Stat 104 Spring 2017: Exam 1 Solutions

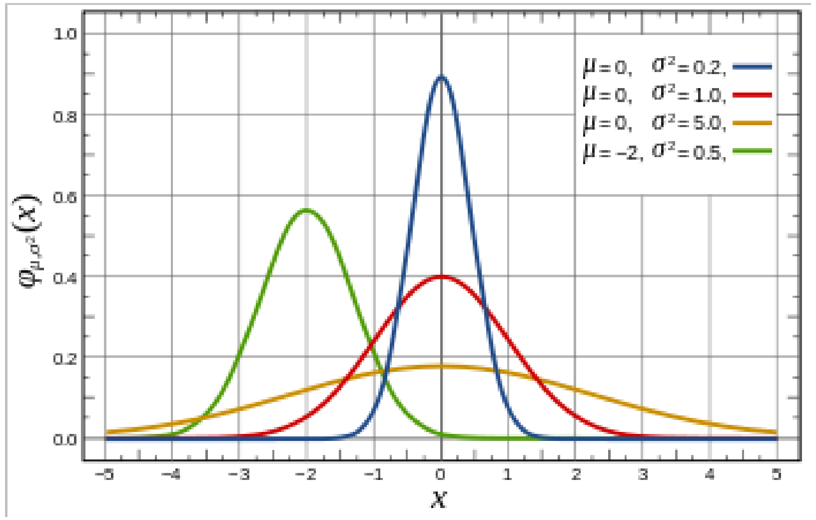
True/False #1

For a continuous random variable, the total area beneath the pdf must be one.

a) True

b) False

The total area under the probability density function=1

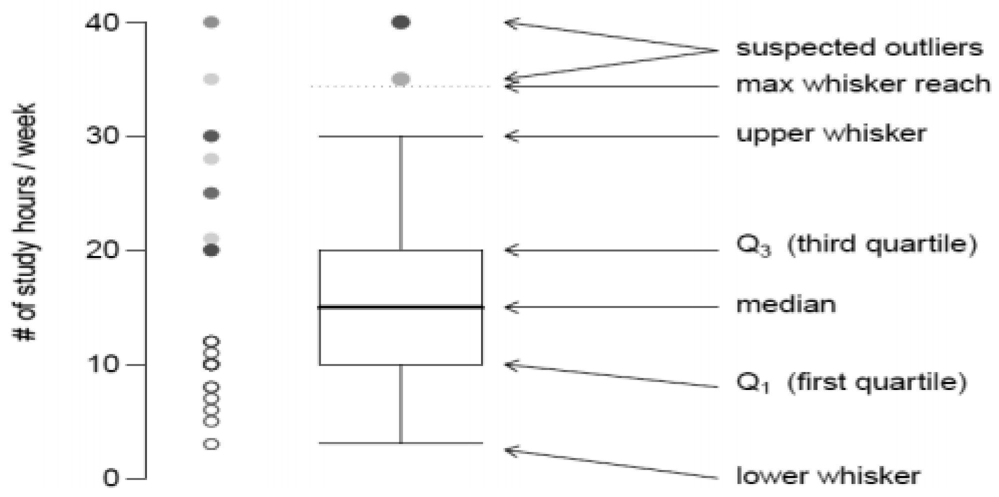


True/False #2

The second quartile is the same as the median

a) True

b) False



True/False #3

Given the data set 10, 5, 2, 6, 3, 4, 20, the median value is 5.

a) True

b) False

Order the data:
2, 3, 4, 5, 6, 10, 20

5 is the middle value

True/ False #4

P(A or B) can exceed one.

a) True

$\boxed{\text{b) False}}$

Probabilities can <u>never</u> be greater than 1.

True/False #5

Of the range, the interquartile range, and the variance, the interquartile range is least influenced by an outlying value in the data set.

a) True

b) False

Measures based on the mean are more susceptible to outliers than measures based on median (or quartiles)

True/False #6

15 cards are selected out of a 52 card deck such that after each card is selected, it is placed back into the deck and the deck is reshuffled. Then the total number of hearts selected follows a binomial distribution.

$\boxed{\text{a) True}}$

b) False

Constant probability of success: 13/52
Trials are independent
Success/failure outcome: Heart or not a heart

True/False #7

If the equation of the least squares regression line was computed to be $y=45.7+3.1x$, then the correlation cannot be less than 0.

a) True

b) False

From the slope (3.1), we can see that there is a positive relationship between x and y (as x increases, y also increases).

Correlation between x and y will be positive.

