Stat 104: Quantitative Methods for Economics
Homework 7: Due Monday, October 30 SOLUTIONS

Hypothesis Testing for a Population Mean

1) A production line produces rulers that are supposed to be 12 inches long. A sample of 49 of the rulers had a mean of 12.1 and a standard deviation of .5 inches. The quality control specialist responsible for the production line decides to do a two-sided hypothesis test to determine whether the production line is really producing rulers that are 12 inches long or not.
   a. What is the null hypothesis?
      Ho : μ = 12

   b. What is the alternative hypothesis?
      Ha : μ ≠ 12

   c. Using whatever method you want, clearly run and summarize the result of the hypothesis test. What does this mean in terms of the problem situation?

Step 1:

Ho : μ = 12
Ha : μ ≠ 12

n = 49
s=0.5
$\bar{x}$= 12.1

Step 2: Significance level .05
Step 3: Decision Rule says to reject the null hypothesis if |tstat| > 1.96

Step 4:

$$\text{tstat} = \frac{(12.1-12)}{\frac{0.5}{\sqrt{49}}}$$

tstat = 1.4

Because our tstat is 1.4, which is less than 1.96, we do not have evidence to reject the null hypothesis.

Step 5: At the 5% level of significance, we do not have sufficient evidence to reject the null hypothesis and suggest that the production line is, on average, producing rulers that are 12 inches long.

2) Snack bags of popcorn are supposed to weigh 5.5 ounces on average. A random sample of 64 bags of popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounces. Test using whatever method you want $H_o: \mu = 5.5$ $H_a: \mu < 5.5$. Be sure to clearly state your conclusion.

Step 1:

Ho : μ = 5.5
Ha : μ < 5.5

n = 64
s=0.24
$\bar{x}$= 5.23

Step 2: Significance level .05
Step 3: Decision Rule says to reject the null hypothesis if |tstat| > 1.64 (one-sided test)

Step 4: tstat = $\frac{(5.23-5.50)}{\frac{0.24}{\sqrt{64}}}$

tstat = -9

Because our |tstat| is 9, which is greater than 1.64, we have evidence to reject the null hypothesis.

Step 5: At the 5% level of significance, we have found sufficient evidence to reject the null hypothesis and suggest that the mean weight of popcorn snack bags is actually less than 5.5 oz.

3) A particular brand of tires claims that its deluxe tire averages more than 50,000 miles before it needs to be replaced. A survey of owners of that tire design is conducted. From the 30 tires surveyed, the mean lifespan was 46,500 miles with a standard deviation of 9800 miles. Do the data support the claim at the 5% level? Test using whatever methods you want $H_o: \mu = 50000$ $H_a: \mu > 50000$. Be sure to clearly state your conclusions.

Step 1:

Ho : μ = 50,000
Ha : μ > 50,000

n = 30
s=9,800
$\bar{x}$= 46,500

Step 2: Significance level .05
Step 3: Decision Rule says to reject the null hypothesis if tstat > 1.64 (one-sided test)

Step 4: tstat = $\frac{(46{,}500-50{,}000)}{\frac{9{,}800}{\sqrt{30}}}$

tstat = -1.956

Because our tstat is -1.956, which is not greater than 1.64, we fail to reject the null hypothesis.

Step 5: At the 5% level of significance, we do not have sufficient evidence to reject the null hypothesis and we cannot conclude that the mean lifespan of the tire is greater than 50,000 miles.

4) A recent study stated that if a person smoked, the average of the number of cigarettes he or she smoked was 14 per day. A researcher wanted to test the claim that the mean number was actually different from 14. A random sample of 40 smokers was obtained and found that the mean number of cigarettes smoked per day was 18. The standard deviation of the sample was 6. Using whatever hypothesis testing method you want, can you conclude that the mean number of cigarettes a person smokes per day actually differed from 14?

