Assignment 7

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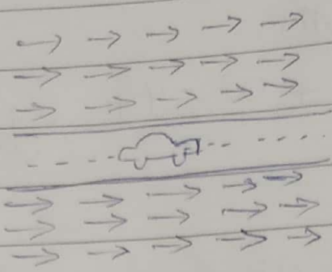
1. Motion:

a) 1) 3D motion vector:
It defines How the object move in world

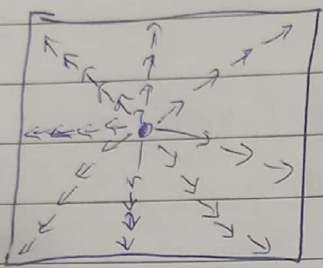
2) 2D projected vector:
It defined, when the projection of 3D motion vector onto 2D, i.e. camera coordinate.
We get these when take picture of or video of 3D objects.

3) 2D Observed motion
: It is what we observed in image coordinate But it includes noisy estimate of 2D projected motion vectors.

Yes, it is possible that motion in 3D will not produce optical flow vector.
Ex. Objects which are far away from camera center, even though it's in motion it don't produce optical flow vector.

b) 

We can say that, projection vector are larger at point closer to the car, & they decrease as the distance to the car increase.

c) 

We can say that, projection motion vector are smaller, closer to the point where plane is trying to land ~~on the~~

The projection vector around that point keep increasing as distance increases, while the projection vector started to decreas as the distance goes much Higher & Higher

d) Fundamental projection Eqn

$$\Rightarrow \quad v = \frac{f}{Z^2}\left(V_Z - V_Z P\right)$$

$$V_Z = \frac{f}{Z^2}\left(V\cancel{Z} - V_Z Z\right) = 0$$

e) Projected Translational motion vector:

$$V_x^{(z)} = \frac{Z_z x - Z_x f}{Z} = \frac{Z_z}{Z}(x - x_0)$$

$$V_y^{(z)} = \frac{Z_z y - Z_y f}{Z} = \frac{Z_z}{Z}(y - y_0)$$

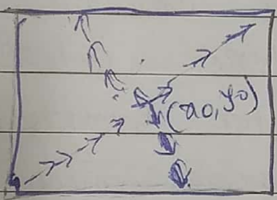
Projection Rotational motion:

$$v_x^{(\omega)} = -\omega_y f + \omega_z y + \frac{\omega_x xy}{f} - \frac{\omega_y x^2}{f}$$

$$v_y^{(\omega)} = \omega_x f \mp \omega_z y - \frac{\omega_y xy}{f} + \frac{\omega_x y^2}{f}$$

f) Projected motion field when it is pure translational motion looks like follow:

Case 1: $T_z \neq 0$ (eg. plane landing

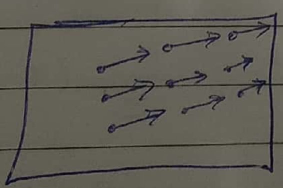


$$v_x = \frac{T_z}{Z}(x - x_0)$$
$$v_y = \frac{T_z}{Z}(y - y_0)$$
as explained in question (c)

(x_0, y_0) ~~epipole (instantaneous)~~

Vector far from (x_0, y_0) are bigger, but the Higher the value Z it's goes decreases i.e. lower the magnitude of optical flow center.

Case 2: $T_z = 0$ eg. driving car



$$v_x = -\frac{T_x}{Z} f \qquad v_y = -\frac{T_y}{Z} f$$

In this case vectors are $||^{al}$ & move in same direction

In this instantaneous epipole is at infinity

Magnitude decrease as moves farther.

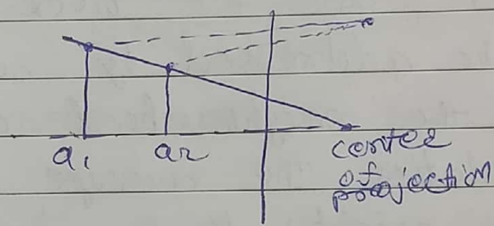
g) Equation of instantaneous pole as follow:

$$x_0 = \frac{T_x}{T_z} f$$

$$y_0 = \frac{T_y}{T_z} f$$

Here,
f is focal length
(T_x, T_y, T_z) are translational motion vector
& (x_0, y_0) instantaneous pole.

h) Motion parallax is created when 2 pts. at same pt in time coincide on image & then they appear to move differently.



It is defined as apparent motion of 2 instantaneous coincident pts.

2. Optical flow

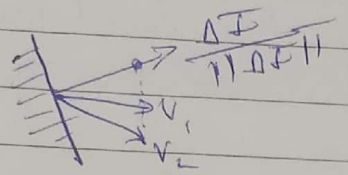
a) Optical flow Constraint Eq^n as follows:

$$\frac{\partial}{\partial t}\left(I(x(t), y(t)), t\right) = 0$$

Basic assumption:
Is that it is used to ~~derive~~ derive the eq^n that "brightness of object is constant throughout the image".

b) Aperture Problem.

Is that, 2 motion vectors having the same projection.



When this happened, we can observe only in the direction of gradient.

Single Point ⇒ we only hope to estimate motion in the direction of gradient (⊥ar vector)

c) Block-based Optical Flow:

In case of block-based the aperture problem be addressed by considering the many pts. in the neighbourhood of the current pixel. & calculating the average of all optical flow in the neighbourhood.

