-Queens matrix having first Queen placed. Use backtracking to place remaining Queens to generate the final n-queen's matrix.

Objective of the Assignment: Students should be able to understand and solve n-Queen Problem,and understand basics of Backtracking

Prerequisite:

1. Basic of Python or Java Programming
2. Concept of backtracking method
3. N-Queen Problem

Contents for Theory:

1. **Introduction to Backtracking**
2. **N-Queen Problem**

Introduction to Backtracking

- Many problems are difficult to solve algorithmically. Backtracking makes it possible to solve at least some large instances of difficult combinatorial problems.

Suppose we have to make a series of decisions among various choices, where

- We don't have enough information to know what to choose
- Each decision leads to a new set of choices.
- Some sequence of choices (more than one choices) may be a solution to your problem.

What is backtracking?

Backtracking is finding the solution of a problem whereby the solution depends on the previous steps taken. For example, in a maze problem, the solution depends on all the steps you take one-by-one. If any of those steps is wrong, then it will not lead us to the solution. In a maze problem, we first choose a path and continue moving along it. But once we understand that the particular path is incorrect, then we just come back and change it. This is what backtracking basically is.

In backtracking, we first take a step and then we see if this step taken is correct or not i.e., whether it will give a correct answer or not. And if it doesn't, then we just come back and change our first step. In general, this is accomplished by recursion. Thus, in backtracking, we first start with a partial sub-solution of the problem (which may or may not lead us to the solution) and then check if we can proceed further with this sub-solution or not. If not, then we just come back and change it.

Thus, the general steps of backtracking are:

- start with a sub-solution
- check if this sub-solution will lead to the solution or not
- If not, then come back and change the sub-solution and continue again

Applications of Backtracking:

- N Queens Problem
- Sum of subsets problem

- Graph coloring
- Hamiltonian cycles.

N queens on NxN chessboard

One of the most common examples of the backtracking is to arrange N queens on an NxN chessboard such that no queen can strike down any other queen. A queen can attack horizontally, vertically, or diagonally. The solution to this problem is also attempted in a similar way. We first place the first queen anywhere arbitrarily and then place the next queen in any of the safe places. We continue this process until the number of unplaced queens becomes zero (a solution is found) or no safe place is left. If no safe place is left, then we change the position of the previously placed queen.

N-Queens Problem:

A classic combinational problem is to place n queens on a n*n chess board so that no two attack, i.,e no two queens are on the same row, column or diagonal.

What is the N Queen Problem?

N Queen problem is the classical Example of backtracking. N-Queen problem is defined as, ―given N x N chess board, arrange N queens in such a way that no two queens attack each other by being in the same row, column or diagonal‖.

- For N = 1, this is a trivial case. For N = 2 and N = 3, a solution is not possible. So we start with N = 4 and we will generalize it for N queens.

If we take n=4then the problem is called the 4 queens problem.

If we take n=8 then the problem is called the 8 queens problem.

Algorithm

1) Start in the leftmost column

2) If all queens are place return true

3) Try all rows in the current column.

Do following for every tried row.

a) If the queen can be placed safely in this row then mark this [row, column] as part of the solution and recursively check if placing queen here leads to a solution.

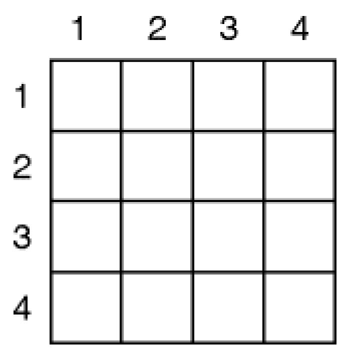
b) If placing the queen in [row, column] leads to a solution then return true.

c) If placing queen doesn't lead to a solution then unmark this [row, column] (Backtrack) and go to step (a) to try other rows.

4) If all rows have been tried and nothing worked,return false to trigger backtracking.

4- Queen Problem

Problem 1 : Given 4 x 4 chessboard, arrange four queens in a way, such that no two queens attack each other. That is, no two queens are placed in the same row, column, or diagonal.



