

# Dynamic Programming - I & Doubt Clearing Session

Foundation Course on Data Structures & Algorithms - Part II

→ Dynamic Programming → (Recursion) → Master

what → ?

~~प्रश्न~~

→ Top-Down [Rec + Mem]

→ Bottom-Up [table]

↳ technique

↳ to solve

problem

0	1	1		
1	1	2		

✓  
✓

$f(3)$

→ Overlapping Subproblem ✓

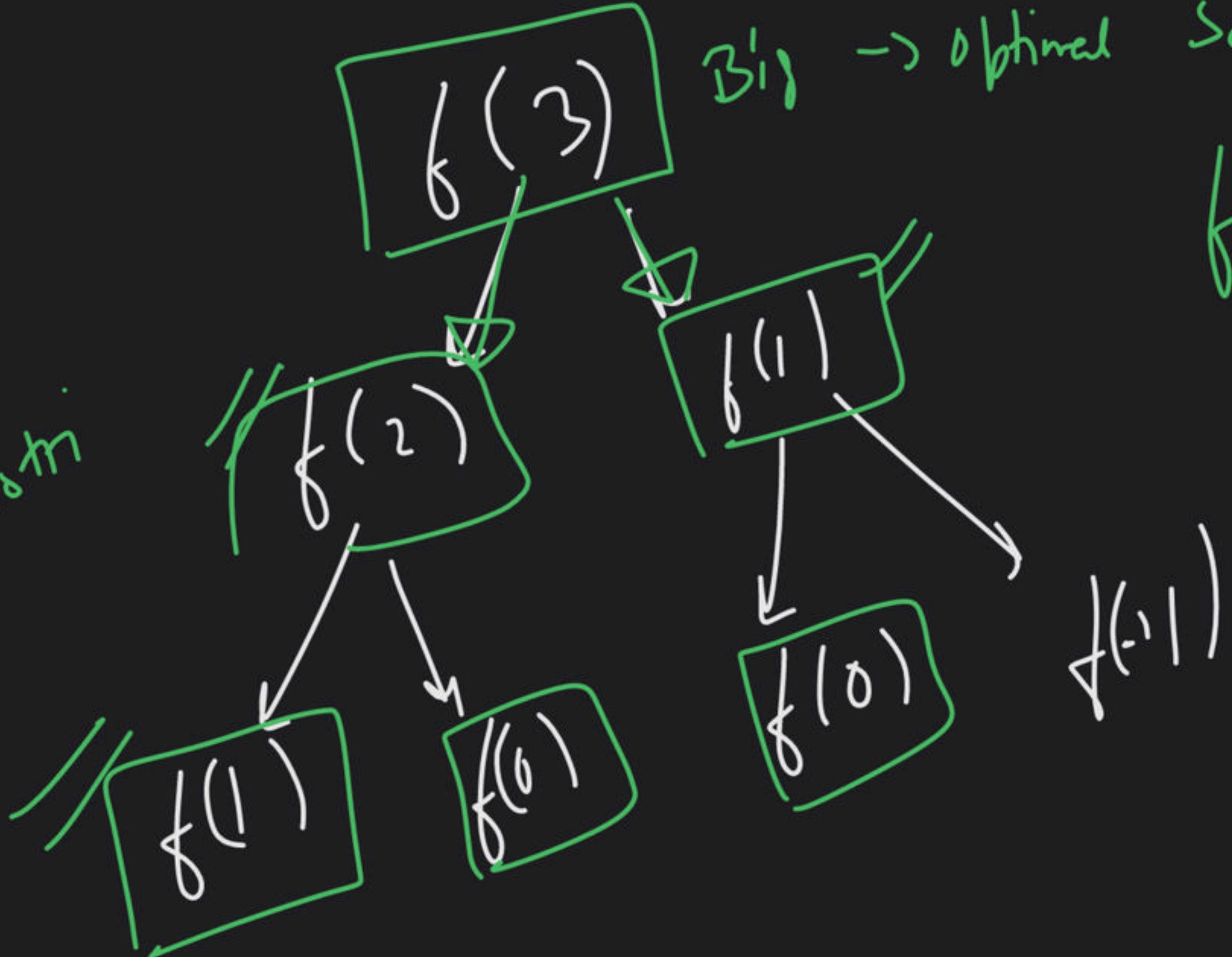
→ Optimal Substructure ✓



Big  $\rightarrow$  optimal solution

$$f(3) = \underline{f(2) + f(1)}$$

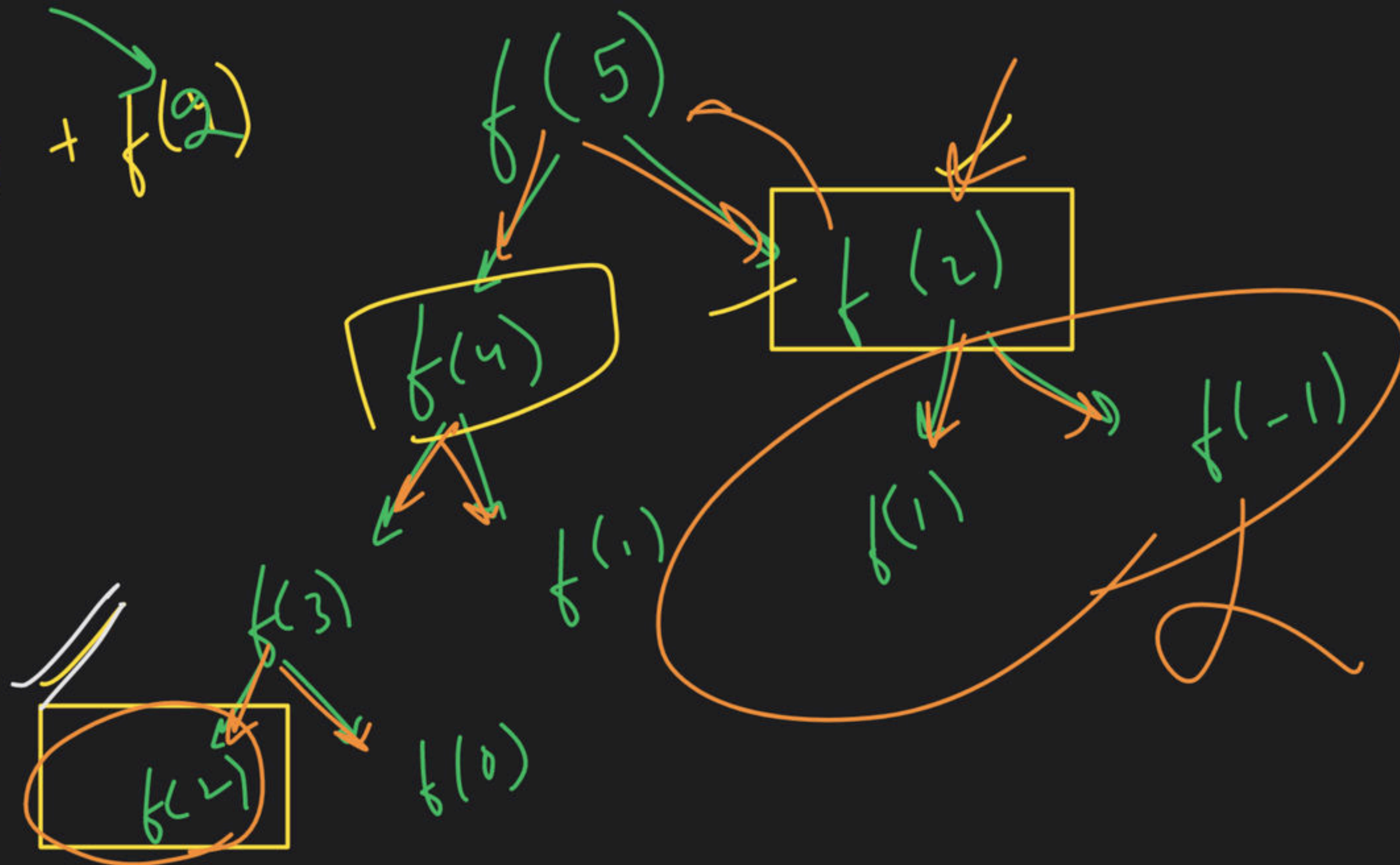
cho tri



R.R  $\rightarrow$

$$f(i) = f(i-1) + f(i-3)$$

$$\underline{\underline{f(5)}} = f(4) + f(2)$$



DP:-

[those who cannot remember the Past  
are condemned to repeat it.]



