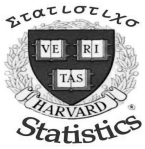


Stat 104: Quantitative Methods for Economists

Class 17:Covariance and Other Topics

Market Model-statistics in finance

$$Stockreturn_t = \alpha + \beta Indexreturn_t$$

Beta=0 : cash under the mattress

Beta=1 : same risk as the market

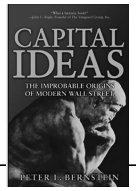
0<Beta<1 : safer than the market

Beta >1: riskier than the market

Beta < 0 : what would this mean???

Portfolios

The word ''portfolio'' always brings back memories of how my father used it in connection with his clients' accounts after he started his investment counseling firm in 1934. My idea of a portfolio was a fancy leather folder with a sheaf of papers inside. In the world of investing, a portfolio has no physical existence. Rather, it represents the investor's total capital.



Portfolios

- A portfolio is a collection of different securities such as stocks and bonds, that are combined and considered a single asset
- The risk-return characteristics of the portfolio is demonstrably different than the characteristics of the assets that make up that portfolio, especially with regard to risk.
- Combining different securities into portfolios is done to achieve *diversification*.

Asset Classes Make-up Portfolios

- Each modern asset class has an underlying general type of risk, return and correlation to the other asset classes.

Large-cap Passive	
Large-cap Value	Large-cap Growth
Small-cap Value	Small-cap Growth
Hedge Funds	
Real Estate	
Private Equity	
Core Fixed Income	
International Equity	
Emerging Markets Equity	
High Yield Bonds	

By no means a complete list.

New Asset Classes Can be Created

Cramer: High-Growth Stocks In An Asset Class of Their Own

As high-growth names like **Netflix**, **Chipotle** or **Apple** (AAPL 359.90 ▲ 0.72 (+0.2%)) continue to climb, Cramer said Monday investors should evaluate these names differently.

Traditionally, he explained, stocks are judged on a **price-to-earning multiple**. On that basis, however, these momentum stocks are too expensive. High-growth names can't be thought of in that way, he said.

Unlike other stocks, Cramer said high-growth names aren't propelled by news. They don't need a news event, like an analyst's upgrade or a change in management, to send the stock higher. These momentum stocks are in an unique asset class, which is drawing a lot of money into the market.

Weird ETFs

iPath Dow Jones-UBS Livestock Subindex Total Return (COW).

Global X Lithium (LIT).

Internet HOLDRS (HHH)

This fund invests in both lithium mining companies and lithium battery manufacturers. This may be one of the best investment plays available on electric cars, and the future potential of a fund like this is obvious.

iPath Global Carbon (GRN).

This fund is in a class by itself as far as strange goes. It isn't even a play on any type of investment, but on European Union Allowances and Certified Emission Reduction Credits. This is purely a political play, and it's subject to all the complications, re-interpretations and delays that affect just about any idea that's hatched by political bodies.

New Bitcoin ETF Effort Launched by Money Management Firm

Aug 14, 2017 at 16:45 UTC by Wolfie Zhao

The "Market"

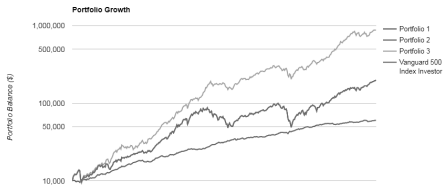
- Before we go any further lets define what we mean by the "Market" generally referred to as the S&P500.
- Two vehicles that represent this are the ETF called SPY and the mutual fund VFINX.

