Numerical Scheme for Planetary Motion

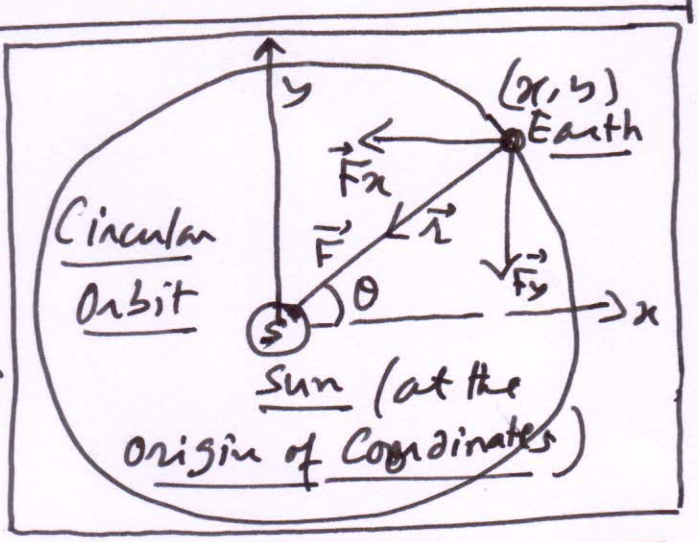
1/ Most planets of the solar system have small eccentricity values (the first place of decimal is zero), except for Mercury and Pluto.

2/. For the Earth $\boxed{\epsilon = 0{\cdot}017}$.

3/. Approximate a circular Earth orbit.

$M_\odot \to$ Solar Mass, $M_E \to$ Earth mass

The Gravitational Force between the Sun and the Earth is



$$\boxed{\vec{F} = -\frac{G M_\odot M_E}{r^2}\,\hat{r}}$$

. Also $\boxed{M_E \frac{d^2\vec{x}}{dt^2} = \vec{F}_x}$

and $\boxed{M_E \frac{d^2\vec{y}}{dt^2} = \vec{F}_y}$. $\boxed{\vec{r} = r\hat{r} = x\hat{x} + y\hat{y}}$

$$\Rightarrow \boxed{\hat{r} = \left(\frac{x}{r}\right)\hat{x} + \left(\frac{y}{r}\right)\hat{y} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}}$$

$\therefore \boxed{|\vec{F}_x| = |\vec{F}|\cos\theta = |\vec{F}|\left(\frac{x}{r}\right)}$ and $\boxed{|\vec{F}_y| = |\vec{F}|\sin\theta = |\vec{F}|\left(\frac{y}{r}\right)}$

(P.T.O.)

The force acting on the Earth is

$$\boxed{\cancel{m_E} \frac{d\vec{v}}{dt} = -\frac{G M_\odot \cancel{m_E}}{r^2}\hat{r}}$$

. In components along x and y ~~components~~ directions,

$$\boxed{\frac{d\vec{v}}{dt} = \left(\frac{dv_x}{dt}\right)\hat{x} + \left(\frac{dv_y}{dt}\right)\hat{y}}.$$

$$\boxed{\vec{F} = \vec{F}_x + \vec{F}_y} \Rightarrow \boxed{-\frac{G M_\odot m_E}{r^2}\hat{r} = -\frac{G M_\odot m_E}{r^2}\times\left(\frac{x}{r}\hat{x} + \frac{y}{r}\hat{y}\right)}$$

Hence, put together components

$$\boxed{\frac{d\vec{v}}{dt} = \left(\frac{dv_x}{dt}\right)\hat{x} + \left(\frac{dv_y}{dt}\right)\hat{y} = -\frac{G M_\odot}{r^2}\left(\frac{x}{r}\hat{x} + \frac{y}{r}\hat{y}\right)}$$

Balancing Components, we write,

$$\boxed{\frac{dv_x}{dt} = -\frac{G M_\odot}{r^3}x} \quad \text{and} \quad \boxed{\frac{dv_y}{dt} = -\frac{G M_\odot}{r^3}y}$$

Also $\boxed{\frac{dx}{dt} = v_x}$, $\boxed{\frac{dy}{dt} = v_y}$ and $\boxed{r = (x^2+y^2)^{1/2}}$

Numerical Scheme: $\boxed{x_{i+1} = x_i + v_{x,i}(\Delta t)}$

$\boxed{y_{i+1} = y_i + v_{y,i}(\Delta t)}$, $\boxed{r_i = \sqrt{x_i^2 + y_i^2}}$,

$\boxed{v_{x,i+1} = v_{x,i} - \frac{G M_\odot x_i}{(x_i^2+y_i^2)^{3/2}}(\Delta t)}$, $\boxed{v_{y,i+1} = v_{y,i} - \frac{G M_\odot y_i}{(x_i^2+y_i^2)^{3/2}}(\Delta t)}$.

