Assignment 3

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Q.1

a) Ans:→

Basic principle of corner detection as follows.

1) Find correlation matrix in local neighbourhood

2) Now, find eigenvalue of correlation matrix.

3) Check

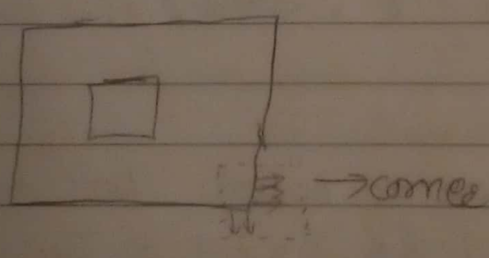
~~if $e(1) \cdot e(2)$ / $e(1)$~~ > T if $d_1 \cdot d_2 > T$

consider $d_1 \Rightarrow$ eigenvalue

$d_2 \Rightarrow$ eigenvalue

d_1, d_2 are the highest eigen value.

If $d_1 > d_2$ then we can detect corner.



Number of principal directions can be assigned :

1) First step is to compute the gradient

2) Now, calculate eigenvalue

~~3) Now check if d_1 & d_2 is large~~

3) Now, if d_1 d_2 is large then corner is detect

b) By finding the direction to minimize the projections of all points, Principal Component Analysis (PCA) is used to find principal directions of gradient orientation in local path

$$E(v) = \sum_{i=1}^{n} (P_i V)^2$$

Where, $E(v)$ is the sum of projections on directions of $\underline{\underline{V}}$

We can also tell that, eigen vector to zero eigen value of correlation Matrix A, as it will be having lowest projection of all points on to V.

c)

∴ Given gradient vector:
$\{(0,0), (0,1), (0,2), (0,3), (0,4), (1,0), (1,1), (1,2), (1,3)\}$

$$C = \sum_{i=1} \underset{2\times1}{P_i} \underset{1\times2}{P_i^T} = \underset{2\times2}{\begin{bmatrix} \sum x_i^2 & \sum x_i y_i \\ \sum x_i y_i & \sum y_i^2 \end{bmatrix}}$$

where $P_i = [x_i, y_i]$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 3 \end{bmatrix} \begin{bmatrix} 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 4 \end{bmatrix} \begin{bmatrix} 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 16 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 1+1+1+1 & 1+2+3 \\ 1+2+3 & 1+4+9+16+1+4+9 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 6 \\ 6 & 44 \end{bmatrix} A$$

d) The condition for corner detection
$d_1 d_2 > T$

where,
$d_1 \Rightarrow$ eigenvalue 1
$d_2 \Rightarrow$ eigenvalue 2
$T \Rightarrow$ Threshold

And, d_1 & d_2 Highest eigenvalue of correlation matrix.

e) Non-Maximum Suppression for corner detect work as follows:

1) First find $d_1 d_2$ for all the location
2) Now sort the pixel based on the value of $d_1 d_2$ in decreasing order

3) Starting from top, select strongest corner
4) Delete all corner's

5) ~~Wen~~ After completion of ~~po~~ step 4, we deleted some percentage i.e. C% of the pixel to have corners.
Then you should stop.

f) Harris Corner detection is represented by following formula

$C(G) = \det(G) - k\,tr^2(G)$
where tr = trace , $\det(G) = d_1 \cdot d_2$

if $C(G)$ is large, then we can tell that we have good quality corner.
$\therefore 0 < k < 0.5 \}$
As mentioned in the formula, we calculate determinant of $d_1 \cdot d_2$ & trace i.e. $d_1 + d_2$ of gradient matrix, ~~wh~~ it avoids direct computing eigen value at gradient calculation matrix.

g) Formula for better localization of corner as follows:

$\nabla E(P) = \sum_i \nabla I(P_i) \cdot \nabla I(P_i)^T P_i$

~~Assump~~ Assume that $V = C\,P$
where C represent $\sum \nabla I(P_i)\,\nabla I(P_i)^T$

& V, C, P are numerical matrices

$$P = C^{-1} P$$

To taking inverse of C, the eigen values should be large
where, C is non-singular.
$d_1 d_2 > \tau$ } corner is detected.

b) ~~Steps~~ We have to apply following steps for feature points ~~the~~ characterized using HOG:

1) First take a window
2) Split it into blocks
3) Now, compute Histogram of gradient orientation in each block
4) Last is we have to concatenate the histogram.

