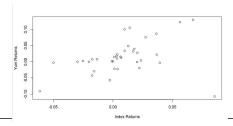




Stat 104: Quantitative Methods Class 8: Basic Probability Theory

#### Quick Beta Review

Regress stock returns on index returns





79.37

# Quick Beta Review

The need for understanding uncertainty is important.

> confint(fit)

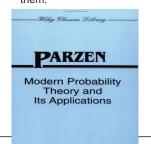
2.5 % 97.5 % (Intercept) -0.007322573 0.02594991 spyret 0.102563508 1.22840375

#### Mike and His Many Issues.....

1,775,592

1y Target Est

I have many issues (surprise) and of course probability is one of them.



Hmmm. My name on a book-maybe I should read it someday??



#### What is probability?

- Probability is a measure of uncertainty.
- We can try to compute the probability of almost any event of interest.
- An event is the basic element to which probability can be applied; it is the result of an observation or experiment, or the description of some potential outcome.

#### Some examples of events

Example events we can assign probability to:

- Getting a head when we flip a coin
- Coca-Cola stock rises 5% next week
- Ford will produce a hybrid SUV that gets 50mpg by 2018.

#### Probability is important

Probability is useful to study since uncertainty plays a major role in economic decisions:

- Will our new product be successful?
- How many yen will a dollar buy in currency markets a month from today?
- Will the manufactured part be defective ?
- Given you took a test drive, will you buy the car?

#### Most people don't get probability

- Unfortunately, most people do not attempt to evaluate probability in a quantitative way. Instead, they rely on intuition, doubtful logic, misinterpreted experience, and emotion
- As the famous American jurist, Oliver Wendell Holmes, put it: Most people think dramatically, not quantitatively.
- This section should help you start thinking about probability (uncertainty) in a clearer fashion and hence make more informed decisions.



# The birthday problem Often times, the results of using probability theory are surprising (to me at least).

- How many people need to be a room together so that there is a more than 50% chance of two people having the same birthday?
  - A) 300
- B) 183
- C) 91
- D) 23

Number of people	Probability that 2 people share a birthday
10	
20	
23	
30	
50	
57	
100	
200	
366	100%

Last 40 Oscar-winning Best Actress Birthdays

Sandra Bullock	Jul 26
Kate Winslet	Oct 5
Marion Cotillard	Sep 30
Helen Mirren	Jul 26
Reese Witherspoon	Mar 22
Hilary Swank	Jul 30
Charlize Theron	Aug 7
Nicole Kidman	Jun 20
Halle Berry	Aug 14
Julia Roberts	Oct 28
Gwyneth Paltrow	Sep 27
Helen Hunt	Jun 15
Frances McDormand	Jun 23
Susan Sarandon	Oct 4
Jessica Lange	Apr 20
Holly Hunter	Mar 20
Emma Thompson	Apr 15
Jodie Foster	Nov 19
Kathy Bates	Jun 28
Jessica Tandy	Jun 7

Cher	May 20
Marlee Matlin	Aug 24
Geraldine Page	Nov 22
Sally Field	Nov 6
Shirley MacLaine	Apr 24
Meryl Streep	May 27
Katharine Hepburn	May 12
Sissy Spacek	Dec 25
Jane Fonda	Dec 21
Diane Keaton	Jan 5
Faye Dunaway	Jan 14
Louise Fletcher	Jul 22
Ellen Burstyn	Dec 7
Glenda Jackson	May 9
Liza Minnelli	Mar 12
Maggie Smith	Dec 28
Barbra Streisand	Apr 24
Elizabeth Taylor	Feb 27
Sophia Loren	Sep 20
Anne Bancroft	Sep 17

