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Stat 104: Quantitative Methods

Class 10: Introduction to Decision Analysis

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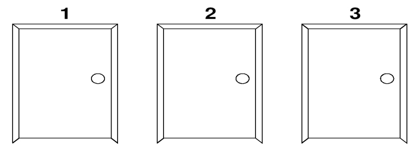
Let's Make a Deal (Monty Hall)

The Rules:

- Three doors – one prize, two blanks
- Candidate selects one door
- Showmaster reveals one losing door
- Candidate may then switch doors



Should you switch ?

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Famous Problem

https://www.youtube.com/watch?v=Zr_xWfThjJ0

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Events of interest:

- A – choose winning door at the beginning
- W – win the prize

Strategy : Switch doors (S)

Know : $P_S(W \mid A) = 0 \quad P_S(A) = 1/3$

$P_S(W \mid \bar{A}) = 1 \quad P_S(\bar{A}) = 2/3$

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Find $P_S(W)$

- Always Draw a Table!

	W	W'	
A	0	1/3	1/3
A'	2/3	0	2/3
	2/3	1/3	

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Events of interest:

- A – choose winning door at the beginning
- W – win the prize

Strategy : Do not switch doors (N)

Know : $P_N(W \mid A) = 1 \quad P_N(A) = 1/3$

$P_N(W \mid \bar{A}) = 0 \quad P_N(\bar{A}) = 2/3$

Find $P_N(W)$

- Always Draw a Table!

	W	W'	
A	1/3	0	1/3
A'	0	2/3	2/3
	1/3	2/3	

So the best strategy is to (choose one)

- Always Switch P(W) =2/3
- Never Switch P(W) =1/3

According to the *New York Times,* this problem and [Marilyn vos Savant's] solution were 'debated in the halls of the Central Intelligence Agency and the barracks of fighter pilots in the Persian Gulf' and 'analyzed by mathematicians at the Massachusetts Institute of Technology and computer programmers at Los Alamos National Laboratory in New Mexico.'"

Most people (including some mathematicians) think it doesn't matter

- "You blew it, and you blew it big! I'll explain: After the host reveals a goat, you now have one-in-two chance of being correct. Whether you change your answer or not, the odds are the same...Shame!"

Scott Smith, Ph.D., University of Florida
(*Parade*, 12/2/90)

Actually, many of Dr. Smith's professional colleagues are sympathetic. **Persi Diaconis**, a former professional magician who was a Harvard University professor specializing in probability and statistics, said there was no disgrace in getting this one wrong.

"I can't remember what my first reaction to it was," he said, "because I've known about it for so many years. I'm one of the many people who have written papers about it. But I do know that my first reaction has been wrong time after time on similar problems. ***Our brains are just not wired to do probability problems very well, so I'm not surprised there were mistakes."***

Summary of Probability Rules

For events A and B,

Rule 1: $P(A^c) = 1 - P(A)$ or $P(A) + P(A^c) = 1$

Rule 2: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If A and B are *mutually exclusive,*

$$P(A \text{ or } B) = P(A) + P(B)$$

Rule 3: $P(A \text{ and } B) = P(A)P(B \mid A) = P(B)P(A \mid B)$

If A and B are *independent,*

$$P(A \text{ and } B) = P(A)P(B)$$

Rule 4:

$$P(A \mid \mathbf{B}) = \frac{P(A \text{ and } B)}{P(\mathbf{B})} \quad \text{or} \quad P(B \mid \mathbf{A}) = \frac{P(A \text{ and } B)}{P(\mathbf{A})}$$

Introduction to Decision Analysis

- The field of decision analysis provides a framework for making important decisions.
- Decision analysis allows us to select a decision from a set of possible decision alternatives when uncertainties regarding the future exist.
- The goal is to optimize the resulting payoff in terms of a decision criterion.

Introduction to Decision Analysis

- The Key Elements of a Decision
- Decisions Based on Extreme Values
- Decision Trees and Expected Monetary Value

Decision Making Overview

- Decision Making
  - Decision Environment
    - Certainty
    - Uncertainty
  - Decision Criteria
    - Nonprobabilistic
    - Probabilistic

The Decision Environment

- Decision Environment
  - Certainty *
  - Uncertainty

Certainty: The results of decision alternatives are known

Example:

Must print 10,000 color brochures

Offset press A: $2,000 fixed cost + $.24 per page

Offset press B: $3,000 fixed cost + $.12 per page

The Decision Environment

- Decision Environment
  - Certainty
  - Uncertainty *

Uncertainty: The outcome that will occur after a choice is unknown

Example: You must decide to buy an item now or wait. If you buy now the price is $2,000. If you wait the price may drop to $1,500 or rise to $2,200. There also may be a new model available later with better features.