Requirement for good characterized feature point as follows:

1) ~~I~~ The feature point should be translation invariant
2) It should be rotation invariant
3) It should be scale invariant too
4) The last is, it should be illumi invariant

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i) SIFT features computed by following steps:

1) We have to detect scale-space extrema. Where, LOG gives good idea for the scale.

2) Now, we have to do keypoint localization as well as filtering.

Q.2

a) The problem is that, slope & y-intercept go upto infinity

$b \in [-\infty, \infty]$ & $a \in [-\infty, \infty)$

so & infinite space can't be allocated.

b) Given: slope $= 45^\circ$

distance from origin $= 10$

$\therefore \; ax + by + c = 0$

Here $d = 10$,

$d = x\cos\theta + y\sin\theta$

$x\cos 45 + y\sin 45 = 10$

$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 10$

$x + y = 10\sqrt{2} = 14.14$

if $a = 1$, $b = 1$

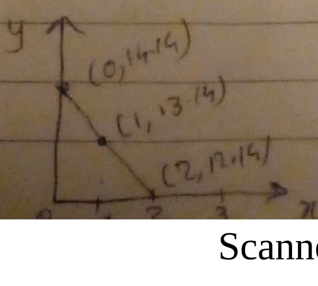
then $c = -14.14$

consider,

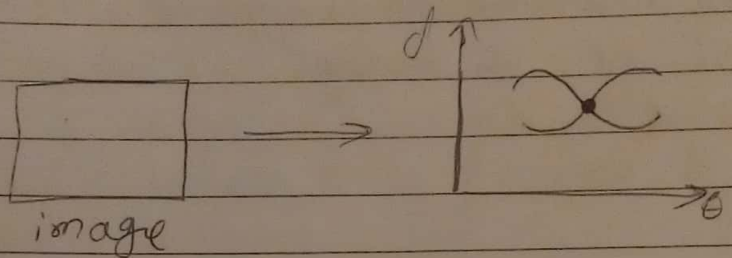
$x = 0$, $y = 14.14$

$x = 1$, $y = 13.14$

$x = 2$, $y = 12.14$



c) In polar co-ordinate, the vote of each point in the image looks like a sinusoidal curve, which is represented as follows:



$d = x\cos\theta + y\sin\theta$

d) 1) First of all, find out the gradient of I i.e. ∇I at voting point.

2) Now, obtain angle i.e. θ

3) Start scanning from $\theta - \Delta, \dots \theta + \Delta$.
Now, we can say that distance & angle of point represent the line.

e) If we use larger bin size, we have to do less computation, but there is High chance that we loss/miss some important information, which effects on accuracy.

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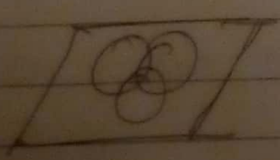
While, if we used smaller bin size, then we have to do High Computation & we get more accuracy bcz we can't loose /miss the information.

f) The main advantage of knowing the normal at each point is that, we don't have to scan the entire ~~image~~ range of θ i.e. from 0 to 180.

Instead, we do scanning only at certain point.

And, it takes place from $[\theta - \Delta, \ldots \theta + \Delta]$. After finding ∇I at voting point

g) The number of dimensions of parameter space, while using Hough transform for circles is $\underline{3}$.

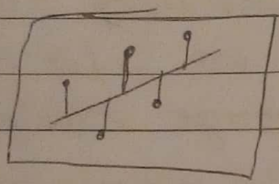


$r = 3$

Q.3

a) $y = ax + b$ $\quad = eq^n$ ①

Disadvantage of using above equation is that the geometric distance betn actual & predicted points are not minimized.



(a)

$y = ax + b$ works ~~for~~ good for figure (a)

The eq^n ① does not always provide Shortest distance to actual line.

b) Normal (1, 2)

$d = 2 \Rightarrow$ distance from origin

$l = 3 \times 1$ vector

$l^T x = 0$

$d = Pn$

$d = \underbrace{ax + by}_{normal} + c = 0$

$l : [1\ 2\ 2]$ $\quad x : [x\ y\ 1]$

$l^T : \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} [x\ y\ 1] = 0$

$x + 2y + 2 = 0$

c) $E(l) = \sum_{i=1}^{n} (l^T x_i)^2$

$= \sum_{i=1}^{n} l^T x_i x_i^T l$

$= l^T \sum_{i=1}^{n} x_i x_i^T l \quad = l^T C l$

$l^* = \underset{l}{\operatorname{argmin}}\, E(l) \qquad \nabla E(l) = 0$

where $E(l) = \sum_{i=1}^{n} (l^T x_i)^2$

To best fit the line, try to minize the geometric distance in the explicit line eq^n by using explicit equation