Great Free Resource

Some Results

Portfolio Returns

Portfolio	Initial Balance	Final Balance	CAGR	Stdev	Best Year	Worst Year	Max. Drawdown	Sharpe Ratio
Portfolio 1	$10,000	$199,707	10.23%	14.83%	37.45%	-37.02%	-50.97%	0.52
Portfolio 2	$10,000	$60,032	6.00%	3.84%	18.18%	-2.66%	-5.86%	0.74
Portfolio 3	$10,000	$884,135	15.69%	14.11%	60.56%	-18.45%	-33.17%	0.89
Vanguard 500 Index Investor	$10,000	$199,707	10.23%	14.83%	37.45%	-37.02%	-50.97%	0.52



Another Great Read

Introduction

- Harry Markowitz's "Portfolio Selection" *Journal of Finance* article (1952) set the stage for modern portfolio theory
  - ❑ The first major publication indicating the importance of security return correlation in the construction of stock portfolios
  - ❑ Markowitz showed that for a given level of expected return and for a given security universe, knowledge of the **covariance and correlation** matrices are required

Modern Portfolio Theory - MPT

- Prior to the establishment of Modern Portfolio Theory (MPT), most people only focused upon investment returns…they ignored risk.
- With MPT, investors had a tool that they could use to *dramatically reduce the risk* of the portfolio *without a significant reduction* in the expected return of the portfolio.

Diversification

Diversification has two faces:

1. Diversification results in an overall reduction in portfolio risk (return volatility over time) with little sacrifice in returns, and
2. Diversification helps to immunize the portfolio from potentially catastrophic events such as the outright failure of one of the constituent investments.

   (e.g. If only one investment is held, and the issuing firm goes bankrupt, the entire portfolio value and returns are lost. If a portfolio is made up of many different investments, the outright failure of one is more than likely to be offset by gains on others, helping to make the portfolio immune to such events.)

Diversification Reduces Risk

- Assume a hypothetical investor had **$100,000** to invest on January 1, 2000, held his investments through December 31, 2009 and reinvested all distributions. We will also assume the use of Vanguard index mutual funds for our examples.
- Investor 1 puts all of her money in the Vanguard 500 Fund which invests in the stocks making up the Standard & Poor's index in their relative weight in the index.
- How much would it be worth by December 31, 2009? This $100,000 investment would have shrunk to **$90,165** for an average annual loss of 1.03 percent. This was truly a lost decade for this investor.

Diversification Reduces Risk

- Investor 2 added the following funds to his portfolio in addition to the Vanguard 500:
- Vanguard Small Cap Index
- Vanguard Mid Cap Index
- Vanguard Total International Stock Index
- How much would an investment of **$100,000** invested equally in each of these four funds have grown to by December 31, 2009? (We are assuming no taxes or rebalancing in this and all examples. The answer is **$137,511**. This is $47,346 or about 52 percent more than an investment of our investor's cash only in the Vanguard 500 Index.

Diversification Reduces Risk

- Investor 3 added some bonds to her mix. In this case let's add the following funds:
- Pimco Total Return
- T. Rowe Price Short-Term Bond
- American Century Inflation Adjusted Bond
- Templeton Global Bond
- If we now divide the investor's **$100,000** investment equally among the four equity funds from the prior example and among these four bond funds, by the end of 2009, $100,000 investment has grown to **$174,506** or almost double what an investment of $100,000 in the Vanguard 500 Index Fund alone would have yielded.

Diversification is not a new idea

Shakespeare "Merchant of Venice"

My ventures are not in one bottom trusted,
Nor to one place; nor is my whole estate
Upon the fortune of this present year;
Therefore, my merchandise makes me not sad.
Act I, Scene 1

Captain Long John Silver in Treasure Island

I puts it all away, some here, some there,
none too much anywheres, by reason of suspicion.