Types:-

$i \rightarrow 0 \rightarrow n$

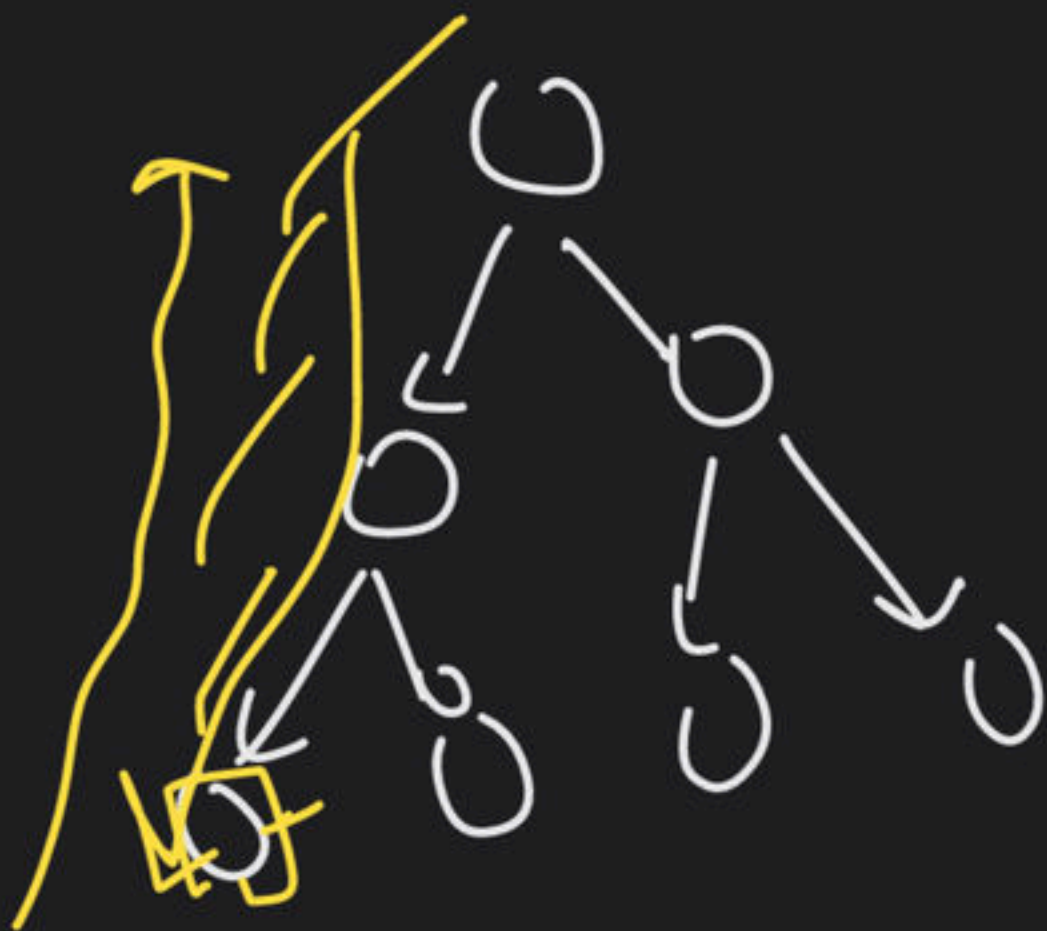
Top-Down Approach

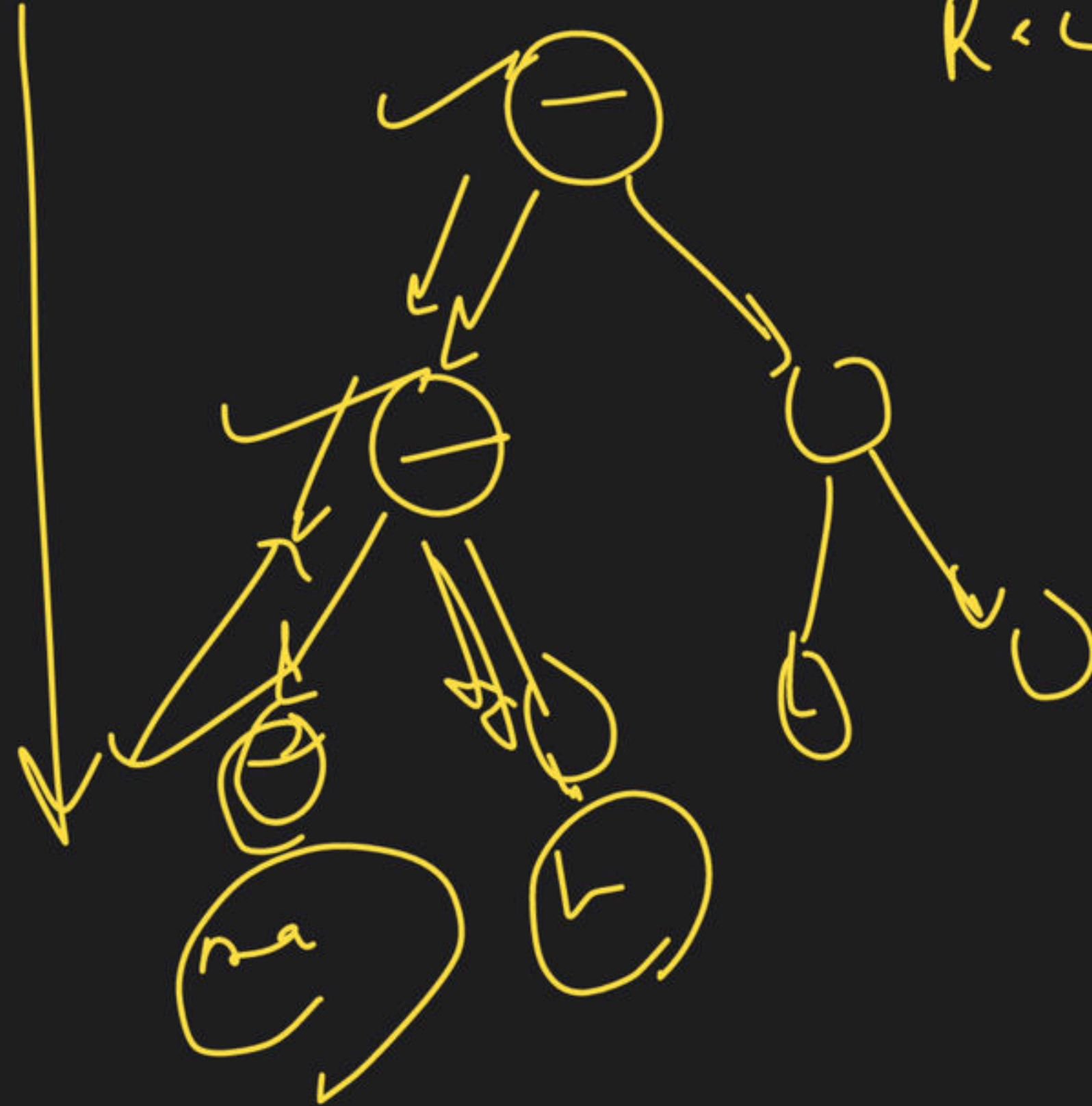
Bottom-Up Approach

$i \rightarrow n \rightarrow 0$

[ Recursion +  
Memoisation ]

↑  
[ Tabulation method ]