$l^T x = 0$

$E(l)$ being a vector sol^n is the eigen vector belonging to zero eigen value.

d) $C = \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i y_i & \Sigma x_i \\ \Sigma x_i y_i & \Sigma y_i^2 & \Sigma y_i \\ \Sigma x_i & \Sigma y_i & n \end{bmatrix}$ Given points $(0,1) \cdot (1,3) \quad (2,6)$

Let $D = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 6 & 1 \end{bmatrix} \quad \therefore D^T = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 6 \\ 1 & 1 & 1 \end{bmatrix}$

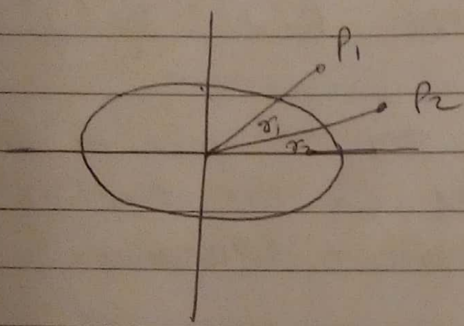
$\therefore D^T D = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 15 & 3 \\ 15 & 46 & 10 \\ 3 & 10 & 3 \end{bmatrix}$

e) $\therefore \; ax^2 + bxy + cy^2 + dx + ey + f = 0$

The above eqn represent conic curve.

$b^2 - 4ac < 0$ is the constraint on the parameters a, b, c, d, e, f that guarantee model will be elipse.

f)



$\sum_{i=1}^{n} (l^T P_i)^2$ on ellipse

$l^T P_i = 0$

Algebric distance

$q_i = l^T P_i$

Now, $q_i \sim \dfrac{d_i}{d_i + r_i}$

$d_1 > d_2 \quad r_2 > r_1$

$\therefore \dfrac{d_1}{d_1 + r_1} > \dfrac{d_2}{d_2 + r_2}$

Points ~~to~~ closer to pt. P_1 will affect more the fitting rather than pt. P_2.

g) $E(l) = \sum_i \dfrac{|f(P_i, l)|}{|\nabla(P_i, l)|}$

$f(P_i, l) = (l^T P_i)^2$

$P_i = (x_i^2, x_i y_i, y_i^2, x_i y_i, 1)$

The above function needs to minimized when fitting an ellipse using geometric distance.

The above, equation is not linear, so we have to do an iterative approach like gradient descent

h) $E[\phi(s)] = \int_{\partial s} (\alpha_{(s)} E_{continuity} + \beta_{(s)} E_{curvature} + \gamma_{(s)} E_{image}) ds$

where,

$E_{continuity} = \left|\frac{\partial \phi}{\partial s}\right|^2$ $E_{curvature} = \left|\frac{\partial^2 \phi}{\partial s^2}\right|^2$

$E_{image} = |\nabla I|^2$ $\alpha(s), \beta(s), \gamma(s) \rightarrow$ coefficient variable

1) $E_{continuity}$ & $E_{curvature}$ = Energy terms.

2) $\alpha_s, \beta_s, \alpha_s$ are the weights of each of them are variable as more importance may be received for either continuity, curvature & or image.

~~they also~~ called coefficients.

i) In discrete space, we use active contours

$\phi(s) \rightarrow \{P_i\}_{i=1}^{n}$

$\therefore E_{continuity} = \left|\frac{\partial \phi}{\partial s}\right|^2 \rightarrow \sum_i |P_i - P_{i-1}|^2$

~~E_{co}~~ $\Rightarrow$ is the distance between neighbouring points

$$E_{curvature} = \left|\frac{\partial^2 \phi}{\partial s^2}\right|^2 = \Sigma \left|(P_{i+1} - P_i) - (P_i - P_{i-1})\right|^2$$
$$= \Sigma \left|P_{i+1} - 2P_i + P_{i-1}\right|^2$$

it is different targets at neighbouring points

i)

~~Place snake~~

$$E_{curvature} = \left|\frac{\partial^2 \phi}{\partial s^2}\right|^2 \rightarrow \Sigma \left|(P_{i+1} - P_i) - (P_i - P_{i-1})\right|$$

the continuity of active contours may be relaxed or to allow discontinuity, we find high curvature point & set

$\beta_i = 0$

i.e. $|P_{i+1} - 2P_i + P_{i-1}| > \gamma$

then $\beta_i = 0$

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