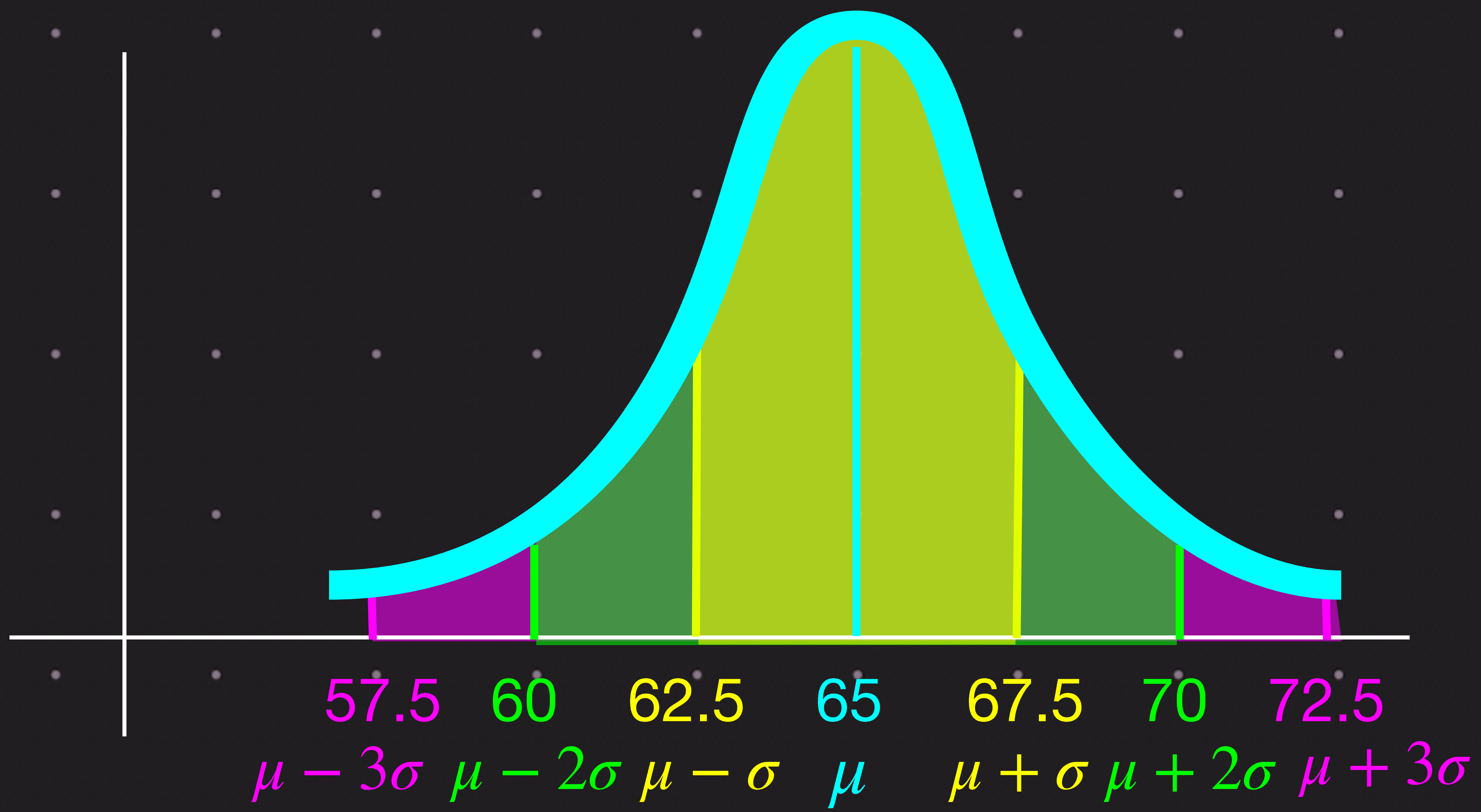
The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