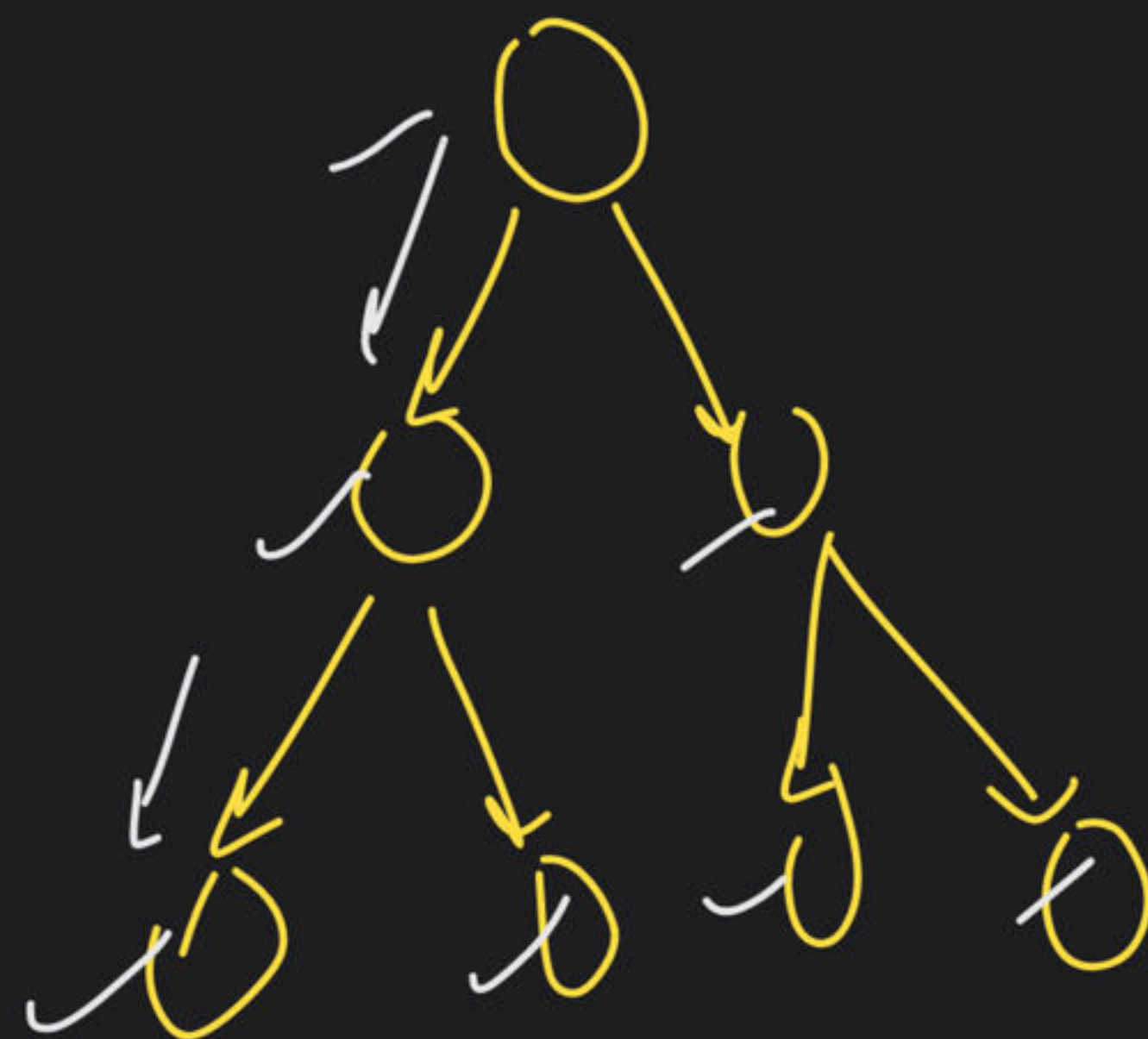




Rec

max Sum

Root  $\rightarrow$  Leaf





# → DP Problem:-

n ✓

CodeChef

DP series

1D DP →



## ① Recursion

Can we apply DP → i.s  
↓  
1D/2D/3D?

②

Recursion + mem

③

Bottom Up soln

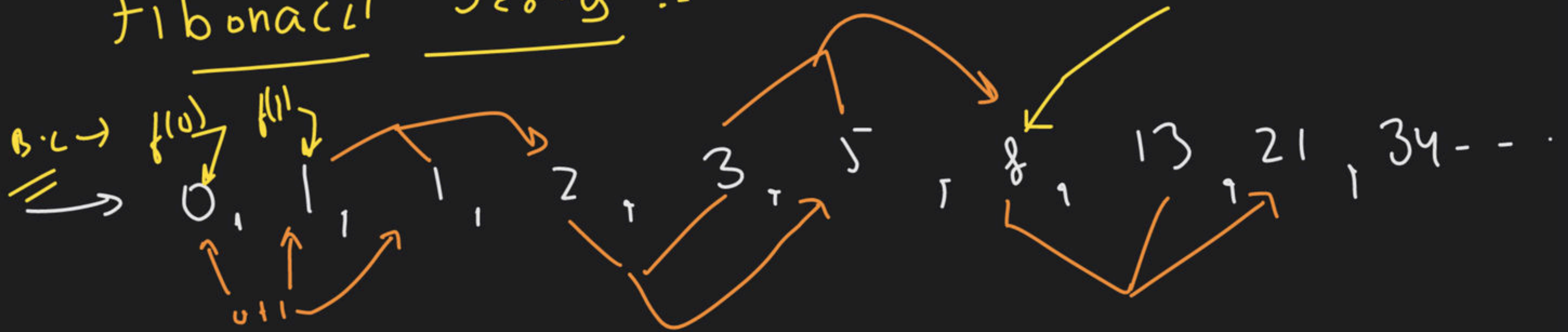
Space Optimisation

2D →

dp array depends



# ① Fibonacci Series :-

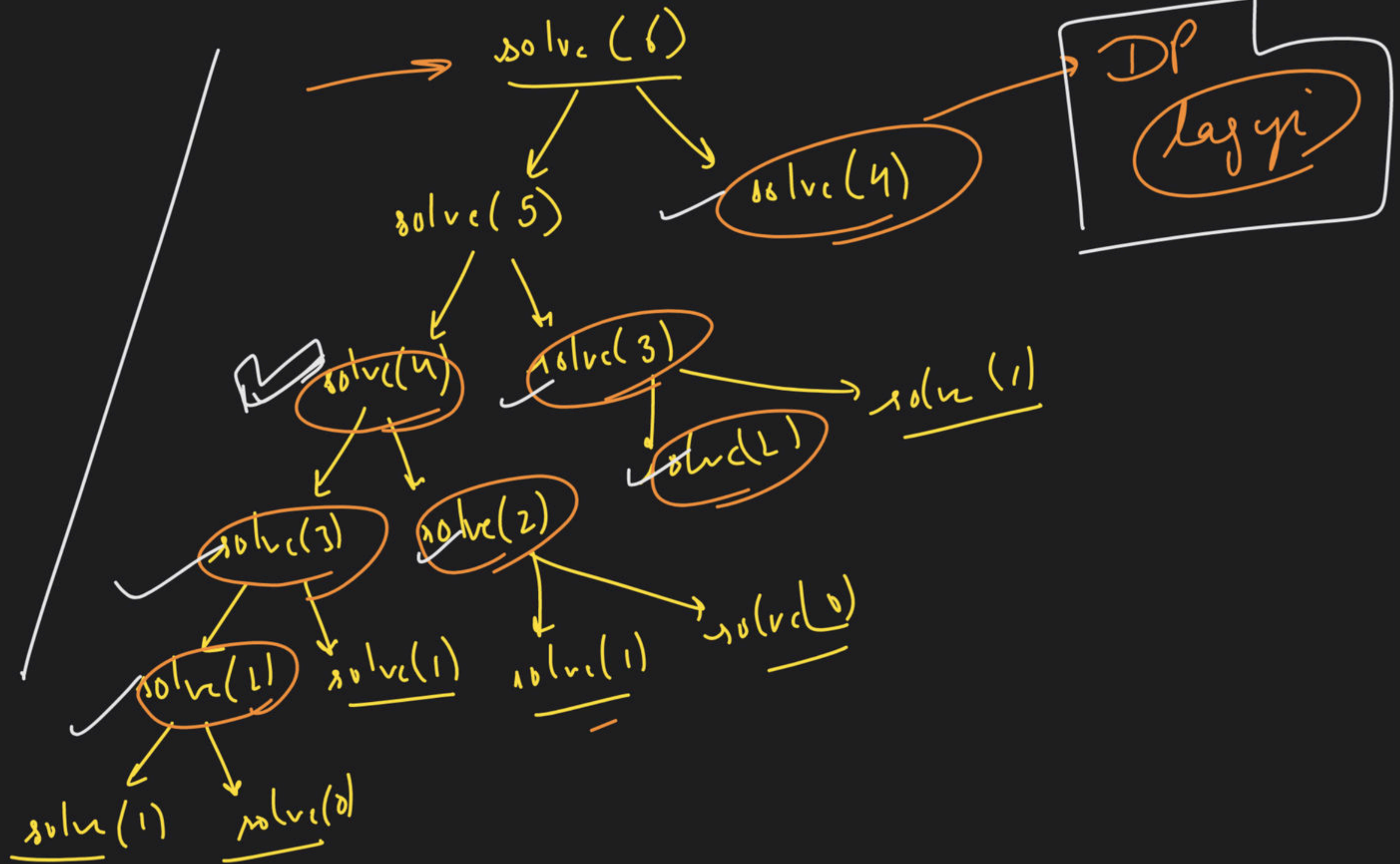


(R.R)

