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Assignment 1

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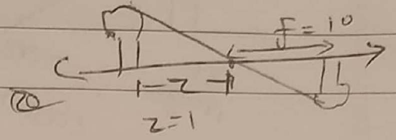
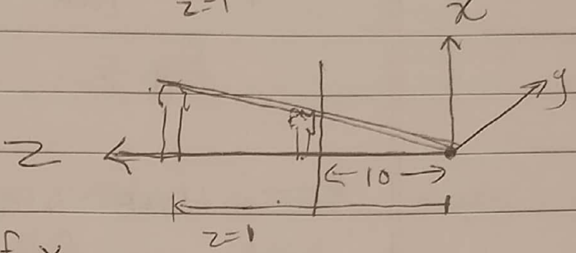
1. a) $f = 10$, $P = (3, 2, 1)$



$\therefore P = (X, Y, Z)$
$= (3, 2, 1)$



$\therefore \dfrac{-u}{f} = \dfrac{X}{Z}$ $u = -f\dfrac{X}{Z}$

$\dfrac{-v}{f} = \dfrac{Y}{Z}$ $v = -f\dfrac{Y}{Z}$

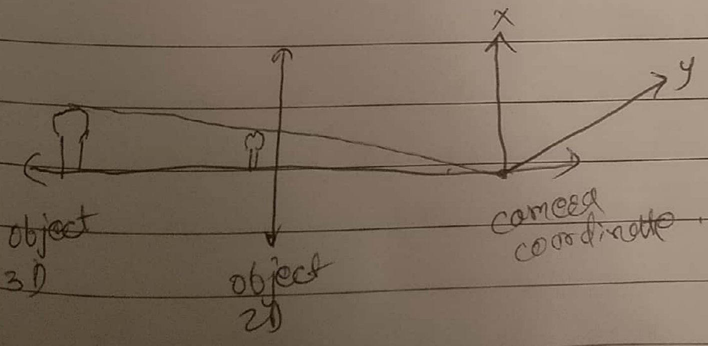
$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} -f & 0 \\ 0 & -f \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{1}\begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$= [-30, -20]$ } it is negative bcz the image is upside down.

To eliminate this effect we will flip the image, so the focal length should be +ve.

$\therefore$ the answer is $P = [30, 20]$

b)



In this case the image plane is ahead of the camera.

fig 1

In this ~~plane~~ model the image plane is behind the camera model.

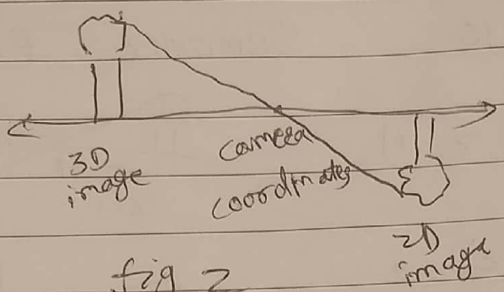


fig 2

Here we can see the image we get in fig 2 is upside down & in fig 1 is not upside down.

So we can say that fig 1 is better to visualize than fig 2.

In real word scenario also we aren't get any images which we have to see as upside down.

So for following two reason
1) No inversion
2) better to visualize

we can justify the pinhole camera model where image plane is infront of camera model is better than another one.

(c) We know the equation
i.e. $u = f\frac{X}{Z}$

$v = f\frac{Y}{Z}$

$\therefore \quad u \propto f$

For focal length. the ~~coordi~~ image coordinates are directly proportional to focal length, so as the focal length increases we get bigger image coordinate i.e. more sharpen image.

$u \propto \frac{1}{Z}$ As the distance between image gets bigger then image going to obtain is small which cause visual image problem.
It happen's because image coordinates is inversely proportional to distance betⁿ objects.

(d) Given
2D ~~p~~ $= (1, 1) = (x, y)$

to make it 2D Homogenous Coordinate
$2DH = (x, y, z) = (1, 1, 1)$
we consider $z = 1$

Another 2DH point ~~a~~ ~~p~~ is $= (2, 2, 2)$
we can see that they are in the multiple
~~we have to make~~ ~~su~~

(e) 2DH point = $(x, y, z) = (1, 1, 2)$

to make it 2D
$(x, y) = (x/z, y/z) = (1/2, 1/2)$

(f) 2DH point $(1, 1; 0)$
These points are called <u>Ideal points</u> whose third coordinate is zero.

