Homework 3
STAT 104 - Introduction to Quantitative Methods for Economics

1) A journal article reported in September 2003 that a Swiss dermatologist established a link between smoking and gray hair in women under 40, with results as shown in this table. Let G denote the event a women in the study has gray hair, and S denote the event a women in the study is a smoker.

	Gray	Not Gray	Total
Smoker	13	10	23
Nonsmoker	0	32	32
Total	13	42	55

a) Was this an observational study or an experiment?

→ This is an **observational study** since subjects were observed and variables were measured without assigning treatments to the subjects

b) FindP(G)

→ P(G) = 13/55 = **0.2363**

c) Find P(S or G)

→ P(S or G) = P(S) + P(G) - P(S and G) = 23/55 + 13/55 - 13/55 = **0.418**

d) Find P(S|G)

→ P(S|G) = P(S and G)/P(G) = (13/55)/(13/55) = **1**

e) Are having Gray hair and being a smoker independent or dependent events? Explain

→ If they are independent,

P(G and S) = P(G)*P(S)

P(G and S) = 13/55

P(G) * P(S) = 13/55 * 23/55

The two are not equal so the two events are **dependent**.

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2) Suppose P(A) = 0.76, P(B | A) = 0.30and P(B | A') = 0.02 . Find P(A') , P(A and B) , and P(A' and B) . Use these to construct a probability table. Now use the table to find the following:

P(A') = 1-P(A) = 1-0.76 = 0.24

P(B|A) = P(A and B)/P(A)
→ P(A and B) = P(B|A) * P(A) = 0.30 * 0.76 = 0.228

P(B|A') = P(A' and B)/P(A')
→ P(A' and B) = P(B|A') * P(A') = 0.02 * 0.24 = 0.0048

	B	B'	TOTAL
A	0.228	0.532	0.76
A'	0.0048	0.2352	0.24
TOTAL	0.2328	0.7672	1

a) P(B'|A)
→ P(B' and A)/P(A) = 0.532/0.76 = 0.7

b) P(B'|A')
→ P(B' and A')/P(A') = 0.2352/0.24 = 0.98

c) P(B)
→ 0.2328

d) P(B')
→ 0.7672

e) P(A|B)
→ P(A and B)/P(B) = 0.228/0.2328 = 0.9794

f) P(A'|B)
→ P(A' and B)/P(B) = 0.0048/0.2328 = 0.0206

g) P(A|B')
→ P(A and B')/P(B') = 0.532/0.7672 = 0.6934

h) P(A'|B')
→ P(A' and B')/P(B') = 0.2352/0.7672 = 0.3066

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3) Suppose the probability of having schizophrenia P(s) = 0.01 in the population, and the conditional probability of "hearing voices" given schizophrenia P(hv | s) = 0.66, and the probability of "hearing voices" P(hv) = 0.75. Find the probability of having schizophrenia given "not hearing voices":

To find: P(s | hv') = P(s and hv')/P(hv')

Given: P(s) = 0.01
P(hv) = 0.75
P(hv | s) = 0.66

	s	s'	TOTAL
hv	0.0066	0.7434	0.75
hv'	0.0034	0.2466	0.25
TOTAL	0.01	0.99	1

P(s | hv') = P(s and hv')/P(hv') = 0.0034/0.25 = **0.0136**

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4) In a class on probability, a statistics professor flips two balanced coins. Both fall to the floor and roll under his desk.

a) A student in the first row informs the professor that he can see both coins. He reports that at least one of them shows tails. What is the probability that the other coin is also tails?
→ The student informs the professor that at least one too them shows tails.
Sample space: {TH, TT, HT}
P(second coin T) = 2/3 = **0.66**

b) Suppose the student informs the professor that he can see only one coin and it shows tails. What is the probability that the other coin is also tails?
→ These are now two independent events. The outcome of the first coin has no impact on the second coin. The probability that the other coin is (also) tails is **0.5**

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5) There are two boxes, Box B1 and Box B2. BoxB1 contains 2 red balls and 8 blue balls. Box B2 contains 7 red balls and 3 blue balls. Suppose Jane first randomly chooses one of two boxes B1 and B2, with equal probability, 1/2, of choosing each. Suppose Jane then randomly picks one ball out of the box she has chosen (without telling you which box she had chosen), and shows you the ball she picked. Suppose you only see that the ball Jane picked is red. Given this information, what is the probability that Jane chose box B1?