$$f(n) = f(n-1) + f(n-2)$$

T.C  $\rightarrow$  exponential







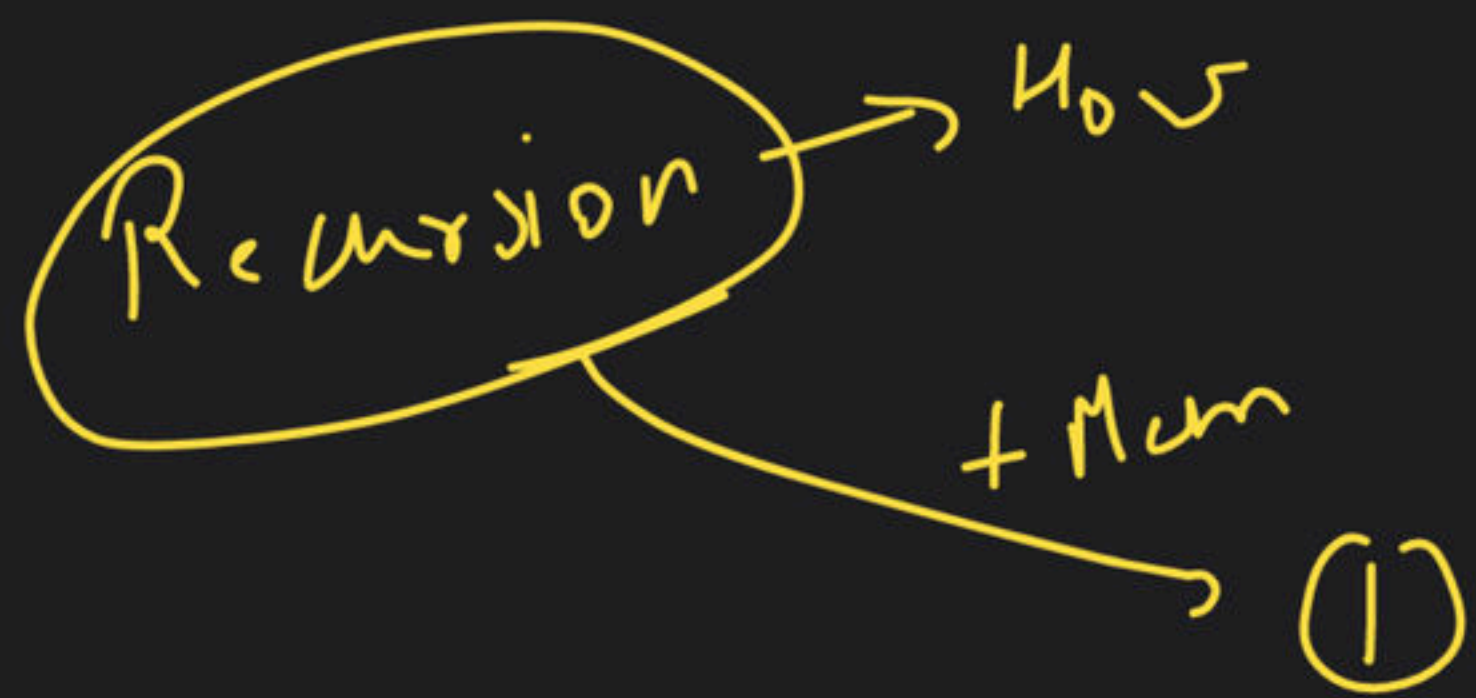


vector<int> dp(n+1)



✓  
solution

~~dp(n)~~

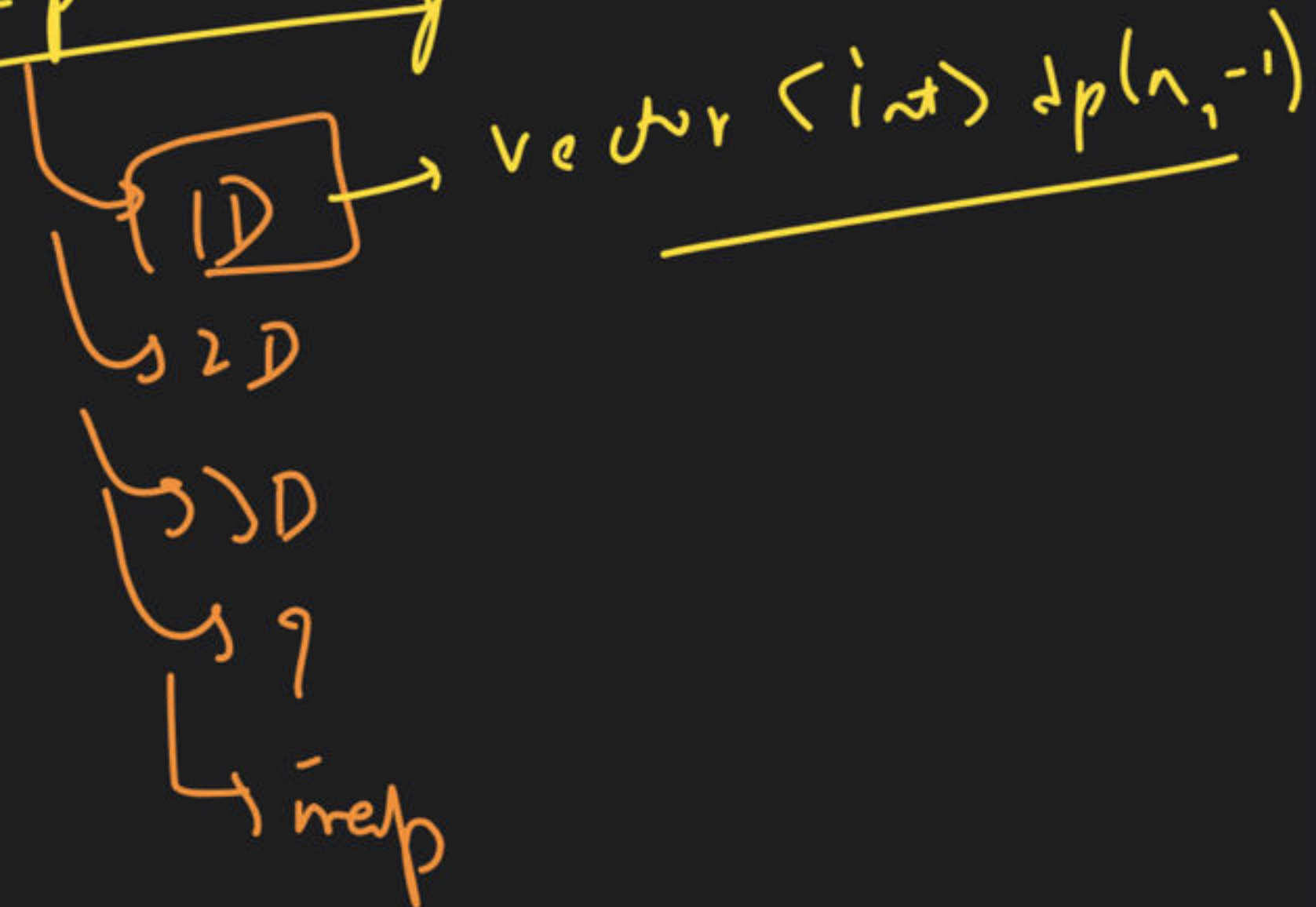


solve(n)

n-1, n-2

solve(— 9 — 9 — 9 —)

create dp array + pass function



(2)

return ans;

return dp[n] = ans;

(3) Base case → Khetam

if (dp[n] != -1)  
return dp[n];



