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Homework 0

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(A)

$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

1) 2A - B

$\rightarrow 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

2) ||A|| and the angle of A relative to positive X axis

$||A|| = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{1+4+9} = \sqrt{14}$

$\cos\alpha = \dfrac{x}{\sqrt{(1)^2+(2)^2+(3)^2}} = \dfrac{1}{\sqrt{14}}$

$\alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{14}}\right) = \cos^{-1}(0.2673) = 74.5^\circ$

3) $\hat{A}$, a unit vector in the direction of A

$||A|| = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{1+4+9} = \sqrt{14}$

$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k} = \dfrac{\hat{i}}{\sqrt{14}} + \dfrac{2}{\sqrt{14}}\hat{j} + \dfrac{3}{\sqrt{14}}\hat{k}$

4) the direction cosine of A

$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$

$D_A = \sqrt{(1-0)^2 + (2-0)^2 + (3-0)^2}$
$= \sqrt{1+4+9} = \sqrt{14}$

$\cos\alpha = \frac{x}{\sqrt{14}}$ $\cos\beta = \frac{y}{\sqrt{14}}$ $\cos\gamma = \frac{z}{\sqrt{14}}$

$\cos\alpha = \frac{1}{\sqrt{14}}$ $\cos\beta = \frac{2}{\sqrt{14}}$ $\cos\gamma = \frac{3}{\sqrt{14}}$

5) A · B and B · A

$A \cdot B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

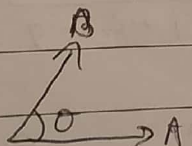
$\bar{a} \cdot \bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\bar{A} \cdot \bar{B} = (1)(4) + (2)(5) + (3)(6)$
$= 4 + 10 + 18 = 32$

$\bar{B} \cdot \bar{A} = (4)(1) + (5)(2) + (6)(3) = 4 + 10 + 18 = 32$

6) the angle between A and B

Suppose θ is the angle betn A & B



$\cos\theta = \frac{\bar{A} \cdot \bar{B}}{\|A\| \, \|B\|}$

$\bar{A}\cdot\bar{B} = 32$

$|A| = \sqrt{(1)^2+(2)^2+(3)^2} = \sqrt{1+4+9} = \sqrt{14}$

$|B| = \sqrt{(4)^2+(5)^2+(6)^2} = \sqrt{16+25+36} = = \sqrt{77}$

$$\cos\theta = \frac{32}{\sqrt{14}\sqrt{77}}$$

$$\theta = \cos^{-1}\left(\frac{32}{\sqrt{14}\sqrt{77}}\right) = 12.9^\circ$$

7) a vector which is perpendicular to A

$\bar{A} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\qquad \bar{A}\cdot\bar{u} = 0$

$\bar{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\qquad \bar{u}\cdot\bar{A} = x + 2y + 3z = 0$

~~Suppose~~ $\cancel{x=1, y=1, z=1}$

Suppose $x=1$, $y=1$, $z=-1$

$1 + 2 + (-3) = 0$

$\underline{\underline{3 - 3 = 0}}$

$\vec{u} = (1, 1, -1)$

8) $A \times B$ and $B \times A$ $\qquad A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$$A \times B = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \bar{i}(12-15) - \bar{j}(6-12) + \bar{k}(5-8)$$

$$= (-3)\bar{i} - \bar{j}(-6) + \bar{k}(-3)$$

$$= -3\bar{i} + 6\bar{j} - 3\bar{k}$$

$$B \times A = \begin{vmatrix} i & j & k \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = \bar{i}(15-12) - \bar{j}(12-6) + \bar{k}(8-5)$$

$$= 3\bar{i} - 6\bar{j} + 3\bar{k}$$

9) a vector which is perpendicular to both A and B

$$\vec{u} = \frac{\bar{A} \times \bar{B}}{|\bar{A} \times \bar{B}|} =$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \bar{i}(12-15) - \bar{j}(6-12) + \bar{k}(5-8)$$

$$= -3\bar{i} + 6\bar{j} - 3\bar{k}$$

$$= (-3, 6, -3)$$

$$|\bar{A} \times \bar{B}| = \sqrt{(-3)^2 + (6)^2 + (-3)^2} = \sqrt{9+36+9} = \sqrt{54}$$

$$\bar{u} = \frac{-3\bar{i} + 6\bar{j} - 3\bar{k}}{\sqrt{54}} = \frac{-3}{\sqrt{54}}\bar{i} + \frac{6}{\sqrt{54}}\bar{j} - \frac{3}{\sqrt{54}}\bar{k}$$