→ Box B1: 2R and 8B (Total = 10)
Box B2: 7R and 3B (Total = 10)

	R	B	TOTAL
B1	0.1	0.4	0.5
B2	0.35	0.15	0.5
TOTAL	0.45	0.55	1

To find: P(B1 | R) = P(B1 and R)/P(R) = 0.1/0.45 = **0.2222**

6) Suppose it has been observed empirically that the word “Congratulations” occurs in 1 out of 10 spam emails (that is, P(congratulations|spam) = 0.1), but that “Congratulations” only occurs in 1 out of 1000 non-spam emails. Suppose it has also been observed empirically that about 4 out of 10 emails are spam. In Bayesian Spam Filtering, these empirical probabilities are interpreted as genuine probabilities in order to help estimate the probability that an incoming email is spam. Suppose we get a new email that contains “Congratulations”. Let C be the event that a new email contains “Congratulations”. Let S be the event that a new email is spam. We have observed C. Calculate P(S | C)

→ Given:

P(C | S) = 0.1
P(C | S’) = 0.001
P(S) = 0.4

To find:

P(S|C) = ?

	S	S’	TOTAL
C	0.04	0.0006	0.0406
C’	0.36	0.5994	0.9594
TOTAL	0.4	0.6	1

P(S|C) = P(S and C)/P(C) = 0.04/0.0406 = **0.9852**

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7) McDonald's is planning on opening a new location. They must decide how big of a restaurant to build at the location: small, medium, or large. Demand for the McDonald's in this location is uncertain, and will affect profitability. They have projected profitability for weak, moderate, and strong demand as shown in the following table:

	Demand		
Size	Weak	Moderate	Strong
Small	400	500	660
Medium	-250	650	800
Large	-400	580	990
Best	400	650	990

a) What is the maximax decision ?

→ Maximum payoff for each option:

Small: 660

Medium: 800

Large: 990

Choose the option with the greater maximum payoff: **Large restaurant**

b) What is the maximin decision?

→ Minimum payoff for each option:

Small: 400

Medium: -250

Large: -400

Choose the option with the greatest minimum payoff: **Small restaurant**

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8) Video Tech is considering marketing one of two new video games for the coming season: Battle Pacific or Space Pirates. Battle Pacific is a unique game and appears to have no competition. Estimated profits (in thousands of dollars) under high, medium, and low demand are as follows:

	Demand		
Battle Pacific	High	Medium	Low
Profit	\$1000	\$700	\$300
Probability	0.2	0.5	0.3

Video Tech is optimistic about its Space Pirates game. However, the concern is that profitability will be affected by a competitor's introduction of a video game viewed as similar to Space Pirates. Estimated profits (in thousands of dollars) with and without competition are as follows