$$dp[2] = 1$$

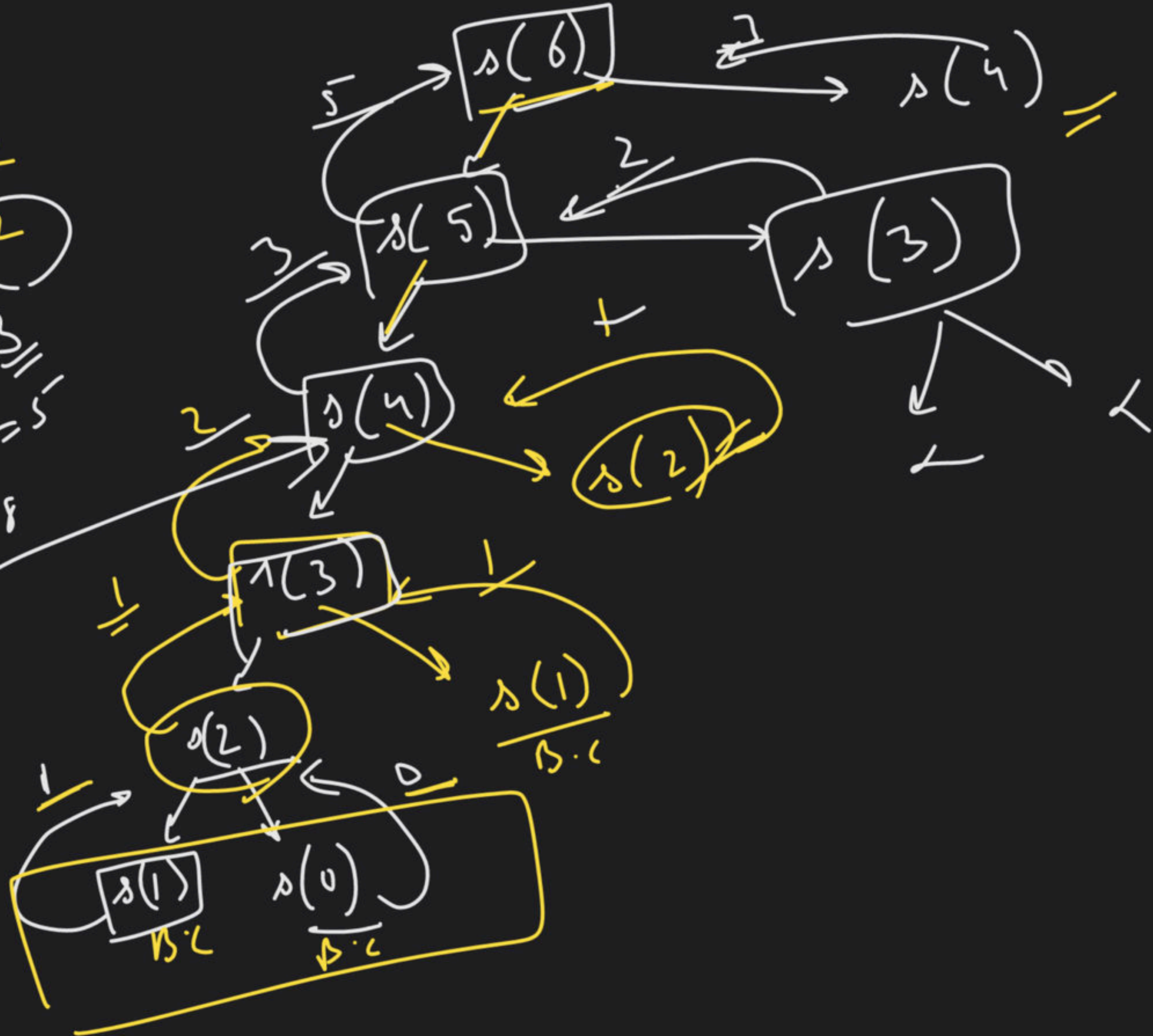
$$dp[3] = 2$$

$$dp[4] = 3$$

$$dp[5] = 5$$

$$dp[6] = 8$$

Top-Down



# Recursion

int solve (int i, dp)

{

    B.C

    if (dp[i] != -1)

        return dp[i];

    R.R

    return ans;

}

1D  
R+M

B.C

if (dp[i] != -1)  
    return dp[i];

R.R

return ~~ans~~;    dp[i] = ans;

}



dp

vector<int> dp ( $\frac{n+1}{2} - 1$ );

solveMem (n, dp)

solveMem (n, dp)

if (

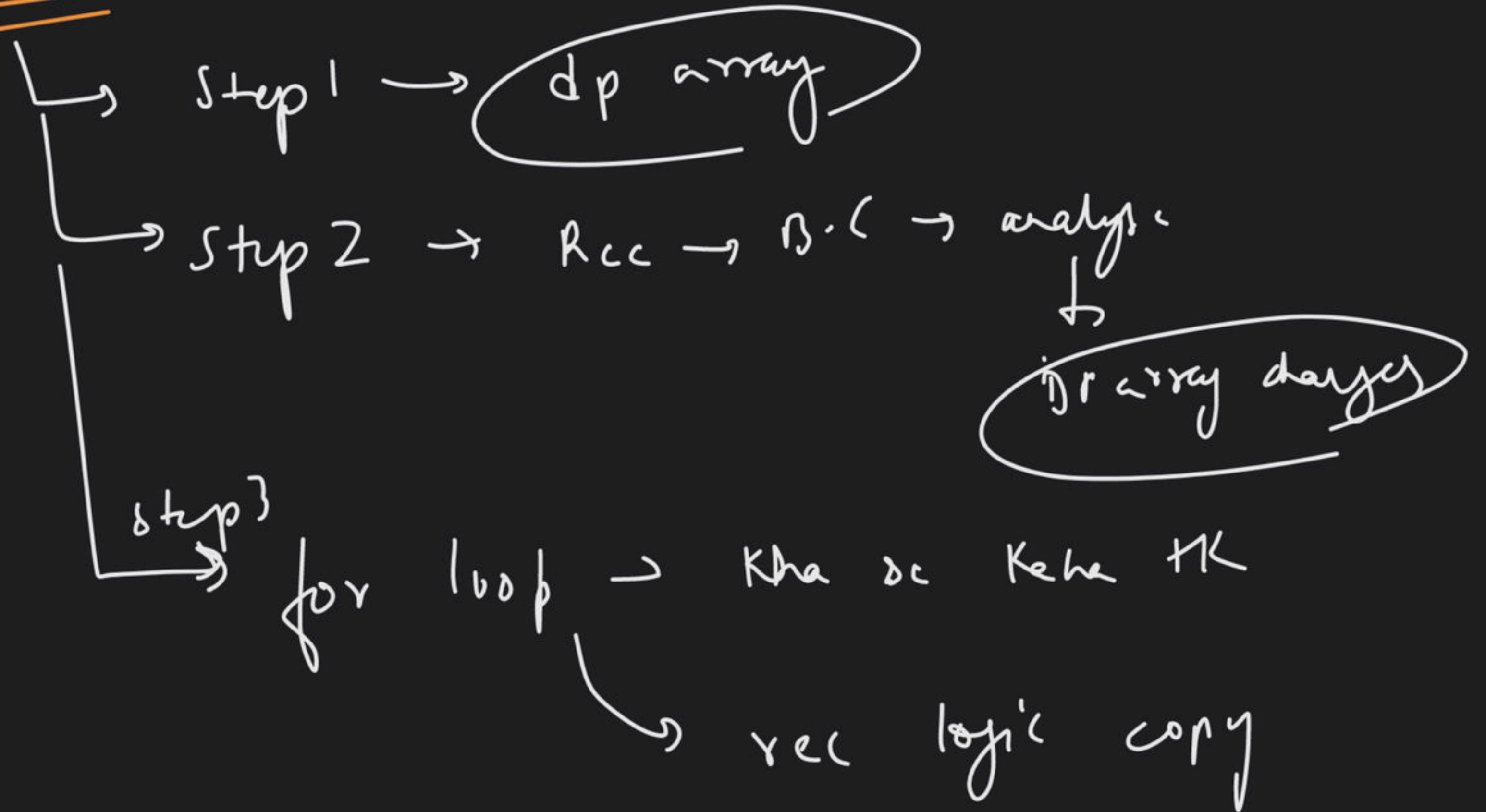
if (dp[n] != -1)

return dp[n]