Last 40	Oscar-	winning Bes	t Actor Birthdays	3
Jeff Bridges	Dec 4		Paul Newman	Jan 26
Daniel Day-Lewis	Apr 29		William Hurt	Apr 20
Forest Whitaker	Jul 15		F. Murray Abraham	Oct 24
Philip Seymour Hoffman	Jul 23		Robert Duvall	Jan 5
Jamie Foxx	Dec 13		Ben Kingsley	Dec 31
Sean Penn	Aug 17		Henry Fonda	May 16
Adrien Brody	Apr 14		Robert De Niro	Aug 17
Denzel Washington	Dec 28		Jon Voight	Dec 29
Russell Crowe	Apr 7		Richard Dreyfuss	Oct 29
Kevin Spacey	Jul 26		Peter Finch	Sep 28
Roberto Benigni	Oct 27		Art Carney	Nov 4
Jack Nicholson	Apr 22		Jack Lemmon	Feb 8
Geoffrey Rush	Jul 6		Marlon Brando	Apr 3
Nicolas Cage	Jan 7		Gene Hackman	Jan 30
Tom Hanks	Jul 9		George C. Scott	Oct 18
Al Pacino	Apr 25		John Wayne	May 26
Anthony Hopkins	Dec 31		Cliff Robertson	Sep 9
Jeremy Irons	Sep 19		Rod Steiger	Apr 14
Dustin Hoffman	Aug 8		Paul Scofield	Jan 21
Michael Douglas	Sep 25		Lee Marvin	Feb 19

#### Random Experiments

- Probability deals with random experiments.
- But we put some structure on the random experiments and assume we know the possible <u>outcomes</u> that can occur on each trial of the phenomena.
- These outcomes can be defined in several ways.
- However, they must be <u>exhaustive</u> and <u>mutually</u> <u>exclusive</u>.

# Properties of experiment outcomes

- <u>Exhaustive</u>: All possible results of the phenomena must be listed
- Mutually exclusive: It cannot be that the outcome of a phenomena can fall into two outcome categories simultaneously
- Sample space (S): A list of all possible outcomes from a random experiment S = {O1, O2,..., Ok}
- Note that the sample space could be infinite (we deal with that in a few classes).

#### Mutually Exclusive Events

- Two events are **mutually exclusive** if, when one event occurs, the other cannot, and vice versa
- •Experiment: Toss a die

Not Mutually Exclusive

- –A: observe an odd number
- -B: observe a number greater than 2
- -C: observe a 6

Mutually Exclusive

–D: observe a 3

# Terminology Review

A(n) (random) <u>experiment</u> is an activity involving chance. Each repetition or observation of an experiment is a <u>trial</u>, and each possible result is an <u>outcome</u>. The <u>sample space</u> of an experiment is the set of all possible outcomes.

	Rolling a number cube	Tossing a coin	Spinning a game spinner
Experiment	5		
Sample Space	{1, 2, 3, 4, 5, 6}	{heads, tails}	{red, blue, green, yellow}

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- We might study the random experiment of how \$AAPL closed today.
- We could define the events
  - ☐ {\$AAPL up, \$AAPL not up}
  - □ Or {\$AAPL up, \$AAPL unchanged, \$AAPL down}
  - □ Or {up more than 2%, up less than 2%, unchanged, down by less than 2%, down more than 2%}
  - ☐ Etc...

Examples

It is useful to review these concepts with the help of the following example:

- \$AAPL rises and \$AAPL declines are mutually exclusive, but not collectively exhaustive outcomes.
- \$AAPL does not rise and \$AAPL does not decline are collectively exhaustive, but not mutually exclusive, outcomes.
- \$AAPL rises and \$AAPL does not rise are mutually exclusive and collectively exhaustive outcomes.

Example, random experiment=grade on an exam

Valid sample spaces:

S = {Pass, Fail, Missed}

 $S = \{A, B, C, D, E, F, Missed\}$ 

 $S = \{[0,50),[50,60),[60,70),[70,80),[80,100], .\}$ 

■ Invalid sample spaces (why?):

 $S = \{Fail\}$ 

 $S = \{[50,60),[60,70),[70,80),[80,100)\}$ 

 $S = \{[0,50),[50,100],[80,100],.\}$ 

Values of probabilities

- We denote the probability of the event A as Prob(A), which we often abbreviate as Pr(A) or simply P(A).
- The probability of the event A is the chance that A will occur. Hence probabilities range from 0 (no chance of occurrence) to 1 (absolute chance of occurrence (a sure thing)).

 $0 \leq \Pr{ob(A)} \leq 1$ 

# How is Probability Interpreted?

Name some events that would go here

By the Way, How is Probability Defined?