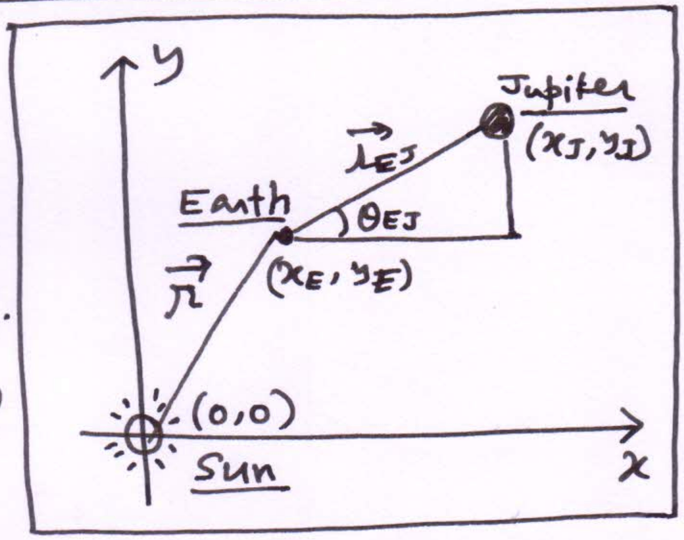
$\Delta t \rightarrow$ Time Step of numerical integration.

Three-Body Problem: Sun-Earth-Jupiter

1/. Apart from the Sun, the planets themselves exert gravity on one another.

2/. Jupiter has the most prominent gravitational effect among planets.

3/. Consider the effect of Jupiter only in a three-body system of the Sun, the Earth and Jupiter.



The Sun is at the Origin of coordinates, in the Solar System, (0,0). The Earth experiences the gravity of both the Sun and Jupiter. The magnitude of the force between the Earth and Jupiter is

$$\boxed{F_{EJ} = -\frac{G m_J m_E}{r_{EJ}^2}}$$

$m_J \rightarrow$ Jupiter mass

$m_E \rightarrow$ Earth mass

$r_{EJ} \rightarrow$ Separation distance between ~~between~~ Jupiter & Earth

(P.T.O)

With respect to Jupiter the gravity felt by the Earth, along the x-component is

$$\boxed{\vec{F}_{EJ,x} = -\frac{G m_E m_J}{r_{EJ}^2}\cos\theta_{EJ}\,\hat{x}} \quad \boxed{\cos\theta_{EJ} = \frac{x_E - x_J}{r_{EJ}}}.$$

$$\Rightarrow \boxed{\vec{F}_{EJ,x} = -\frac{G m_E m_J}{r_{EJ}^3}(x_E - x_J)\,\hat{x}}.$$

Similarly the y-component is

$$\boxed{\vec{F}_{EJ,y} = -\frac{G m_E m_J}{r_{EJ}^2}\sin\theta_{EJ}\,\hat{y}} \quad \boxed{\sin\theta_{EJ} = \frac{y_E - y_J}{r_{EJ}}}.$$

$$\Rightarrow \boxed{\vec{F}_{EJ,y} = -\frac{G m_E m_J}{r_{EJ}^3}(y_E - y_J)\,\hat{y}}$$

These are in addition to the Sun's gravity.

The x-component of the Earth's motion under the total gravity of the Sun and Jupiter is

$$\boxed{\frac{dv_{x,E}}{dt} = -\frac{G M_\odot x_E}{r^3} - \frac{G m_J (x_E - x_J)}{r_{EJ}^3}}$$

Likewise, the y-component is, $\boxed{r^2 = x_E^2 + y_E^2}$

$$\boxed{\frac{dv_{y,E}}{dt} = -\frac{G M_\odot y_E}{r^3} - \frac{G m_J (y_E - y_J)}{r_{EJ}^3}}$$

Note: $\boxed{x_E < x_J}$, $\boxed{y_E < y_J}$, (Jupiter's orbit is larger).

$\therefore$ Jupiter contributes positive terms to the components