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Assignment 6

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1. Stereo

(a) Sparse stereo matching:
- We do correspondence with just few points in the image
- We try to find matching pts. for specific pts.

Advantage:
1) It is efficient
2) unambigious
3) used with far views

Disadvantage:
i) may not find correspondance due to change in intensity.

Dense Stereo Matching:
- We take many pixels and try to find the correspondance for each of the pixels.
- Instead of feature pts. we compare all patches.

Advantage:
i) Accurate Matching

Disadvantage:
i) It can be ambigious
2) we required close view of the images

1) Normalized Cross Correlation:

- Take a window in each view & multiply corresponding pixels in them

$$\cancel{\phi(w_1, w_2) = \sum_i \frac{(w_1(x_i, y_i) - \mu_1)}{\sigma} \cdot (w_2(x_i}$$

$$\phi(w_1, w_2) \quad \sum_i \frac{(w_1(x_i, y_i) - \mu_{w_1})\,(w_2(x_i, y_i) - \mu_{w_2})}{\sigma_{w_1}\,\sigma_{w_2}}$$

- we first normalize each pixel with centre of mean & variance to overcome the problem of just ~~correspondance~~ considering the value of pixels by the approach of correlation.

- Higher the value NCC, Higher correspondence i.e. Higher matching similarity.

2) Sum of Squared Distance (SSD):
In this we took two window, take the corresponding difference of the pixels in different views & square them

$$\phi(w_1, w_2) = \sum_i (w_1(x_i, y_i) - w_2(x_i, y_i))^2$$

The lower the SSD, Higher the correspondance

By allowing search space to be the entire image, we are prone to mistake of making an incorrect correspondance betn pts & inform making wrong reconstruction.
And so we find ambigious problem, where

there are more than one suitable match in other view for the point in current view.

c) Given:

$f = 10$ $\quad x_l = (100, 200)$

$T = 100$ $\quad x_r = (103, 200)$

$\therefore z = f\frac{T}{d}$

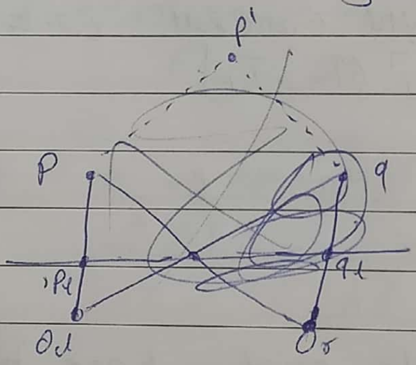
$d = \sqrt{\cancel{x_r - x_l}\ \sqrt{(103-100)^2}}$

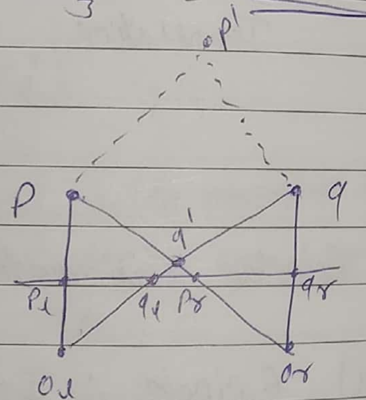
$d = \sqrt{(103-100)^2 - (200-200)^2}$

$= \sqrt{9-0} = \sqrt{9} = 3$

$z = \frac{10 \times 100}{3} = \frac{1000}{3} = \underline{333.33}$

d)





In this, there are two pts p & q & p_l, p_r & q_l, q_r } ⇒ corresponding pts on left & right image

If we wrongly correspond p_l to q_r for point p & p_r to q_l for q then it ~~gives~~ we get reconstruction at p' & q' but the actual reconstruction is p & q which are totally wrong.

This is the when we choose wrong matches or if there are many ambigious pts. that match for a single pt. It leads to ambigious.

e) $R_l, T_l \Rightarrow$ Rotation & Translation of left cameras with respect to world

$R_r, T_r \Rightarrow$ Rotation & Translation of right cameras. w.r.t. world.

