DAA assignment - 1

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Ans. 1.

Asymptotic Notation :

These notation are used to tell the complexity of an algorithm with respect to the input size.

Different asymptotic notation :

1. **Big oh (O) :**

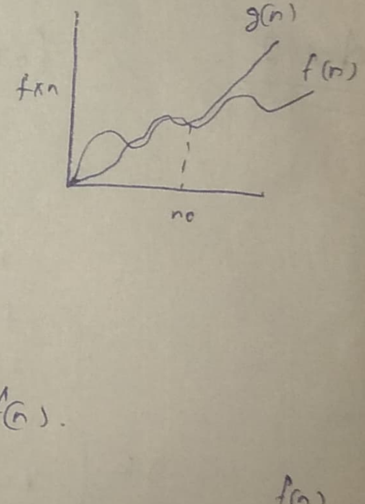
It is a upper tight bound of $f(n)$.

$f(n) \le c \cdot g(n)$

$f(n) = O(g(n))$

Ex: $g(n) = n^2 + n + 5$

$\boxed{f(n) = O(n^2)}$



2. **Big omega (Ω) :**

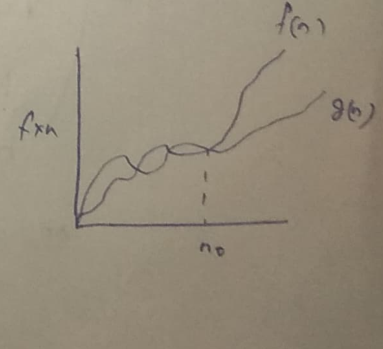
It is a lower tight bound of $f(n)$.

$f(n) \ge c \cdot g(n)$

$f(n) = \Omega(g(n))$

Ex: $g(n) = n^2 + \log n$

$\boxed{f(n) = \Omega(\log n)}$

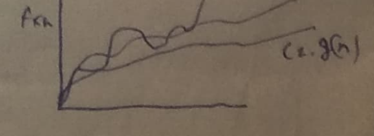


3. **Theta (Θ) :**

It give us a range of $f(n)$.

$c1 \cdot g(n) \le f(n) \le c2 \cdot g(n)$

$\boxed{f(n) = \Theta(g(n))}$

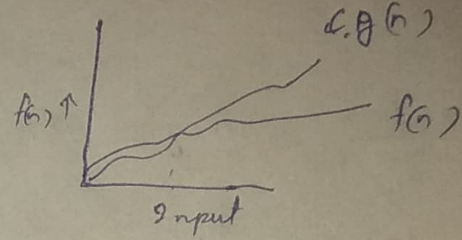


4. small oh (o) :

It is a upper bound of $f(n)$.

$f(n) = o(g(n))$

$f(n) < c.g(n)$

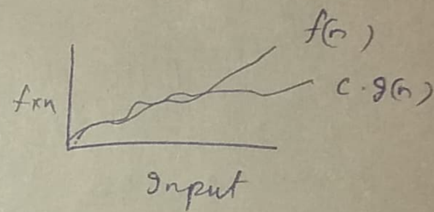


5. Small omega (ω)

It is a lower bound of $f(n)$.

$f(n) = \omega(g(n))$

$f(n) > c \cdot g(n)$



Ans. 2 .

for (i=1 ; i<=n; i=i+~~1~~)

i = i*2;

i

$$K\begin{cases}1\\ 2+1 = 3\\ 3*2+1 = 7\\ 7*2+1 = 15\\ \vdots\\ n\end{cases}$$

so, $2^K - 1 = n$

$2^K = n+1$

$\log(n+1) = K$

so, $\boxed{f(n) = O(\log n)}$

So, It has logarithmic complexity.