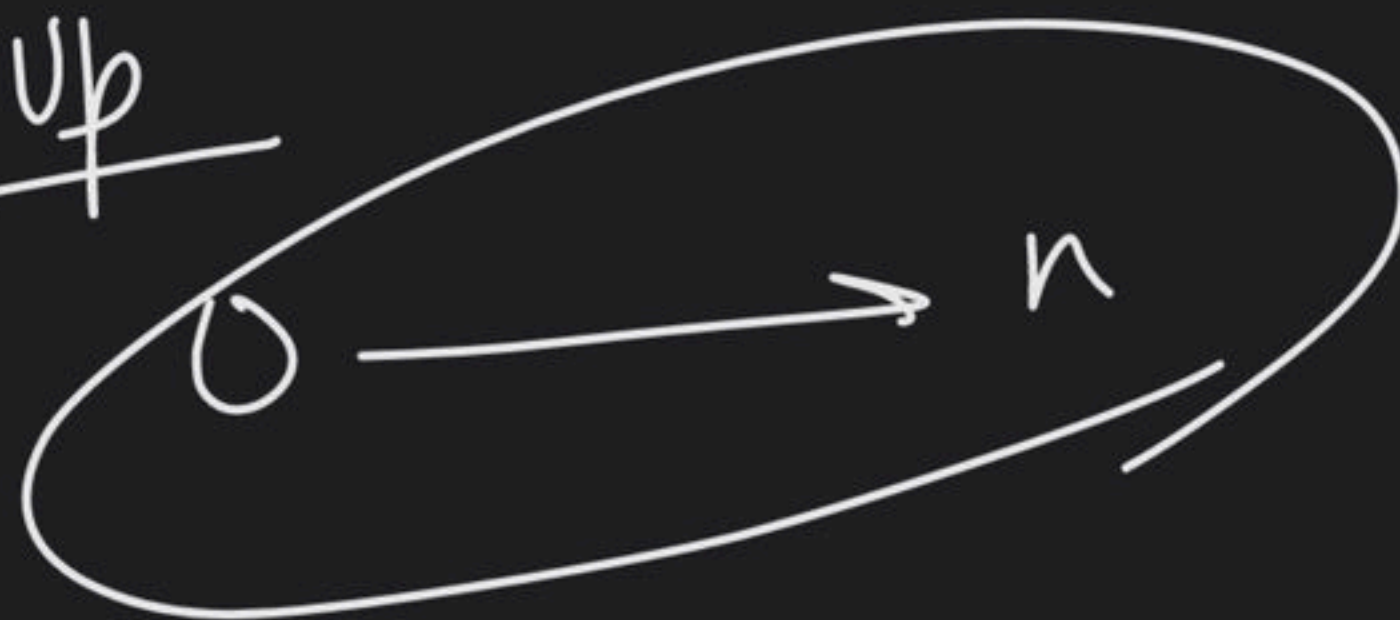
### (3) Bottom - Up



Top-Down



Bottom-Up



$R \rightarrow \text{Exps}$

$R + M \rightarrow O(n) =$

$\rightarrow S.C \rightarrow \underbrace{O(n)}_{\substack{\uparrow \\ \text{dp}}} + \underbrace{O(n)}_{\substack{\uparrow \\ \text{Recursion}}}$