Chapter 11. Robert Louis Stevenson

Risk Aversion

Portfolio theory assumes that investors are averse to risk

- Given a choice between two assets with equal expected rates of return, risk averse investors will select the asset with the lower level of risk
- It also means that a riskier investment has to offer a higher expected return or else nobody will buy it

Markowitz Portfolio Theory

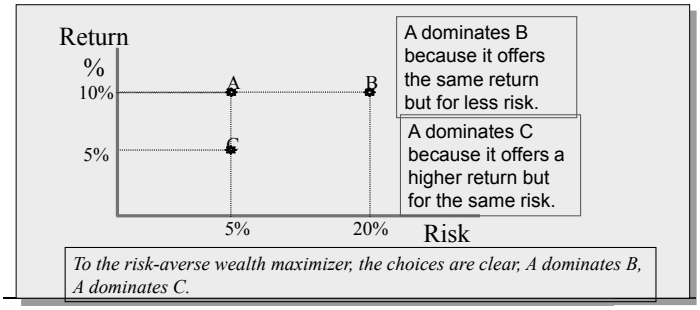
- Derives the expected rate of return for a portfolio of assets and an expected risk measure
- Markowitz demonstrated that the variance of the rate of return is a meaningful measure of portfolio risk under reasonable assumptions
- The portfolio variance formula shows how to effectively diversify a portfolio

Markowitz Portfolio Theory

Assumptions

- Investors base decisions solely on expected return and risk.
- For a given risk level, investors prefer higher returns to lower returns.
- Similarly, for a given level of expected returns, investors prefer less risk to more risk.

Investment Choices



Combinations of Random Variables

- If X and Y are independent

$$E(X+Y) = E(X) + E(Y)$$
$$Var(X+Y) = Var(X) + Var(Y)$$

- If X and Y are not independent

$$E(X+Y) = E(X) + E(Y)$$
$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$$

- The most general case

$$E((a+bX)+(c+dY)) = a + bE(X) + c + dE(Y)$$
$$Var((a+bX)+(c+dY)) = b^2Var(X) + d^2Var(Y) + 2bdCov(X,Y)$$

Std Dev as a measure of risk

The standard deviation is often used by investors to measure the risk of a stock or a stock portfolio. The basic idea is that the standard deviation is a measure of volatility: the more a stock's returns vary from the stock's average return, the more volatile the stock.

Consider the following two stocks and their respective returns (in per cent) over the last six months.

	Stock A			Stock B		
Month	Value	Return (%)	Final value	Value	Return (%)	Final value
June	$1,000.00	0.75	$1,007.50	$1,000.00	1.50	$1,015.00
July	$1,007.50	1.00	$1,017.58	$1,015.00	5.00	$1,065.75
Aug	$1,017.58	3.00	$1,048.10	$1,065.75	12.00	$1,193.64
Sept	$1,048.10	-1.50	$1,032.38	$1,193.64	-9.00	$1,086.21
Oct	$1,032.38	0.50	$1,037.54	$1,086.21	-4.00	$1,042.76
Nov	$1,037.54	2.00	$1,058.29	$1,042.76	1.50	$1,058.41

Which stock would you say is more volatile ?

Descriptive Statistics: Returns1, Returns2

Variable	N	Mean	Median	TrMean	StDev
Returns1	6	0.958	0.875	0.958	1.520
Returns2	6	1.17	1.50	1.17	7.24

Both portfolios end up increasing in value from $1,000 to $1,058. However, they clearly differ in volatility.

Portfolio A's monthly returns range from -1.5% to 3% whereas Portfolio B's range from -9% to 12%.

This volatility is represented by the large differences in standard deviations; the standard deviation of the returns for Portfolio A is 1.52; for Portfolio B it is 7.24.

Forming portfolios

Suppose you have $100 to invest.

Let R_A be the return on asset A.
If R_A = .1, and you put all your money into asset A you will have $110 at the end of the period.

Let R_B be the return on asset B.
If R_B = .15, and you put all your money into asset B you will have $115 at the end of the period.

Suppose you put 1/2 your money into A and 1/2 into B.

How much will you make ?

Portfolio weights

At the end of the period you will have

(100)*.5*(1+.1) + (100)*.5*(1+.15)
=100*(1+.5*.1+.5*.15)
so the return is .5*.1 + .5*.15=.125. The average of the two returns

To generalize, let w_A be the fraction of your wealth you invest in asset A. Let w_B be the fraction of your wealth you invest in asset B.