$\mu = 65$

$\sigma = 2.5$

Fraction of people whose height is between 62.5 and 67.5 is 68%

$$P[62.5 < X < 67.5] = 0.68$$

$$P[\mu - \sigma < X < \mu + \sigma] = 0.68$$

Fraction of people whose height is between 60 and 70 is 95%

$$P[60 < X < 70] = 0.95$$

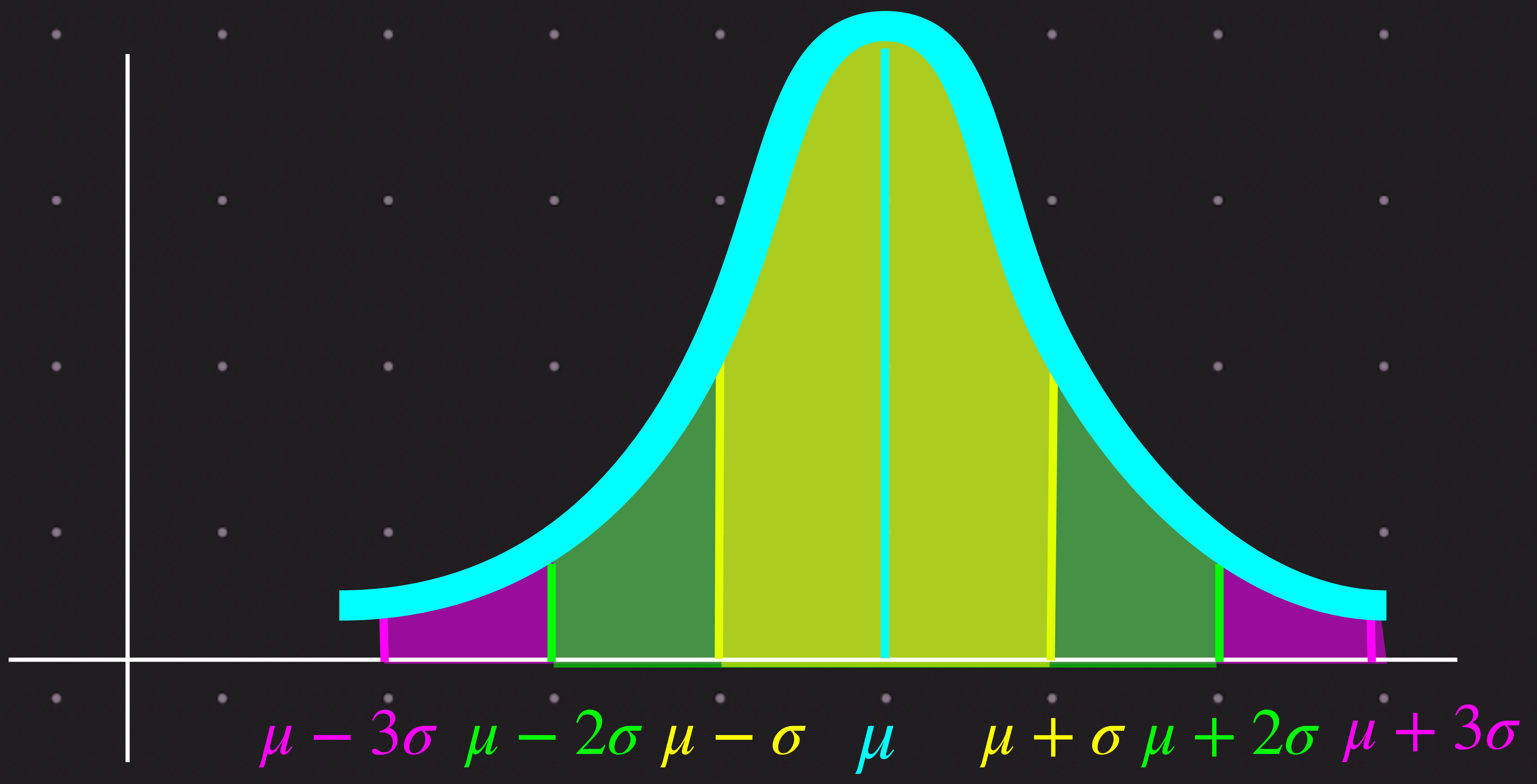
$$P[\mu - 2\sigma < X < \mu + 2\sigma] = 0.95$$