Rotation of right cameras w.r.t. to left camera

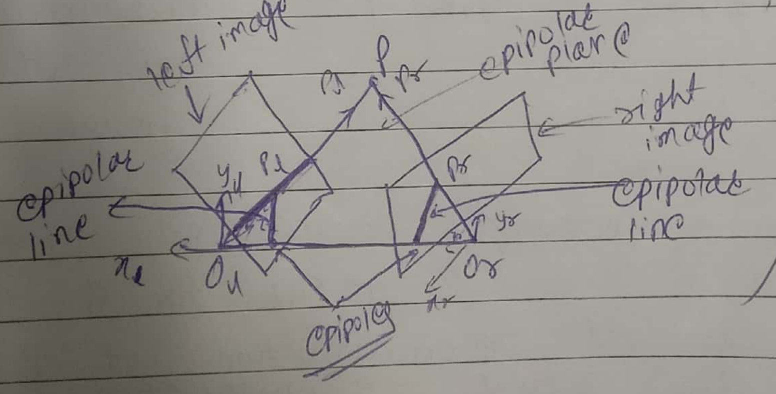
$$\Rightarrow R_r = R_l^T R_r$$

Translation of right cameras w.r.t. to left

$$\Rightarrow T = R_l^T (T_r - T_l)$$

2) Epipolar Geometry :→

a) Epipole : It is the point where baseline i.e. T of epipolar plane intersects the image.



It shows that epipolar lines are formed when epipolar plane intersects the image.

b) $\therefore$ Essential Matrix
$$E = R^T [T]_x$$

$R \Rightarrow$ Rotational matrix
$[T]_x \Rightarrow$ skew symmetric matrix

$$T = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \quad \text{then } [T]_x = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix}$$

P_l & $P_r \Rightarrow$ camera corresponding pts.

Epipolar Constraint
$$\Rightarrow \boxed{P_r^T E P_l = 0}$$

c) Fundamental matrix:
$$F = (K_r^{*-1^T}) E (K_l^*)^{-1}$$

where, $E \Rightarrow$ essential matrix with ~~essential~~ external parameter
K_r^* & $K_l^* \Rightarrow$ internal camera coordinates of left & right

Epipolar Constraint
$$\Rightarrow \boxed{P_r^T F P_l = 0}$$

d) Rank of both Essential (E) & Fundamental (F) matrix $\Rightarrow$ 2

bcz $[T]_x \Rightarrow$ has only 2 independents rows
$\therefore$ T is rank 2 matrix
that's why E & F are Rank 2

~~e) Given:~~

~~$P_t \Rightarrow$ left image point~~

e) The corresponding right epipolar line ~~plane~~ given by:

$$\boxed{F P_l}$$

f) The corresponding left epipolar line given by:

$$\boxed{F^T P_r}$$

g) **Weak Calibration:**

Is an approach to find E & F using 8 point Correspondance algo & Epipolar Constraint.

$\therefore$ Given $\{p_i'\}_{i=1}^{n} \longleftrightarrow \{p_i\}_{i=1}^{m}$ $\quad m \geq 8$

left pts $\quad$ right pts.

$p_i' \Rightarrow$ left image pts

$p_i \Rightarrow$ right image pts.

h) Given. $x_1' = 200 \quad y_1' = 200$ } left image pts

$x_1 = 50 \quad y_1 = 100$ } right img. pts

$$\begin{bmatrix} x_1 x_1' & x_1 y_1' & x_1 & y_1 x_1' & y_1 y_1' & y_1 & x_1' & y_1' & 1 \\ \vdots & & & & & & & & \\ \vdots & & & & & & & & \end{bmatrix} \Big[\quad \Big]$$

$$= \begin{bmatrix} 50\times100 & 50\times200 & 100 & \cancel{100}\times100 & 100\times200 & 100 & 100 & 200 & 1 \\ \vdots \\ \vdots \end{bmatrix}$$

$$= \begin{bmatrix} 5000 & 10000 & 100 & 10000 & 20000 & 100 & 100 & 200 & 1 \\ \vdots \end{bmatrix}$$

i) Normalize:

Normalize the pt. in 8-point algo. to the center of mean & std (μ) by follow.

