Tutorial - 6

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section : F

class Roll no : 11

Ans. 1

→ Out of all the possible spanning tree of a graph, the spanning tree with minimal sum of weights is called minimum spanning tree.

-> Application :

1. Consider n station are to be linked using a communication network & lying of communication link between any two station involved a cost. The ideal solution would be to extract a subgraph termed as MST.

2. To build railway or highway concept of MST is used.

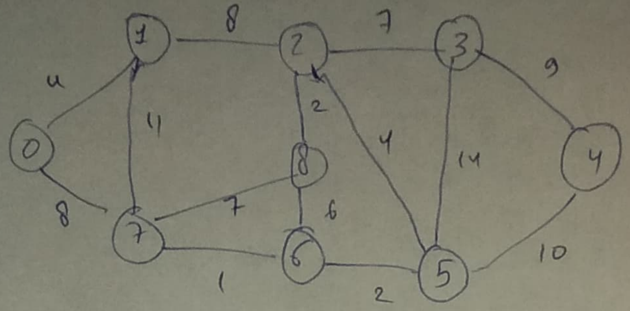
3. Designing LAN.

4. To find path (efficient) on a graph MST is used.

Ans. 2

	Time Complexity	space complexity
Prism algo	O(E log|V|)	O(V)
kruskal algo	O(E log|V|)	O(V)
Dijkstra algo	$O(V^2)$	$O(V^2)$
Bellman ford	O(VE)	O(E)

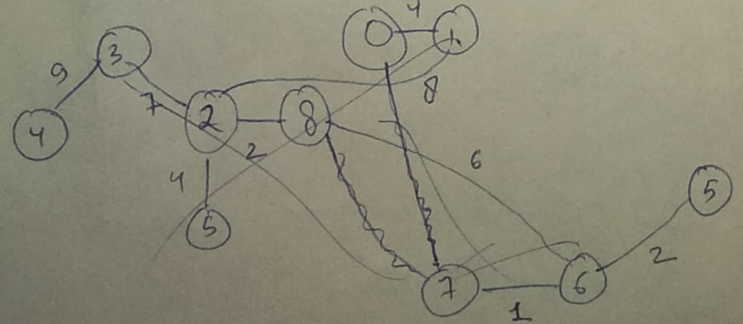
Ans. 3



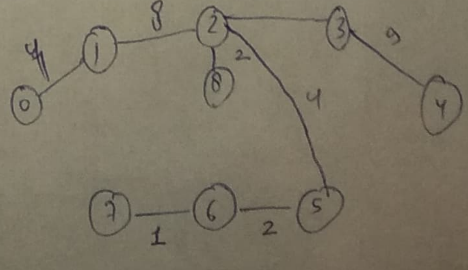
Kruskal

weight	source	destination	
1	7	6	✓
2	6	5	✓
2	2	8	✓
4	0	1	✓
4	2	5	✓
6	8	6	X
7	7	8	X
7	2	3	✓
8	1	2	✓
8	0	7	X
9	3	4	✓
10	5	4	X
11	1	7	X
14	3	5	X

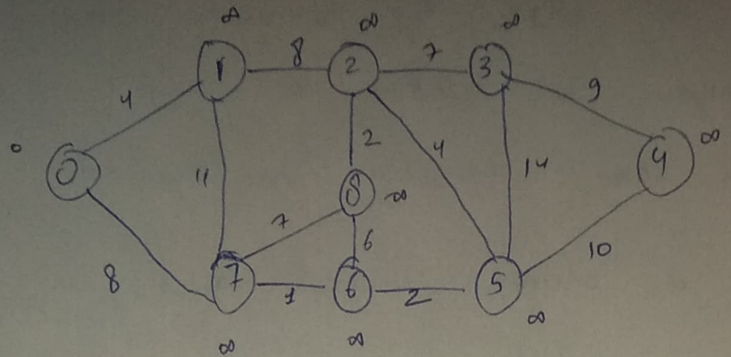
Now add edges if cycle not formed by it.



$\boxed{\text{Weight sum : 3[illegible]}}$



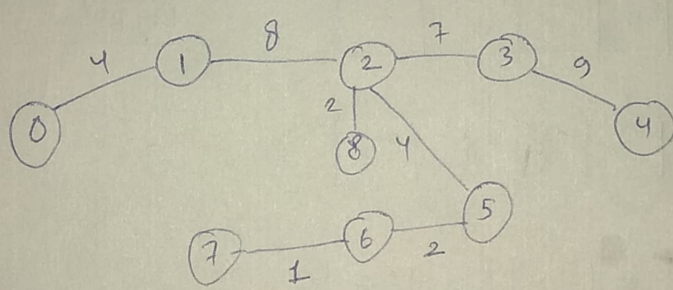
Prism



	0	1	2	3	4	5	6	7	8
parent	-1	0	1	2	~~5~~ 3	2	~~5~~ 8	~~8~~ ~~6~~ 8	2

	0	1	2	3	4	5	6	7	8
value	0	4 ~~∞~~	8 ~~∞~~	7 ~~∞~~	~~10~~ 9	4 ~~∞~~	~~6~~ 2 ~~∞~~	~~11~~ ~~8~~ 1 ~~∞~~	2 ~~∞~~

