Stat 104: Quantitative Methods
Homework 4: Due Sunday, October 1

Homework policy: Homework is due by **5:00PM** (EST) on the due date. Homework is to be handed in via the course website in pdf format. You do not need to type the homework; there are many ways (scanner in the library or phone apps) to convert written homework into a pdf file. Ask the teaching staff if you need assistance.

Late homework will not be accepted. You are encouraged to discuss homework problems with other students (and with the instructor and TFs, of course), but you must write your final answer in your own words. Solutions prepared "in committee" or by copying someone else's paper are not acceptable.

- Please submit your homework in pdf format; this can be done in Word, or OpenOffice or via cellphone apps that will scan and turn into pdf.
- Please make your homework solutions legible by **bolding** or using circles to identify your solution.
- Since we are not printing out anything, use lots of s p a c e for your solutions, and put each answer on a different page if it makes the solution easier to read.
- Please make sure your submitted solutions are in numerical order [problem 1, problem 2 and so on].
- Please keep your computer output to a minimum and focus on the required answer. The easiest way to put your computer output into your homework is to cut and paste it into a Word file and use the font "courier new".
- Please keep in mind the course rules on Academic Honesty and Collaboration

Binomial Problems [feel free to use R when possible]

1) The random variable *X* has a binomial distribution with E(X)=18 and Var(X)=7.2. Find *n* and *p* for this distribution [use the formulas for mean and variance].

E(X) = np = 18
Var(X) = np(1-p)=7.2.

Divide Var/E(X) to get (1-p). 7.2/18 = 0.4 so p=0.6. n = 18/.6 = n=30

2) Suppose *X* is a binomial random variable with *n*=15 and *p*=0.3. Feel free to use a computer to answer the following:

   a) P(X=0) = 0.004748
   b) P(X=2) = 0.09156
   c) P(X<2) = 0.03527
   d) P(X>8) = 0.00365
   e) E(X) = 4.5
   f) Var(X) = 3.15

3) A school newspaper reporter decides to randomly survey 12 students to see if they will attend YardFest ™ festivities this year. Based on past years, she knows that 18% of students attend YardFest ™ festivities. We are interested in the number of students who will attend the festivities.

   a) How many of the 12 students do we expect to attend the festivities?
   .18 * 12 = 2.16
   b) Find the probability that at most 4 students will attend.
   pbinom(4,12,.18) = 0.9511
   c) Find the probability that more than 2 students will attend.
   1- pbinom(3,12,.18) = 0.1552

4) Suppose that about 85% of graduating students attend their graduation. A group of 22 graduating students is randomly chosen.

   a) How many are expected to attend their graduation?
   np = 22 * 0.85 = 18.7
   b) Find the probability that 17 or 18 attend.
   P(X = 18) + P(X = 17)

   dbinom(17,22,.85) + dbinom(18,22,.85) = .3249

   c) Based on numerical values, would you be surprised if all 22 attended graduation? Justify your answer numerically.

   P(X = 22) = .85^22 = 0.028

I would be very surprised if all 22 attended graduation because the probability of 22 students attending graduation is only 2.8%.

5) A new drug named CURAIDS that is 60% effective in extending the average life of an AIDS patient by twenty years. Five randomly selected AIDS patients are treated with this new drug. Answer the following questions based on the above information.

(a) What is the probability that no more than 4 patients are cured?
$1 - P(X=5) = .9222$

(b) Find the probability that more than 2 patients or less than or equal to 5 patients are cured.

OR is a key word here, probability = 1

6) According to flightstats.com, American Airlines flights from Dallas to Chicago are on time 70% of the time. Suppose 15 flights are randomly selected, and the number of on-time flights is recorded.

a) Explain why this is a binomial experiment.
Independent trials, set number of n trials, success or failure of landing on time, and number of successful trials sought after.
b) Find and interpret the probability that exactly 10 flights are on time.
P(X=10) =0.2061, 20.61% chance that exactly 10 flights will land on time.
c) Find and interpret the probability that between 8 and 10 flights, inclusive, are on time.
P(X=8) + P(X=9) + P(X=10) = 0.4345, 43.45% probability that there will be exactly 8,9, or 10 flights landing on time.
d) Find the mean and variance of the number of on time flights.
np = 10.5, var = npq = 15*.7*.3 = 3.15

Continuous Random Variables [feel free to use R when possible]

7) Let X be a uniformly distributed random variable on the interval 0 to 1

a) Calculate P(X = 0.25) = 0, it's continuous
b) Calculate P(0.7 < X < 1) = .3
c) Calculate the expected value of X = .5
d) Calculate $E(X^2)$ = ((1-0)^2)/12 + (1/2)^2 = 1/3

8) For the standard normal random variable Z, compute the following:

a) $P(0 \le Z \le 0.73)$

0.7673-0.5=0.2673

b) $P(-1.50 \le Z \le 0)$

1 – 0.0668 = 0.4332

c) $P(Z \ge 0.44)$

1 -0.67 =0.33

d) $P(-1.50 \le Z \le 0.40)$
0.6654 – 0.0668 = 0.5886

e) $P(Z \le 5.23)$
0.9999

f) $E(3-4Z)$
3 –(4*0) = 3

g) $Var(4 - 3Z)$
9*1 = 9

9) The time needed to hand stitch a Swoosh soccer ball is normally distributed with mean 41 minutes and standard deviation 3 minutes. If 4 workers start at the same time, what is the probability that at least one of them will complete their soccer ball in under 44 minutes?
Probability of not finishing is =.1587
1-(.1587)^4 = 0.9994

10). The weight of reports produced in a certain department has a Normal distribution with mean 60g and standard deviation 12g. What is the probability that the next report will weigh less than 445g?

z = ((45-60)/12) = -1.25, P(Z < -1.25) = .1056

11). The times taken to complete an introduction to business statistics exam have a normal distribution with a mean of 65 minutes and standard deviation of 7 minutes. There are 150 students who took the exam and students are allowed a total of 75 minutes to take the exam.

a) What is the chance that Mike finished his exam in 63 to 72 minutes?

72-65/7 = 1 = Z, 0.84134. 63-65/7 = -0.29 = z, 0.38591. 0.84134-0.38591 = 0.45543

b) What is the expected number of students who finished in less than 75 minutes?

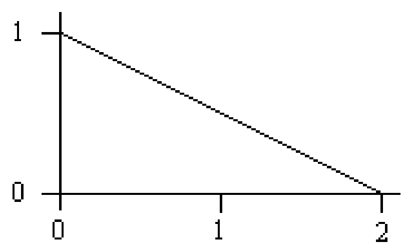
75-65/7= 1.43,Z 0.92364 *150 = 138.5 so 138 students.

c) As some students were not able to finish the exam in time, the instructor allowed 6 more minutes. Given he already spent 75 minutes on the exam, what is the chance that Chris finished his exam in extended time, that is between 75 and 81 minutes

P((75<x<81) | X>75)

1-0.92364 = 0.07636
81-65/7 = 2.29 P(Z<2.29) = 0.98899
75-65/7 = 1.43 P(Z>1.43) = 0.92364
P(Z<2.29) - P(Z>1.43) = 0.06535 = P(75<x<81)
0.06535/0.07636 = 0.8558 (Divide by this number because it is given that it is above 75 mins)

12) Suppose X is a continuous random variable taking values between 0 and 2 and having the probability density function below. Calculate $P(1 \le X \le 2)$



$P(1 \le X \le 2) = 1*0.5*0.5 =$ 0.25