Homework 2
Project Report
Epipolar geometry from F-matrix

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1 Objective

The objective is to establish a relationship between the pixels of a point in one image and the pixels of the same point in the second image.

2 Experimental Procedure

2.1 Data Capture

An object was kept on a table, in my case, its a book. First image was taken from the camera of iPhone 6S, and then the phone was moved to the left and rotated to point towards the object. The following figure shows the two images that were taken (Figure 1).

The points to be measured are shown with green circles in the Figure 2. Total eight points are chosen in each of the image. All points are measured in the pixel coordinates of the images.

This step gives us the coordinates of 8 points with their corresponding points in the image. We need to estimate the fundamental matrix, hence, these 8 points are sufficient for the calculation.

The camera used was of the phone iPhone 6S. It is 12MP and we can see it in the picture resolution as specified above, and it uses "Stacked back-illuminated CMOS image sensor", and its size is 4.80 x 3.60 mm (1/3"). The pixel size is 1.22 μm. The aperture is f/2.2. The focal length is 4.15 mm.

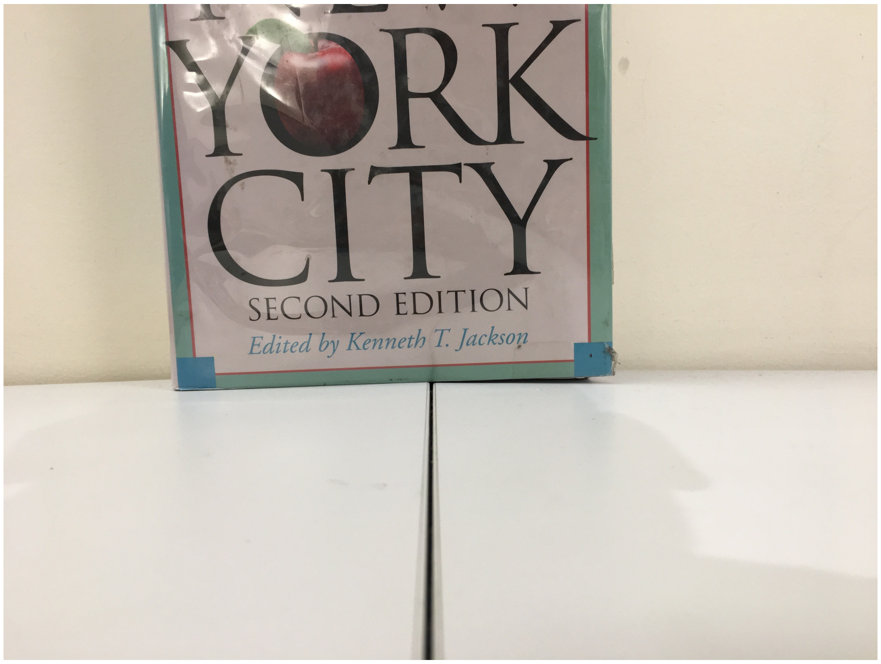
2.2 Estimation of the Fundamental Matrix

Fundamental Matrix is providing us the epipolar relation betwen the two camera. Here rather than using the svd function of MATLAB to find the matrix, I have used the function *estimateFundamentalMatrix* from the MATLAB inbuilt libraries to achieve more accurate results. The technique that this method uses is 'Norm 8 Point'.

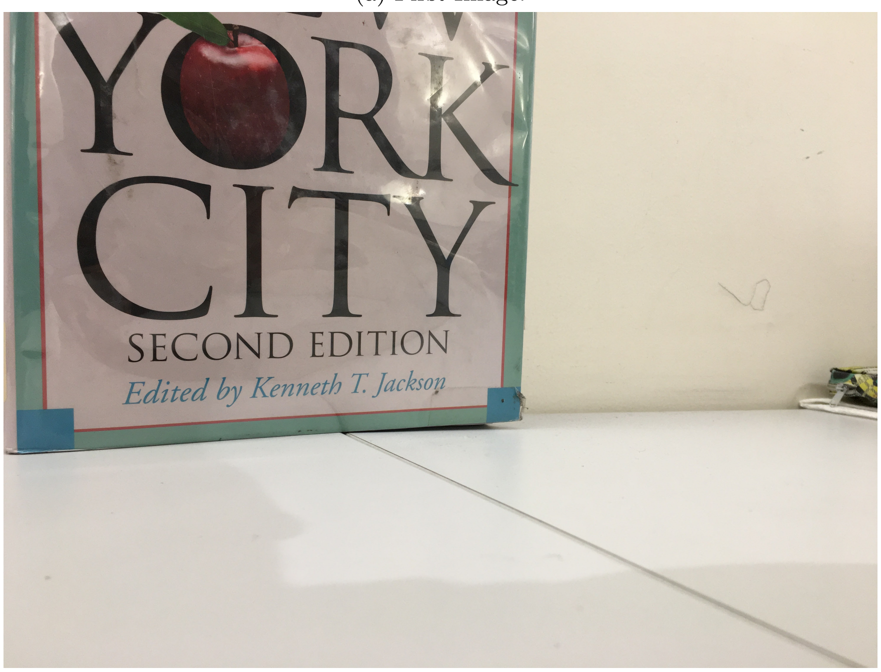
I tried using the *svd* function but I was not able to. I guess there was some error which I could not debug. I also plotted the epipolar lines on the image, but they did not meet anywhere at one single point. It was all variable.