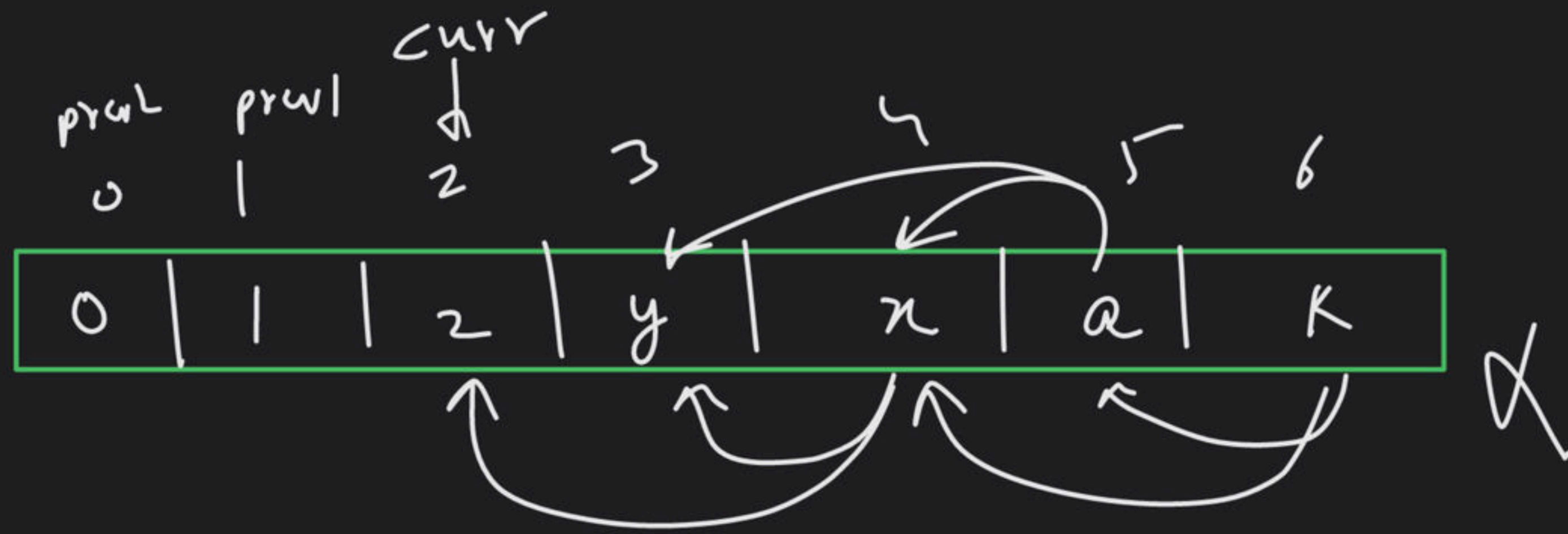
$B.V \rightarrow T.C = O(n)$

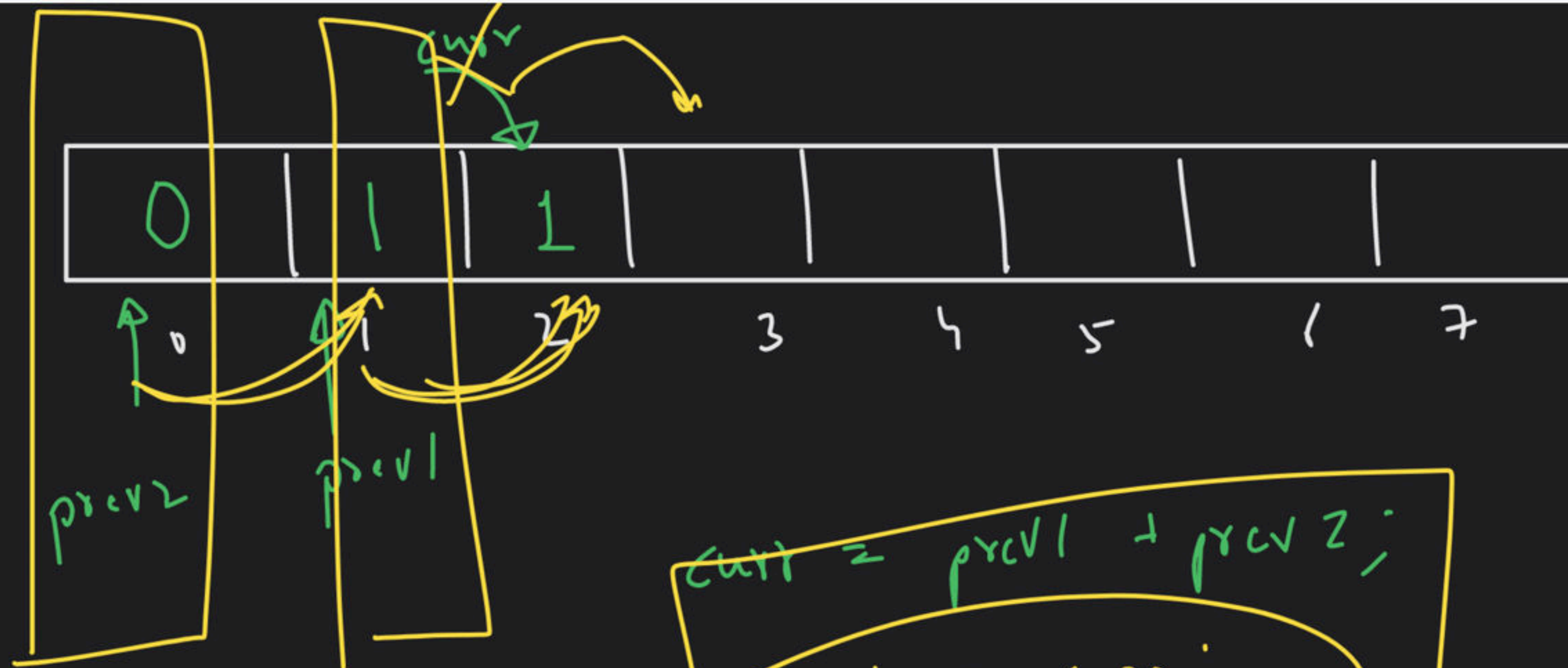
$\rightarrow S.C = O(n)$

S.D

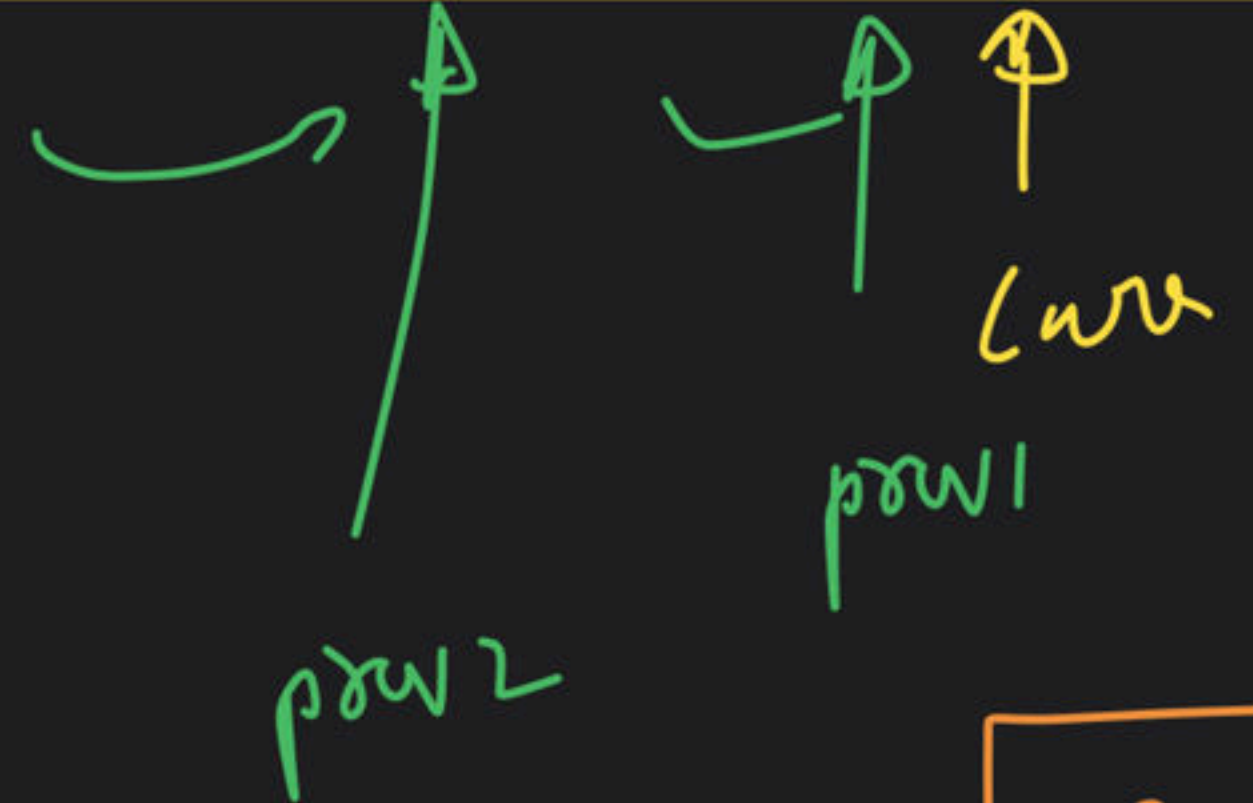
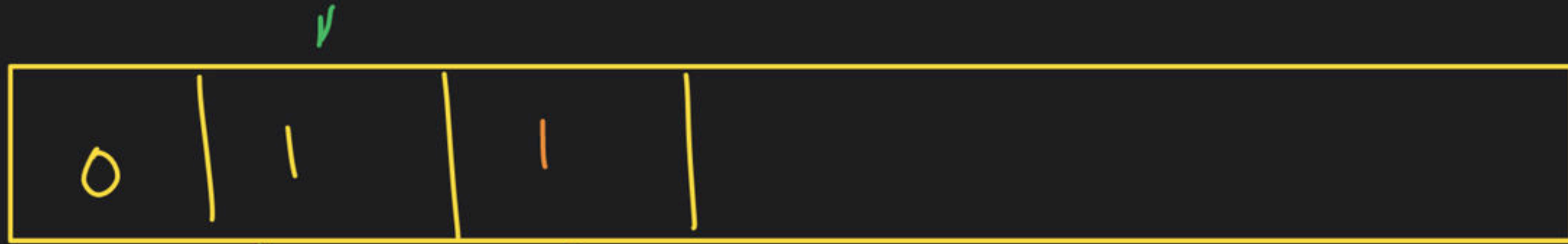








$$\text{curr} = \text{prev1} + \text{prev2};$$
$$\text{prev1} = \text{curr};$$
$$\text{prev2} = \text{prev1};$$



int curr = prev1 + prev2

→

prev1 = curr

prev2 = prev1

✓

prev2 = prev1

prev1 = curr

✓



return ans

1D

Recursion

(n)

$\frac{n-1}{1}$  ,  $\frac{n-2}{1}$

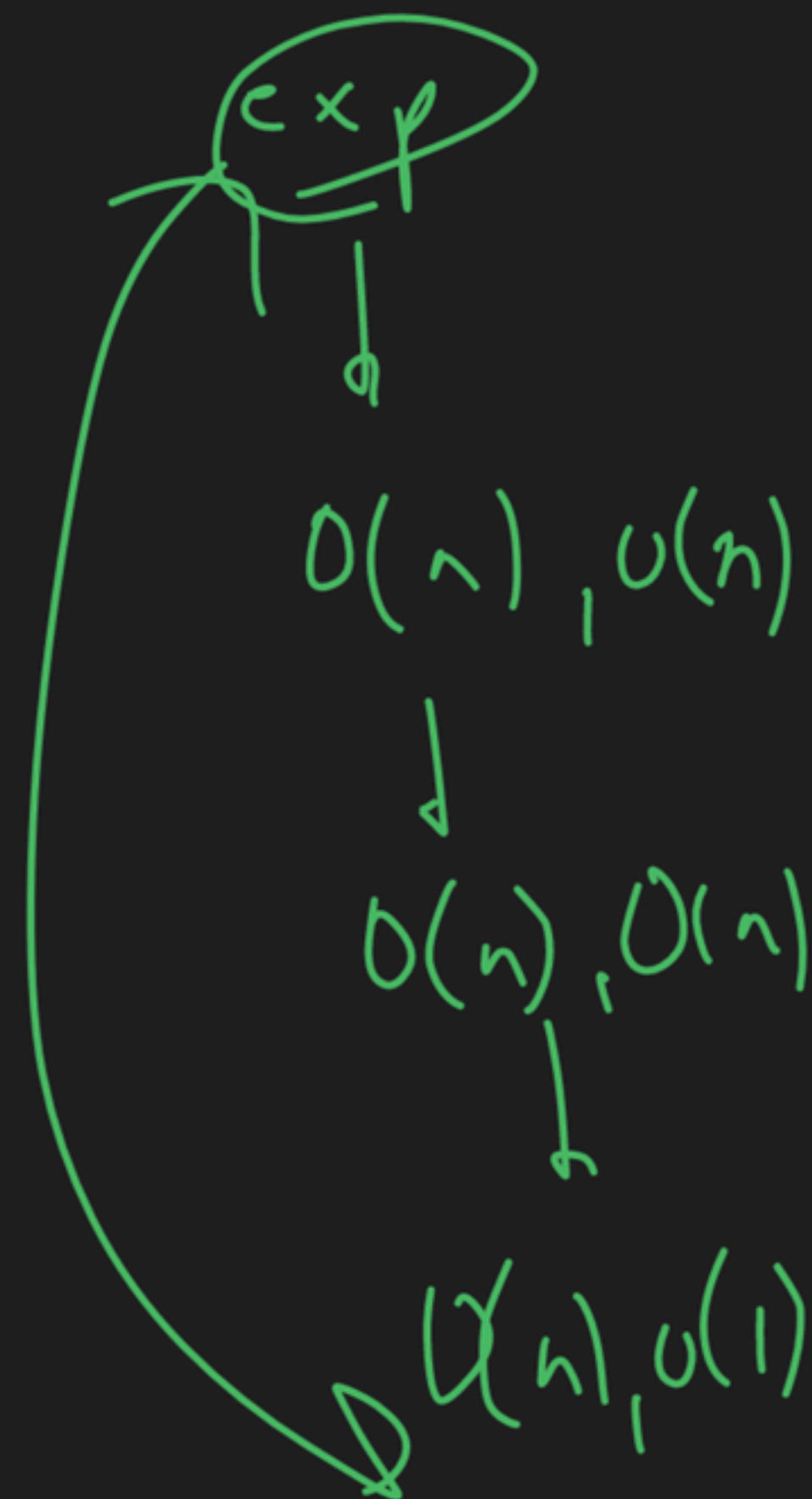
}

dp[] , dp[][] , dp[][][]

return dp[n] - ans

dp[n] = ans  
return ans

Question :-



dp[i]

it's fib town

Rec  $\rightarrow$

fib

$\text{if}(n == 0)$   
return 0

$\text{if}(n \geq 1)$   
return 1;

$\text{dp}[0] = 0$

$\text{dp}[1] = 1$

$n$   $ans$   $n$   $ans$



Ques → Recursion → Rec tree

↓  
Overlapping Subproblems

int solve (i - (dp))

↓  
ans

if (dp[i] != -1)  
return dp[i]