Fraction of people whose height is between 57.5 and 72.5 is 99.7%

$$P[57.5 < X < 72.5] = 0.997$$

$$P[\mu - 3\sigma < X < \mu + 3\sigma] = 0.997$$

Gaussian Empirical Rule or 68/95/99 Rule



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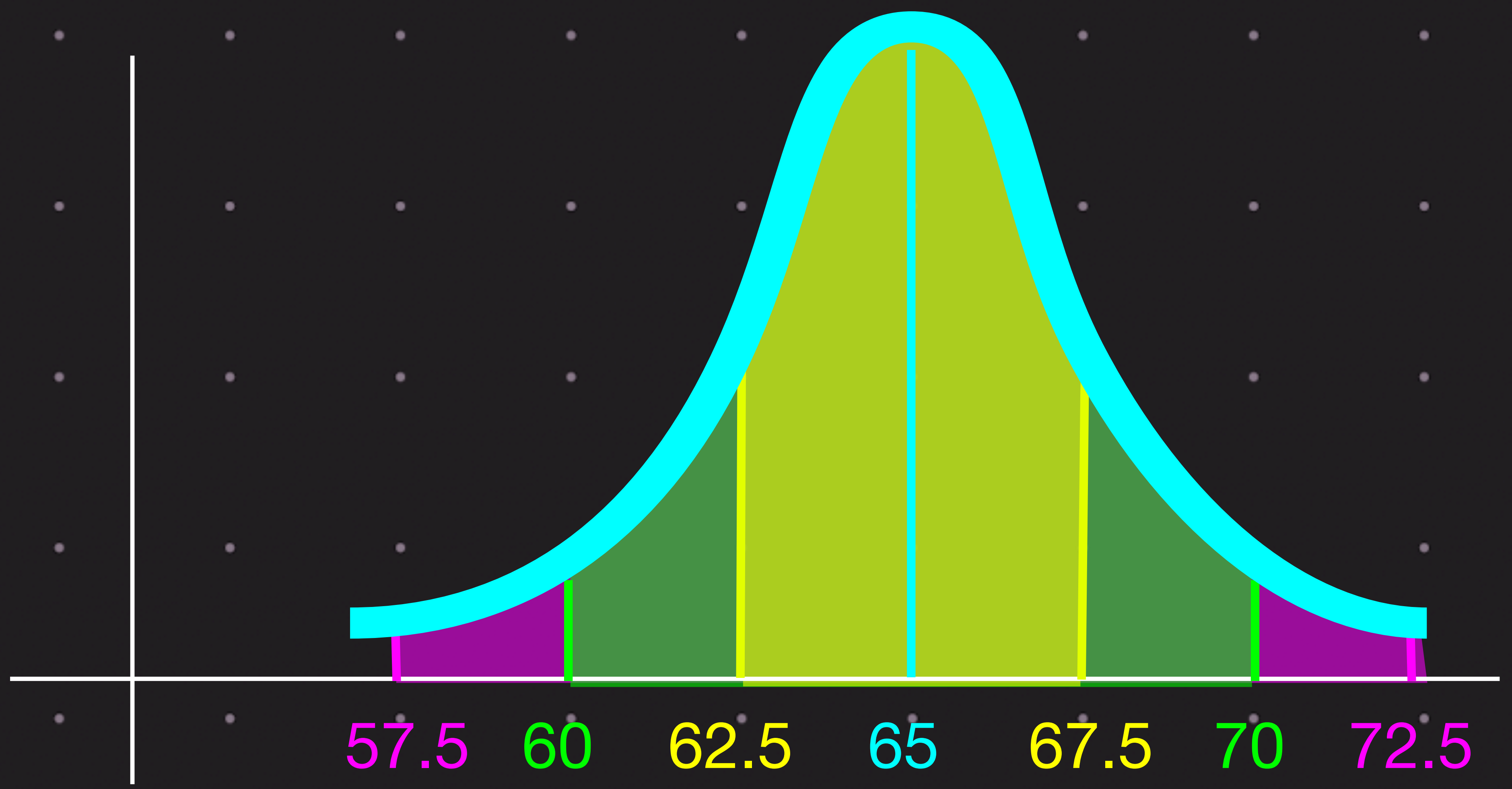
The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



$$\mu = 65$$

$$\sigma = 2.5$$

$$P[62.5 < X < 67.5] = 0.68$$

$$P[60 < X < 70] = 0.95$$

$$P[57.5 < X < 72.5] = 0.997$$

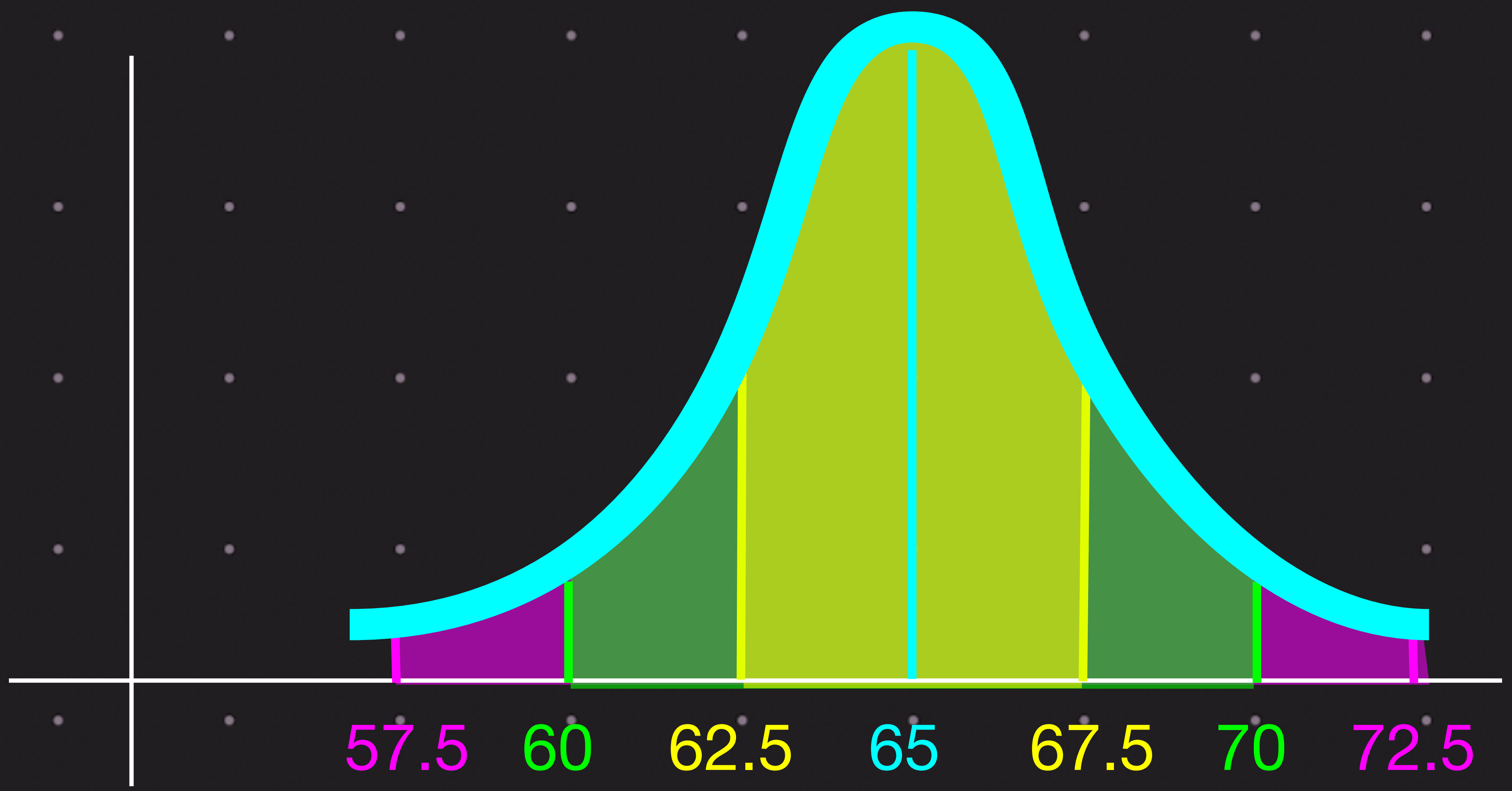
What is the fraction of people whose height is between 60 and 72.5?