The w's are called the portfolio weights, and we usually require that they sum to 1.

Return on a portfolio

Hence the return on a portfolio is given by

$$R_p = w_A R_A + w_B R_B$$

(a weighted sum of the two different returns)

People like to study the mean and variance of portfolio returns, and in the next few slides we give you the formulas to do so.

Example

- Let's use country data and suppose that we had put .5 of our money into USA and .5 into Hong Kong.
- What would our returns have been ?

```
. generate portf = .5*honkong + .5*usa
. list honkong usa portf
```

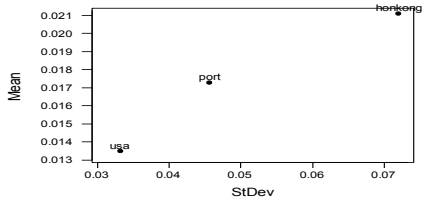
	honkong	usa	portf
1.	.02	.04	.03
2.	.06	-.03	.015
3.	.02	.01	.015
4.	-.03	.01	-.01
5.	.08	.05	.065
6.	.02	0	.01
7.	-.08	-.03	-.055
8.	.01	.04	.025

Comparing returns

How do the returns on this portfolio compare with those of hongkong and usa?

It looks like the mean for my portfolio is right in between the means of usa and hongkong.

What about the sd?

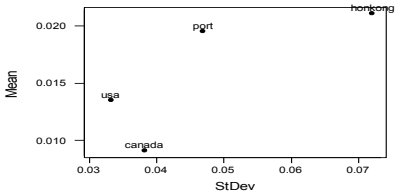


Now try three stocks

Let's try a portfolio with three stocks. The weights must add up to one, but they can be negative, this is called going short in the asset.

Clearly, forming portfolios is an interesting thing to do!!

```
port = -.5*canada+usa+.5*honkong
```



The Set up

- Stock 1 has random return R_1 with mean μ_1 and standard deviation σ_1
- Stock 2 has random return R_2 with mean μ_2 and standard deviation σ_2
- The covariance between the two returns is σ_{12}

The Portfolio

- The weight w_1 is what is put in stock 1
- The weight w_2 is what is put in stock 2
- The portfolio P is given by

$$P = w_1 R_1 + w_2 R_2$$

- The portfolio is random so it has a mean and a variance.

Portfolio Expected Value

- We want to find E(P)
- By rules for combining random variables

$$E(P) = w_1 E(R_1) + w_2 E(R_2)$$
$$= w_1 \mu_1 + w_2 \mu_2$$

Portfolio Variance

- We want to find Var(P)
- By rules for combining random variables

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{12}$$

Total Risk | Risk from A | Risk from B | Interactive Risk

- Using correlation

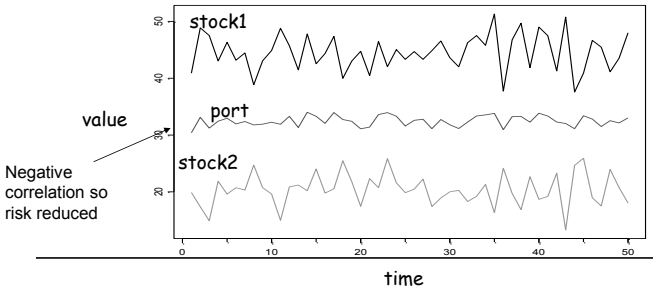
$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho_{12} \sigma_1 \sigma_2$$

Variance of A Linear Combination (cont)

- If two securities have low correlation, the interactive risk will be small
- If two securities are uncorrelated, the interactive risk drops out
- If two securities are negatively correlated, interactive risk would be negative and would reduce total risk

