Maxwell's Equations of Electrostatics and Magnetostatics

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0} \leftarrow \text{Gauss's Law} \qquad \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\boxed{\vec{\nabla}\times\vec{E} = \vec{0}} \qquad \text{Ampere's Law} \rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}}$$

1/. $\rho \rightarrow$ Charge density, $\vec{J} \rightarrow$ Cross-Sectional Current density

$\rho \rightarrow \dfrac{\text{Charge}}{\text{Volume}} \rightarrow \dfrac{\text{coulomb}}{m^3}$ (S.I. unit)

$\vec{J} \rightarrow \dfrac{\text{Current}}{\text{Area}} = \dfrac{\text{ampere}}{m^2}$ (S.I. unit)

2/. Physical "sources" are electric in nature $\rightarrow \boxed{\rho, \vec{J}}$. There is no physical element of magnetism.

3/. All magnetic effects arise due to electric phenomena. $\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$ (Ampere)

4/. Magnetic fields have neither a source nor a sink. $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$.
There are No magnetic monopoles.

5/. Now $\boxed{\vec{\mathcal{E}} = \mathcal{E}(r)\,\hat{r}}$ → A central field.

$\boxed{\mathcal{E}(r) \propto \frac{1}{r^2}}$ ⇒ $\boxed{\vec{\mathcal{E}} \propto \frac{1}{r^2}\,\hat{r}}$ → Coulomb Inverse Square field.

Since $\vec{\mathcal{E}}$ is a central vector field,

⇒ $\boxed{\vec{\nabla}\times\vec{\mathcal{E}} = \vec{0}}$ for the electrostatic field.

6/. $\boxed{\vec{\nabla}\cdot\vec{\mathcal{E}} = \frac{\rho}{\epsilon_0}}$ is the Consequence of Coulomb's law force

of a $\boxed{\frac{1}{r^2}\,\hat{r}}$ ~~[illegible]~~ radial field.

Integrate (volume integral) $\boxed{\int_V (\vec{\nabla}\cdot\vec{\mathcal{E}})\,d\tau = \frac{1}{\epsilon_0}\int_V \rho\, d\tau}$

But $\boxed{\int_V (\vec{\nabla}\cdot\vec{\mathcal{E}})\,d\tau = \oint_S \vec{\mathcal{E}}\cdot d\vec{a}}$ → Gauss Divergence Theorem

and $\boxed{\int_V \rho\, d\tau = q_{enc}}$ → Total charge enclosed in the volume

Hence, $\boxed{\oint_S \vec{\mathcal{E}}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}}$ → Gauss's law of electrostatics in the integral form

$\boxed{\oint_S \vec{\mathcal{E}}\cdot d\vec{a} \longrightarrow \text{Flux}}$ (P.T.O.)

$$\boxed{\vec{\nabla}\cdot\vec{\mathcal{E}} = \frac{\rho}{\epsilon_0}}$$ → Gauss's law of electrostatics in the differential form.

And, Electric Flux → $\boxed{\oint_s \vec{\mathcal{E}}\cdot d\vec{a}}$ through a closed surface is (charge enclosed)/ϵ_0.

7/. $\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$ → Ampere's law

Integrate (surface integral) $\boxed{\int_s (\vec{\nabla}\times\vec{B})\cdot d\vec{a} = \mu_0\int_s \vec{J}\cdot d\vec{a}}$

But $\boxed{\int_s (\vec{\nabla}\times\vec{B})\cdot d\vec{a} = \oint_p \vec{B}\cdot d\vec{l}}$ → The Stokes Curl Theorem

and $\boxed{\int_s \vec{J}\cdot d\vec{a} = I_{enc}}$ → Total current enclosed through a full cross-sectional area.

Hence, $\boxed{\oint_p \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}}$ → Ampere's law in the integral form.

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$$ → Ampere's law in the differential form.

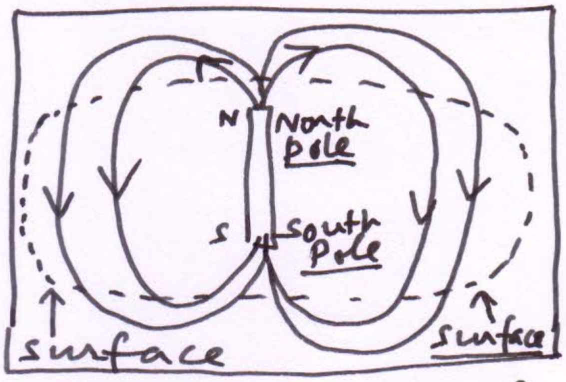
8/. $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$ ⇐ $\boxed{\int_V (\vec{\nabla}\cdot\vec{B})\,d\tau = \oint_s B\cdot d\vec{a} = 0}$

By the Gauss Divergence Theorem ⇒

(P.T.O.)

$\therefore$ Magnetic Flux vanishes through a closed surface.

$$\boxed{\oint_s \vec{B} \cdot d\vec{a} = 0}$$



Field lines close upon themselves.

Total field lines entering a surface equals total field lines exiting it.

Hence, net magnetic flux is zero.

i) Magnetic field lines have no starting point or finishing point. They always close upon themselves. They have no source point or a sink (point). Zero Divergence

ii) Electric field lines start at a positive charge, and end on a negative charge. They have sources or sinks. Hence they have non-zero divergence.

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

But $\boxed{\vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = 0}$

$\Rightarrow \boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$

$\vec{A} \rightarrow$ Magnetic vector Potential.

$$\boxed{\vec{\nabla} \times \vec{E} = 0}$$

But $\boxed{\vec{\nabla} \times (\vec{\nabla} V) = \vec{0}}$

$\Rightarrow \boxed{\vec{E} = -\vec{\nabla} V}$

$V \rightarrow$ Electrostatic Scalar Potential

Maxwell's Equations of Electrodynamics

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0} \qquad \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}} \qquad \boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$

1/. Only the "curl" ($\times$) equations are modified. Their right-hand sides now have time-varying $\vec{E}$ and $\vec{B}$ fields. No longer "static" ("Electro-dynamics")

2/. $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$ → Faraday's law. A changing magnetic field induces an electric field.

3/. In $\boxed{\vec{\nabla}\times\vec{B} = \cdots}$, Maxwell introduced a time-varying correction, $\boxed{\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$ to the static [redacted] Ampere law. $\boxed{\vec{J}_D = \epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}}$, is known as the displacement current

4/. In free-space, $\boxed{\rho = 0,\ \vec{J} = 0}$, no charge and no current. There, just a time-varying electric field $\frac{\partial\vec{E}}{\partial t}$, induces a magnetic field.