- We all know that the probability of getting a head when I flip a coin is 0.5.
- What about the probability of a 35 year old white male who hasn't smoked in 10 years of being diagnosed with lung cancer this year?
- Where do probabilities come from ?

- There are three basic ways of classifying probability. These three represent rather different conceptual approaches to the study of probability theory; in fact, experts disagree about which approach is the proper one to use. The three approaches are
  - 1) Subjective
  - 2) Logical
  - 3) Experimental

#### But the rules are the same......

- Lucky for us, though, no matter how the probabilities we work with are assigned, be it objective or subjective, the rules governing the manipulation of the values are the same.
- So in some sense you can say forget this-but it is helpful to go over these different concepts once, just to understand some of the subtleties of probability theory.

# Subjective Probabilities

- Subjective probability is the probability we assign to the outcome of an experiment that cannot be repeated. The growth of GNP next year is an experiment that can be observed only once, and thus cannot be assigned an objective probability.
- The importance of subjective probability stems from the fact that it is often an important input into decision making by groups and individuals.

# Opinions on \$ANF are Subjective

Upgrades & Downgrades History							
Date	Research Firm	Action	From	То			
Aug 23, 2013	Robert W. Baird	Downgrade	Outperform	Neutral			
Aug 22, 2013	Janney	Downgrade	Buy	Neutral			
Apr 2, 2013	Barclays	Initiated		Underweight			
Dec 10, 2012	Robert W. Baird	Upgrade	Neutral	Outperform			
Nov 14, 2012	Janney Mntgmy Scott	Upgrade	Neutral	Buy			
Oct 3, 2012	Avondale	Initiated		Mkt Perform			
Aug 2, 2012	Robert W. Baird	Downgrade	Outperform	Neutral			
Aug 2, 2012	Caris & Company	Downgrade	Buy	Average			
May 1, 2012	UBS	Upgrade	Neutral	Buy			
Jan 9, 2012	Brean Murray	Downgrade	Buy	Hold			

☐Subjective probabilities are based on the belief of the person making the probability assessments.

□In fact, subjective probability can be defined as the probability assigned to an event by an individual, based on whatever evidence is available. This evidence may be in the form of relative frequency of past occurrences, or it may just be an educated guess.

□Subjective assessment of probability permits the widest flexibility of the three concepts we have discussed.

#### Subjective probabilities:

Rank	Statement	И	Mean	Median	α	Range	Rank	Statement	N	Mean	Median	a	Range
1	Highly probable	187	0.89	0.90	0.04	.6099	21	Slightly less than	188	0.45	0.45	0.04	.0550
2	Very likely	185	0.87	0.90	0.06	.4599		half the time					
3	Very probably	187	0.87	0.89	0.07	.6099	22	Slight odds against	185	0.45	0.45	0.11	.1099
4	Quite likely	188	0.79	0.80	0.10	.3099	23	Not quite even	180	0.44	0.45	0.07	.0560
5	Usually	187	0.77	0.75	0.13	.1599	24	Inconclusive	153	0.43	0.5	0.14	.0175
6	Good Chance	188	0.74	0.75	0.12	.2596	25	Uncertain	173	0.4	0.5	0.14	.0190
7	Predictable	146	0.74	0.75	0.20	.0199	26	Possible	178	0.37	0.49	0.23	.0199
8	Likely	188	0.72	0.75	0.11	.2599	27	Somewhat unlikely	186	0.31	0.33	0.12	.0380
9	Probably	188	0.71	0.75	0.17	.0199	28	Fairly unlikely	187	0.25	0.25	0.11	.0275
10	Rather likely	188	0.69	0.70	0.09	.1599	29	Rather unlikely	187	0.24	0.25	0.12	.0179
11	Pretty good chance	188	0.67	0.70	0.12	.2595	30	Not very probably	187	0.21	0.2	0.12	.0160
12	Fairly likely	188	0.66	0.70	0.12	.1595	31	Unlikely	188	0.18	0.16	0.1	.0145
13	Somewhat likely	187	0.59	0.60	0.18	.2092	32	Not much chance	186	0.16	0.15	0.09	.014
14	Better than even	187	0.58	0.60	0.06	.4589	33	Seldom	188	0.16	0.15	0.08	.0147
15	Rather	124	0.58	0.60	0.11	.1080	34	Barely possible	180	0.13	0.05	0.17	.0160
16	Slightly more than	188	0.55	0.55	0.06	.4580	35	Fairly possible	184	0.13	0.05	0.16	.016
	half the time						36	Improbably	187	0.12	0.1	0.09	.0141
17	Slight odds in favor	187	0.55	0.55	0.08	.0575	37	Quite unlikely	187	0.11	0.1	0.08	.015
18	Fair chance	188	0.51	0.50	0.13	.2085	38	Very unlikely	186	0.09	0.1	0.07	.015
19	Tossup	188	0.5	0.50	0.00	.4552	39	Rare	187	0.07	0.05	0.07	.0130
20	Fighting chance	186	0.47	0.55	0.17	.0590	40	Highly improbable	181	0.06	0.05	0.05	.013