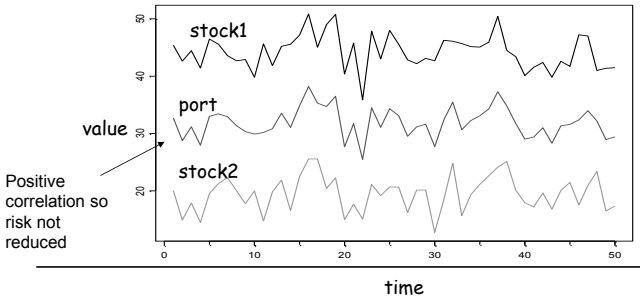
Risk is reduced when stocks are negatively correlated

Example: Cor(stock1,stock2)=-.82



Risk is reduced when stocks are negatively correlated

Example: Cor(stock1,stock2)= .57



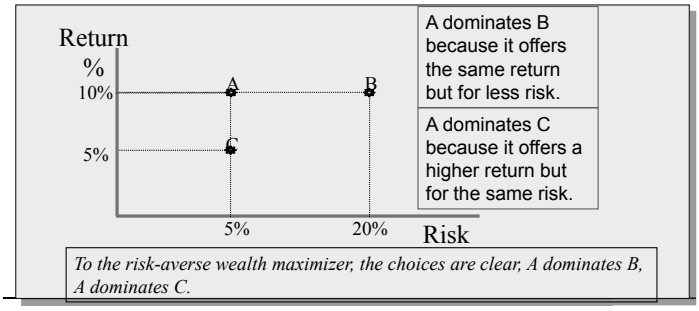
Portfolio Construction in A Nutshell

- Various portfolio combinations may result in a given return
- The investor wants to **choose the portfolio** combination that provides the least amount of variance (for a given level of return).

Portfolio Dominance

- It is assumed that investors are rational, wealth-maximizing and risk averse.
- If so, then some investment choices dominate others.

Investment Choices

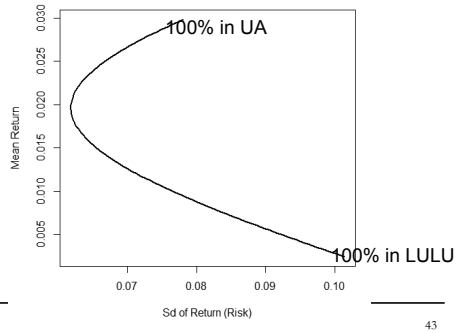


Example

- Suppose LULU returns have monthly mean return 0.24% and sd 10.14%
- Suppose UA returns have monthly mean returns 0.29% and sd 7.8%
- Suppose the correlation between LULU and UA is -0.0021

Resulting Risk/Return Graph

Using the values for the means, variances and covariance we can create the following risk/return graph. We cycle through weights from 0 to 100% in UA



Using the portfolio formulas

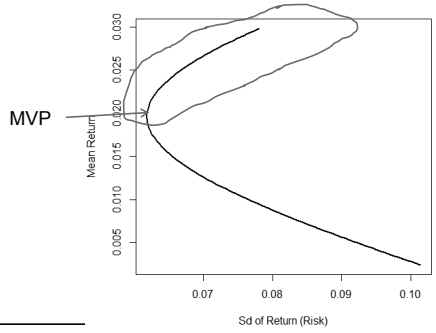
Average portfolio return

$$E(R_P) = 0.5\mu_{UA} + 0.R\mu_{LULU} = 0.5(.0029) + (0.5)(0.029) = 0.016$$

$$\sigma^2_{R_P} = (0.5)^2\sigma^2_{R_{UA}} + (0.5)^2\sigma^2_{R_{LULU}} + 2(0.5)(0.5)\sigma_{R_{UA},R_{LULU}} = (0.5)^2(.1014)^2 + (0.5)^2(.0078)^2 + 2(0.5)(0.5)(.1014)(.078)(-0.0027) = 0.004096$$

$$\sigma_{R_P} = \sqrt{s^2_{R_P}} = \sqrt{0.004096} = 0.064$$ ← portfolio standard deviation