11)

10) Linear dependency between A, B, C

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

$$C_1\bar{A} + C_2\vec{B} + C_3\bar{C} = \bar{0}$$

$$C_1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + C_2\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + C_3\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{bmatrix} = 1\begin{bmatrix} 5 & 1 \\ 6 & 3 \end{bmatrix} - 4\begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix} + (-1)\begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$= 1(15-6) - 4(6-3) - 1(12-15)$$

$$= (9) - 4(3) - 1(-3)$$

$$= 9 - 12 + 3 = 0$$

$\therefore$ It's linear dependent

11) A^TB and AB^T

A^TB = $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

~~A^TB~~ $A^TB = \underset{1\times3}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}} = [4 + 10 + 18] = \underline{32}$

$AB^T =$ $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $B^T = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$

$AB^T = \underset{3\times1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} \underset{1\times3}{\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}}$ ~~$= 4 + 10 + 18$~~

$$= \underset{3\times3}{\begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}}$$

(B) $A \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$

1) $2A - B$

$$2\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (2-1) & (4-2) & (6-1) \\ (8-2) & (-4-1) & (6-(-4)) \\ (0-3) & (10-(-2)) & ((-2)-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$$

2) AB and BA

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+4+9) & (2+2-6) & (1-8+3) \\ (4-4+9) & (8-2-6) & (4+8+3) \\ (0+10-3) & (0+(-5)+(2)) & (0-20-1) \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & +7 & -21 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+8+0) & (2-4+5) & (3+6-1) \\ (2+4+0) & (4-2-20) & (6+3+4) \\ (3-8+0) & (6+4+5) & (9-6-1) \end{bmatrix} = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

3) $(AB)^T$ and $B^T A^T$

$$AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & -7 & -21 \end{bmatrix}$$ from previous example

$$(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & -7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$B^T A^T = \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} (1+4+9) & (4-4+9) & (0+10-3) \\ (2+2-6) & (8-2-6) & (0+5+2) \\ (1-8+3) & (4+8+3) & (0-20-1) \end{bmatrix} = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

4) $|A|$ and $|C|$ (note $A - 10$)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$|A| = 1 \begin{bmatrix} -2 & 3 \\ 5 & -1 \end{bmatrix} - 2 \begin{bmatrix} 4 & 3 \\ 0 & -1 \end{bmatrix} + 3 \begin{bmatrix} 4 & -2 \\ 0 & 5 \end{bmatrix}$$

$$= 1(2-15) - 2(-4-0) + 3(20-0)$$

$$= -13 - 2(-4) + 3(20)$$

$$= -13 + 8 + 60 = 55$$

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$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$$

$$|C| = 1\begin{bmatrix} 5 & 6 \\ 1 & 3 \end{bmatrix} - 2\begin{bmatrix} 4 & 6 \\ -1 & 3 \end{bmatrix} + 3\begin{bmatrix} 4 & 5 \\ -1 & 1 \end{bmatrix}$$

$$= 1(15-6) - 2(12-(-6)) + 3(4-(-5))$$

$$= 9 - 2(18) + 3(9)$$

$$= 9 - 36 + 27 = \cancel{29+27=0}$$

$$= 36 - 36 = 0$$

5) The matrix (A, B or C) in which the row vectors form an orthogonal set

A $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & -3 \\ 0 & 5 & -1 \end{bmatrix}$ Row vector