Between 60 and 65? $\frac{95}{2} = 47.5$

Between 65 and 72.5? $\frac{99.7}{2} = 49.85$

Totally, 47.5 + 49.85 = 97.35

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



$$\mu = 65$$

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$$P[62.5 < X < 67.5] = 0.68$$

$$P[60 < X < 70] = 0.95$$

$$P[57.5 < X < 72.5] = 0.997$$

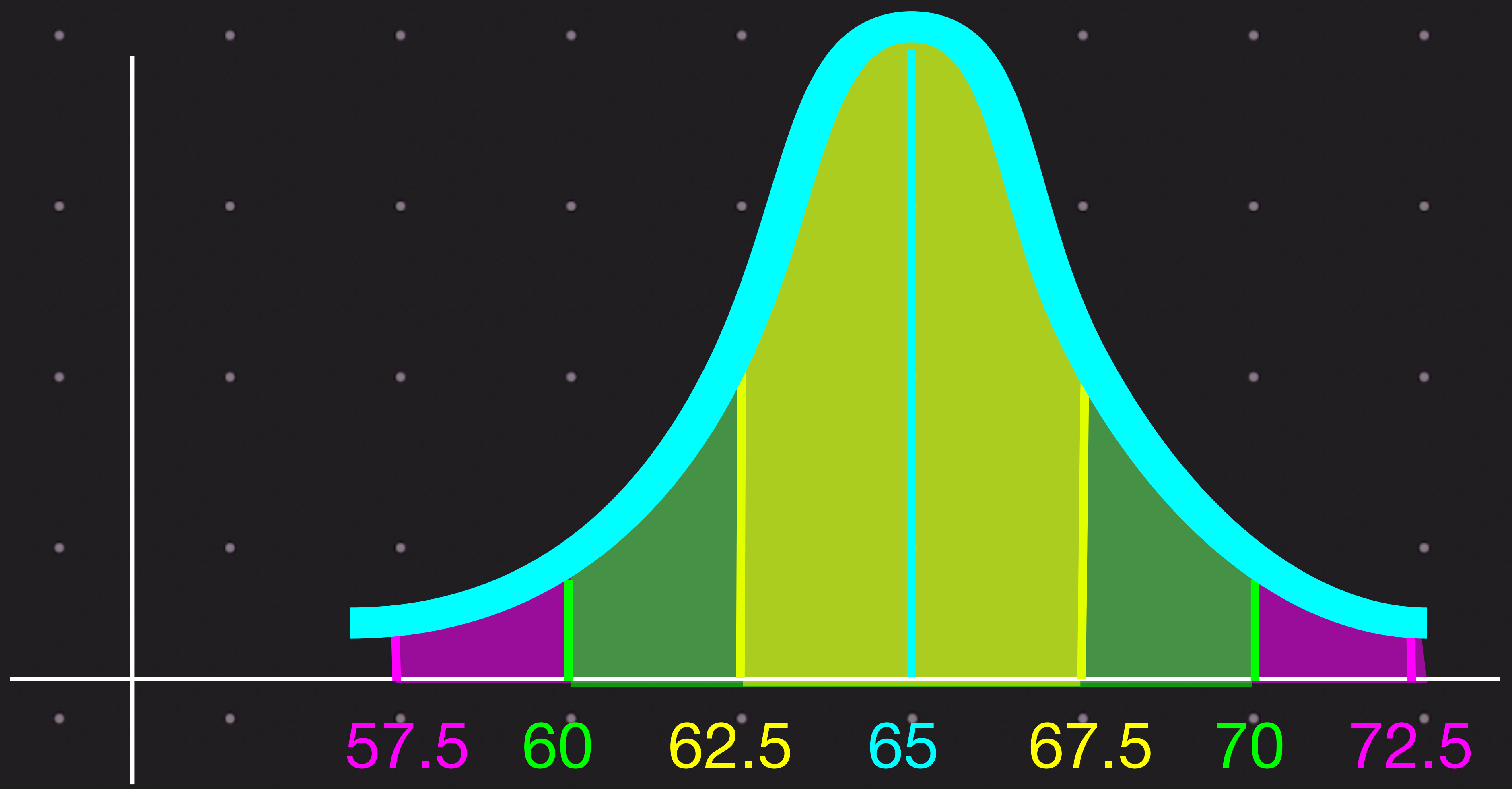
What fraction of people are shorter than 67.5?