Decision Criteria

Nonprobabilistic Decision Criteria: Decision rules that can be applied if the probabilities of uncertain events are not known.

- Decision Criteria
  - * Nonprobabilistic
  - Probabilistic

Optimistic ▪ maximax criterion

Pessimistic ▪ maximin criterion

Decision Criteria

Probabilistic Decision Criteria: Consider the probabilities of uncertain events and select an alternative to maximize the expected payoff of minimize the expected loss

- Decision Criteria
  - Nonprobabilistic
  - * Probabilistic

- maximize expected value

A Payoff Table

A payoff table shows alternatives, states of nature, and payoffs

What to do

Investment Choice (Alternatives)	Profit in $1,000's (States of Nature)		
	Strong Economy	Stable Economy	Weak Economy
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20

We don't have probabilities here

Maximax Solution

The maximax criterion (an optimistic approach):

1. For each option, find the maximum payoff

Investment Choice (Alternatives)	Profit in $1,000's (States of Nature)			1. Maximum Profit
	Strong Economy	Stable Economy	Weak Economy	
Large factory	200	50	-120	200
Average factory	90	120	-30	120
Small factory	40	30	20	40

Maximax Solution

The maximax criterion (an optimistic approach):

1. For each option, find the maximum payoff
2. Choose the option with the greatest maximum payoff

Investment Choice (Alternatives)	Profit in $1,000's (States of Nature)			1. Maximum Profit
	Strong Economy	Stable Economy	Weak Economy	
Large factory	200	50	-120	200
Average factory	90	120	-30	120
Small factory	40	30	20	40

2. Greatest maximum is to choose Large factory

The MaxiMax Criterion

- This criterion is based on the best possible scenario. It fits both an optimistic and an aggressive decision maker.
- An optimistic decision maker believes that the best possible outcome will always take place regardless of the decision made.
- An aggressive decision maker looks for the decision with the highest payoff (when payoff is profit).

Maximin Solution

The maximin criterion (a pessimistic approach):

1. For each option, find the minimum payoff

Investment Choice (Alternatives)	Profit in $1,000's (States of Nature)			1. Minimum Profit
	Strong Economy	Stable Economy	Weak Economy	
Large factory	200	50	-120	-120
Average factory	90	120	-30	-30
Small factory	40	30	20	20

Maximin Solution

The maximin criterion (a pessimistic approach):

1. For each option, find the minimum payoff
2. Choose the option with the greatest minimum payoff

Investment Choice (Alternatives)	Profit in $1,000's (States of Nature)			1. Minimum Profit
	Strong Economy	Stable Economy	Weak Economy	
Large factory	200	50	-120	-120
Average factory	90	120	-30	-30
Small factory	40	30	20	20

2. Greatest minimum is to choose Small factory

The MaxiMin Criterion

- This criterion is based on the worst-case scenario.
  - It fits both a pessimistic and a conservative decision maker's styles.
  - A pessimistic decision maker believes that the worst possible result will always occur.
  - A conservative decision maker wishes to ensure a guaranteed minimum possible payoff

Decision Making Under Risk

- The probability estimate for the occurrence of each state of nature (if available) can be incorporated in the search for the optimal decision.
- For each decision calculate its expected payoff (expected value).

Expected Value Solution

- The expected value is the weighted average payoff, given specified probabilities for each state of nature

Investment Choice (Alternatives)	Profit in $1,000's (States of Nature)		
	Strong Economy (.3)	Stable Economy (.5)	Weak Economy (.2)
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20

Suppose these probabilities have been assessed for these states of nature

Expected Value Solution

Investment Choice (Alternatives)	Profit in $1,000's (States of Nature)			Expected Values
	Strong Economy (.3)	Stable Economy (.5)	Weak Economy (.2)	
Large factory	200	50	-120	61
Average factory	90	120	-30	81
Small factory	40	30	20	31

Maximize expected value by choosing Average factory

Example: EV (Average factory) = 90(.3) + 120(.5) + (-30)(.2) = 81

Investing - The Expected Value Criterion

Decision	The Expected Value Criterion					Expected Value
	Large rise	Small rise	No change	Small fall	Large fall	
Gold	-100	100	200	300	0	100
Bond	**250**	**200**	**150**	**-100**	**-150**	**130**
Stock	500	250	100	-200	-600	125
C/D	60	60	60	60	60	60
Prior Prob.	0.2	0.3	0.3	0.1	0.1	

EV = (0.2)(250) + (0.3)(200) + (0.3)(150) + (0.1)(-100) + (0.1)(-150) = 130

Problem: Jenny Lind

Jenny Lind is a writer of romance novels. A movie company and a TV network both want exclusive rights to one of her more popular works. If she signs with the network, she will receive a single lump sum, but if she signs with the movie company, the amount she will receive depends on the market response to her movie. What should she do?

