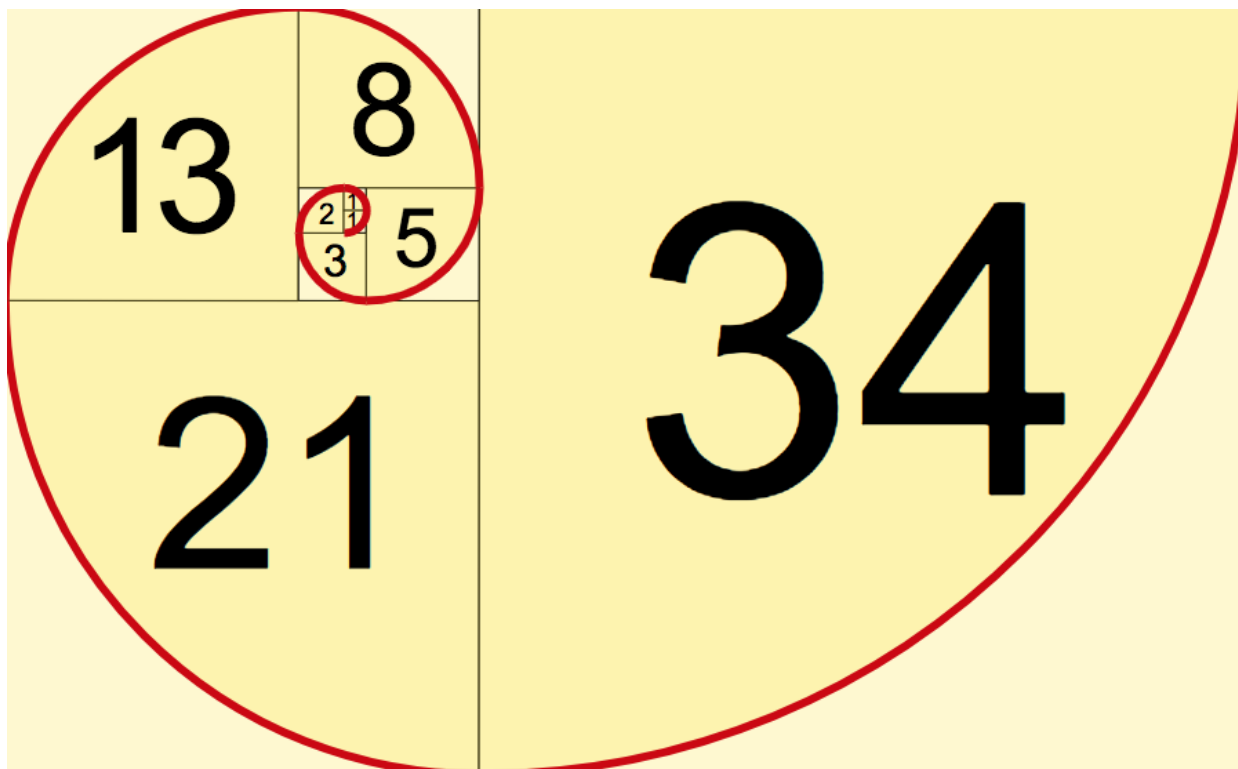


Fibonacci Sequence in TensorFlow

Fibonacci sequence is a series of numbers where a number is found by adding up the two numbers before it. Starting with 0 and 1, the sequence goes: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so on.

Each computed value in this series depends on previously computed values



Our goal is to calculate the Fibonacci sequence using tensors, where

$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$, and $\text{fib}(0) = 0$ and $\text{fib}(1) = 1$.

In the next, first exercise we will use matrix multiplication. We will start with an initial vertical vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

That vector represents the zero-th and the first Fibonacci numbers.

That vector is multiplied from the left by a 2x2 matrix:

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

This gives us a new vertical vector with the first and the second Fibonacci number. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Every subsequent multiplication by the matrix give us the next Fibonacci number, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and so forth.

Fibonacci Numbers as a Matrix Calculation

- The difference equation describing Fibonacci numbers could be expressed as a matrix sequence:

$$\begin{pmatrix} F_{k+2} \\ F_{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_{k+1} \\ F_k \end{pmatrix}$$

- The above means that we can also write:

$$\begin{pmatrix} F_{k+1} \\ F_k \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^k \begin{pmatrix} F_1 \\ F_0 \end{pmatrix}$$

```
In [10]: import tensorflow as tf

with tf.Graph().as_default() as g:

    # We want to calculate and output the Fibonacci sequence using TF. We will use
    # tf.assign() to perform the multiplications and assign the result
    # back to the variable fib_seq.

    fib_matrix = tf.constant([[1.0, 1.0],
                              [1.0, 0.0]], name='matrix')
    print (fib_matrix)

    # Initialize fib_seq to a 2x1 TensorFlow tensor *Variable* with
    # the initial values of 1.0 and 0.0.
    # Below, we make sure we specify a 2D tensor of shape 2x1, and
    # not a 1D tensor or a tensor of dimension 1x2.
    # See fib_matrix above (a 2x2 2D tensor)
    fib_sequence = tf.Variable([[1.0], [0.0]], name='fib_sequence')
    print (fib_sequence)
    # We need to multiply fib_matrix and fib_sequence using tf.matmul()
    next_fib = tf.matmul(fib_matrix, fib_sequence)

    # And assign the result back to fib_sequence using tf.assign()
    assign_op = tf.assign(fib_sequence, next_fib)

    init = tf.global_variables_initializer() # an operation, initializes all
    with tf.Session() as sess:
        sess.run(init) # run init operation, initialize all
        print(sess.run(fib_sequence)) # show that the initial fib_sequence #this
        print(sess.run(fib_matrix))
        print('*****') # is a 2x1 matrix
        for step in range(30):
            sess.run(assign_op)
            print(sess.run(fib_sequence [0]))

    file_writer = tf.summary.FileWriter("fibonacci01", sess.graph)
    # file_writer.add_graph(sess.graph)
    file_writer.close()
    sess.close()
```

```
Tensor("matrix:0", shape=(2, 2), dtype=float32)
<tf.Variable 'fib_sequence:0' shape=(2, 1) dtype=float32_ref>
[[ 1.]
 [ 0.]]
[[ 1.  1.]
 [ 1.  0.]]
*****
[ 1.]
[ 2.]
[ 3.]
[ 5.]
[ 8.]
[ 13.]
[ 21.]
[ 34.]
[ 55.]
[ 89.]
```

```
[ 144.]  
[ 233.]  
[ 377.]  
[ 610.]  
[ 987.]  
[ 1597.]  
[ 2584.]  
[ 4181.]  
[ 6765.]  
[ 10946.]  
[ 17711.]  
[ 28657.]  
[ 46368.]  
[ 75025.]  
[ 121393.]  
[ 196418.]  
[ 317811.]  
[ 514229.]  
[ 832040.]  
[ 1346269.]
```