There is another name called as <u>points at infinity</u>.

These points lie on a line also called as <u>ideal line</u>.

(g) Pinhole Projection model

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z} \begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Here $\frac{1}{z}$ makes it non-linear

If the above equation is converted to homogenous like

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

These represent it as a linear equation

(h) $M = K [I] [0]$

$K = 3 \times 3$ matrix
$I =$ Identity matrix $= 3 \times 3$
$0 =$ zero vector $= 3 \times 1$

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(i) m = projection vector

P = 3D point

$$m = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \quad P = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 3D point

to make it 3D H = $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$u = 1 + 4 + 9 + 1 = 15$

$v = 5 + 12 + 21 + 8 = 46$

$w = 1 + 4 + 3 + 1 = 9$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 15 \\ 46 \\ 9 \end{bmatrix}$$ ⟹ 2D H point

to make it 2D = $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u/w \\ v/w \end{bmatrix} = \begin{bmatrix} 15/9 \\ 46/9 \end{bmatrix}$

Q-2

a) point $A = (1, 1) = (a, b)$

$T = (2, 3) = (tx, ty)$

$\therefore \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$

$u = a + tx = 1 + 2 = 3$

$v = b + ty = 1 + 3 = 4$

$\therefore \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

b) $A = (1, 1) = (a, b)$

$S = (2, 2) = (Sx, Sy)$

$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix}$ $\therefore$ $u = a\,Sx = 1 * 2$

$v = b\,Sy = 1 * 2$

$[u, v] = [2, 2]$

(c) $A = (a, b) = (1, 1)$

$\theta = 45^\circ$

$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1/\sqrt{2} - 1/\sqrt{2} \\ 1/\sqrt{2} + 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix} =$

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$u = 0$

$v = \sqrt{2}$

(d) $A = (x, y) = (1, 1)$ $\theta = 45$ point $P = (2, 2) = (t_x, t_y)$

1) Translate the point to origin

$\therefore$ $A = x - t_x = 1 - 2 = -1$ $A = (-1, -1)$

$y - t_y = 1 - 2 = -1$

2) rotate by 45^0

$$\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\sqrt{2} \end{bmatrix}$$

3) translate it back to origin

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -\sqrt{2} + 2 \\ 1 \end{bmatrix}$$ $(\bar{x}, \bar{y}) = (2, 0.585)$

(e) R = rotation matrix

T = translation matrix

$\therefore$ $x' = \underline{TRx}$ Here the formula represents we first rotate the matrix then we translate it.

(f) $$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore x' = M x = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 3x \\ 2y \\ 1 \end{bmatrix}$$ $\therefore x' = 3x$

$y' = 2y$

~~$z' = z$~~

This represents its scaled by $\underline{(3, 2)}$

(g) $m = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$x' = Mx = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$x' = x + 1$
$y' = y + 2$

$\therefore$ It represents ~~tr~~ translate by $(1,2)$

$\therefore \left.\begin{matrix} t_x = 1 \\ t_y = 2 \end{matrix}\right\}$

(h) $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ As per the answer in Q:2(~~e~~) it is scaled by $(3,2)$

to reverse the effect we have to do inverse scaling.

$= \begin{bmatrix} 1/s_x & & \\ & 1/s_y & \\ & & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & & \\ & 1/2 & \\ & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$x' = x/3 \qquad y' = y/2$

(i) $M = R(45)\ T(1,2)$

$= (RT)^{-1} = T^{-1} R^{-1}$

$= T^{-1} R^{T} \quad \therefore R^{-1} \cong R^{T}$

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$$R = \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$R^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\cancel{T = \begin{bmatrix} tx \\ ty \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \therefore T^{-1} = \begin{bmatrix} -tx \\ -ty \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}}$$

$$\cancel{M = T^{-1} R^T = \begin{bmatrix} -1 \\ -2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}}$$

$$T = \begin{bmatrix} tx & 0 \\ 0 & ty \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = T^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$M = T^{-1} R^T = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