True/False #8

A researcher found the correlation between the age of death and number of cigarettes smoked per day to be -0.95. Based just on this information, the researcher can justly conclude that smoking causes early death.

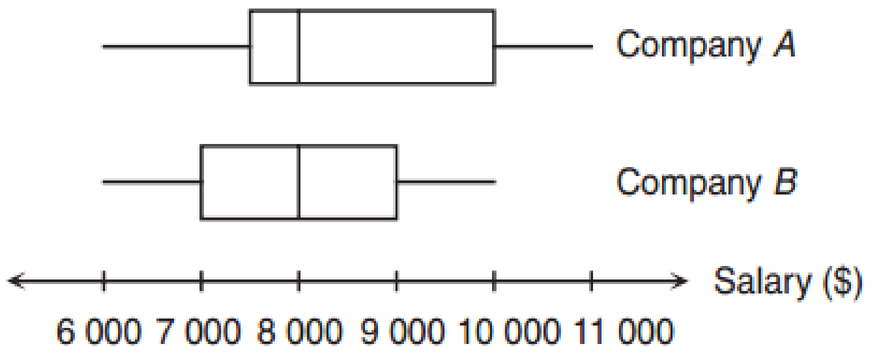
a) True

b) False

Correlation ≠ Causation

Multiple Choice #9

The box-and-whisker diagrams below show the salary distributions of two companies. Which of the following measures are the same for these two companies?



a) Range

b) Median

c) Lower quartile

d) Inter-quartile range

Multiple Choice #10

Information was collected on those who attended the opening of a new movie. The analysis found that 56 percent of the moviegoers were female, 26 percent were under age 25, and of those under the age of 25, 17 percent were females. Find the probability that a moviegoer is either female or under age 25.

a) $\boxed{0.78}$
b) 0.82
c) 0.65
d) 0.50
e) 0.35

P(F)=0.56
P(under 25)=0.26
P(F|under 25)=0.17

Want to find P(F or under 25)

P(F or under 25)= P(F) + P(under 25) - P(F and under 25)

P(F and under 25)= P(F|under 25)*P(under 25)= 0.17*0.26= 0.0445

P(F or under 25)= 0.56 +0.26 -0.0445= 0.78

Multiple Choice #11

There are two traffic lights on the route used by Pikup Andropov to go from home to work. Let E denote the event that Pikup must stop at the first light and F in a similar manner for the second light. Suppose that P(E)=0.4 and P(F)=0.3 and P(E and F)=0.15. What is the probability that he doesn't stop at either light.

P(E)=0.4
P(F)=0.3
P(E and F)=0.15

Want to find P(E' and F')

	E	E'	
F	0.15	0.15	0.3
F'	0.25	0.45	0.7
	0.4	0.6	1

a) 0.66
b) 0.27
c) $\boxed{0.45}$
d) 0.80
e) 0.20

Multiple Choice #12

The mean and the standard deviation of the ages of a group of children are 7 and 1.31 respectively. If a child of age 7 joins the group of children, what are the effects on the mean and the standard deviation of the ages of the group of children?

a) Mean Unchanged, Standard deviation Increased

b) Mean Unchanged, Standard deviation Decreased

c) Mean Increased, Standard deviation Increased

d) Mean Decreased, Standard deviation Decreased

Multiple Choice #13

A certain model car comes in a two-door version, a four-door version, and a hatchback version. Each version can be equipped with either an automatic transmission or a manual transmission. The accompanying table gives the relevant proportions for purchases.

	Two-Door	Four-Door	Hatchback
Automatic	.32	.27	.18
Manual	.08	.04	.11

A customer who has purchased one of these cars was randomly selected. Given that a customer did not purchase a hatchback, what is the probability that the car has a manual transmission?

a) 0.64
b) 0.83
c) 0.18
d) 0.08
e) 0.38

Want to find P(Manual|not a hatchback)

= P(Manual and not a hatchback)/P(not a hatchback)
= (0.08 +0.04)/ (0.32+0.27+0.08+0.04)= 0.17

Multiple Choice #14

Leah is flying from Boston to Denver with a connection in Chicago. The probability her first flight leaves on time is 0.15. If the flight is on time, the probability that her luggage will make the connecting flight is 0.95, but if the first flight is delayed, the probability that the luggage will make it is only 0.65. What is the probability that her luggage arrives in Denver with her?

a) 0.40
b) 0.20
c) 0.70
d) 0.80
e) 0.15

P(1^{st} on time)=0.15
P(L|1^{st} on time)=0.95
P(L|1^{st} delayed)= 0.65

We want to find P(L)