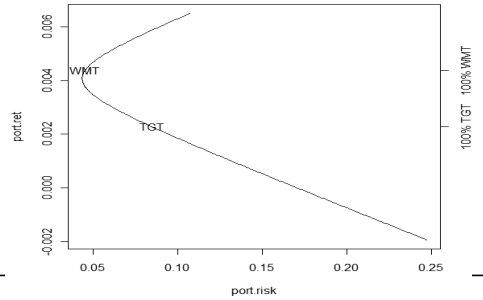
The Efficient Frontier



In R (for advanced students)

```
myport = function(ticker1,ticker2) {

s1 = getSymbols(ticker1,auto.assign=FALSE)
s2 = getSymbols(ticker2,auto.assign=FALSE)

r1 = monthlyReturn(Ad(s1))
r2 = monthlyReturn(Ad(s2))

w=seq(-1.5,1.5,.01)
port.ret = w*mean(r1)+(1-w)*mean(r2)

port.risk=sqrt(w^2*var(r1)+(1-w)^2*var(r2)+2*w*(1-w)*cov(r1,r2))
plot(port.risk,port.ret,type="l")

text(sd(r1),mean(r1),ticker1)
text(sd(r2),mean(r2),ticker2)
axis(side=4,at=c(mean(r1),mean(r2)),labels=c(paste("100%",ticker1),paste("100%",ticker2)))

}
```


More Examples



Example: The Caffeine Portfolio

- Consider Coke(KO) and Starbucks (SBUX)

```
> getSymbols("SBUX")
[1] "SBUX"
> getSymbols("KO")
[1] "KO"
> r1=monthlyReturn(SBUX)
> r2=monthlyReturn(KO)
> cov(r1,r2)
                  monthly.returns
monthly.returns      0.001526567
> cor(r1,r2)
                  monthly.returns
monthly.returns       0.2649362
> mean(r1)
[1] 0.004292857
> mean(r2)
[1] 0.006915436
> var(r1)
                  monthly.returns
monthly.returns      0.01141654
> var(r2)
                  monthly.returns
monthly.returns     0.002908136
>
```

The 50/50 Portfolio by hand (argh!)

Average portfolio return

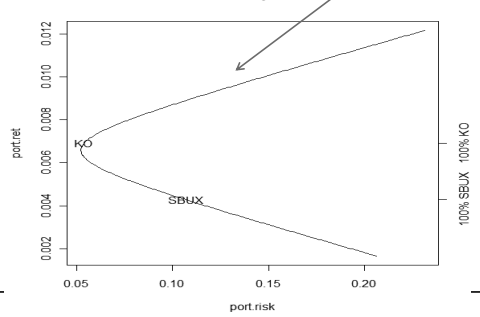
$$\bar{R}_P = 0.5\bar{R}_{SBUX} + 0.5\bar{R}_{KO} = 0.5(.0043) + (0.5)(0.0069) = 0.056$$

$$s^2_{R_P} = (0.5)^2 s^2_{R_{SBUX}} + (0.5)^2 s^2_{R_{KO}} + 2(0.5)(0.5)s_{R_{SBUX},R_{KO}} = (0.5)^2(.01141)^2 + (0.5)^2(.0029)^2 + 2(0.5)(0.5)(.0015) = 0.0043$$

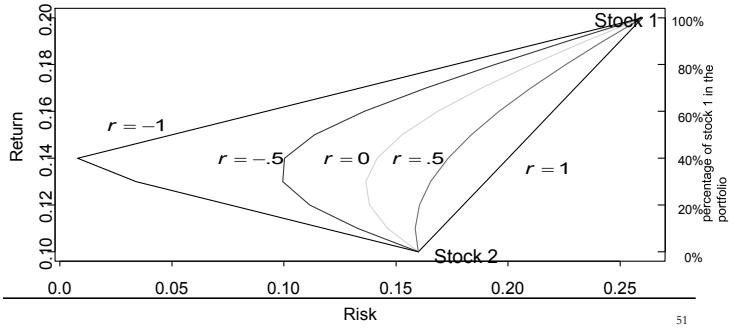
$$s_{R_P} = \sqrt{s^2_{R_P}} = \sqrt{0.0043} = 0.0655$$ ← portfolio standard deviation