DP

Rec + Mem

① create dp array  
= vector<int> dp(n, -1)

↓  
R.R (dp)  
return dp[i] = ans;

②  
2<sup>nd</sup> step



Bottom Up

dp array

vector<int> dp(n, 0);

dp[0] = 0;

for {

n -> 0

Recall

dp[]

dp[i] = dp[i-1] + dp[i-2]

~~return dp[n];~~

return dp[n]; }

S.O

①

dp array &  
int row =  
int prevL =  
    curr

②

for

dp() & dp()

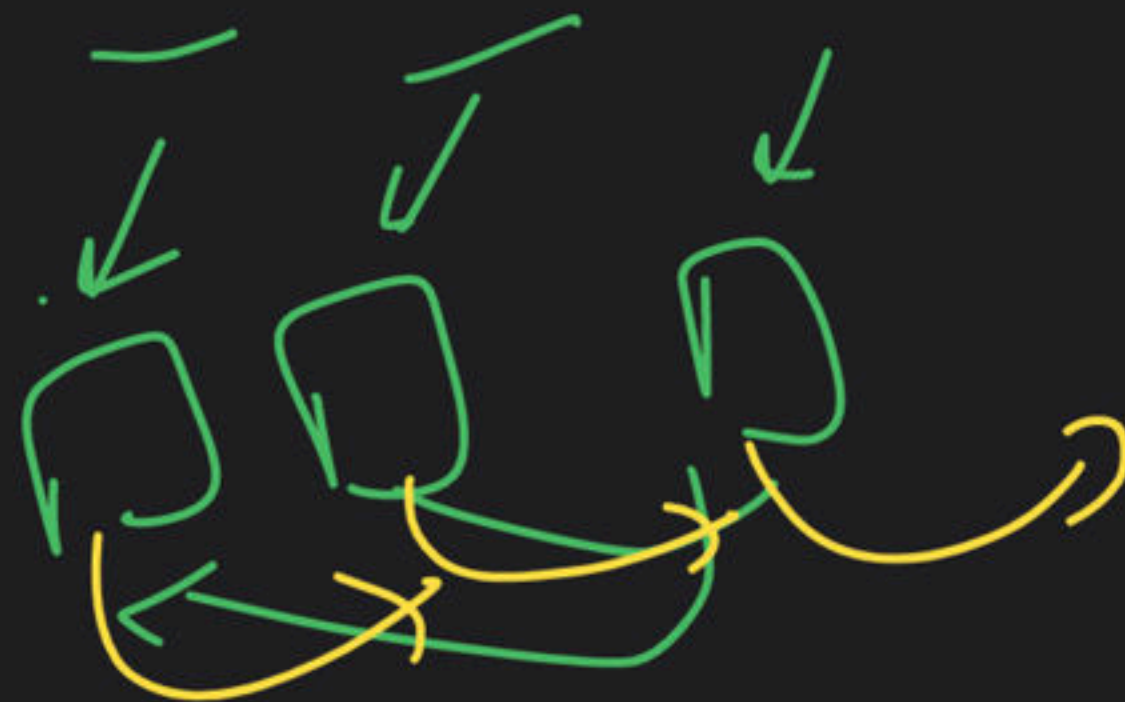
(curr, row, prev2)

③

movement



return row;





Video

10 min

15 min

3-5 code

DP

Rec

Rec + Mem

B. Up

S. U

3 line

5 min

6 min

10 min



Rec: - `int solve (vector<int> nums, int n)`  $\sum$  `vector<int> &dp`

①  
1D or 2D

$n \rightarrow 0$

`vector<int> dp ( $n+1$ )`

`if ( $n == 0$ )  
    return 0;`  
`if ( $n < 0$ )  
    return INT_MAX;`

`if ( $dp[n] \neq -1$ )  
    return  $dp[n]$`

`int mini = INT_MAX;  
for (int i = 0; i < nums.size(); i++)  
{  
    int ans = solve (nums,  $n - \text{nums}[i]$ , dp)  
    if ( $ans \neq \text{INT\_MAX}$ )  
        mini = min (mini, ans);  
}  
return ans; dp [ $n$ ] = ans;`



T.P

$n \rightarrow 0$

D.V

$0 \rightarrow n$

D.C

Wild Card

Pattern Matching

H/W  $\rightarrow$  DP

int solveTab ( — )

{ ① return <int> dp(n+1, 0)

②  $dp[0] = 0;$

③ for (int i = 0; i < n; i++)

part

~~solve(nums, n - nums[i], dp)~~

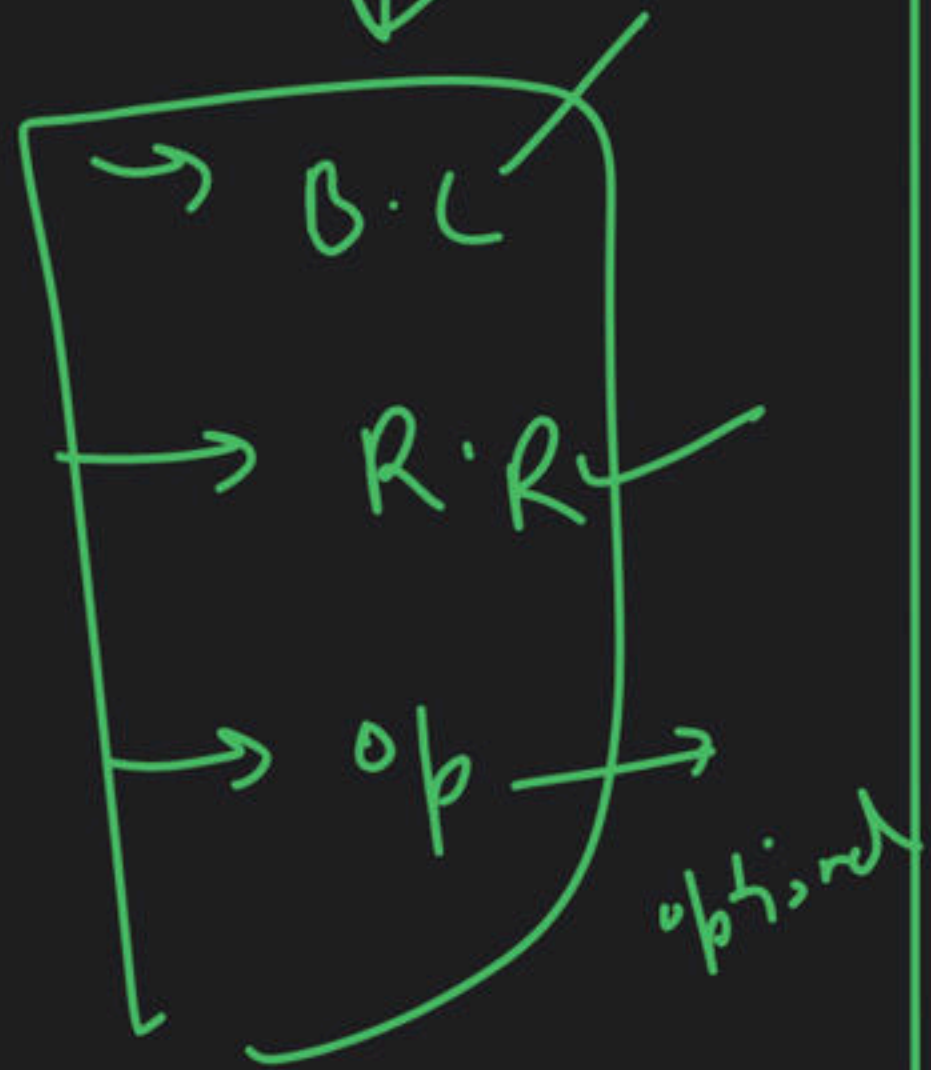
$dp[n - nums[i]]$

④ return dp[n];



1 → (51) → 800

Rec



(Top-Down)  
Rec + Mem

- Step 1 → DP array create  
↓  
pass function
- Step 2 → ans store  
dp array &  
return
- Step 3 → after B.C,  
if ans already present  
, return it (No need  
of Rec calls)

(Tabulation)  
Bottom-UP

- (51)  
→ Create DP Array  
↳ (initialisation)
- Rec → Base Cases  
↓  
analyse  
dp array update
- for loop → (reverse-  
T-D  
range)
  - ↳ copy paste  
↳ Rec logic  
↳ replace Rec  
calls → dp

Space  
Optimisation

- (check  
dependency  
of current ans)
- ↳ soln exist
- ↳ remove dp,  
apply D.S/var
- ↳ for(dp[1], dp[2])  
↓  
replace  
↳ movement of ans  
↳ extra code

