We get the $\mathcal{F}$ matrix as:

$$\mathcal{F} = \begin{bmatrix} -3.59771146663311e-08 & -2.15949483390579e-06 & 0.00318981196127254 \\ 1.99944563167745e-06 & -9.66167427408277e-08 & -0.00426927270475672 \\ -0.00315874109425805 & 0.00414093720903172 & 0.999972236701263 \end{bmatrix} \tag{1}$$

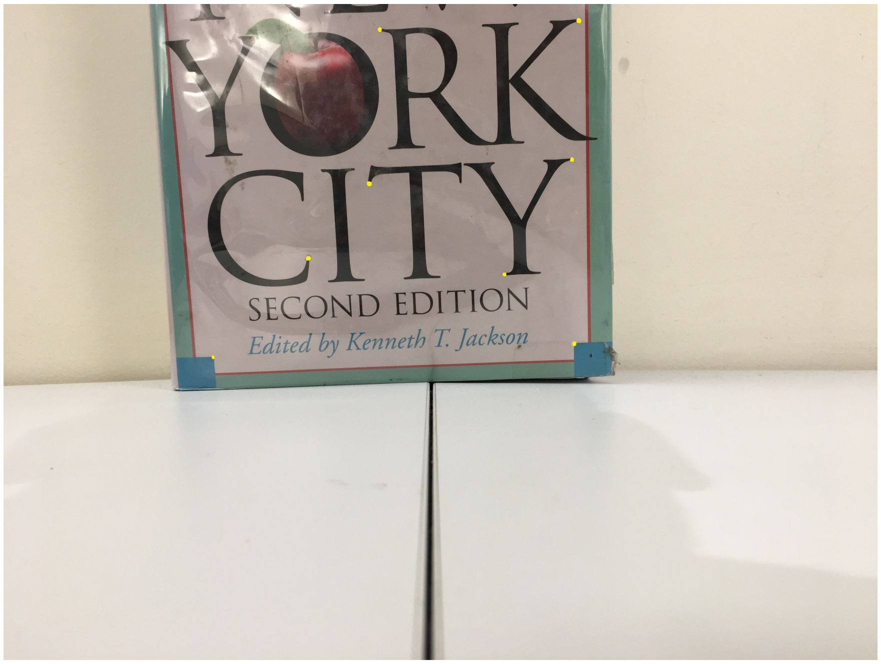


(a) First Image.