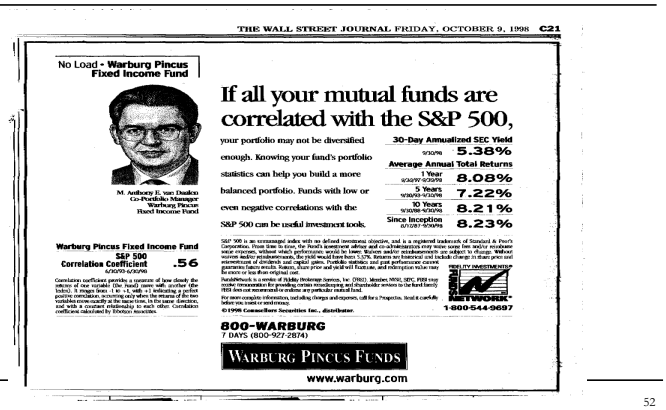
The Graph

Some leveraged coke portfolios (weights more than 1)



Correlation drives portfolio risk





THE WALL STREET JOURNAL FRIDAY, OCTOBER 9, 1998 C21

No Load • Warburg Pincus Fixed Income Fund

If all your mutual funds are correlated with the S&P 500,

your portfolio may not be diversified enough. Knowing your fund's portfolio statistics can help you build a more balanced portfolio. Funds with low or even negative correlations with the S&P 500 can be useful investment tools.

M. Anthony E. van Daalen Co-Portfolio Manager Warburg Pincus Fixed Income Fund

30-Day Annualized SEC Yield
9/30/98 5.38%

Average Annual Total Returns
1 Year 8.08%
5 Years 7.22%
10 Years 8.21%
Since Inception 8.23%

Warburg Pincus Fixed Income Fund
S&P 500 Correlation Coefficient .56
6/30/93-6/30/98

800-WARBURG
7 DAYS (800-927-2874)

WARBURG PINCUS FUNDS

www.warburg.com

Zero Risk Portfolio

- We can calculate the portfolio that removes all risk.
- When ρ = -1, then

$$\sigma_p = \sqrt{(w_A)^2(\sigma_A)^2 + (w_B)^2(\sigma_B)^2 + 2(w_A)(w_B)(\rho_{A,B})(\sigma_A)(\sigma_B)}$$

- Becomes:

$$\sigma_p = w\sigma_A - (1-w)\sigma_B$$

- Solve this equation for 0.

The Zero Risk Portfolio

- As you can see from the previous slide, if you can find two stocks that have a correlation of -1, you can build a portfolio with 0 risk!
- Mathematically it can be shown that this will happen with

$$w_1 = \frac{\sigma_B}{\sigma_A + \sigma_B}$$

- However, will the portfolio have positive return? Unfortunately, usually no!

Example

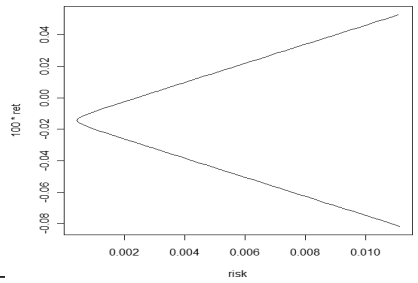
- Consider
  - RYURX (rydex ursa mutual fund)
  - VFINX (Vangaurds S&P 500 mutual fund)

```
cor(vfinx,ryurx)
                RYURX.Adjusted
VFINX.Adjusted     -0.9968647
```


No Free Lunch

- Darn!

As is typical, the zero risk portfolio has a negative expected return!



Things you should know

- Conditional Distributions
- Conditional Expectation
- Covariance and Correlation
- Expectation and variance of a sum