Payouts and Probabilities

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- Movie company Payouts
  - Small box office - $200,000
  - Medium box office - $1,000,000
  - Large box office - $3,000,000
- TV Network Payout
  - Flat rate - $900,000
- Probabilities
  - P(Small Box Office) = 0.3
  - P(Medium Box Office) = 0.6
  - P(Large Box Office) = 0.1

Jenny Lind - Payoff Table

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Decisions	States of Nature		
	Small Box Office	Medium Box Office	Large Box Office
Sign with Movie Company	$200,000	$1,000,000	$3,000,000
Sign with TV Network	$900,000	$900,000	$900,000
Prior Probabilities	0.3	0.6	0.1

Using Expected Return Criteria

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EV_{movie}=0.3(200,000)+0.6(1,000,000)+0.1(3,000,000)
= <u>$960,000</u>

EV_{tv} =0.3(900,000)+0.6(900,000)+0.1(900,000)
= $900,000

Therefore, using this criteria, Jenny should select the movie contract.

Something to Remember

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Jenny's decision is only going to be made one time, and she will earn either $200,000, $1,000,000 or $3,000,000 if she signs the movie contract, not the calculated EV of $960,000!!

Nevertheless, this amount is useful for decision-making, as it will maximize Jenny's expected returns in the long run if she continues to use this approach.

Decision Tree Analysis

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- A Decision tree shows a decision problem, beginning with the initial decision and ending will all possible outcomes and payoffs.
  - Use a square to denote decision nodes
  - Use a circle to denote uncertain events

Characteristics of a decision tree

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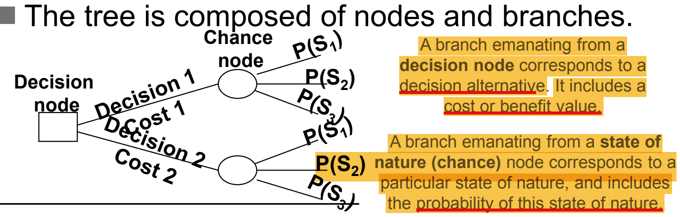
- A Decision Tree is a chronological representation of the decision process.
- The tree is composed of nodes and branches.



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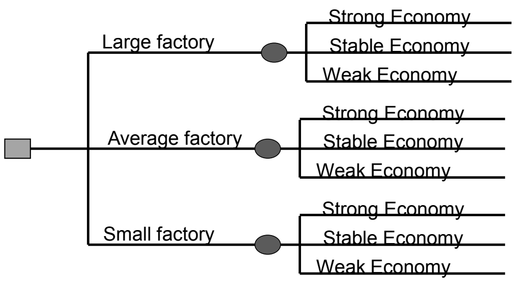
Sample Decision Tree



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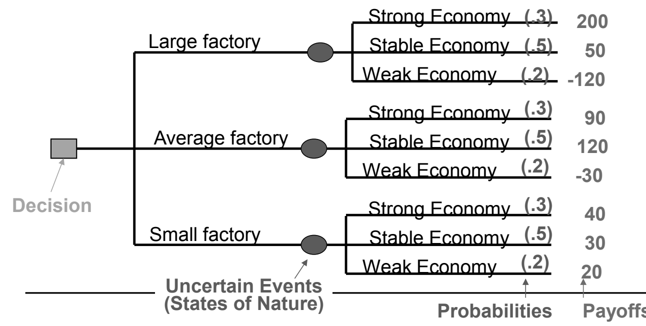
Add Probabilities and Payoffs



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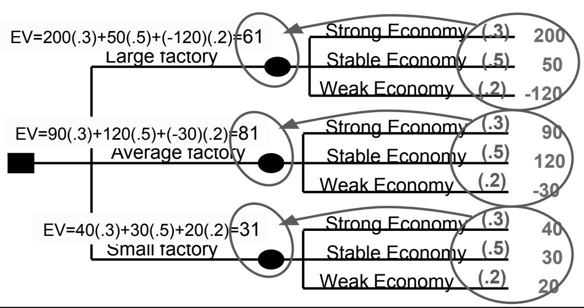
Fold Back the Tree