Step 1:

Ho : μ = 14
Ha : μ ≠ 14

n = 40
s=6
$\bar{x}$= 18

Step 2: Significance level .05
Step 3: Decision Rule says to reject the null hypothesis if |tstat| > 1.96

Step 4: tstat = $\frac{(18-14)}{\frac{6}{\sqrt{40}}}$

tstat = 4.2163

Because our tstat is 4.2163, which is greater than 1.96, we have evidence to reject the null hypothesis

Step 5: At the 5% level of significance, we have found sufficient evidence to reject the null hypothesis and suggest that the mean number of cigarettes a person smokes per day actually differs from 14.

5) In this exercise we show the relationship between sample size and sample evidence. Suppose the nationwide average for the math SAT is 480 but we think UCLA students are smarter than the average. We want to test $H_o: \mu = 480$ $H_a: \mu > 480$.

   a. Using R what is the p-value for the test if n=100, $\bar{x} = 483$, s=100

   If n=100, $\bar{x}$= 483, s=100, then tstat = $\frac{(483-480)}{\frac{100}{\sqrt{100}}}$ = 0.3

   1-pnorm(0.3,0,1)

[1] 0.3820886

b. Using R what is the p-value for the test if n=1000, $\bar{x} = 483$, s=100

If n=1000, $\bar{x}$= 483, s=100, then tstat = $\frac{(483-480)}{\frac{100}{\sqrt{1000}}}$ = 0.95

1-pnorm(0.95,0,1)
[1] 0.1710561

c. Using R what is the p-value for the test if n=10000, $\bar{x} = 483$, s=100

If n=10000, $\bar{x}$= 483, s=100, then tstat = $\frac{(483-480)}{\frac{100}{\sqrt{10000}}}$ = 3

1-pnorm(3,0,1)
[1] 0.001349898

d. What happens to the p-value as the sample size increased?

As the sample size increases, the p-value decreases.

6) Suppose the same set up as above with the same hypothesis to test.

a. Test $H_o: \mu = 480$ $H_a: \mu > 480$ assuming n=100, $\bar{x} = 496.4$, s=100

If n=100, $\bar{x}$= 496.4, s=100, then tstat = $\frac{(496.4-480)}{\frac{100}{\sqrt{100}}}$ = 1.64

1-pnorm(1.64,0,1)
[1] 0.05050258

b. Test $H_o: \mu = 480$ $H_a: \mu > 480$ assuming n=100, $\bar{x} = 496.7$, s=100

If n=100, $\bar{x}$= 496.7, s=100, then tstat = $\frac{(496.7-480)}{\frac{100}{\sqrt{100}}}$ = 1.67

1-pnorm(1.67,0,1)
[1] 0.04745968

c. In practical terms should there be a difference between (a) and (b)? Discuss briefly.

In practical terms, there should not be a difference between (a) and (b) because the x-bar value only differs by 0.3 points and 0.3 points on the math SAT is negligible w hen we are testing whether the average UCLA student's score is greater than 480. However, because (b)'s p-value is marginally less than 0.05, we reject the null hypot hesis, but in (a) we fail to reject.

Hypothesis Test for a population proportion

7) A survey of 4000 people in the US finds that 2856 of them believe that daily weather reports are totally useless because meteorology is not really a science. Given this data perform a hypothesis test to see if more than half of the people in the US believe that weather reports are useless.
   a. What is the null hypothesis?
      Ho: p = 0.5

   b. What is the alternative hypothesis?
      Ha: p > 0.5

   c. Using whatever method you want, clearly run and summarize the result of the hypothesis test. What does this mean in terms of the problem situation?

Step 1

Ho: p = 0.5
Ha: p $>$ 0.5
N = 4000
p0*(1-p0) = .5 * .5 = 0.25
p^ = 2856/4000 = 0.714

Step 2

Significance Level .05

Step 3

Decision Rule says to reject the null hypothesis if the t-stat > 1.64

t-stat = $\frac{.714 - .5}{\sqrt{.25/4000}}$ = 27.07

Step 4

Because our tstat is 27.07, which is more than 1.64, we do have evidence to reject the null hypothesis.