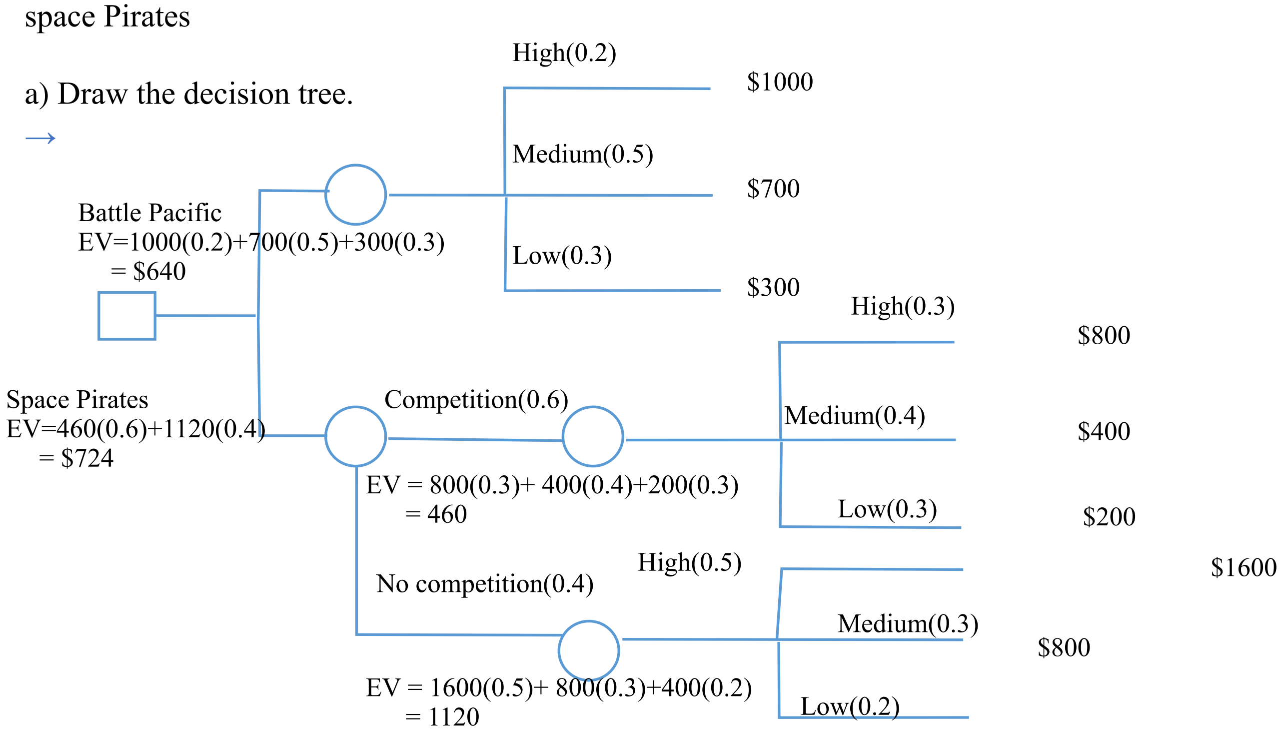
	Demand		
Space Pirates **With Competition**	High	Medium	Low
Profit	\$800	\$400	\$200
Probability	0.3	0.4	0.3

Space Pirates **Without Competition**	High	Medium	Low
Profit	\$1600	\$800	\$400
Probability	0.5	0.3	0.2

Video Tech believes there is a 0.6 probability that its competitor will produce a new game similar to space Pirates

a) Draw the decision tree.

→



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b) What is the optimal decision?

→ The optimal decision, considering the decision tree, is to go ahead with **Space Pirates**.

c) Right now the chance of competition is 60%. How much higher or lower would this probability need to be in order for your decision to change?

→ We want to understand how far we can let the competition increase. Let the maximum chance of competition be 'p'

EV for Battle Pacific = 640
EV for Space Pirates = 460p + 1120(1-p)

For our decision not to change, EV for Space pirates needs to be higher than EV for Battle Pacific
$460p+1120(1-p) > 640$
$460p+1120-1120p > 640$
$-660p > -480$
$p < 0.7273$

For our decision not to change, the competition needs to be lower than 75%. As soon as the competition increases beyond 72.73%, our decision will change.

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9) Let X be a discrete random variable with PMF (probability mass function) given by

$$p_X(x) = \begin{cases} x^2/a, & if\ x = -3, -2, -1, 0, 1, 2, 3 \\ 0, & otherwise \end{cases}$$

a) Find the value of a

→ Sum of probability mass function must be 1

$(-3)^2/a + (-2)^2/a + (-1)^2/a + (0)^2/a + (1)^2/a + (2)^2/a + (3)^2/a = 1$

$9/a + 4/a + 1/a + 1/a + 4/a + 9/a = 1$

$a = 28$

b) Calculate E(X)

→ $-3\ (9/28) + (-2)(4/28) + (-1)(1/28) + 0 + 1(1/28) + 2(4/28) + 3(9/28)$

$= 0$

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10) The night watchman in a factory cannot guard both the safe in back and the cash register in front. The safe contains $6000, while the register has only $1000. Tonight the guard fears a robbery; the probability that the thief will try the cash register is 0.8 and the probability the thief will try the safe is 0.2. If the guard is not present, the thief will take all the money. If the guard is present, the thief will go away empty handed. Where should the guard be positioned in order to minimize the thief's gains?