It provide us more smooth sol^n that satisfies the entire neighbourhood.

d) Objective function of block-based optical flow:

$$\sum_{(x,y)\in patch} \left(\nabla I(x,y)\cdot V + I_t\right)^2 = E(v)$$

$$V^* = \underset{v}{\arg\min}\ E(v)$$

$$\nabla E(v) = 0$$

$$\therefore \quad \frac{\partial}{\partial x_t} E = 0 \qquad \frac{\partial}{\partial y_t} E = 0$$

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$\Rightarrow (I_x x_t + I_y y_t + I_t) I_x = 0$

& $(I_x x_t + I_y y_t + I_t) I_y = 0$

The main purpose of weighted block $\Rightarrow$ is to give importance to the pixel close to the center rather than just having same weights for all pixels for the consideration window.

$$E(v) = \sum_{(x,y) \in patch} w(x,y) (I_x x_t + I_y y_t + I_t)^2$$

$$\therefore w(x,y) = \frac{1}{\|(x,y) - (x_c, y_c)\| + 1}$$

where $(x_c, y_c) \Rightarrow$ center

This leads us to the better solution.

$$\therefore sol^n = \begin{bmatrix} \sum w I_x^2 & \sum w I_x I_y \\ \sum w I_x I_y & \sum w I_y^2 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} -\sum w I_x I_t \\ -\sum w I_y I_t \end{bmatrix}$$

c) Advantage of Affine Motion Model
$\Rightarrow$ We compute the optical flow vector at each pt in the window, rather than consider that it is constant in the ~~cor~~ window. Which lead us to more accurate solution

Objective function.

$$E(a) = \sum_{(x,y) \in patch} \big(I_x(a_1 + a_2 x + a_3 y) + I_y(a_4 + a_5 x + a_6 y) + I_t\big)^2$$

Sol^n is given by

$$\nabla E(a) = 0$$

Once we find model parameter $(a_1, a_2, a_3, a_4, a_5 \& a_6)$ we can recover motion vector by using following:

$$v(x, a) = \begin{bmatrix} a_1 + a_2 x + a_3 y \\ a_4 + a_5 x + a_6 y \end{bmatrix}$$

We will find model parameter as follow

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{array}{cc} \begin{bmatrix} \sum I_x^2 & \sum I_x^2 x & \sum I_x^2 y & \sum I_x I_y & \sum I_x I_y x & \sum I_x I_y y \\ \sum I_x^2 x & \sum I_x^2 x^2 & \sum I_x^2 xy & \sum I_x I_y x & \sum I_x I_y x^2 & \sum I_x I_y xy \\ \sum I_x^2 y & \sum I_x^2 xy & \sum I_x^2 y^2 & \sum I_x I_y y & \sum I_x I_y xy & \sum I_x I_y y^2 \\ \sum I_x I_y & \sum I_x I_y x & \sum I_x I_y y & \sum I_y^2 & \sum I_y^2 x & \sum I_y^2 y \\ \sum I_x I_y x & \sum I_x I_y x^2 & \sum I_x I_y xy & \sum I_y^2 x & \sum I_y^2 x^2 & \sum I_y^2 xy \\ \sum I_x I_y y & \sum I_x I_y xy & \sum I_x I_y y^2 & \sum I_y^2 y & \sum I_y^2 xy & \sum I_y^2 y^2 \end{bmatrix} & M \end{array}$$

$$M = \begin{bmatrix} -\sum I_x I_t \\ -\sum I_x I_t x \\ -I_x I_t y \\ -I_y I_t \\ -I_y I_t x \\ -I_y I_t y \end{bmatrix}$$

) Objective ~~of~~ for Global motion estimation (Horn - Schunk)

$\longrightarrow$

$$E(v(x,y,t)) = \int_{image} \underbrace{E_{OF}^2 (v(x,y,t))}_{\text{optical flow}} + \underbrace{\lambda^2}_{\text{user selectable smoothing importance}} \underbrace{E_S^2 (v(x,y,t))}_{\text{smoothness}}$$

Advantage:
We can define how much to smooth the optical flow vector

$$\therefore E_{OF}^2 = (I_x \cdot U + I_y \cdot V + I_t)^2$$

$$E_S^2 = U_x^2 + U_y^2 + V_x^2 + V_y^2$$

U : image formed by x components of OFV
V = image formed by v components of OFV

But, to calculate U & V, is very difficult, bcz we have to take iterative approach.

By using this, we are regularizing the OFV & removing irregularities.

Q) H-S iterative optical flow algorithms as follows:

1) Start with initial guess value of U & V

2) Iterate to refine U & V

$$U^{n+1} = \bar{U}^{(n)} - \frac{(I_x \bar{u}^{(n)} + I_y \bar{v}^{(n)} + I_t) I_x}{I_x^2 + I_y^2 + \alpha^2}$$

$$V^{n+1} = \bar{V}^{(n)} - \frac{(I_x \bar{U}^{(n)} + I_y \bar{V}^{(n)} + I_t) I_y}{I_x^2 + I_y^2 + \alpha^2}$$

3) Stop the iterative when there is not much change in U & V

$$\therefore \max(\|U^{n+1} - U^n\|, \|V^{n+1} - V^n\|) < \tau \Big\} \text{ stopping condition}$$

We can use Lukas-Kanade / affine flow estimation methods to find initial value of u & v. it is the very 1st iterative of the algo.