	1^{st} on time	1^{st} delayed	
L	=0.95*0.15 =0.1425	=0.65*0.85 =0.5525	=0.1425+ 0.5525=0.7
L’			
	0.15	1-0.15= 0.85	1

Multiple Choice #15

Foresters use regression to predict the volume of timber in a tree using easily measured quantities such as diameter. Let y be the volume of timber in cubic feet and x be the diameter in inches. One set of data gives y=-30+60x. The predicted volume for a tree of 18 inches is:

a) 1050 cubic feet
b) 600 cubic feet
c) 104 cubic feet
d) 90 cubic feet
e) 60 cubic feet

-30+60(18)= 1050

Multiple Choice #16

The probability that an engine will not start is 0.04. A rocket has four independent engines. What is the probability that at least one of the engines does not start?

a) 0.82

b) 0.0006

c) $\boxed{0.15}$

d) 0.04

e) 0.001

Binomial with $p=0.04$ and $n=4$

We want to find $P(X\geq 1)$

$1-P(X=0)=1-(1-0.04)^4= 0.15$

Multiple Choice # 17

Consider a Poisson random variable with rate parameter 7. Then $E[X^2]$ equals

a) 7

b) 42

c) 27

$\boxed{\text{d) 56}}$

e) 3.52

$Var(X)=E[X^2]-[E(X)]^2$

$E(X)=7$
$Var(X)=7$

$7=E[X^2]-[7]^2$
$E[X^2]=56$

Multiple Choice # 18

Which of the following is not a requirement of a binomial distribution?

a) constant probability of success

b) only two possible Bernoulli outcomes

c) fixed number of trials

d) equally likely outcomes

Multiple Choice # 19

A company Orange manufactures a hand-held device uPhone. Each uPhone has a probability 10% of being defective, independently of other units. Let X be the number of defective uPhones in a batch of 900 uPhones. The variance of X is

a) 3
b) 9
c) $\boxed{81}$
d) 90
e) 56

Binomial with $p=0.1$ and $n=900$

$Var(X)=np(1-p)=900(0.1)(1-0.1)=81$

Multiple Choice # 20

Suppose the time (in minutes) Lisa spends grading an Ec 10 quiz is uniformly distributed in the interval (5,15). If Lisa already spent 9 minutes grading a quiz, what is the probability she will spend a total of at least 12 minutes grading the quiz?

a) $\boxed{3/6}$
b) 3/10
c) 12/15
d) 3/15
e) 4/5

We want to find $P(X>12|X>9)$

$P(X>12)=(15-12)*0.1= 0.3$
$P(X>9)=(15-9)*0.1=0.6$

$P(X>12|X>9)=0.3/0.6= 3/6$

Multiple Choice # 21

The speed of cars traveling on a stretch of Interstate 5 in California is approximately normally distributed with a mean of 72.6 miles/hour and a standard deviation of 4.78 miles/hour. The legal speed limit posted on this stretch of Interstate 5 is 70 miles/hour. 60% of all cars on this stretch of Interstate 5 are going at most how fast?

a) 73.8 mph
b) 71.4 mph
c) 68.8 mph
d) 69.2 mph
e) 72.7 mph

$X \sim N(72.6, 4.78^2)$

We want to find X so that there is 60% in the lower tail
Find z-score: 0.26
$0.26=(X-72.6)/4.78$
$X=73.8$

Multiple Choice # 22

The regression line $\hat{y} = 3 + 2x$ has been fitted to the data points (4, 8), (2, 5), and (1, 2). The sum of the squared residuals will be:

a) 7

b) 15

c) 8

d) 22

e) 17

3+2(4)= 11, residual= 8-11= -3
3+2(2)= 7, residual= 5-7= -2
3+2(1)= 5, residual= 2-5= -3

Sum of squared residuals= $(-3)^2+(-2)^2+(-3)^2=22$

Multiple Choice # 23

A regression analysis between sales (y in $1000) and advertising (x in $) resulted in the following least squares line: $\hat{y} = 80{,}000 + 5x$. This implies that an:

a) increase of $1 in advertising is expected, on average, to result in an increase of $5 in sales

b) increase of $5 in advertising is expected, on average, to result in an increase of $5000 in sales

c) increase of $1 in advertising is expected, on average, to result in an increase of $80,005 in sales

d) increase of $1 in advertising is expected, on average, to result in an increase of $5,000 in sales

Multiple Choice # 24

Let X represent a random variable whose distribution is normal with mean 100 and standard deviation of 10. Which of the following is equivalent to $P(X>115)$?