I he main worry is that this person could be

#### The Logical Approach

 $P(\text{event occurs}) = \frac{\text{number of outcomes where the event occurs}}{\text{total number of possible outcomes}}$ 

□ It must be emphasized that in order for this equation to work, each of the possible outcomes must be equally likely. This is a rather complex way of defining something that may seem intuitively obvious to us. □ For example, when flipping a coin, we obtain

 $P(Head) = \frac{1}{1}$ 

 $\Box \text{When rolling a die, we obtain}_{P(roll\ 5) = \frac{1}{1+1+1+1+1+1}}$ 

#### Example:

■ Probability of throwing a 5 with a pair of dice

	•	•	••	• •	::	::
•	2	3	4	5	6	7
	3	4	5	6	7	8
••	4	5	6	7	8	9
• •	5	6	7	8	9	10
:•:	6	7	8	9	10	11
::	7	8	9	10	11	12

The logical approach is useful when dealing with card games, dice games, coin tosses and the like, but has serious problems when we try to apply it to the less orderly decision problems we encounter in actual real-world applications.

The Experimental Approach

- How do we determine the probability that a person lives past the age of 85?
- We could collect a lot of data on people and count how many actually live that long.
- The experimental method defines probability as essentially the observed relative frequency of an event in a very large number of trials.

Probability = number of trials in which event did result total number of trials

#### A Caveat on Relative Frequency

- One difficulty with the relative frequency approach is that people often use it without evaluating a sufficient number of outcomes.
- If you heard someone say "My aunt and uncle got the flu this year and both are over 65, so everyone in that age bracket will probably get the flu", you would know that your friend did not base his assumptions on enough

evidence.

#### Probability Rules

- There are some formal rules that govern how to manipulate and work with probabilities.
- Luckily, these rules work no matter if we are dealing with subjective, classical or experimental probability.



#### Rule 1



- We've seen this one already; probabilities are always between 0 and 1 (inclusive).
- Even if you think an event can't happen, its probability can't be less than 1. And a sure thing doesn't have a probability larger than 1.

A probability is a number between 0 and 1. For any event A,  $0 \le P(A) \le 1$ .

 As you probably realize by know we use <u>capital letters</u> to denote events.

#### Rule 2



- The probability of the set of all outcomes must be 1 (denoted O<sub>1</sub>, O<sub>2</sub>, etc..)
- This means that on each trial of the random experiment we're studying, *something* has to happen.
- We can denote this as

 $\sum_{i=1}^{k} P(O_i) = 1$ 

- If this sum < 1 we are missing a possible outcome
- If this sum > 1 then 2 outcomes can happen at once which is a no-no.

#### Rule 3: The complement rule

- Suppose the probability that you get to class on time is 0.8
- What's the probability that you don't get to class on time? Yes, it's 0.2.
- We denote not(A) as A, and its called the complement of A.
- In symbols,

$$P(\overline{A}) = 1 - P(A)$$

# Birthday Problem [23 people]

- Let A = event no one in group shares same birthday.
- Then not(A) = event at least 2 people share same birthday

Assuming independence and using classical approach:

P(A) = 365/365 × 364/365 × ... × 343/365 = 0.493

So, then P(not(A)) = 1 - 0.493 = 0.507

# Contingency Tables

- Suppose that, over a period of 4 years, that is, 1,000 market days, \$AAPL and \$GPS have been observed to rise, remain unchanged, and decline.
- For somewhat obscure reasons such a table is referred to as a contingency table.