The Fibonacci sequence could be calculated by direct calculation, i.e. a recursion, where we start with values: $\text{fib}(0)=0$ and $\text{fib}(1)=1$, and repeatedly apply formula $f(n) = f(n-1) + f(n-2)$. The recursive relationship is naturally expressed as a computational recursion or recursive function.

```

In [11]: # Pure Python
# define the recursive function fib() which invokes itself
def fib(n):
    if n <= 1:
        return n
    else:
        return(fib(n-1) + fib(n-2))
# find Fibbinacci number with index nn
nn = 30
if nn <= 0:
    print("The input must be a positive integer")
else:
    print("Fibonacci sequence:")
    for y in range(30):
        print("Fibonacci(%d) = %d" % (y,(fib(y))))
exit()

```

```

Fibonacci sequence:
Fibonacci(0) = 0
Fibonacci(1) = 1
Fibonacci(2) = 1
Fibonacci(3) = 2
Fibonacci(4) = 3
Fibonacci(5) = 5
Fibonacci(6) = 8
Fibonacci(7) = 13
Fibonacci(8) = 21
Fibonacci(9) = 34
Fibonacci(10) = 55
Fibonacci(11) = 89
Fibonacci(12) = 144
Fibonacci(13) = 233
Fibonacci(14) = 377
Fibonacci(15) = 610
Fibonacci(16) = 987
Fibonacci(17) = 1597
Fibonacci(18) = 2584
Fibonacci(19) = 4181
Fibonacci(20) = 6765
Fibonacci(21) = 10946
Fibonacci(22) = 17711
Fibonacci(23) = 28657
Fibonacci(24) = 46368
Fibonacci(25) = 75025
Fibonacci(26) = 121393
Fibonacci(27) = 196418
Fibonacci(28) = 317811
Fibonacci(29) = 514229

```

Let us do the same in TensorFlow

```
In [1]: import tensorflow as tf

#create variables to compute Fibonacci sequence; Fn_2 is Fibonacci (n-2), and Fn_1
#and so Fn is Fibonacci (n)
#add Fn_2 and Fn_1 to get Fn
Fn_2=tf.Variable(0, name='Fn_2')
Fn_1=tf.Variable(1, name = 'Fn_1')
temp=tf.Variable(0, name='temp')
Fn = tf.Variable(0,name='Fn')
Fn = tf.add(Fn_2,Fn_1)

#Constantly moving up: what was Fn in the last iteration, becomes Fn_1, and so on

#create tensors
update1=tf.assign(temp,Fn)
update2=tf.assign(Fn_2,Fn_1)
update3=tf.assign(Fn_1,temp)

#run session and print 30 numbers
init = tf.global_variables_initializer() #tensorflow assigns values; we execute
#this is all just painting of the graph of the operations we will execute
with tf.Session() as sess:
    sess.run(init)
    #We tell Session to execute init
    #Now we do the loop from 0 to 29, and print a; step is integer
    for step in range(30):
        print('Fibonacci('+str(step)+')= '+ str(sess.run(Fn_2))) #adding string
        sess.run(update1) #see above: what ever was in Fn (the most recently cal
        sess.run(update2) #whatever was in Fn_1 is pushed into Fn_2; whatever wa
        sess.run(update3) #calculate Fn
        #we always print the Lowest Fibonacci we have -- Fn_2

#create graph
file_writer = tf.summary.FileWriter("fibonacci02", sess.graph)
file_writer.add_graph(sess.graph)
file_writer.close()
sess.close()

#to see graph, we run on Command Prompt tensorboard --logdir fibonacci02
```

```
Fibonacci(0)= 0
Fibonacci(1)= 1
Fibonacci(2)= 1
Fibonacci(3)= 2
Fibonacci(4)= 3
Fibonacci(5)= 5
Fibonacci(6)= 8
Fibonacci(7)= 13
Fibonacci(8)= 21
Fibonacci(9)= 34
Fibonacci(10)= 55
Fibonacci(11)= 89
Fibonacci(12)= 144
Fibonacci(13)= 233
Fibonacci(14)= 377
Fibonacci(15)= 610
Fibonacci(16)= 987
```

```
Fibonacci(17)= 1597
Fibonacci(18)= 2584
Fibonacci(19)= 4181
Fibonacci(20)= 6765
Fibonacci(21)= 10946
Fibonacci(22)= 17711
Fibonacci(23)= 28657
Fibonacci(24)= 46368
Fibonacci(25)= 75025
Fibonacci(26)= 121393
Fibonacci(27)= 196418
Fibonacci(28)= 317811
Fibonacci(29)= 514229
```

In []: