Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

1) The random variable X has a binomial distribution with E(X)=18 and Var(X)=7.2. Find n and p for this distribution [use the formulas for mean and variance].

E(X) = np = 18
Var(X) = npq = 7.2

$$\frac{Var(X)}{E(x)} = \frac{npq}{np} = \frac{7.2}{18} = 0.4 \therefore q = 0.4$$

q = 1-p ∴ 1- p = 0.4 ∴ p = 0.6
npq = n*0.6*0.4 = 7.2 ∴ n = 30

Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

2) Suppose X is a binomial random variable with n=15 and p=0.3. Feel free to use a computer to answer the following:

```
> n = 15
> p = 0.3
```

a) P(X=0)

```
> dbinom(0,n,p)
[1] 0.004747562
```

b) P(X=2)

```
> dbinom(2,n,p)
[1] 0.09156011
```

c) P(X<2)

This is equivalent to P(X<=1) since we're dealing with discrete random variables.

```
> pbinom(1,n,p)
[1] 0.0352676
```

d) P(X>8)

P(X>8) = 1-P(X<=8) = 1-0.9847575 = 0.0152425

```
> pbinom(8,n,p)
[1] 0.9847575
```

e) E(X)

```
> n*p
[1] 4.5
```

f) Var(X)

```
> q = 1-p
> n*p*q
[1] 3.15
```

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Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

3) A school newspaper reporter decides to randomly survey 12 students to see if they will attend YardFest TM festivities this year. Based on past years, she knows that 18% of students attend YardFest TM festivities. We are interested in the number of students who will attend the festivities.

n = 12
success = Attend YardFest TM festivities
P(success) = 0.18

a) How many of the 12 students do we expect to attend the festivities?

→ E(X) = n*p = 2.16

b) Find the probability that at most 4 students will attend.

→ P(0 attend or 1 attends or 2 attend or 3 attend or 4 attend)
= P(0 attends) + P(1 attends) + P(2 attend) + P(3 attend) + P(4 attend)
= P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)

```
> dbinom(0,12,0.18)+ dbinom(1,12,0.18)+ dbinom(2,12,0.18) + dbinom(3,12,0.18) + dbinom(4,12,0.18)
[1] 0.9510694
```

c) Find the probability that more than 2 students will attend.

→ P(more than 2 students will attend) = 1 - P(up to 2 students will attend)
= 1-[P(no student will attend)+P(1 student will attend)+P(2 students will attend)]
= 1-[P(X=0) + P(X=1) + P(X=2)]

```
> 1-(dbinom(0,12,0.18)+ dbinom(1,12,0.18)+ dbinom(2,12,0.18))
[1] 0.3702131
```

Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

4) Suppose that about 85% of graduating students attend their graduation. A group of 22 graduating students is randomly chosen.

n = 22
Success = Attend their graduation
P(success) = 0.85

a) How many are expected to attend their graduation?

→ E(X) = n*p = 18.7

b) Find the probability that 17 or 18 attend.

→ P(X=17) + P(X=18)
> dbinom(17,n,p) + dbinom(18,n,p)
[1] 0.3248747

c) Based on numerical values, would you be surprised if all 22 attended graduation? Justify your answer numerically.

→ > p^n
[1] 0.02800376

The probability that all 22 attended graduation is 0.028. This is very low so I will be surprised.

> q = 0.15
> sqrt(n*p*q)
[1] 1.674813

The 22 is beyond 2 standard deviation away from the mean so it would truly be surprising.

Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

5) A new drug named CURAIDS that is 60% effective in extending the average life of an AIDS patient by twenty years. Five randomly selected AIDS patients are treated with this new drug. Answer the following questions based on the above information.

$n = 5$
Success = cured
P(success) = 0.6

(a) What is the probability that no more than 4 patients are cured?

→ P(no more than 4 patients are cured) = P(at least one is not cured)
= 1 - P(all are cured) = $1 - p^n$
= $1 - (0.6)^5$
= 0.9222

(b) Find the probability that more than 2 patients or less than or equal to 5 patients are cured.

→ P(more than 2 patients or less than or equal to 5)
= P(less than or equal to 5 patients) - P(more than 2 patients)
For the example, $n = 5$ i.e. total number of samples.
P(less than or equal to 5 patients are cured) = 1

$P(x \leq 2)$

```
> pbinom(2,5,.6)
[1] 0.31744
```

= 1 – 0.31744 = 0.6825

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Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

6) According to flightstats.com, American Airlines flights from Dallas to Chicago are on time 70% of the time. Suppose 15 flights are randomly selected, and the number of on-time flights is recorded.

n = 15
Success = Flight is on time
P(success) = 0.7

a) Explain why this is a binomial experiment.

→ This is a binomial experiment because:
1. 15 independent trials are taking place
2. The flights are either on time or not i.e. success or failure on each trial
3. The probability of success is a constant 0.7
4. Our interest is in the number of on-time flights i.e. successes

b) Find and interpret the probability that exactly 10 flights are on time.

→ P(exactly 10 flights are on time) = P(X=10)

```
> dbinom(10,n,p)
[1] 0.2061304
```

Out of a randomly selected 15 flights, there's a 20.613% chance that exactly 10 flights are on time.

c) Find and interpret the probability that between 8 and 10 flights, inclusive, are on time.

→ P(between 8 and 10 flights are on time)
= P(8 flights are on time or 9 flights are on time or 10 flights are on time)
= P(8 flights are on time) + P(9 flights are on time) + P(10 flights are on time)

```
> dbinom(8,n,p) + dbinom(9,n,p) + dbinom(10,n,p)
[1] 0.4344964
```

Out of a randomly selected 15 flights, there's a 43.45% chance that between 8 and 10 flights are on time.

d) Find the mean and variance of the number of on time flights.

→ Mean = E(X) = n*p = 10.5
Variance = Var(X) = n*p*q = 3.15

Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

7) Let X be a uniformly distributed random variable on the interval 0 to 1

a) Calculate $P(X = 0.25)$

→ For continuous random variables, probability that X is exactly equal to a value is 0.

b) Calculate $P(0.7 < X < 1)$

→ $P(0.7 < X < 1) = 1 - P(X < 0.7) = 1 - 0.7 * \frac{1}{1-0} = 1 - 0.7 = 0.3$

c) Calculate the expected value of X

→ $E[X] = \frac{a+b}{2} = \frac{1+0}{2} = 0.5$

d) Calculate $E(X^2)$

→ $Var(X) = E(X^2) - E(X)^2$

$E(X^2) = Var(X) + E(X)^2$

$$= \frac{(b-a)^2}{12} + (0.5)^2 = \frac{1}{12} + (0.5)^2 = 0.3333$$

Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

8) For the standard normal random variable Z, compute the following:

For a standard normal random variable, $\mu = 0$ and $\boldsymbol{\sigma}^2 = 1$

a) $P(0 \leq Z \leq 0.73)$

```
> pnorm(0.73,0,1) - pnorm(0,0,1)
[1] 0.2673049
```

b) $P(-1.50 \leq Z \leq 0)$

```
> pnorm(0,0,1) - pnorm(-1.5,0,1)
[1] 0.4331928
```

c) $P(Z \geq 0.44)$

```
> 1 - pnorm(0.44,0,1)
[1] 0.3299686
```

d) $P(-1.50 \leq Z \leq 0.40)$

```
> pnorm(0.4,0,1) - pnorm(-1.5,0,1)
[1] 0.5886145
```

e) $P(Z \leq 5.23)$

```
> pnorm(5.23,0,1)
[1] 0.9999999
```

f) $E(3\text{-}4Z)$

$\rightarrow E(3\text{-}4Z) = 3 - 4 * E(Z) = 3 - 4 * (0.5) = 1$

g) $Var(4 - 3Z)$

$\rightarrow Var(4\text{-}3Z) = (-3)^2 * Var(Z) = 9 * 1 = 9$

Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

9) The time needed to hand stitch a Swoosh soccer ball is normally distributed with mean 41 minutes and standard deviation 3 minutes. If 4 workers start at the same time, what is the probability that at least one of them will complete their soccer ball in under 44 minutes?

→ $\mu = 41$, $\boldsymbol{\sigma} = 3$

P(at least one will complete in under 44 minutes) = 1 - P(None complete in under 44 minutes)

P(None complete in under 44 minutes)

$= P(X1 > 44 \text{ and } X2 > 44 \text{ and } X3 > 44 \text{ and } X4 > 44)$

$= P(X1>44) * P(X2>44) * P(X3>44) * P(X4>44)$

$= [P(X>44)]^4$

$= [1 - P(X<44)]^4$

P(X < 44) =

```
> pnorm(44,mu,sigma)
[1] 0.8413447
```

P(None complete in under 44 minutes) $= [1 - 0.8413447]^4 = 0.0006$

P(at least one will complete in under 44 minutes) = 1 - 0.0006 = **0.9994**

Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

10) The weight of reports produced in a certain department has a Normal distribution with mean 60g and standard deviation 12g. What is the probability that the next report will weigh less than 45g?

→ $\mu = 60$, $\boldsymbol{\sigma} = 12$

$P(X<45) =$

```
> mu = 60
> sigma = 12
> pnorm(45,mu,sigma)
[1] 0.1056498
```

Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

11) The times taken to complete an introduction to business statistics exam have a normal distribution with a mean of 65 minutes and standard deviation of 7 minutes. There are 150 students who took the exam and students are allowed a total of 75 minutes to take the exam.

$\mu = 65$, $\boldsymbol{\sigma} = 7$

a) What is the chance that Mike finished his exam in 63 to 72 minutes?

P(X<72) - P(X<63)

```
> pnorm(72,mu,sigma) - pnorm(63,mu,sigma)
[1] 0.4537963
```

There's a 45.38% chance that Mike finished his exam ins 63 to 672 minutes.

b) What is the expected number of students who finished in less than 75 minutes?

P(X<75)

```
> pnorm(75,mu,sigma)
[1] 0.9234363
```

92.34% of students are expected to finish in less that 75 minutes
i.e. 0.9234 * 150
~ 139 students

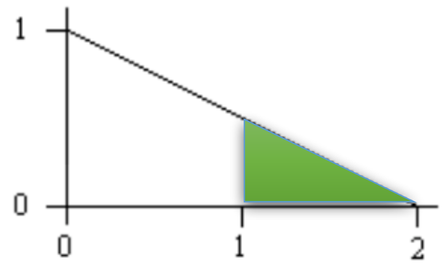
c) As some students were not able to finish the exam in time, the instructor allowed 6 more minutes. Given he already spent 75 minutes on the exam, what is the chance that Chris finished his exam in extended time, that is between 75 and 81 minutes

```
> pnorm(81,mu,sigma)
[1] 0.9888645
```

There is a 98.89% chance Chris finished his exam in the extended time.

Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

12) Suppose X is a continuous random variable taking values between 0 and 2 and having the probability density function below. Calculate $P(1 \le X \le 2)$



Equation of line

$y = mx + c$

$y = -(1/2)x + 1$

$P(1 \le X \le 2)$ = Area under the line

$$= \frac{1}{2} * base * height = \frac{1}{2} * 1 * height = \frac{height}{2}$$

For height, x = 1

Substituting in equation for line,

$y = -(1/2)+1 = 1/2$

$$P(1 \le X \le 2) = \text{Area under the line} = \frac{1}{2} * \frac{1}{2} = \frac{1}{4} = 0.25$$