4 x 4 Chessboard

- We have to arrange four queens, Q1, Q2, Q3 and Q4 in 4 x 4 chess board. We will put with queen in ith row. Let us start with position (1, 1). Q1 is the only queen, so there is no issue. partial solution is <1>
- We cannot place Q2 at positions (2, 1) or (2, 2). Position (2, 3) is acceptable. the partial solution is <1, 3>.
- Next, Q3 cannot be placed in position (3, 1) as Q1 attacks her. And it cannot be placed at (3, 2), (3, 3) or (3, 4) as Q2 attacks her. There is no way to put Q3 in the third row. Hence, the algorithm backtracks and goes back to the previous solution and readjusts the position of queen Q2. Q2 is moved from positions (2, 3) to
(2, 4). Partial solution is <1, 4>

- Now, Q3 can be placed at position (3, 2). Partial solution is <1, 4, 3>.
- Queen Q4 cannot be placed anywhere in row four. So again, backtrack to the previous solution and readjust the position of Q3. Q3 cannot be placed on (3, 3) or(3, 4). So the algorithm backtracks even further.
- All possible choices for Q2 are already explored, hence the algorithm goes back to partial solution <1> and moves the queen Q1 from (1, 1) to (1, 2). And this process continues until a solution is found.

All possible solutions for 4-queen are shown in fig (a) & fig. (b)

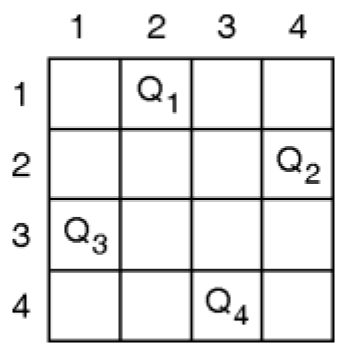


Fig. (a): Solution – 1

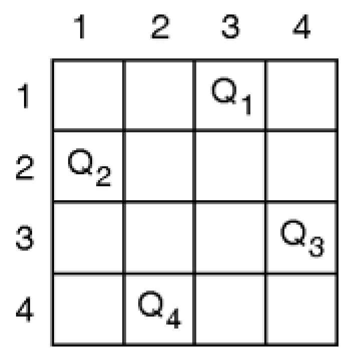
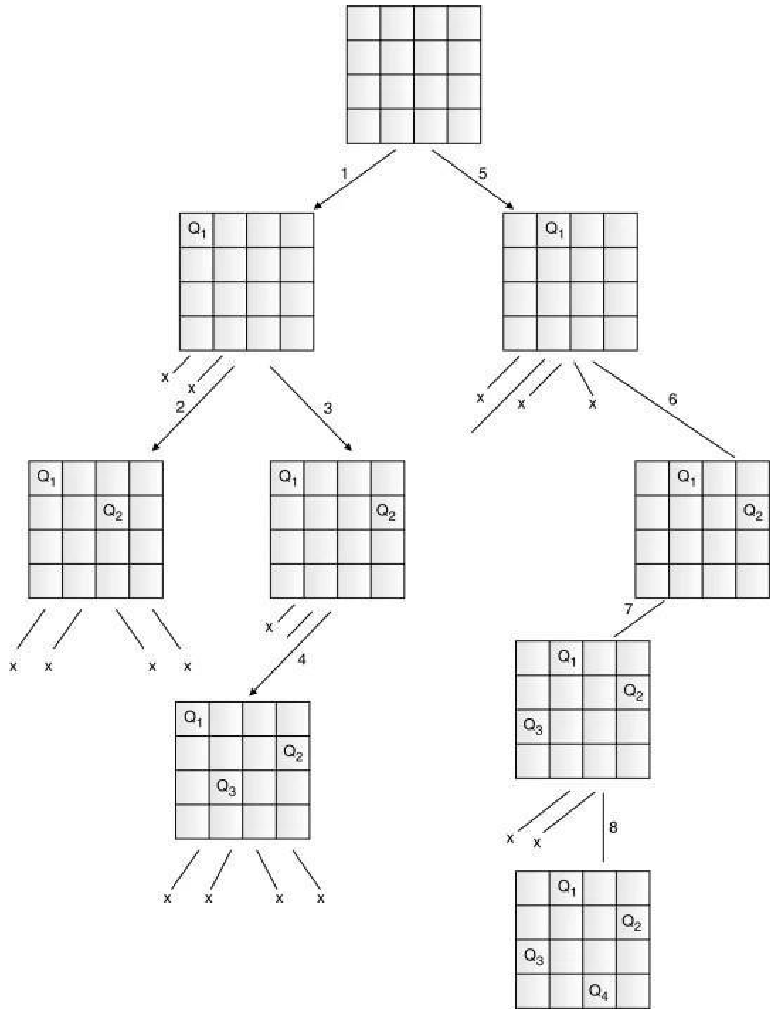


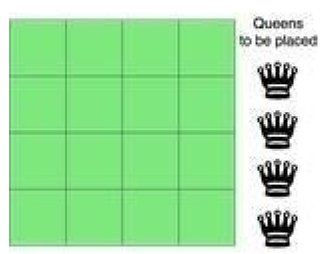
Fig. (b): Solution – 2

Fig. (d) describes the backtracking sequence for the 4-queen problem.