Step 5

At the 5% significance level, we have found sufficient evidence to reject the null hypothesis and conclude that the true proportion of people in the United States who believe that weather reports are useless is greater than 0.5.

8) Your statistics instructor claims that 60 percent of the students who take her Elementary Statistics class go through life feeling more enriched. For some reason that she can't quite figure out, most people don't believe her. You decide to check this out on your own. You randomly survey 64 of her past Elementary Statistics students and find that 34 feel more enrich as a result of her class. Now, what do you think?

Step 1

Ho: $p = .6$
Ha: $p \neq .6$
$N = 64$
$p0*(1-p0) = .40 * .60 = 0.24$
$p^\wedge = 0.53$

Step 2

Significance Level .05

Step 3

Decision Rule says to reject the null hypothesis if the |t-stat| > 1.96
t-stat = $\frac{.53-.60}{\sqrt{.24/64}}$ = -1.14

Step 4

Because our |tstat| is 1.04, which is not more than 1.96, we do not have evidence to reject the null hypothesis

Step 5

At the 5% significance level, we have not found sufficient evidence to reject the null hypothesis and conclude that the true proportion of students who take this teacher's Elementary school Statistics class and feel more enriched is different from .60.

9) A poll done for Newsweek found that 13% of Americans have seen or sensed the presence of an angel. A contingent doubts that the percent is really that high. It conducts its own survey. Out of 76 American surveyed, only 2 had seen or sensed the presence of an angel. As a result of the contingent's survey, would you agree with the Newsweek poll?

Step 1

Ho: $p = .13$
Ha: $p < .13$
$N = 76$
$p0*(1-p0) = .13 * .87 = 0.113$
$p^\wedge = 0.026$

Step 2

Significance Level .05

Step 3

Decision Rule says to reject the null hypothesis if the t-stat < -1.64

t-stat = $\frac{.026-.13}{\sqrt{.113/76}}$ = -2.69

Step 4

Because our tstat is -2.69, which is less than -1.64, we have enough evidence to reject the null hypothesis

Step 5

At the 5% significance level, we have found significant evidence to conclude that the percentage of Americans who have seen or sensed the presence of an Angel is less than 13%.

10) According to the 2010 Census, 58.5% of women worked. A county commissioner feels that more women work in his county, so he conducts a survey of 1000 randomly selected women and finds that 622 work. Is he correct?

Step 1

Ho: p = .585

Ha: p > .585

N = 1000

p0*(1-p0) = .585 * .415 = 0.243

p^ = 0.622

Step 2

Significance Level .05

Step 3

Decision Rule says to reject the null hypothesis if the t-stat > 1.64

t-stat = $\frac{.622-.585}{\sqrt{.243/1000}}$ = 2.37

Step 4

Because our tstat is 2.37, which is more than 1.64, we have enough evidence to reject the null hypothesis

Step 5

At the 5% significance level, we have found significant evidence to conclude that the proportion of women who work in this county commissioner's county is higher than the census average.

Confidence Intervals and Hypothesis Tests for Two Samples

11) Two groups of students are given a problem-solving test, and the results are compared. Find and interpret the 95% confidence interval of the true difference in means. Feel free to use R.

Mathematics majors	Computer science majors
$\bar{X}_1 = 83.6$	$\bar{X}_2 = 79.2$
$s_1 = 4.3$	$s_2 = 3.8$
$n_1 = 36$	$n_2 = 36$

$$\bar{x}_1 - \bar{x}_2 + 1.96\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (83.6 - 79.2) + 1.96\sqrt{\frac{4.3^2}{36} + \frac{3.8^2}{36}} = (2.52543, 6.274565)$$

We observe that the calculated 95% confidence interval is (2.52543, 6.274565). We are thus 95% confident that the true difference in means between Mathematics majors and Computer science majors on the test lies between(2.52543, 6.274565).

12) Many doctors believe that early prenatal care is very important to the health of a baby and its mother. Efforts have recently been focused on teen mothers. A random sample of 52 teenagers who have birth revealed that 32 of them began prenatal care in the first trimester of pregnancy. A random sample of 209 women in their twenties who have birth revealed that 163 of them began prenatal care in the first trimester of their pregnancy.

a. Construct a 95% confidence interval for the difference between the proportion of teen mothers who get early prenatal care and the proportion of mothers in their twenties who get early prenatal care (you may do this using R).

```
> prop.test(c(32,163), c(52,209), alternative = "two.sided")

        2-sample test for equality of proportions with continuit
y correction

data:  c(32, 163) out of c(52, 209)
X-squared = 5.1265, df = 1, p-value = 0.02356
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.320193854 -0.008845527
sample estimates:
   prop 1    prop 2
0.6153846 0.7799043
```

95% CI: (-0.32, -0.01)

b. Briefly interpret the confidence interval.

From the confidence interval above, we are 95% confident that the true difference between the proportion of teen mothers who get early prenatal care and the proportion of mothers in their twenties who get early prenatal care is between -0.32 and -0.01. We have enough evidence to conclude that there is indeed a significant difference.

13) In a sample of 80 Americans, 55% wished that they were rich. In a sample of 90 Europeans, 45% wished that they were rick. Run a two sided hypothesis test to see if there is a difference in the proportions against the null that they are equal.

Step 1

Ho: p1=p2
Ha: p1≠p2

Step 2

Significance level= .05

Step 3

Decision rule is that we will reject the null hypothesis if the p-value >0.05

Step 4

```
> prop.test(c(44,40.5), c(80,90), alternative = "two.sided")

        2-sample test for equality of proportions with
        continuity correction

data:  c(44, 40.5) out of c(80, 90)
X-squared = 1.3178, df = 1, p-value = 0.251
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.06163387  0.26163387
sample estimates:
prop 1 prop 2
  0.55   0.45
```

Step 5

Since the p-value>0.05, we do not have enough evidence to reject the null hypothesis and we thus we do not have evidence to conclude that the two proportion differ.

14) Suppose in a survey of college students, 1630 out of 7180 men responded Yes to being frequent binge drinkers and 1684 out of 9916 women responded yes. Find a 95% confidence interval for the difference between the proportions of men and women who are frequent binge drinkers. Interpret the interval.

```
> prop.test(c(1630,1684), c(7180,9916), alternative = "two.sided")

        2-sample test for equality of proportions with
        continuity correction

data:  c(1630, 1684) out of c(7180, 9916)
X-squared = 86.806, df = 1, p-value < 2.2e-16
alternative hypothesis: two.sided
95 percent confidence interval:
 0.04488666 0.06949925
sample estimates:
   prop 1    prop 2
0.2270195 0.1698265
```

We are thus 95% confident that the true proportion of the difference between men and women who frequently binge drink lies between .045 and .069. This means that at the .05 significance level, we can conclude that men frequently binge drink more than women do.

15) In an effort to increase production of an automobile plant, the factory manager decides to play music in the manufacturing area. Eight workers are selected, and the number of items each produced for a specific day is recorded. After one week of music, the same workers are monitored again. The data are given below. Can the manager conclude that the music has increased?

Worker	1	2	3	4	5	6	7	8
Before	6	8	10	9	5	12	9	7
After	10	12	9	12	8	13	8	10

Step 1

Ho: $\mu_{after} - \mu_{before} = 0$

Ha: $\mu_{after} - \mu_{before} > 0$

Step 2

.05 Significance Level

Step 3

Reject if t-stat > 1.645 or p-value <.05

Step 4

```
> before=c(6,8,10,9,5,12,9,7)
> after=c(10,12,9,12,8,13,8,10)
> t.test(after,before,alt="greater", paired=TRUE)

        Paired t-test

data:  after and before
t = 2.7325, df = 7, p-value = 0.01462
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 0.6133105       Inf
sample estimates:
mean of the differences
                      2
```

Our p-value is .01462, which is less than .05 so we reject the null hypothesis.

Step 5

At the .05 level of significance, we have enough evidence to conclude that the music has increased production.