→ P(register) = 0.8
P(safe) = 0.2

Expected value lost if watchman is at safe:
$0.8 * 1000 + 0 = 800$

Expected value lost if watchman is at register:
$0.2 * 6000 + 0 = 1200$

To minimize the thief's gain, the watchman should be positioned at the safe.

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11) The probability that a cellular phone company kiosk sells X number of new phone contracts per day is shown below.

x	4	5	6	8	10
P(x)	0.3	0.15	0.35	0.15	0.05

a) Find the mean, variance, and standard deviation for this probability distribution.
→ mean = 4(0.3) + 5(0.15) + 6(0.35) + 8(0.15) + 10(0.05) = **5.75**

variance = $(0.3)(4 - 5.75)^2 + (0.15)(5 - 5.75)^2 + (0.35)(6 - 5.75)^2 + (0.15)(8 - 5.75)^2 + (0.05)(10 - 5.75)^2$
= 0.9188 + 0.0844 + 0.0219 + 0.7594 + 0.9031 = **2.6876**

standard deviation = sqrt(variance) = **1.6394**

b) Suppose the kiosk salesperson makes $80/day (8 hours at $10/hour), plus a $25 bonus for each new phone contract sold. What is the mean and variance of the salesperson's daily salary?
→ First, calculate the expected value of the bonus:
(0.3)(25*4) + (0.15)(25*5) + (0.35)(25*6) + (0.15)(25*8) + (0.05)(25*10) = 143.75

By the properties of mean, E[V+c] = c + E[V]
mean = 143.75 + 80 = **223.75**

Same for standard deviation, first calculate of the bonus:
variance = $(0.3)(25*4-223.75)^2 + (0.15)(25*5-223.75)^2 + (0.35)(25*6-223.75)^2 + (0.15)(25*8-223.75)^2 + (0.05)(25*10-223.75)^2$
= (4594.21875) + (1462.734375) + (1903.671875) + (84.609375) + (34.453125)
= 8079.6875

Var(c+X) = Var(X)
variance of salary = 8079.6875

Standard deviation = **89.8871**

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12) In a population of students, the number of calculators owned is a random variable X with $P(X = 0) = 0.2$, $P(X = 1) = 0.6$, $P(X = 2) = 0.2$.

a) Find E(x)

→ $E(x) = 0*0.2 + 1*0.6 + 2*0.2 = 1$

b) Find Var(X)

→ $Var(X) = (0.2)(0-1)^2 + (0.6)(1-1)^2 + (0.2)(2-1)^2$
$= 0.2 + 0 + 0.2$
$= 0.4$

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13) You roll two dice.

a) What is the probability of two sixes? Of exactly one 6? Of no sixes?

→ P{6,6} = 1/6 * 1/6 = **0.0278**

P(exactly one 6) = P(first is six and second is not) + P(first is not six and second is)
= (1/6*5/6) + (5/6*1/6) = **0.2778**

P(no sixes) = P(first is not 6) * P(second is not six)
= 5/6 * 5/6 = **0.6944**

b) What is the expected number of sixes that will show?

→
A : No six will show
B : 1 six will show
C : 2 sixes will show

P(A) = 0.6944
P(B) = 0.2778
P(C) = 0.0278

E[no. of 6] = (0)(0.6944) + (1)(0.2778) + (2)(0.0278) = **0.3334**

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14) We can simulate the expected value result in part (b) above. Follow the following steps in R:

i. Simulate two dice rolls using (use similar code for die2)
die1=sample(1:6,10000,replace=TRUE)

ii. Combine the two dice rolls into a matrix using
dicerolls=cbind(die1,die2)

iii. Each row of dicerolls represents the outcome of rolling two dice. We want to count how many 6's appear each time we roll two dice. We do that as follows.
num6=head(rowSums(dicerolls==6))

iv. Take the mean of the num6 variable and compare it to part (b) above. How does this mean change if we instead use 1000000 rolls?