(j) $\cancel{u \cdot v = |u||v|}$ $\quad u = i + 3j \quad \vec{u} = \frac{u}{|u|}$

$|\bar{u}| = \sqrt{1+9} = \sqrt{10} \quad \therefore \vec{u} = \frac{i+3j}{\sqrt{10}}$

$\vec{u}$ is the vector $\perp^{ar}$ to $\underline{u}$

(k) $u \cdot v = |u|\,|v| \cos\theta \qquad \cos\theta = \frac{u \cdot v}{|u||v|}$

$= \frac{17}{\sqrt{10}\ \sqrt{29}} \qquad = \frac{2+15}{\cancel{\sqrt{10}\ \sqrt{29}}} =$

$\cos\theta = 17/\sqrt{290}$

$\theta = \cos^{-1}(17/\sqrt{290}) = \underline{3.43^{\circ}}$

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Q.3

a) We use different coordinate system for both camera and image because they are independent of each other.

Why we need them independently, bcz we would like to percept or speak about the positions of objects independently where the camera is.

b) $$M_{c \leftarrow w} = \tilde{R}^{-1} \tilde{T}^{-1}$$

$$= \left[\begin{array}{c|c} R & 0 \\ \hline 0 & 1 \end{array}\right]^{-1} \left[\begin{array}{c|c} I & T \\ \hline 0 & 1 \end{array}\right]^{-1}$$

$\tilde{R}^{-1} \Rightarrow$ cancel rotation of the camera

$\tilde{T}^{-1} \Rightarrow$ align camera with world

$$= \left[\begin{array}{c|c} R^T & 0 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} I & -T \\ \hline 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} R^T & -R^T T \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} R^* & T^* \\ \hline 0 & 1 \end{array}\right]$$

c) ~~Rotation Matrix~~ $= \begin{bmatrix} \hat{x}_c^T \\ \hat{y}_c^T \\ \hat{z}_c^T \end{bmatrix}$ ~~It represent~~ the rotation of the camera with ~~respect to world~~

~~$\hat{x}_c \Rightarrow$ to~~

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c) Rotation matrix $= \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix}$ - 4x4 rotation matrix

$$\underline{\underline{R}} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ z \\ 1 \end{bmatrix}$$

d) $M = \begin{bmatrix} R^* & T^* \\ 0 & 1 \end{bmatrix}$

$R^* \Rightarrow$ Rotation of the world with respect to camera

$T^* \Rightarrow$ Translation of the world with respect to ~~H~~ camera

e) $M_{i \leftarrow c} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} k_u & 0 & 512 \\ 0 & k_u & 512 \\ 0 & 0 & 1 \end{bmatrix}$

f) $M = K^* [R^* | T^*]$

$K^* \Rightarrow$ represents instrinsic parameters of the world

$[R^* | T^*] \Rightarrow$ represents extrinsic parameters of the world.

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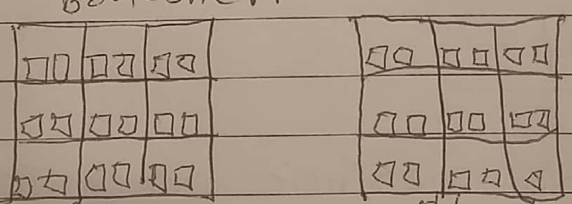
g) We include a 2D skew parameter in the camera model to get the accurate result with not any error in the model.

for example:
Industrial Inspection - In this ~~we~~ case we want accurate result for better performance of model. So, to make the model accurate we required to add some 2D skew parameter to make the model 100%. Correct.

h) Radial lens distortion shrink the image more as you are away from the center. The camera lenses causes distortion image.