$AR_1 = [1 \ 2 \ 3]$ $AR_2 = [4 \ -2 \ -3]$

$AR_3 = [0 \ 5 \ -1]$

if ~~two~~ dot product of two vectors is zero then it's orthogonal

$AR_1 . AR_2 = (4) + (-4) + 9 = 9$ (not orthogonal)

$AR_2 . AR_3 = (0 - 10 - 3) = -13$ (not orthogonal)

$AR_3 . AR_1 = (0 + 10 - 3) = 7$ (not orthogonal)

$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$ Row vector

~~RB~~ $BR_1 = [1 \ 2 \ 1]$

$BR_2 = [2 \ 1 \ -4]$

$BR_3 = [3 \ -2 \ 1]$

$BR_1 \cdot BR_2 = [(2) + (2) - 4] = 0$

$\therefore$ orthogonal to each other

$BR_2 \cdot BR_3 = (6 - 2 - 4) = 0$

$\therefore$ orthogonal set

$BR_1 \cdot BR_3 = (3 - 4 + 1) = 0$

$\therefore$ orthogonal set

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$$

Row vector

$CR_1 = [1 \; 2 \; 3]$ $\quad CR_2 = [4 \; 5 \; 6]$

$CR_3 = [-1 \; 1 \; 3]$

$CR_1 \cdot CR_2 = (4 + 10 + 18) = 32$

$\therefore$ not orthogonal set

$CR_2 \cdot CR_3 = (-4 + 5 + 18) = 19$

$\therefore$ not orthogonal set

$CR_3 \cdot CR_1 = (-1 + 2 + 9) = 10$

$\therefore$ not orthogonal set

6) A^{-1} and B^{-1}

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \qquad A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$\cancel{\operatorname{adj}(A)} \begin{bmatrix} (2 - 15) & (-4 - 0) & (20 - 0) \\ (-2 - 15) & (-1 - 0) & (5 - 0) \\ (6 + 6) & (3 - 12) & (-2 - 8) \end{bmatrix}$$

matrix of minors

$$\cancel{adj(A)} = \begin{bmatrix} -13 & -4 & 20 \\ -17 & -1 & 5 \\ 12 & -9 & -10 \end{bmatrix}$$

cofactor

$$\therefore \cancel{adj(A)} \qquad = \begin{bmatrix} -13 & 4 & 20 \\ 17 & -1 & -5 \\ 12 & 9 & -10 \end{bmatrix}$$

$$\therefore adj(A) = \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$|A| = 1(2-15) \quad -2(-4-0) \quad +3(20-0)$$
$$= -13 + 8 + 60$$
$$= \underline{55}$$

$$A^{-1} = \frac{1}{55}\begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|}\, adj(B) \qquad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1-8) & (2+12) & (-4-3) \\ (2+2) & (1-3) & (-2-6) \\ (-8-1) & (-4-2) & (1-4) \end{bmatrix} = \begin{bmatrix} -7 & 14 & -7 \\ 4 & -2 & -8 \\ -9 & -6 & -3 \end{bmatrix}$$ matrix of minors

$$\text{cofactor} = \begin{bmatrix} -7 & -14 & -7 \\ -4 & -2 & 8 \\ -9 & 6 & -3 \end{bmatrix} = adj(B) = \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$$

$$|B| = (1-8) - 2(2+12) + 1(-4-3)$$
$$= -7 - 28 - 7 = -42$$

$$B^{-1} = \frac{-1}{42}\begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$$

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c) $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

1. the eigenvalues and corresponding eigenvector of A

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = 0$$

$$= (1-\lambda)(2-\lambda) - 6 = 0$$

$$= 2 - \lambda - 2\lambda + \lambda^2 - 6 = 0$$

$$= 2 - 3\lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0 \quad \therefore \lambda = 4 \text{ or } \lambda = -1$$

$\therefore$ eigenvalues of A
$\lambda = 4$ or $\lambda = -1$

eigenvector for $\lambda = 4$

$$\begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = \begin{bmatrix} 1-4 & 2 \\ 3 & 2-4 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_1 + R_2 \rightarrow R_1$

$$= \begin{bmatrix} 0 & 0 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$3x_1 - 2x_2 = 0$

