There are 2 main types of philosophies in probability, Bayesian & Frequentist. Frequentist formulates their hypotheses on the basis of sampling/experiments. Bayesian formulate their hypothesis on the basis of underlying beliefs, concepts & assumptions of distribution governing the data.

The foundation of Bayesian decision theory lies in the Bayes theorem. The foundation of the Bayes theorem lies in the 2 rules of probability:

1) Sum Rule: $p(Y) = \sum_{Y} p(X, Y)$

2) Product Rule: $p(X, Y) = p(Y|X)p(X)$

Bayes theorem calculates the conditionals probability $p(Y|X)$ based on the likelihood & prior $p(Y|X) = p(X|Y)p(Y) \div p(X)$

Bayesian decision theory uses Bayes theorem to decide(choose) the optimal action based on the minimum conditional(overall) risk for each action (or maximum conditional posterior for the action):

$$\alpha(x) = argmin_{\alpha} R(\alpha|x) \;\; where \; R(\alpha|x) = -p(\alpha|x)$$

Hence, using Bayesian decision theory we classify the best action that gives us the minimum risk (Bayesian Risk, R*) for the given ‘x’.