What fraction of people are shorter 65? 50%

What fraction of people are in between 65 and 67.5? 68/2 = 34%

Totally 50 + 34 = 84% $P[X < 67.5] = P[X < 65] + P[65 < X < 67.5] = 0.5 + 0.34 = 0.84$

The height of people is Gaussian with mean 65 inches and standard deviation 2.5 inches



$$\mu = 65$$

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What fraction of people are shorter than 69.1?

How many σ (std devs) away from 65 is this number?

$$65 + z\,(2.5) = 69.1$$

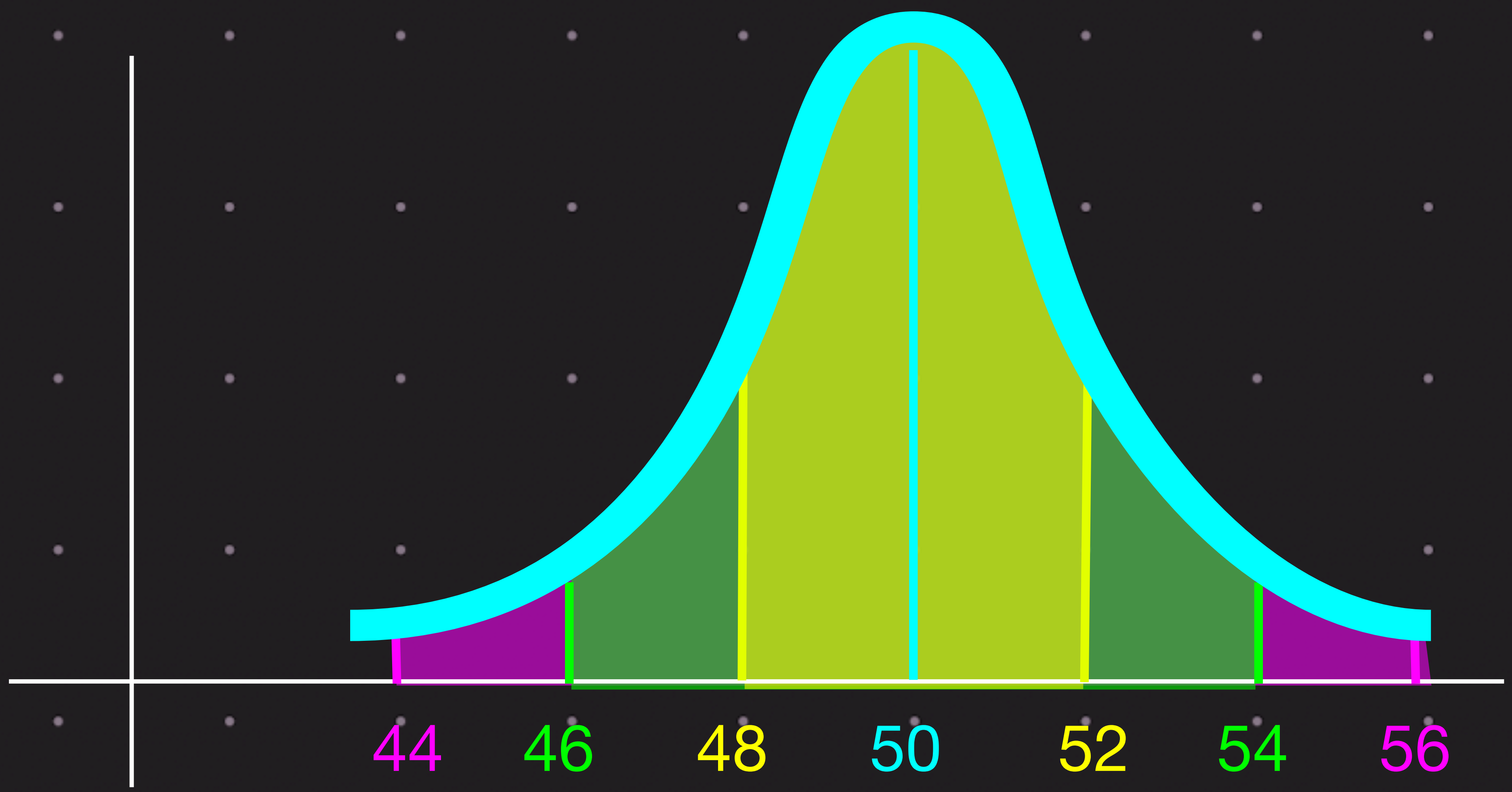