$3x_1 = +2x_2$

$\therefore$ Suppose $x_2 = 3$

$\therefore 3x_1 = +2 \times 3$

~~$x_1 = +2/3$~~

$x_1 = 2$

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$$\text{Eigenvector} = \begin{bmatrix} \cancel{2/3} \\ \cancel{1} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Now for $\lambda = -1$

$$\begin{bmatrix} 1-(-1) & 2 \\ 3 & 2-(-1) \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\frac{3}{2} R_1 - R_2 \rightarrow R_1$

$$\begin{bmatrix} 0 & 0 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$3x_1 + 3x_2 = 0$

$3x_1 = -3x_2$

$x_1 = -x_2$

$\therefore$ eigenvector $= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

2) the matrix $V^{-1}AV$ where V is composed of eigenvectors of A

$A \Rightarrow$ ~~$V = \begin{bmatrix} 2/3 & -1 \\ 1 & 1 \end{bmatrix}$~~ $V = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$

$V^{-1} = \frac{1}{|V|} V_{adj}$

~~$|V| = \sqrt{\ }$~~

$|V| = (2) - (-3)$

$= 2 + 3 = 5$

$|V| = 5$

$V_{adj} = \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} = \cancel{\ }$

$V^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}$

$$V^{-1} A V = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 & +1 \\ 12 & -1 \end{bmatrix} = \cancel{\frac{1}{5}\begin{bmatrix} 20 & +2 \\ 0 & -5 \end{bmatrix}}$$

$$= \frac{1}{5}\begin{bmatrix} 20 & 0 \\ 0 & -5 \end{bmatrix} = \cancel{\begin{bmatrix} 4 & -2/5 \\ 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$$

3) dot product betn eigenvectors of A

$$\bar{x}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \bar{x}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\bar{x}_1 . \bar{x}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = 2 \times (-1) + 3(1)$$

$$= -2 + 3 = 1$$

4) dot product betn eigenvectors of B

$$B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$$

$$\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$B \cdot \lambda I = 0 \qquad \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix}$$

$$(2-d)(5-d) - 4 = 0$$

$$10 - 2d - 5d + d^2 - 4 = 0$$

$$d^2 - 7d + 6 = 0$$

$$d^2 - 6d - d + 6 = 0$$

$$d(d-6) - 1(d-6) = 0$$

$$d = 6 \text{ or } d = 1$$

eigenvector for $d = 6$

$$\begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$R_1 - 2R_2 \rightarrow R_2$

$$\begin{bmatrix} -4 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-4x_1 - 2x_2 = 0$$

$$-4x_1 = 2x_2$$

$$x_2 = -2x_1$$

$$\therefore \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

eigenvector for $d = 1$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$2R_1 + R_2 \rightarrow R_1$

$$\begin{bmatrix} 0 & 0 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -2x_1 + 4x_2 = 0$$

$$4x_2 = 2x_1$$

$$x_1 = 2x_2$$

$$\therefore \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 \\ -2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \quad 2 - 2 = 0$$

5) the propeety of eigenvector at B & it's reson

As their dot product is zero so they are orthogonal

D) $f(x) = x^2 + 3$ $\qquad g(x, y) = x^2 + y^2$

1) first & second derivatives of f(x) with respect to $f'(x)$ and $f''(x)$

$\therefore f'(x) = 2x + 3$

$f''(x) = 2$

2) the partial derivative of $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$

$\frac{\partial g}{\partial x} = \underline{\underline{2x}}$ $\qquad \frac{\partial g}{\partial y} = \underline{\underline{2y}}$

3) the gradient vector $\nabla g(x, y)$

$$\nabla g(x, y) = \begin{bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

5) the probability density function (pdf) of univariate Gaussian (normal) distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-u}{\sigma}\right)^2}$$

$\therefore$ where u = mean

σ = standard deviation