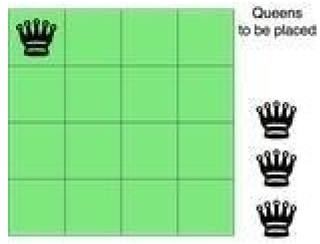


The solution of the 4-queen problem can be seen as four tuples (x_1, x_2, x_3, x_4), where x_i represents the column number of queen Q_i. Two possible solutions for the 4-queen problem are (2, 4, 1, 3) and (3, 1, 4, 2).

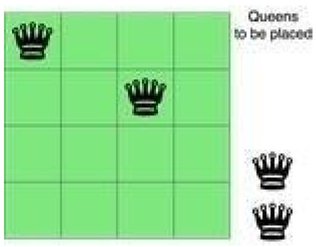
Explanation :

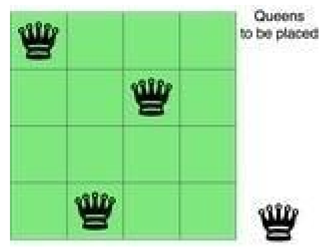


The above picture shows an NxN chessboard and we have to place N queens on it. So, we will start by placing the first queen.



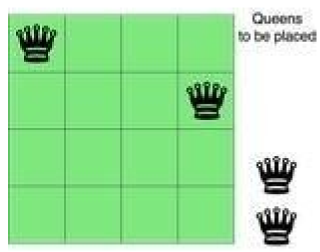
Now, the second step is to place the second queen in a safe position and then the third queen.



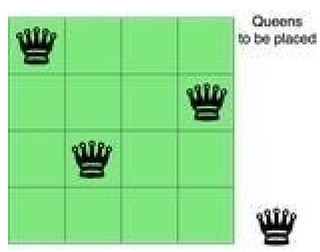


Now, you can see that there is no safe place where we can put the last queen. So, we will just change the position of the previous queen. And this is backtracking.

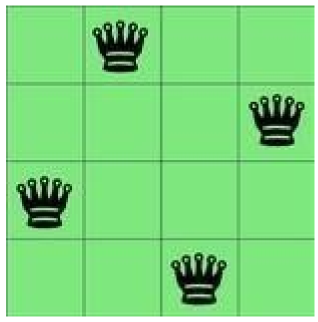
Also, there is no other position where we can place the third queen so we will go back one more step and change the position of the second queen.



And now we will place the third queen again in a safe position until we find a solution.

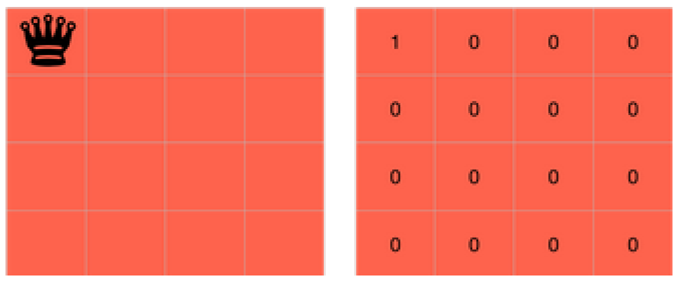


We will continue this process and finally, we will get the solution as shown below.

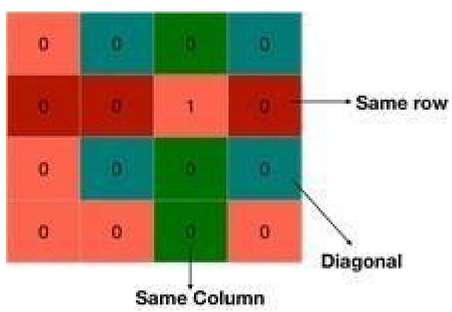


We need to check if a cell (i, j) is under attack or not. For that, we will pass these two in our function along with the chessboard and its size - IS-ATTACK(i, j, board, N).