$$z = \frac{(69.1 - 65)}{2.5} = 1.64$$

Z-Score

$$z = \frac{(x - \mu)}{\sigma}$$

To find this probability, we use the Z-table 94.9%

Balls produced by manufacturer have mean 50 mm and std dev 2 mm

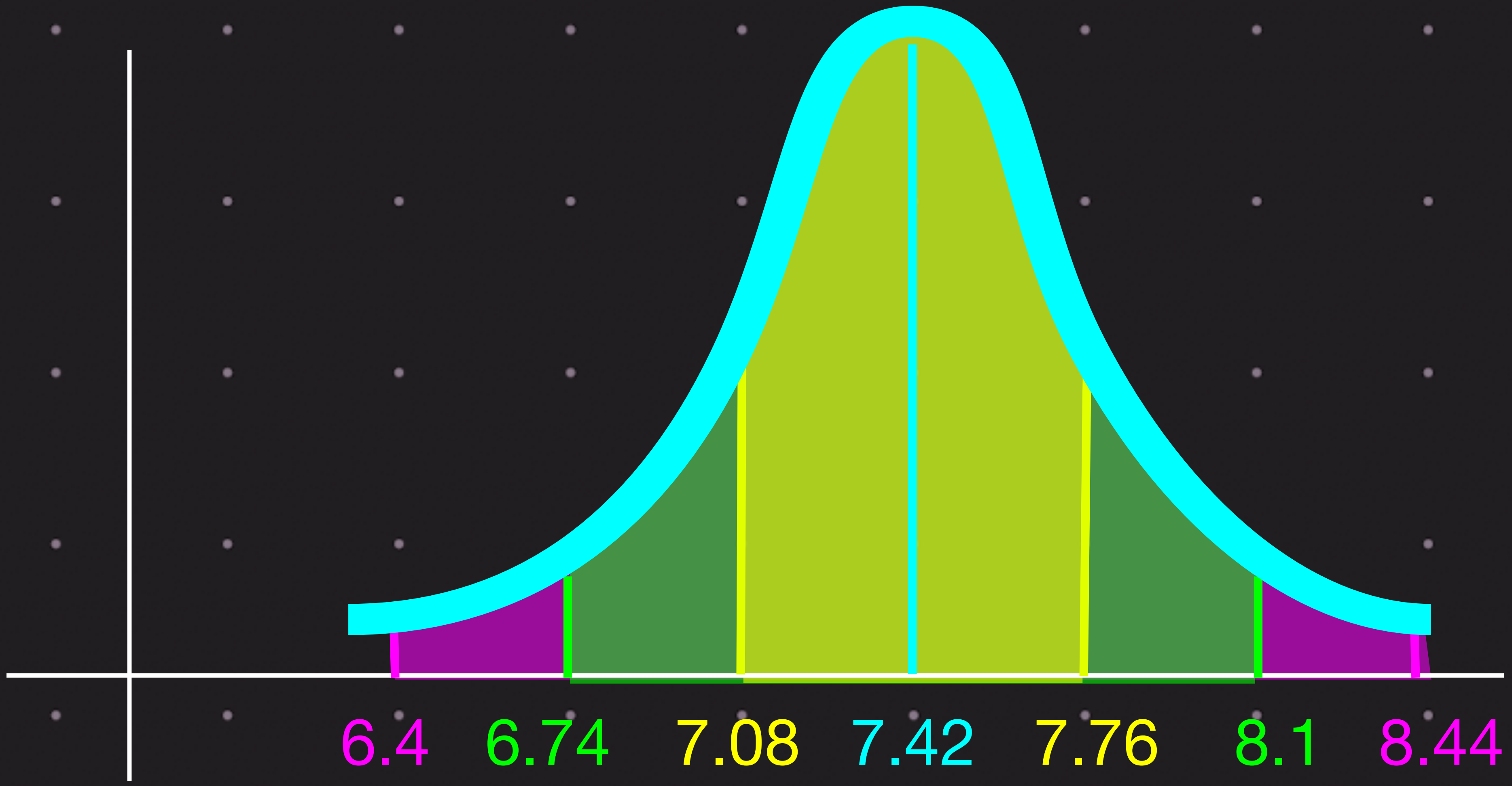


What fraction of balls are smaller than 53 mm?

$$z = \frac{(53 - 50)}{2} = 1.5$$

From Z-table, we see that the answer is 93.32%

Skaters take a mean of 7.42 seconds and std dev of 0.34 seconds for 500 meters.
What should his speed be such that he is faster than 95% of his competitors?