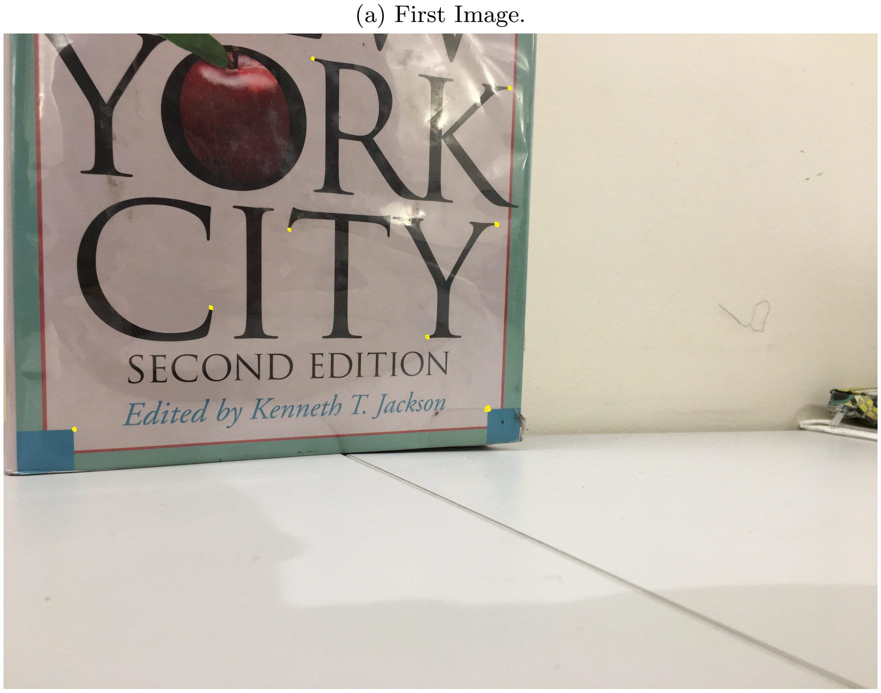


(b) Second Image, move camera to left and rotate it.

Figure 1: Images taken to collect the data.



(a) First Image.



(b) Second Image, move camera to left and rotate it.

Figure 2: Points in the images taken to collect the data.

As we can se, our F_{33} is really close to 1 and all the other values are also very small.

2.3 Computation of the Epipolar Lines and the Epipoles

A point in the first camera is located on a special line called the epipolar line on the second camera. It works both ways from one camera to the other.

The equation of epipolar line is given by:

$$l_1 = \mathcal{F}p' \tag{2}$$

The equation of epipolar line in the other image is given by:

$$l_2 = p^T \mathcal{F} \tag{3}$$

We can then using the information contained int the result, we obtain in homogeneous coordinates, we can extract the lines equation as:

$$au + bv + p = 0 \tag{4}$$

and then we get,

$$v = -\frac{a}{b}u - \frac{p}{b} \tag{5}$$

Once we have the epipolar lines associated with both the images, we can plot the results in the images. Same has been shown in the figure 3.

2.4 Estimation of the Epipoles

The epipoles are the null spaces of the matrix $\mathcal{F}$, and can be determined using:

$$e_1 \mathcal{F} = 0 \tag{6}$$

$$\mathcal{F} e_2 = 0 \tag{7}$$

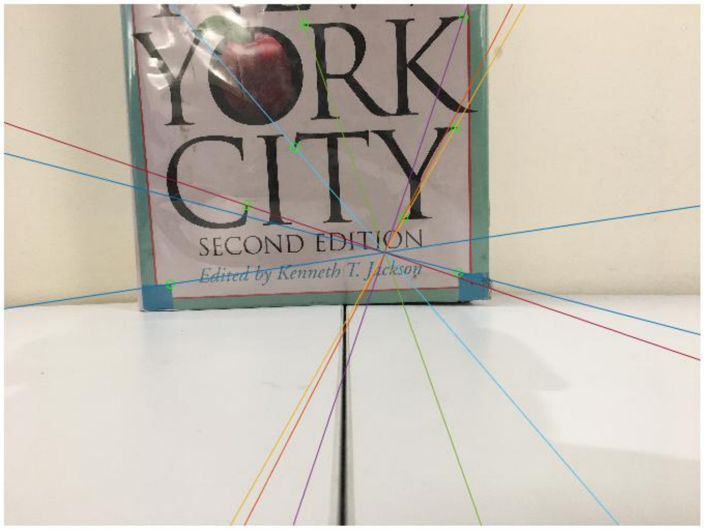
To do so, we can rely on the computation of the normalized eigenvector associated to the smallest eigenvalue fo the matrix $\mathcal{F}$'$\mathcal{F}$.