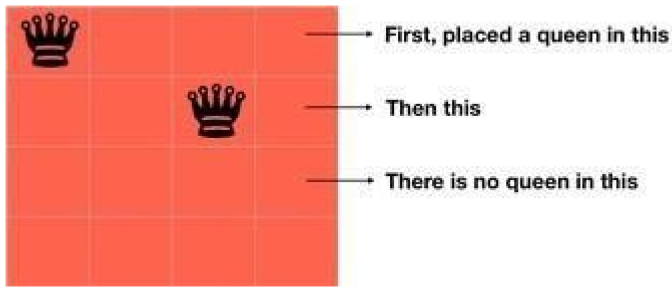
If there is a queen in a cell of the chessboard, then its value will be 1, otherwise, 0.



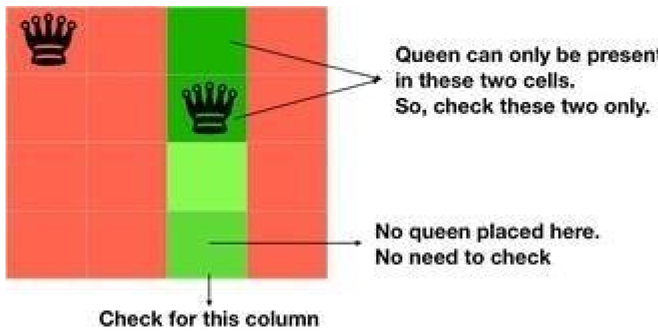
The cell (i,j) will be under attack in three condition - if there is any other queen in row i, if there is any other queen in the column j or if there is any queen in the diagonals.



We are already proceeding row-wise, so we know that all the rows above the current row(i) are filled but not the current row and thus, there is no need to check for row i.



We can check for the column j by changing k from 1 to i-1 in board[k][j] because only the rows from 1 to i-1 are filled.



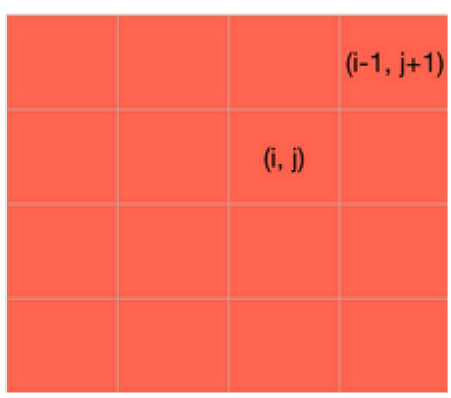
for k in 1 to i-1

if board[k][j]==1

return TRUE

Now, we need to check for the diagonal. We know that all the rows below the row i are empty, so we need to check only for the diagonal elements which above the row i.

If we are on the cell (i, j), then decreasing the value of i and increasing the value of j will make us traverse over the diagonal on the right side, above the row i.



k = i-1

l = j+1

while k>=1 and l<=N

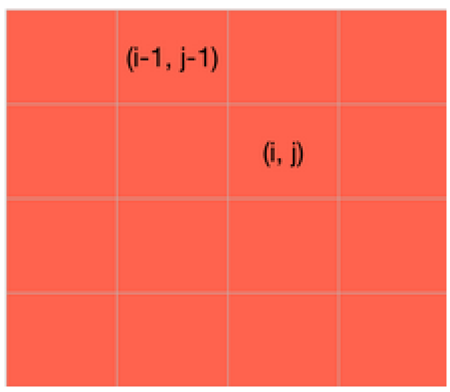
if board[k][l] == 1

return TRUE

k=k-1

l=l+1

Also if we reduce both the values of i and j of cell (i, j) by 1, we will traverse over the left diagonal, above the row i.



k = i-1

l = j-1

while k>=1 and l>=1

if board[k][l] == 1

return TRUE

k=k-1

l=l-1

At last, we will return false as it will be return true is not returned by the above statements and the cell (i,j) is safe.

We can write the entire code as:

```
IS-ATTACK(i, j, board, N)
 // checking in the column j
 for k in 1 to i-1
  if board[k][j]==1
   return TRUE

 // checking upper right diagonal
 k = i-1
 l = j+1
 while k>=1 and l<=N
  if board[k][l] == 1
   return TRUE
  k=k+1
  l=l+1

 // checking upper left diagonal
 k = i-1
 l = j-1
```

```
while k>=1 and l>=1
  if board[k][l] == 1
    return TRUE
  k=k-1
  l=l-1

return FALSE
```

Now, let's write the real code involving backtracking to solve the N Queen problem.