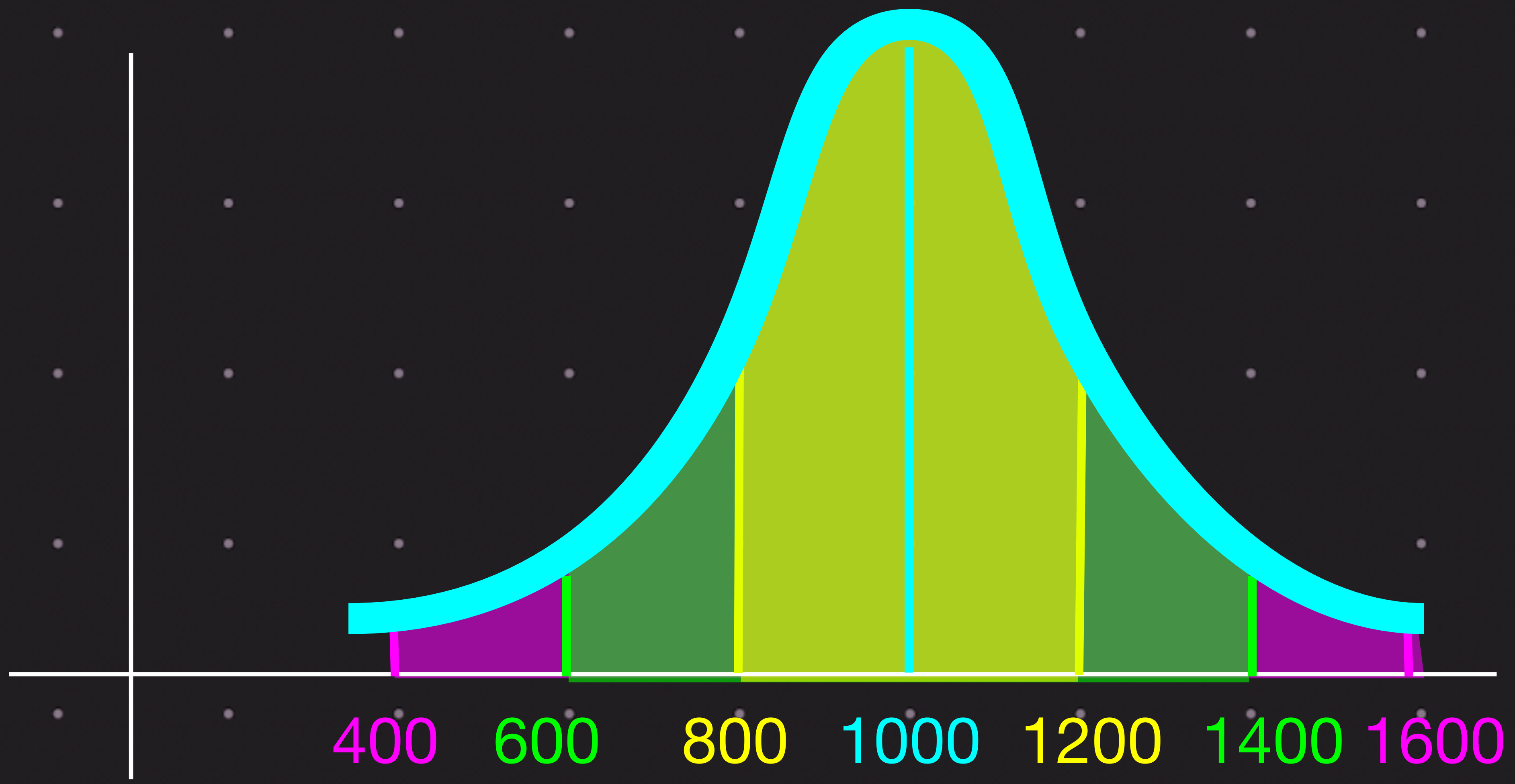
Unlike earlier examples, here the fraction is given, and we have to find Z-score

Let us use the Z-table   We need the Z-score of the area corresponding to 0.05

From Z-table, z-score is -1.65

$$z = \frac{(x - \mu)}{\sigma} \qquad x = \sigma z + \mu = (0.34)\,(-1.65) + 7.42 \; = 6.859$$

A retail outlet sells around 1000 toothpastes a week, with std dev = 200.
If the on-hand inventory is 1300, what is the need for replenishment within the week?



Let X denote the weekly sales. The questions asks for the probability that X > 1300

What is the Z-score of 1300?