3 Results

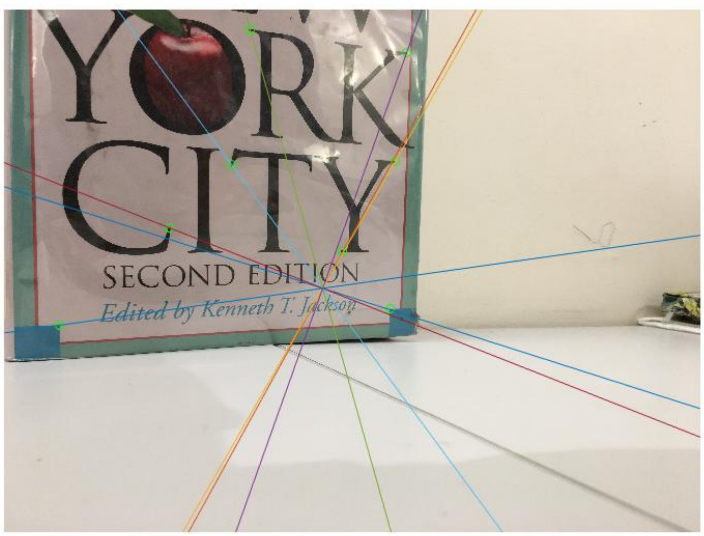
3.1 Funamental Matrix

The estimated Fundamental matrix using the above described method obtained is,

$$\mathcal{F} = \begin{bmatrix} -3.59771146663311e-08 & -2.15949483390579e-06 & 0.00318981196127254 \\ 1.99944563167745e-06 & -9.66167427408277e-08 & -0.00426927270475672 \\ -0.00315874109425805 & 0.00414093720903172 & 0.999972236701263 \end{bmatrix} \tag{8}$$



(a) First Image.



(b) Second Image, move camera to left and rotate it.

Figure 3: Epipolar lines reconstruction on left and right image.

3.2 Epipole Location

The calculated epipoles for the respective images are:

$$e_1 = \begin{bmatrix} 1845.38 & 1613.01 \end{bmatrix}^T \tag{9}$$

and the epipoles for the respective images are:

$$e_2 = \begin{bmatrix} 2204.82 & 1440.37 \end{bmatrix}^T \tag{10}$$

The two estimated values make sense because we can see these lines intersecting at a point. If we try to read the results from the image intself where are the epipolar lines are intersecting, then we can clearly see that the oberserved pixels have almost the same values as the calculated values.
The observed values are:

$$e_1 = \begin{bmatrix} 1845.5 & 1611.5 \end{bmatrix}^T \tag{11}$$

$$e_2 = \begin{bmatrix} 2205.15 & 1441.41 \end{bmatrix}^T \tag{12}$$

And the corresponding error for e_1 is :

$$error_1 = \begin{bmatrix} -0.12 & 1.51 \end{bmatrix}^T \tag{13}$$

and for e_2 is:

$$error_2 = \begin{bmatrix} -0.33 & -1.03 \end{bmatrix}^T \tag{14}$$

4 MATLAB Code

The pixel coordinates were stored in *x1* for the first image and in *x2* for the second image. And following that the code is self explanatory.

4.1 The Code

```
%Open the first image and take 8 points
orig = imread('orig.jpg');
imshow(orig);
[X,Y] = ginput(8);
x1 = [X,Y];

%Open the second image and take 8 points
tr = imread('tr.jpg');
imshow(tr)
[X,Y] = ginput(8);
x2 = [X,Y];

%%
%Make the x1 and x2 Homogeneous
homo_x2 = [x2, ones(8,1,'double')];
homo_x1 = [x1, ones(8,1,'double')];

%%
%Compute the F matrix
[F,inliers] = estimateFundamentalMatrix(x1,x2,'Method','Norm8Point');

%%
```

```
%Display the second image
imshow(tr);
hold on;

%Plot the taken points on the image
plot(x2(inliers,1),x2(inliers,2),'go')

%Find the epipolar lines
epiLines1 = [F * homo_x1']';
points = lineToBorderPoints(epiLines1,size(tr));

%Plot the epipolar lines on the image
line(points(:,[1,3])',points(:,[2,4])');
truesize;

%Find the epipole using the formula
[D1,E1] =eigs(F*F');
e1calculated = D1(:,1)./D1(3,1);

%%
%I found that the epipole lies in the image itself, hence I obeserved the
    values from the image.
e1observed  = ginput(1);
e1observed = [e1observed 1]';

%Find the difference in observed and calculated epipoles
error1 = e1calculated - e1observed;

%%
%Display the first image
imshow(orig);
hold on;

%Plot the taken points on the image
plot(x1(inliers,1),x1(inliers,2),'go')

%Plot the
epiLines2 = homo_x2 * F;
points = lineToBorderPoints(epiLines2,size(tr));

%Plot the epipolar lines on the image
line(points(:,[1,3])',points(:,[2,4])');
truesize;

%Find the epipole using the formula
[D2,E2] =eig(F'*F);
e2calculated = D2(:,1)./D2(3,1);

%%
%I found that the epipole lies in the image itself, hence I obeserved the
    values from the image.
e2observed  = ginput(1);
e2observed = [e2observed 1]';

```

```
%Find the difference in observed and calculated epipoles
error2 = e2calculated - e2observed;
```