Our function will take the row, number of queens, size of the board and the board itself - N-QUEEN(row, n, N, board).

If the number of queens is 0, then we have already placed all the queens.

```
if n==0
  return TRUE
```

Otherwise, we will iterate over each cell of the board in the row passed to the function and for each cell, we will check if we can place the queen in that cell or not. We can't place the queen in a cell if it is under attack.

```
for j in 1 to N
  if !IS-ATTACK(row, j, board, N)
    board[row][j] = 1
```

After placing the queen in the cell, we will check if we are able to place the next queen with this arrangement or not. If not, then we will choose a different position for the current queen.

```
for j in 1 to N
  ...
```

```
  if N-QUEEN(row+1, n-1, N, board)
    return TRUE


  board[row][j] = 0
```

if N-QUEEN(row+1, n-1, N, board) - We are placing the rest of the queens with the current arrangement. Also, since all the rows up to 'row' are occupied, so we will start from 'row+1'. If this returns true, then we are successful in placing all the queen, if not, then we have to change the position of our current queen. So, we are leaving the current cell board[row][j] = 0 and then iteration will find another place for the queen and this is backtracking.

Take a note that we have already covered the base case - if n==0 → return TRUE. It means when all queens will be placed correctly, then N-QUEEN(row, 0, N, board) will be called and this will return true.

At last, if true is not returned, then we didn't find any way, so we will return false.

```
N-QUEEN(row, n, N, board)
...
 return FALSE
```

```
N-QUEEN(row, n, N, board)
 if n==0
  return TRUE


 for j in 1 to N
  if !IS-ATTACK(row, j, board, N)
   board[row][j] = 1
```

if N-QUEEN(row+1, n-1, N, board)

return TRUE

board[row][j] = 0 //backtracking, changing current decision

return FALSE

Code :-

```
# Python3 program to solve N Queen
# Problem using backtracking
global N
N = 4

def printSolution(board):
	for i in range(N):
		for j in range(N):
			print(board[i][j], end = " ")
		print()

# A utility function to check if a queen can
# be placed on board[row][col]. Note that this
# function is called when "col" queens are
# already placed in columns from 0 to col -1.
# So we need to check only left side for
# attacking queens
def isSafe(board, row, col):

	# Check this row on left side
	for i in range(col):
		if board[row][i] == 1:
			return False

	# Check upper diagonal on left side
	for i, j in zip(range(row, -1, -1),
					range(col, -1, -1)):
		if board[i][j] == 1:
			return False

	# Check lower diagonal on left side
	for i, j in zip(range(row, N, 1),
					range(col, -1, -1)):
		if board[i][j] == 1:
			return False

	return True

def solveNQUtil(board, col):

	# base case: If all queens are placed
	# then return true
	if col >= N:
		return True

	# Consider this column and try placing
	# this queen in all rows one by one
	for i in range(N):

		if isSafe(board, i, col):

			# Place this queen in board[i][col]
```

```
                board[i][col] = 1

                # recur to place rest of the queens
                if solveNQUtil(board, col + 1) == True:
                        return True

                # If placing queen in board[i][col
                # doesn't lead to a solution, then
                # queen from board[i][col]
                board[i][col] = 0

        # if the queen can not be placed in any row in
        # this column col then return false
        return False

# This function solves the N Queen problem using
# Backtracking. It mainly uses solveNQUtil() to
# solve the problem. It returns false if queens
# cannot be placed, otherwise return true and
# placement of queens in the form of 1s.
# note that there may be more than one
# solutions, this function prints one of the
# feasible solutions.
def solveNQ():
        board = [ [0, 0, 0, 0],
                [0, 0, 0, 0],
                [0, 0, 0, 0],
                [0, 0, 0, 0] ]

        if solveNQUtil(board, 0) == False:
                print ("Solution does not exist")
                return False

        printSolution(board)
        return True

# Driver Code
solveNQ()
```

Output:-

```
0 0 1 0
1 0 0 0
0 0 0 1
0 1 0 0
```

Conclusion- In this way we have explored Concept of Backtracking method and solve n-Queen problem using backtracking method

Assignment Question

1. **What is backtracking? Give the general Procedure.**
2. **Give the problem statement of the n-queens problem. Explain the solution**
3. **Write an algorithm for N-queens problem using backtracking?**
4. **Why it is applicable to N=4 and N=8 only?**

Reference link

- https://www.codesdope.com/blog/article/backtracking-explanation-and-n-queens-problem/