a) $P(X<115)$

b) $\boxed{P(X<85)}$

c) $P(X>15)$

d) $P(85<X<115)$

e) $1-P(X<85)$

Use symmetry of normal curve

115 is +15 units from the center
85 is -15 units from the center

Tail above 115=tail below 85

Multiple Choice # 25

The mean annual salary of employees at a company is $40,000 with a standard deviation of $3500. At the end of the year, each employee receives a $2000 bonus and a 4% raise (based on salary). What is the standard deviation of the new salaries?

a) 3360

b) 3872

c) 3546

d) $\boxed{3640}$

e) 3905

1.04*3500=3640

Multiple Choice # 26

Let X and Y be independent random variables with X~N(0,1) and Y~N(0,2). The value of P(X>Y) is

a) 0

b) 0.05

c) $\boxed{0.50}$

d) 0.95

e) 0.45

P(X>Y)=P(X-Y>0)
X-Y~(0-0=0, 1+2=3)
Z=0-0/3=0
P(Z>0)=0.5

Multiple Choice # 27

The statistics below provide a summary of the distribution of heights, in inches, for a simple random sample of 200 young children.

Mean: 46 inches

Median: 45 inches

Standard Deviation: 3 inches

First Quartile: 43 inches

Third Quartile: 48 inches

About 100 children in the sample have heights that are

a) Less than 43 inches

b) Less than 48 inches

c) Between 43 and 48 inches

d) Between 40 and 52 inches

e) More than 46 inches

Multiple Choice # 28

A delivery service places packages into large containers before flying them across the country. These filled containers vary greatly in their weight. Suppose the delivery service's airplanes always transport two such containers on each flight. The two containers are chosen so their combined weight is close to, but does not exceed, a specified weight limit. A random sample of flights with these containers is taken, and the weight of each of the two containers on each selected flight is recorded. The weights of the two containers on the same flight

a) will have a correlation of 0

b) will have a negative correlation

c) will have a positive correlation that is less than 1

d) will have a correlation of 1

e) cannot be determined from the information given

Multiple Choice # 29

Let X be a random variable with the following probability mass function; $P(X = -1) = 0.2, P(X = 0) = 0.4, P(X = 1) = 0.4.$ Compute $P(X = 0|X \leq 1)$.

a) 0.2

b) 0.4

c) 0.5

d) 0.3

e) None of the above

P(X=0|X≤1)= P(X=0)/P(X≤1)

P(X=0)=0.4
P(X≤1)= 0.2+0.4+0.4=1

P(X=0|X≤1)=0.4/1= 0.4

Multiple Choice # 30

Let X and Y be two random variables with the following joint probability distribution; $P(X = -1\ and\ Y = -1) = \frac{1}{3}, P(X = 0\ and\ Y = 0) = \frac{1}{3}$ and $P(X = 1\ and\ Y = 1) = \frac{1}{3}$. Compute $P(XY = 1)$.

a) 0
b) 1/9
c) 1/3
d) 2/3
e) 2/9

The following events satisfy P(XY=1):
P(X=-1 and Y=-1)
P(X=1 and Y=1)
P(XY=1)=1/3+1/3=2/3

Multiple Choice # 31

You have an equally-weighted portfolio of two stocks, Microsoft and McDonald's (the weights are 0.5 in each asset). The standard deviations of Microsoft and McDonald's are 20 and 15 respectively. Your portfolio standard deviation is 15. What is the covariance of Microsoft and McDonald's?

a) 0.62

$\boxed{\text{b) 138}}$

c) 92

d) 100

e) 0.46

SD of portfolio= $\sqrt{b^2 s_x^2 + d^2 s_y^2 + 2bd cov_{xy}}$

$$SD = \sqrt{0.5^2 20^2 + 0.5^2 15^2 + 2 * 0.5 * 0.5 cov_{xy}} = 15$$

$$cov_{xy} = 138$$

Multiple Choice # 32

Suppose that X is a normal random variable with mean 5. If $P(X > 9)=.2$, approximately, what is Var[X]?