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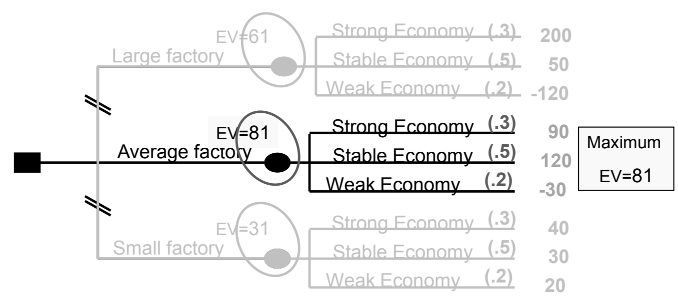
Make the Decision



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Example : The inventor's problem

An engineer has invented a new device and has to decide whether to manufacture the device himself or to sell the patent rights to an established company. If he manufactures the device himself and sales are high, he estimates that he will earn $80,000, but if sales are low, he will lose $5000.

Alternatively, if he sells the patent rights, he will earn $40,000 in royalties if sales are high, but only $1000 if sales are low. The engineer estimates that the probability of high sales is 0.4, irrespective of whether he sells the patent rights or manufactures the device himself.

If risk averse - sell patent rights

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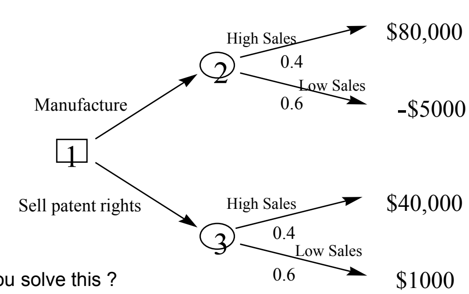
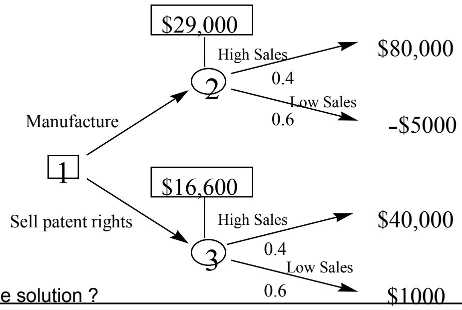
A decision tree for this problem is shown below:



How do you solve this ?

Decision tree with EMV's:

80k*0.4 - 5k*0.6



What is the solution ?

Text

Some limitations of the Decision Tree method

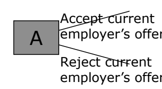
- since an expected value represents the average payoff which would accrue if the decision was repeated a large number of times, is it reasonable to apply it to a one-off decision?
- the EMV criterion does not take into account the attitude to risk of the decision maker
- the EMV criterion assumes that the decision involves only one objective, namely maximisation of monetary returns -many decisions involve several, often conflicting, objectives which cannot be easily assessed in monetary terms, such as conservation of the environment, aesthetic and social factors.
- the probabilities and payoffs were only guestimates - however, note that sensitivity analysis (see next section) often shows that these guestimates do not need to be exact and it sometimes takes large changes in these figures before an alternative option has a higher expected payoff

Decision Tree Models Involving a Sequence of Decisions

- Bill Sampras is a first year MBA student thinking about summer employment.
  - On a flight to school he meets Vanessa, VP for a major banking firm
    - She tells him she'd like to discuss summer employment opportunities in mid-November
  - The company Bill left promised him a summer position for $12,000
    - The offer is only good until the end of October
  - Alternatively he could seek other employment
  - He feels all available opportunities would offer similar learning and networking experiences
  - His only decision criterion therefore is salary

Decision Trees

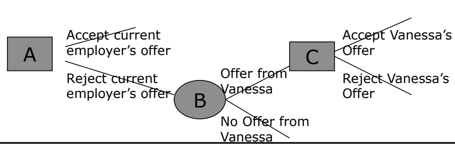
- Building the decision tree - Bill's initial decision node:



It's his decision - square

Decision Trees

- If Bill doesn't accept his current employer's offer, he should next see whether Vanessa offers him a job.
  - This results in event node B
  - If Vanessa offers him a job he must decide whether to accept it, i.e., decision node C