→ R Studio execution:

```
> # Simulate two dice rolls using (use similar code for die2)
> die1=sample(1:6,10000,replace=TRUE)
> die2=sample(1:6,10000,replace=TRUE)
>
> # Combine the two dice rolls into a matrix using
> dicerolls=cbind(die1,die2)
>
> # Each row of dicerolls represents the outcome of rolling two dice.
> # We want to count how many 6's appear each time we roll two dice.
> # We do that as follows.
> num6=rowSums(dicerolls==6)
>
> # Take the mean of the num6 variable and compare it to part (b) above.
> mean(num6)
[1] 0.3268
```

If we increase samples to 1000000, the simulated value gets closer to the calculate value.

```
> # Simulate two dice rolls using (use similar code for die2)
> die1=sample(1:6,1000000,replace=TRUE)
> die2=sample(1:6,1000000,replace=TRUE)
>
> #Combine the two dice rolls into a matrix using
> dicerolls=cbind(die1,die2)
>
> # Each row of dicerolls represents the outcome of rolling two dice.
```

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```
> # We want to count how many 6's appear each time we roll two dice.
> # We do that as follows.
> num6=rowSums(dicerolls==6)
>
> # Take the mean of the num6 variable and compare it to part (b) above.
> mean(num6)
[1] 0.333284
```

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15) If random variable X has mean μ and variance $\boldsymbol{\sigma}^2$, show (using the a+bX rule) what the mean and variance of $Z = (X - \mu) / \boldsymbol{\sigma}$ are.

→

Mean(Z) = Mean((X - μ)/$\boldsymbol{\sigma}$) = (1/$\boldsymbol{\sigma}$) (mean(X -μ)) = (1/$\boldsymbol{\sigma}$) (mean(X) - μ) = (1/$\boldsymbol{\sigma}$) (μ - μ) = 0

Var(Z) = Var((X - μ)/$\boldsymbol{\sigma}$) = (1/$\boldsymbol{\sigma}^2$) (Var(X - μ)) = (1/$\boldsymbol{\sigma}^2$)Var(X) = (1/$\boldsymbol{\sigma}^2$)($\boldsymbol{\sigma}^2$) = 1

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16) Find the variance of each of the following bets from the class notes. Which bet is riskiest and which best is safest?

a) You get \$5 with probability 1.0.

→ mean = 5
variance = 1(5-5) = 0
This bet is the safest bet

b) You get \$10 with probability 0.5, or \$0 with probability 0.5.

→ mean = 0.5*10 + 0.5*0 = 5
variance = $0.5(10-5)^2 + 0.5(0-5)^2 = (0.5)(25) + (0.5)(25) = 25$

c) You get \$5 with probability 0.5, \$10 with probability 0.25 and \$0 with probability 0.25.

→ mean = 0.5*5 + 0.25*10 + 0.25*0 = 5
variance = $0.5(5-5)^2 + 0.25(10-5)^2 + 0.25(0-5)^2 = (0.5)(0) + (0.25)(25) + (0.25)(25) = 12.5$

d) You get \$5 with probability 0.5, \$105 with probability 0.25 or lose \$95 with probability 0.25.

→ mean = 0.5*5 + 0.25*105 + 0.25*95 = 52.5
variance = $0.5(5-52.5)^2 + 0.25(105-52.5)^2 + 0.25(-95-52.5)^2 = (1128.125)+(689.0625)+(5439.0625) = 7256.25$
This is the riskiest bet

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17) Let X be a random variable with E(X) = 120 and Var(X) = 20. Find the following.

a) $E(X^2)$

→ $Var(X) = E(X^2) - [E(X)]^2$
$E(X^2) = Var(X) - [E(X)]^2$
$= 20 - [120]^2 = 20 - 14400 = -14380$

b) E (3X + 10)

→ E(3X+10) = 3*E(X) + 10 = 3*120 + 10 = 370

c) E (-X)

→ E(-X) = E(-1 * X) = -1*E(X) = -120

d) Standard deviation of –2X ?

→ $Var(-2X) = (-2)^2 * Var(X) = 4 * 20 = 80$
$SD(-2X) = \sqrt{80} = 8.9443$

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