a) 20.6

b) 18.9

c) 16.2

d) 23.8

e) $\boxed{22.7}$

Find z-score that equals a probability of 0.2 in the upper tail
$z= 0.84$

$0.84=(9-5)/SD$
$SD=4.76$
$Var=4.76^2=22.7$

Multiple Choice # 33

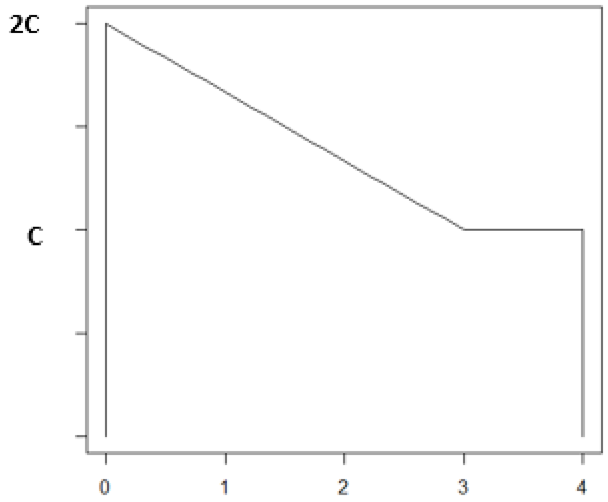
A councilman claims that 40% of residents support a tax increase on alcoholic beverages. A polling agent is going to sample 30 residents. Let X be the number of people out of 30 who are in favor of the tax. If *F* represents the cumulative distribution function for the appropriate binomial distribution, what is the probability that at least ten but less than twenty-five of the residents sampled would support such a tax increase on alcoholic beverages?

We want all events from P(X=10)…P(X=24)

a) F(25)-F(10)
b) F(24)-F(9)
c) F(24)-F(10)
d) F(25)-F(9)

Multiple Choice # 34

A continuous random variable X has the following density function.



Find the value of c.

a) 0.32
b) 0.20
c) 0.18
d) 0.25
e) 0.21

(4*C)+(3*C*0.5)=1
4C+1.5C=1
C=0.18

Multiple Choice # 35

Below are summary statistics for two salespersons, John and Sue. The data are based on weekly sales for 52 weeks.

Variable	N	Mean	StDev
John	52	967.3	97.3
Sue	52	1063.3	171.4

Covariances

	John	Sue
John	???????	
Sue	7459.97	29364.36

What is the correlation between John's sales and Sue's sales?

a) 0.87

b) 0.57

c) 0.24

d) 0.63

e) 0.45

$$r_{js} = \frac{cov_{js}}{s_j s_s}$$

$$r_{js} = \frac{7459.97}{97.3 * 171.4} = 0.45$$

Multiple Choice # 36

For a given high school basketball team, the number of baskets (X) for the leading scorer has E(X)=8.3 with standard deviation 1.25 and the number of baskets for the second leading scorer has E(Y)=6.6 with standard deviation 2.31. Hypothetically, let's say the leading scorer shoots only 3-point baskets and the second leading scorer only shoots 2-point baskets and they shoot independently of each other. What is the standard deviation of the difference in points scored between the leading scorer and the second leading scorer?

a) 5.95
b) 2.67
c) 3.87
d) 7.02
e) 4.51

$Var=(3^2*1.25^2)+(2^2*2.31^2)=35.41$

$SD=sqrt(35.41)=5.95$

Multiple Choice # 37

Nicole adores chocolate! She often purchases chocolate when she is out shopping at the grocery store. Let X be the number of chocolate bars that Nicole purchases on one visit to the grocery store. The following describe Nicole's chocolate purchasing habits.

- 89.6% of the time Nicole purchases at least two chocolate bars — $P(X \geq 2)=0.896$
- She **never** purchases just one chocolate bar — $P(X=1)=0$
- She **never** purchases more than 4 chocolate bars — $P(X \geq 4)=0$
- 25.5% of the time, Nicole purchases at most 2 bars — $P(X \leq 2)=0.255$
- 42.8% of the time, Nicole purchases more than 3 chocolate bars — $P(X>3)=0.428$
- 10.4% of the time, Nicole does not purchase any chocolate bars — $P(X=0)=0.104$

What is the expected number of chocolate bars she will purchase?

a) 2.61
b) 3.41
c) 2.11
d) $\boxed{2.97}$
e) 3.07

0	1	2	3	4
0.104	0	=0.255-.104 =0.151	=1-(0.104+0.428+0.151) =0.317	0.428

$$E(X)=(0*0.104)+(1*0)+(2*0.151)+(3*0.317)+(4*0.428)=2.97$$

Multiple Choice # 38

The lengths of brook trout caught in a certain Colorado stream are normally distributed with a mean of 14 inches and a standard deviation of 3 inches. What proportion of brook trout caught will be between 12 and 18 inches in length?

- $\boxed{\text{a) } 0.66}$
- b) 0.87
- c) 0.27
- d) 0.41
- e) 0.33

$P(12<X<18)$
$Z=12-14/3=-0.67$
$Z=18-14/3=1.33$
$P(-0.67<Z<1.33)=0.9082-0.2514=0.66$