Ans. 3 $T(n) = 3T(n-1)$ if $n > 0$

1 if $n \le 0$

$T(n) = 3T(n-1)$

$T(0) = 1$

$T(n-1) = 3T(n-2)$

$T(n) = 9T(n-2)$

$T(n-2) = 3T(n-3)$

$T(n) = 27T(n-3)$

$T(n) = 3^k T(n-1-k)$

at $T(0) = 1$

$n-1-k = 0$

$k = n-1$

so, $T(n) = 3^{n-1}$

$$\boxed{T(n) = \Theta(3^n)}$$

Ans 4. $T(n) = 2T(n-1) - 1$ if $n > 0$

1 if $n \le 0$

$T(n) = 2T(n-1) - 1$

$T(n-1) = 2T(n-2) - 1$

$T(n) = 2(2T(n-2) - 1) - 1$

$T(n) = 4T(n-2) - 2 - 1$

$T(n-2) = 2T(n-3) - 1$

$T(n) = 4(2T(n-3) - 1) - 2 - 1$

$T(n) = 8T(n-3) - 4 - 2 - 1$

$+n^2$

$T(n) = 2^k (T(n-k) - 2^{k-1} - 2^{k-2} - 2^{k-3} - \dots - 2 - 1$

$T(n) = 2^k - (1 + 2 + 4 + \dots + 2^{k-1})$

$T(n) = 2^k - \left[\frac{(1)[2^k - 1]}{2-1} \right]$

$T(n) = 2^k - 2^k + 1$

$\boxed{T(n) = 1}$

Ans. 5

```
int i=1, s=1;          — (2)
while ( s<=n) {
    i++;               — (n)
    s = s+i;           — (n)
    printf("#");       — (n)
}
```

$T(n) = 3n + 2$

$\boxed{T(n) = O(n)}$

Ans. 6

```
void function ( int n)            — (1)
{                                 — (2)
    int i, count=0;
    for(i=1; i*i<=n; i++)         O(√n)
        count++
}
```

i

1

2*2 = 4

5*5 = 25

k

⋮

n

$(k+1)^2 = n$

$\boxed{k = \sqrt{n} - 1}$

Time Complexity $[T(n) = O(\sqrt{n})]$

Ans. 7.

```
void function (int n)                          - 1
{
   int i, j, k, count = 0;                     - 4
   for( i = n/2 ; i <= n; i++)                 - n/2
     for(j=1 ; j<=n ; j=j*2)                   - log n
       for (k=1; k<=n; k=k*2)                  - log n
         count++;
}
```

$$T(n) = \frac{n}{2} \log n \log n + 5$$

$$\boxed{T(n) = O(n (\log n)^2)}$$

Ans. 8.

```
function (int n) {                 <- T(n)
   if(n==1)
      return;                      - 1
   for(i=1 to n) {
      for (j=1 to n) {
         printf("*");              - n^2
      }
   }
   function (n-3);                 <- T(n-3)
}
```

$$T(n) = T(n-3) + n^2$$

$$T(0) = 1$$

$$T(n-3) = T(n-6) + (n-3)^2$$

$$T(n) = T(n-6) + (n-3)^2 + n^2$$

$$T(n-6) = T(n-9) + (n-6)^2$$

$$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n^2$$

$$T(n) = T(n-3-3k) + (n-3k)^2 + (n-3(k-1))^2 + \dots + n^2$$

$T(n) = T(n-3) + n^2$

$T(n) = T(n-3(k+1)) + (n-3k)^2 + (n-3(k-1))^2 + (n-3(k-2))^2 + \dots + (n-3(k-k))^2$

$T(n) = 1 + [n-(n-3)]^2 + [n-(n-3)+3]^2 + (n-(n-3)+6]^2 + [n-(n-3)+9]^2 + \dots + (n-3(k-k))^2$

$T(n) = 1 + 3^2 + 6^2 + 9^2 + 12^2 + \dots + (3(k+1))^2$

$T(n) = 1 + \sum_{i=0}^{k} [3(i+1)]^2$

$T(n) = 1 + \sum_{i=0}^{\frac{n-3}{3}} [3(i+1)]^2$

$\boxed{T(n) = \Theta(n^3)}$

Ans. 9.

```
void function (int n)          — 1
{
   for( i= 1 to n)             — n
   {
      for( j=1; j<=n; j=j+1)   — n*n
         printf("*");
   }
}
```

$\boxed{T(n) = O(n^2)}$

Ans. 10

$n^k = O(c^n)$

$n^k \le c_1 . c^n$

let, $n_0^k \ne c_1 . c^{n_0}$