	0	1	2	3	4	5	6	7	8
set list	T ~~F~~	T ~~F~~	T ~~F~~	T ~~F~~	F	T ~~F~~	T ~~F~~	T ~~F~~	T ~~F~~



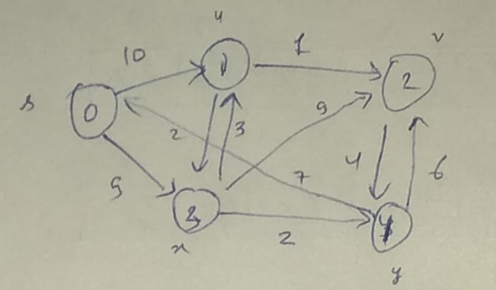
Weight sum = 38

Ans. 4. (i) The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'. for ex, let shortest path ~~weight~~ weight is 15 & has 5 edges. let there be another path with 2 edges & total weight 25. The weight of shortest path increased by 5×10 & becomes 15+50 = 65 whereas the weight of 2 edged path become 25 + 20 = 45 so, the shortest path changes there.

(ii).

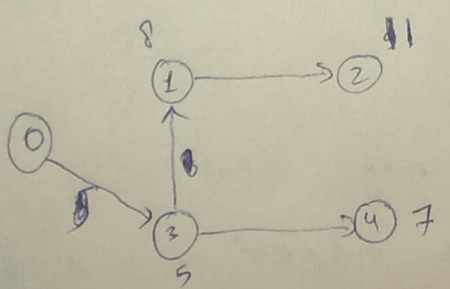
If we multiply all edges by 10 unit, the shortest path does'nt change. The reason is simple, as all the weights are equally multiplied, so the differente is alway ~~increase~~ goes in the ratio of previous difference.

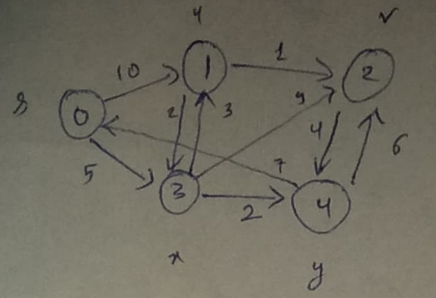
Ans. 5.



	0	1	2	3	4
parent :	-1	3 ~~0~~	~~4~~ 1 ~~3~~	0	3
value :	0	~~10~~ 8 ~~∞~~	~~14~~ ~~13~~ 11 ~~∞~~	5 ~~∞~~	7 ~~∞~~
processed :	T ~~F~~	~~F~~	F	T ~~F~~	T ~~F~~

Dijkstra algorithm





Edges:

(0, 1) (3, 1) (4, 2)
(0, 3) (3, 2) (4, 0)
(1, 2) (3, 4)
(1, 3) (2, 4)

First iteration

value:

0	1	2	3	4
0	~~10~~ 8 ~~∞~~	11 ~~∞~~	~~∞~~ 5	7 ~~∞~~

parent:

-1	~~0~~ 3	1	0	3

second iteration

value:

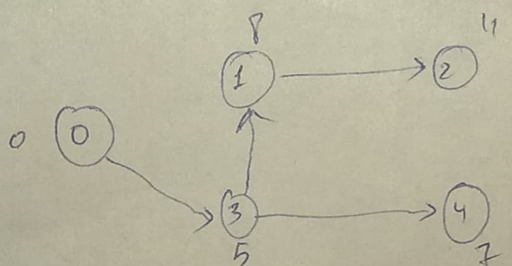
0	1	2	3	4
0	8	11	5	7

parent:

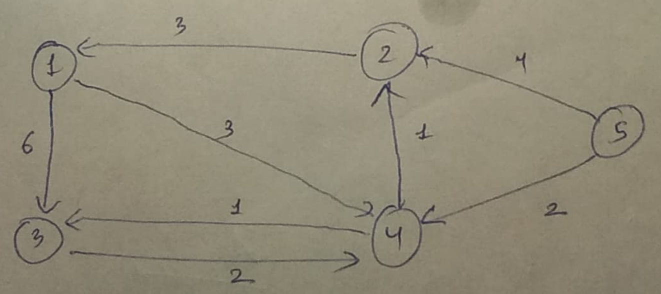
0	1	2	3	4
-1	3	1	0	3

as no update occur.

so, our graph is now become single source shortest path graph.



Ans. 6



$$D^0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & \infty & \infty & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$$D^1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$d[1][3] = \min(d[1][3],$
$d[1][1] + d[1][3]$

$$D^2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & 4 & 1 & 1 & 0 & \infty \\ 5 & 7 & 4 & \infty & 2 & 0 \end{array}$$

$$D^3 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & 4 & 1 & 1 & 0 & \infty \\ 5 & 7 & 4 & \infty & 2 & 0 \end{array}$$

$$D^4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 4 & 3 & \infty \\ 2 & 3 & 0 & 7 & 6 & \infty \\ 3 & 6 & 3 & 0 & 2 & \infty \\ 4 & 4 & 1 & 1 & 0 & \infty \\ 5 & 6 & 3 & 3 & 2 & 0 \end{array}$$

Ans