Given:

$$\{p_i'\}_{i=1}^{m} \Longleftrightarrow \{p_i\}_{i=1}^{m} \Rightarrow \{q_i'\}_{i=1}^{m} \Longleftrightarrow \{q_i\}_{i=1}^{m}$$

$$q_i = \frac{p_i - \mu_p}{\sigma_p} \qquad q_i' = \frac{p_i' - \mu_p'}{\sigma_p'}$$

μ = mean
σ = std

$\left.\begin{matrix}\mu_p' \\ \sigma_p'\end{matrix}\right\}$ mean & std. of points in left image

$\left.\begin{matrix}\mu_p \\ \sigma_p\end{matrix}\right\}$ mean & std. of points in right image

- We apply normalize to make it consistent & work well otherwise it won't be consistent i.e. sometimes it works but sometimes not.

$\therefore$ In Matrix form:

$$q_i = \underbrace{\begin{bmatrix} 1/\sigma_p & 0 & 0 \\ 0 & 1/\sigma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\mu_x \\ 0 & 1 & -\mu_y \\ 0 & 0 & 1 \end{bmatrix}}_{M} p_i$$

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$\therefore q_i = M p_i \iff q_i' = M p_i'$

$\therefore$ So, the fundamental matrix (F) can be recovered from fundamental matrix (F') as follows

$F = (M')^T F' . M$

j) All epipolar line must pass through the epipole

~~$\therefore$ the epipolar constraint $P_r^T F P_l = 0$~~

becomes

$e_r^T F P_l = 0 \quad \therefore e_r \Rightarrow$ ~~right epipole.~~

but it is true only if

$e_r^T F = 0 \quad \therefore \Rightarrow F^T e_r = 0$

Now Do SVD on F^T & right epipole plane is the last column of U

$F^T = U D^T V$

where e_r is also called as left null space of F

Similarly, for the left epipole

$P_r^T F e_l = 0 \quad \forall P_r \rightarrow F e_l = 0$

$\rightarrow$ ~~e_l is the~~ last column of V ~~on SVD~~ after applying SVD on F

$e_l \Rightarrow$ right null space of F.

Scanned by CamScanner

3) Reconstruction:

a) stereo pairs can be rectified using following steps.

i) align right img to left img
ii) align both img with baseline
iii) make the img coplanar

After rectification, corresponding pts of both right & left are aligned Horizontally in the same line

b) Different Approaches for reconstructions are as follows:

i) Absolute Reconstruction:
use when both intrinsic & extrinsic parameters are known

ii) Eucledian Reconstruction:
used when we only known intrinsic & extrinsic unknown (upto scale)

iii) None Parameters known:
Recons. upto unknown 3D projective map is performed.

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c) $\begin{bmatrix} | & | & | \\ P_l & -RP_r & P_l \times RP_r \\ | & | & | \end{bmatrix}$ is the matrix

to find coefficients (a, b, c)

$\therefore \; m \begin{bmatrix} a \\ b \\ c \end{bmatrix} = T \Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = M^{-1} T$

d) $P = aP_l + \frac{1}{2} c\omega$

or we can also write as

~~$P = a P_l + \frac{1}{2}$~~ $P = \frac{1}{2}(aP_l + bRP_r + T)$

$\omega = P_l \times RP_r$

P_l & P_r $\Rightarrow$ points at left & right image coordinate

$R \Rightarrow$ Rotation of right image w.r.t. to left image

$T \Rightarrow$ Translation of right image w.r.t. to left image

e) We don't know the scale, b/c we only known intrinsic parameter & extrinsic parameters are unknown to us, due to which we don't know the baseline.

Unknown scale can be removed using following steps:

1) Estimate fundamental F by weak calliboration

2) Find E from F

3) Normalize E

4) Recover T upto unknown scale sign

5) Recover R upto unknown sign

6) Resolve ambiguity in T & R

7) Reconstruct upto unknown scale.

f) $\hat{E} = \frac{2}{tr(E^T E)} E$ } This eq^n can be used to normalize to get baseline of 1

g) Unknown sign ~~can be~~ of R & T can be determined by extensive search using euclidian Reconstruction.

As the possible sign of
TR = (++, +-, -+, --)

We reconstruct (using Triangularity) each option & choose to solution where all z are positive.