Decision Trees

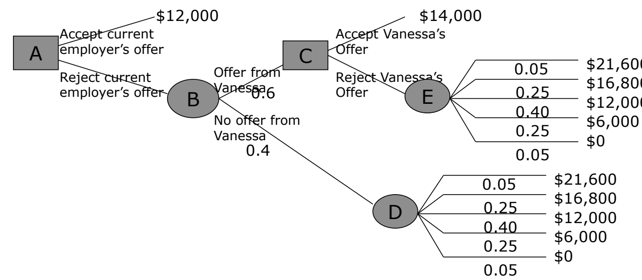
- If either Vanessa doesn't offer Bill a job or he rejects Vanessa's offer, he is in the same boat:
  - He will need to seek other employment
  - Bill's school keeps past records on summer employment, which are represented by the table below
  - Bill feels that these statistics fairly represent the probability he will achieve each given salary level

Weekly Salary	Total Summer Pay	% Students at this salary
$1800	$21600	5%
$1400	$16800	25%
$1000	$12000	40%
$500	$6000	25%
$0	$0	5%

Decision Trees

- Bill feels that the probability he will get an offer from Vanessa is 0.6 and he estimates that the summer salary at Vanessa's firm will equal $14,000.
- We assign a probability to each branch emanating from an event node.
- To each final outcome we assign the value of that outcome.

Bill's Complete Decision Tree



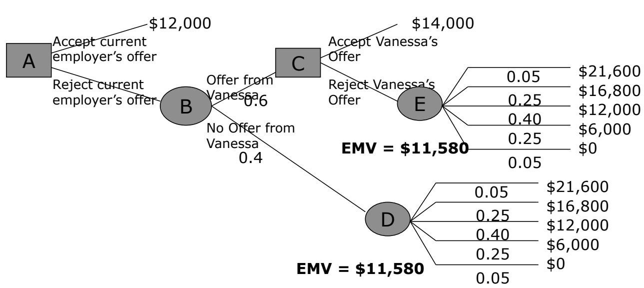
Decision Tree Analysis

- Now that we have a decision tree, what can we do with it?
  - Common approach is to assess the Expected Monetary Value (EMV) of uncertain events
  - EMV is the weighted average of all possible outcomes; outcome values are weighted by probabilities and summed to give EMV
  - EMV of participating in school's summer recruiting:
    - EMV = 0.05 × $21,600 + 0.25 × $16,800 + 0.40 × $12,000 + 0.25 × $6,000 + 0.05 × $0 = $11,580

Expected Monetary Value

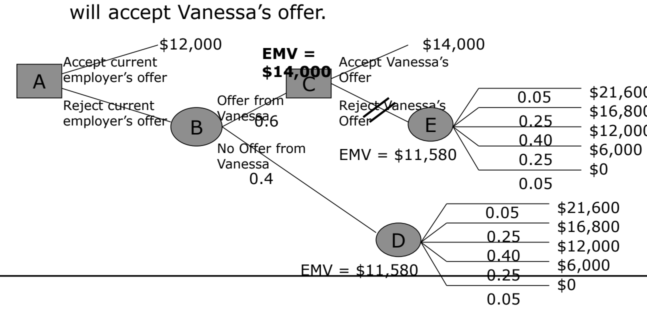
- The EMV of a certain event is just the outcome value, e.g., EMV of taking his old firm's offer is $12,000.
- We can use EMV to "solve" the decision tree.
  - Called "folding back" the decision tree or "backwards induction"
  - We start at the ends of branches and move backwards

Folding Back the Decision Tree



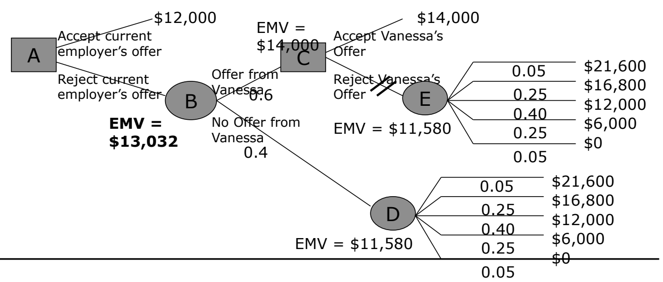
Folding Back the Decision Tree

Since C is a decision node, if we get to this point, we will accept Vanessa's offer.