			\$AAPL		
		Rise	Decline	Unchanged	Total
	Rise	338	158	1	497
\$GPS	Decline	202	293	1	496
	Unchanged	3	4	0	7
	Total	543	455	2	1000

#### Contingency Tables

Notice that the events listed horizontally, as well as those listed vertically, are mutually exclusive and collectively exhaustive. This must always be the case in contingency tables.

		Rise	\$AAPL Decline	Unchanged	Total
		Rise	Decline	Unchanged	Total
	Rise	338	158	1	497
\$GPS	Decline	202	293	1	496
	Unchanged	3	4	0	7
	Total	543	455	2	1000

			\$AAPL		
		Rise	Decline	Unchanged	Prob
	Rise	0.338	0.158	0.001	0.497
\$GPS	Decline	0.202	0.293	0.001	0.496
	Unchanged	0.003	0.004	0	0.007
	Prob	0.543	0.455	0.002	1

#### Joint Probabilities

- The numbers inside the table are called joint probabilities, because they refer to two events happening at the same time.
- These nine joint probabilities are associated with nine mutually exclusive and collectively exhaustive events, so the sum of these probabilities has to equal 1.

			\$AAPL		
		Rise	Decline	Unchanged	Prob
	Rise	0.338	0.158	0.001	0.497
\$GPS	Decline	0.202	0.293	0.001	0.496
	Unchanged	0.003	0.004	0	0.007
	Prob	0.543	0.455	0.002	1

# Joint Probabilities

- P(\$GPS rises AND \$AAPL rises)= 0.338
- P(\$GPS rises AND \$AAPL declines) = 0.158
- Etc....
- P(A and B) is called a joint probability.

#### Marginal Probabilities

- The entries in the right column and bottom row (both labeled total) give the so-called marginal probabilities. These probabilities are called marginal probabilities for the simple reason that they are written in the margin. We see, for example,
- P(\$AAPL rises) = 0.543
- P(\$GPS decline) = 0.496
- Marginal probabilities give only information concerning the behavior of one stock, either \$AAPL or \$GPS.

#### Marginal Probabilities

- Marginal probabilities give only information concerning the behavior of one stock, either \$AAPL or \$GPS.
- The marginal row and the marginal column both sum to 1, and this again is as it should be, for the events "rises," "is unchanged," and "declines" are mutually exclusive and collectively exhaustive.

# Marginal Probabilities (cont)

	)	- 0 10 001	3,5 50,5 === 5 (5 5 ==			
			\$AAPL			
		Rise	Decline	Unchanged		Prob
	Rise	0.338	0.158	0.001		0.497
\$GPS	Decline	0.202	0.293	0.001		0.496
	Unchanged	0.003	0.004	0		0.007
	Prob	0.543	0.455	0.002		1

#### Obtain the marginal distribution

\$AAPL	Prob	\$GPS	Prob
Rise	0.543	Rise	0.497
Decline	0.455	Decline	0.496
Unchanged	0.002	Unchanged	0.007

#### The Addition Rule

■ The addition rule is helpful when we have two events and are interested in knowing the probability that at least one of the events occurs.

The addition rule of probability states that for any two events A and B

P(A or B) = P(A) + P(B) - P(A and B)

The probability P(A or B) is the probability that A occurs, or that B occurs, or that both A and B occur.

Probability "or" is defined a bit weird......

#### Example

Look at the probability table

	Cut this pr		SAAPL		
		Rise	Decline	Unchanged	Prob
	Rise	0.338	0.158	0.001	0.497
\$GPS	Decline	0.202	0.293	0.001	0.496
	Unchanged	0.003	0.004	0	0.007
		\	/		
	Prob	0.543 /	0.455	0.002	1
	FIOD	0.543	0.433	0.002	1

P(\$AAPL rise OR \$GPS rise) = 0.543 + 0.497 - 0.338 = 0.702

We need to subtract off the intersection so we don't double count.

