DAA

Tutorial 4

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Ans. 1. $T(n) = 3T(n/2) + n^2$

$a = 3, \; b = 2, \; f(n) = n^2$ $\quad a \geq 1$ & $b > 1$

$c = \log_2 3 \simeq 1.2$

$n^c = n^{1.2} < n^2$

So, $T(n) = \Theta(n^2)$

Ans. 2 $T(n) = 4T(n/2) + n^2$

$a = 4, \; b = 2, \; f(n) = n^2$ $\quad a \geq 1$ & $b > 1$

$c = \log_2 4 = 2$

$f(n) = n^c = n^2$

So, $\boxed{T(n) = \Theta(n^2 \log n)}$

Ans. 3 $T(n) = T(n/2) + 2^n$

$a = 1, \; b = 2, \; f(n) = 2^n$ $\quad a \geq 1$ & $b > 1$

$c = \log_2 1$

$n^c < f(n)$

So, $\boxed{T(n) = \Theta(2^n)}$

Ans. 4 $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$, $b = 2$, $f(n) = n^n$ $\quad a \geq 1$ & $b > 1$

$c = \log_2 2^n = n$

$f(n) = n^c = n^n$

so, $\boxed{T(n) = \Theta(n^n \log n)}$

Ans. 5 $T(n) = 16\,T(n/4) + n$

$a = 16$, $b = 4$, $f(n) = n$ $\quad a \geq 1$ & $b > 1$

$c = \log_4 16 = 2$

$n^c > f(n)$

so, $\boxed{T(n) = \Theta(n^2)}$

Ans. 6 $T(n) = 2\,T(n/2) + n \log n$

$a = 2$, $b = 2$, $f(n) = n \log n$ $\quad a \geq 1$ & $b > 1$

$c = \log_2 2 = 1$

$n^1 < n \log n$

so, $\boxed{T(n) = \Theta(n \log n)}$

Ans. 7 $T(n) = 2\,T(n/2) + n/\log n$

$a = 2$, $b = 2$, $f(n) = \dfrac{n}{\log n}$ $\quad a \geq 1$ & $b > 1$

$c = \log_2 2 = 1$

$n > n/\log n$

so, $\boxed{T(n) = \Theta(n)}$

Ans. 8

Ans. 8 $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \quad b = 4, \quad f(n) = n^{0.51}$

$c = \log_4 2 = 0.5$

$n^c < n^{0.51}$

so, $\boxed{T(n) = O(n^{0.51})}$

Ans. 10. $T(n) = 16T(n/4) + n!$

$a = 16, \quad b = 4, \quad f(n) = n!$

$c = \log_4 16 = 2$

$n^2 < n!$

so, $\boxed{T(n) = O(n!)}$

Ans. 09. $T(n) = 0.5\,T(n/2) + \frac{1}{n}$

$a = 0.5$ as $a \neq 1$ so, it is not possible to solve this using master's method.

Ans. 18. $T(n) = 4T(n/2) + \log n$

$a = 4, \quad b = 2, \quad f(n) = \log n$

$c = \log_2 4 = 2$

$n^c > \log n$

so, $\boxed{T(n) = O(n^2)}$

Ans. 12 $T(n) = sqrt(n)\,T(n/2) + \log n$

$a = sqrt(n)$, $b = 2$, $f(n) = \log(n)$

$c = \log_2 \sqrt{n} = \frac{1}{2} \log n$

as, $n^{\log \sqrt{n}} > \log(n)$

so, $\boxed{T(n) = \Theta(n^{\log \sqrt{n}})}$

Master's method is not applicable as a is not constant.

Ans. 13. $T(n) = 3T(n/2) + n$

$a = 3$, $b = 2$, $f(n) = n$

$c = \log_2 3 \simeq 1.2$

$n^c > n$

so, $\boxed{T(n) = \Theta(n^{\log_2 3})}$

Ans. 14. $T(n) = 3T(n/3) + sqrt(n)$

$a = 3$, $b = 3$, $f(n) = \sqrt{n}$

$c = \log_3 3 = 1$

$n \not> \sqrt{n}$

so, $\boxed{T(n) = \Theta(n)}$

Ans. 15. $T(n) = 4T(n/2) + cn$

$a = 4$, $b = 2$, $f(n) = cn$

$c = \log_2 4 = 2$

$n^2 > cn$

so, $\boxed{T(n) = \Theta(n^2)}$

16. $T(n) = 3T(n/4) + n \log n$

$a = 3, \quad b = 4, \quad f(n) = n \log n$

$c = \log_4 3 = 0.7924$

$n^c \not\leq n \log n$

So, $\boxed{T(n) = \Theta(n \log n)}$

Ans. 17. $T(n) = 3T(n/3) + n/2$

$a = 3, \quad b = 3, \quad f(n) = n/2$

$c = 1$

$n^1 > n/2$

So, $\boxed{T(n) = \Theta(n)}$

Ans. 18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3, \quad f(n) = n^2 \log n$

$c = \log_3 6 = 1.43$

$n^c < n^2 \log n$

$\boxed{T(n) = \Theta(n^2 \log n)}$

Ans. 19. $T(n) = 4T(n/2) + \frac{n}{\log n}$

$a = 4, \quad b = 2, \quad f(n) = n/\log n$

$c = \log_2 4 = 2$

$n^2 > n/\log n$

$\boxed{T(n) = \Theta(n^2)}$ ~~$\log n$~~

Ans. 20 $T(n) = 64T(n/8) - n^2 \log n$

$a = 64, \quad b = 8, \quad f(n) = n^2 \log n$

$c = \log_8 64 = 2$

$n^2 < n^2 \log n$

So, $\boxed{T(n) = \theta(n^2 \log n)}$

Master theorem not applicable as $f(n)$ is decreasing function.

Ans. 21. $T(n) = 7T(n/3) + n^2$

$a = 7, \quad b = 3, \quad f(n) = n^2$

$c = \log_3 7 = 1.4$

$n^c < n^2$

So, $\boxed{T(n) = O(n^2)}$

Ans. 22.

$T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \quad b = 2, \quad f(n) = n(2 - \cos n)$

$c = \log_2 1 = 0$

$1 < n(2 - \cos n)$

So, $\boxed{T(n) = O(n(2 - \cos n))}$

Master theorem is not applicable since regularity condition is isolated in case 3.