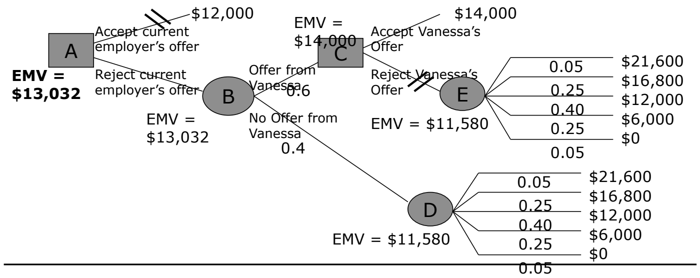
Folding Back the Decision Tree

We can now calculate the EMV of node B:
= 0.6 × \$14,000 + 0.4 × \$11,580 = \$13,032



Folding Back the Decision Tree

Since A is a decision node, if we get to this point, we will reject current employer's offer.



Bill's Optimal Decision Strategy

- Bill should reject his current employer's offer.
- If Vanessa offers him a job he should take it; If not, he will have to participate in recruiting.
- Bill's EMV is \$13,032.

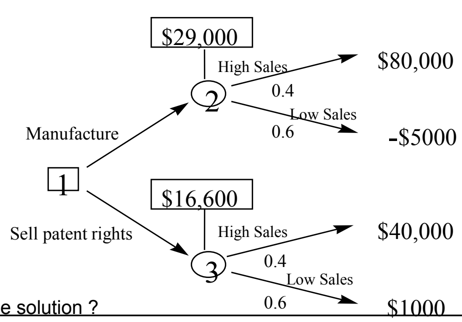
Recall : The inventor's problem

An engineer has invented a new device and has to decide whether to manufacture the device himself or to sell the patent rights to an established company. If he manufactures the device himself and sales are high, he estimates that he will earn \$80,000, but if sales are low, he will lose \$5000.

Alternatively, if he sells the patent rights, he will earn \$40,000 in royalties if sales are high, but only \$1000 if sales are low. The engineer estimates that the probability of high sales is 0.4, irrespective of whether he sells the patent rights or manufactures the device himself.

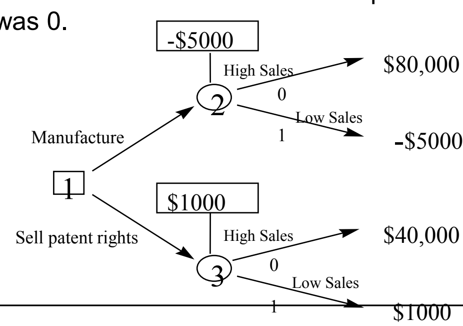
Which option should the engineer choose?

Decision tree with EMV's:

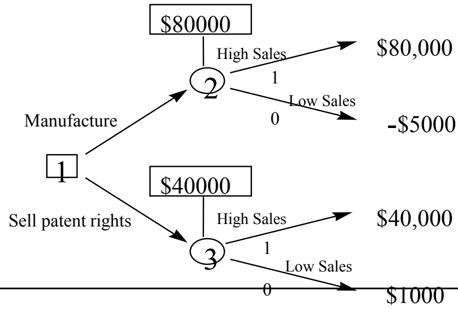


What is the solution ?

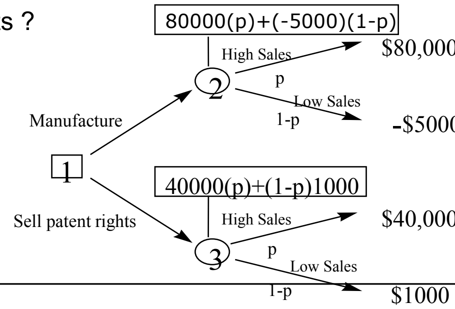
Now applying sensitivity analysis, we first examine which option would have been selected if the probability of high sales was 0.



Then we examine which option would have been selected if the probability of high sales was 1.0.



What is the value of the probability of high sales at which it makes no difference whether the engineer should manufacture the device himself or should sell the patent rights ?



The inventor should manufacture the device himself if

$$
\begin{aligned}
&80000(p) - 5000(1-p) > 40000(p) + (1-p)1000 \\
&\Rightarrow 40000(p) > 6000(1-p) \\
&\Rightarrow \frac{p}{(1-p)} > 0.15 \\
&\Rightarrow p > 0.15(1-p) \\
&\Rightarrow p > 0.13
\end{aligned}
$$

Conclusion: if p>.13 then he should manufacture the device himself.

Things you should know

- ❑The payoff table and decision trees
- ❑Criteria for decision making
  - o If no probabilities are known: maximin, minimax regret
  - o When probabilities are known: expected monetary value
- ❑ Sensitivity Analysis