#### Example

Consider the following probability table

	Sierra Mist	Sprite	
Male	0.4	0.15	
Female	0.22	0.23	

What is the probability that someone is a male or likes Sprite?

# Conditional Probability

- Random events are random (duh!) but sometimes it happens that we can get some information that sheds light on the issue.
- For example, suppose we want to know the probability that an 8 (as a sum)will turn up when we roll two dice.

		SECOND DIE						
				٠.	::	::	::	
	$\overline{}$	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	
	٠.	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	
D	٠.	(3, 1)	(3, 2)	(1, 3) (2, 3) (3, 3) (4, 3) (5, 3)	(3, 4)	(3, 5)	(3, 6)	
£	::	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	
_	::	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	_
	E E	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	

Pr(8)=5/36

# Conditional Probability

- Now suppose we roll one of the dice first (rather than at the same time). Then we'll have a better idea of getting an 8.
- That is, if our first roll is a 1, what is the chance the sum of the two dice will be 8?
- If our first roll is a 3, what do we need for the second roll? What is the chance of that happening?

#### Conditional Probability

- Conditional probabilities ask, what is the chance that A happens, given that we know B happened?
- If the package doesn't make it to Chicago by 9am tomorrow, what's the chance we lose the deal ?"
- "given we spend 10 million on research, what is the probability of inventing a successful competitor to the Ipod?
- Note the use of the words "if" and "given"
- The notation is P(A|B) and we say the probability of event A given event B has happened.

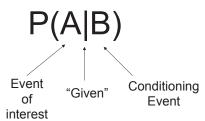
52

4

#### Three DIFFERENT probability statements

- If a person has HIV, the probability that they will test positive on a screening test is 90%.
- If a person tests positive on a screening test, the probability that they have HIV is 90%.
- The probability that a person has HIV and tests positive on a screening test is 90%.

# Conditional Probability



The probability that A occurs given that B occurs

# Definition of conditional probability

# $P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$

Out of all the times B happens, how often does A also happen ?

Note that P(B) is assumed to not be zero.

#### Example

- 70% of your friends like Chocolate, and 35% like Chocolate AND like Strawberry.
- What percent of those who like Chocolate also like Strawberry?
- P(Strawberry|Chocolate)
  - = P(Chocolate and Strawberry) / P(Chocolate)

#### Example

- A board game comes with a special deck of cards, some of which are black, and some of which are gold. If a card is randomly selected, the probability it is gold is 0.20, while the probability it gives a second turn is 0.16. Finally, the probability that it is gold and gives a second turn is 0.08.
- Suppose that a card is randomly selected, and it allows a player a second turn. What is the probability it was a gold card?

#### Example

 A survey asked full time and part time students how often they had visited the college's tutoring center in the last month. The results are shown below.

	Number of times students visited tutoring					
	One or fewer times	Two to three times	Four or more times	Total		
Full time student	12	25	8	45		
Part time student	2	5	6	13		
Total	14	30	14	58		

What is the probability the student visited the tutoring center four or more times, given that the student is full time?

8

# Final Example

A survey of middle school students asked: What is your favorite winter sport? The results are summarized below:

Grade	Snowboarding	Skiing	Ice Skating	TOTAL	
6th	68	41	46	155	
7th	84	56	70	210	
8th	59	74	47	180	
TOTAL	211	171	162	EAE	

Grade	Snowboarding	Skiing	Ice Skating	
6th	0.12	0.08	0.08	0.28
7th	0.15	0.10	0.13	0.39
8th	0.11	0.14	0.09	0.33
	0.39	0.31	0.30	

- · What is the probability of selecting a student whose favorite sport is skiing?
- What is the probability of selecting a 6th grade student?

  If the student selected is a 7th grade student, what is the probability that the student prefers ice-skating?
- If the student selected prefers snowboarding, what is the probability that the student is a 6th grade student?

#### Things you should know

- □What is probability
- □Random phenomena, events
- □ Joint Probability, Marginal Probability
- ■Addition rule
- □Conditional Probability