Ex.
Book shelf



(a) without radial lens distortion

(d) with radial lens distortion

As you seen in image (b) the lines are curve which represent radial lens distortion

Complications: $p(i) = \begin{bmatrix} 1/d & & \\ & 1/d & \\ & & 1 \end{bmatrix} K^* [R^* | T^*] P^{(w)}$

d is not constant it depends on following eqn

$d = 1 + k_1 d + k_2 d$ (d ⇒ distance from center)

As d↑ d↑ ⇒ so the it becomes more scale
we get more distorted image

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~~Q4~~

ii) weak peeceptive Cameea : is a case when we look at the image we don't see much perceptive distortion (i.e. distance object appears to be smaller)

$$M\infty = \begin{bmatrix} \text{———} \\ \text{———} \\ 0 \quad 0 \quad 0 \quad 1 \end{bmatrix} \Rightarrow \text{this row shows no peeceptive division}$$

i.e. we don't see much peeceptive

Affine Cameea : This is a cameea that we don't see an real. Mattrice = $\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix}$ used for computational purpose

Q.4

a) Sueface Radiance ⇒ powee of light pee unit aeea reflection from sueface

Image irradiance ⇒ powee of light pee unit aeea received at the image.

b) radiosity equation

$$E(P) = L(P) \frac{\pi}{4} \left(\frac{d}{f}\right)^2 (\cos\alpha)^4$$

∴ f ⇒ focal length

d ⇒ diametee of lens

$\cos\alpha$ ⇒ angle betn principal axis & sueface normal

c) $\rho \Rightarrow$ ~~at~~ it denotes as reflection coefficient which varies between 0 & 1 (i.e. $\in (0,1)$).

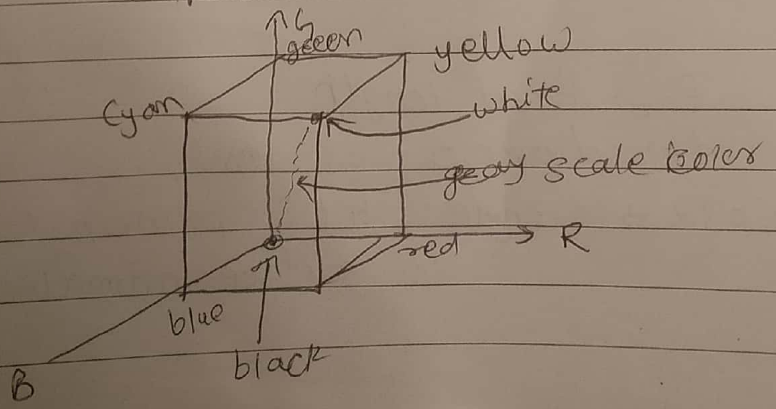
If $\rho = 0 \Rightarrow$ poor reflection
$\rho = 1 \Rightarrow$ very good reflection / accurate reflection

d) We use RGB color model, because ~~at~~ all wavelength reconstructed by the other color using combination of R, G, B color.
Ex. Brown = (165, 42, 42) Here value of R = 165, G = 42, B = 52

e) In RGB color Cube,

(0, 0, 0) ⇒ Represents black color
(1, 1, 1) ⇒ Represents white color

f) Mixture of ~~the~~ light of these primary colors (RGB color model) covers all the part of the human color space & thus it produce large amount of human color experience.



The cube in the diagram represents How it map to real world:

We see that

R, G, B with values (0, 0, 0) ⇒ black color

with values (1, 1, 1) ⇒ white color

The dotted line shows they all are grey scale color.

~~We also see that, How ~~full~~ amount of ~~bto~~ blue & green~~

R G B

(0, 1, 1) ⇒ cyan color

(1, 1, 0) ⇒ yellow color

(1, 0, 1) ⇒ violet color

RGB is also called as additive.

g)
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.123 & 0.465 & 0.210 \\ 0.313 & 0.261 & 0.100 \\ 0.145 & 0.232 & 0.444 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

The use of luminance Component Y is to get black & white of color image.
In case of black & white we only see luminance.

h) The advantage of LAB color space is that the eucledian distance is human perceptive.

It means that if we want to compare two color then human eye ~~tells~~ able to tell that they are similar or not.

If the eucledian distance is small then they are very similar, while if the eucledian distance is large i.e. they are not similar color.

~~The~~ The human perception didn't occur in RGB color space. Means it's not Human perceptive.

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