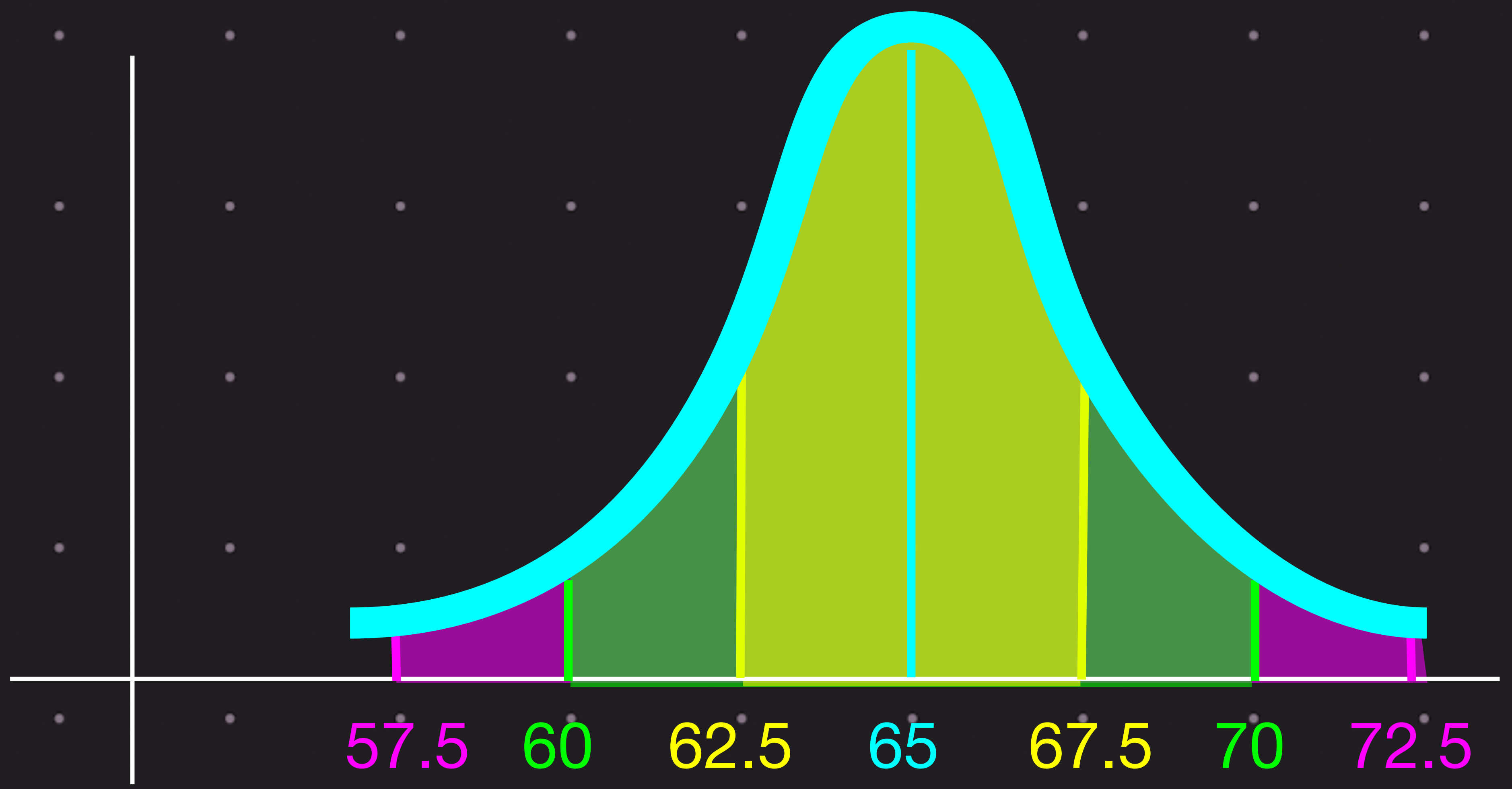
$$z = \frac{1300 - 1000}{200} = 1.5$$

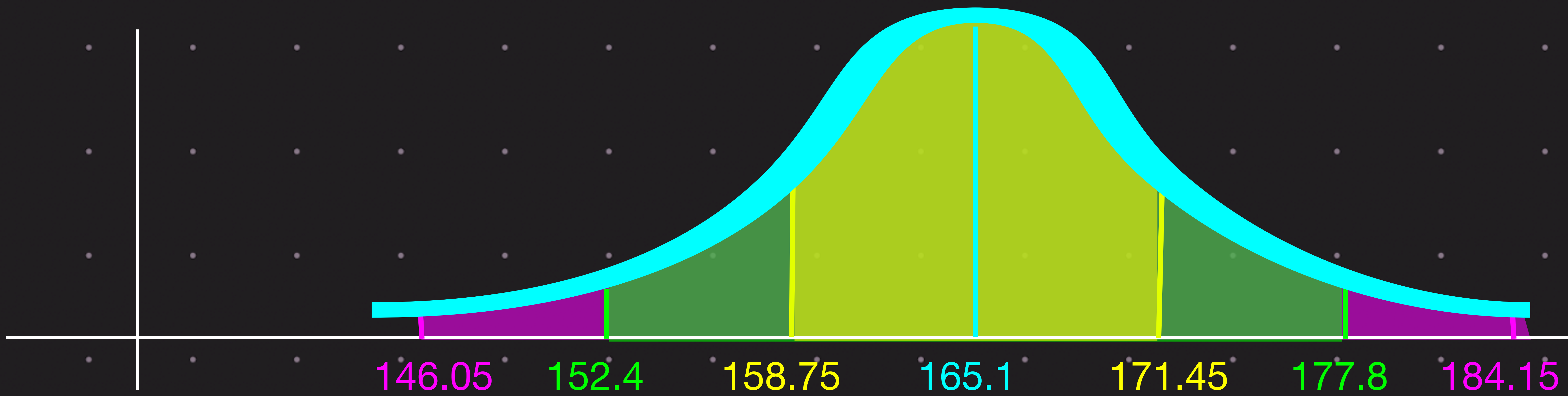
From Z-table, we see that $P[X \leq 1300] = 0.933$

$$P[X > 1300] = 1 - 0.933 = 0.067$$

Let us go back to the heights example. Mean is 65, std dev is 2.5

How would this look like if we look at heights in centimetres?





Casino case study

A bag has 3 red and 2 blue balls.

You pick a ball, write its colour, and put it back in the bag. This is done 4 times in total.

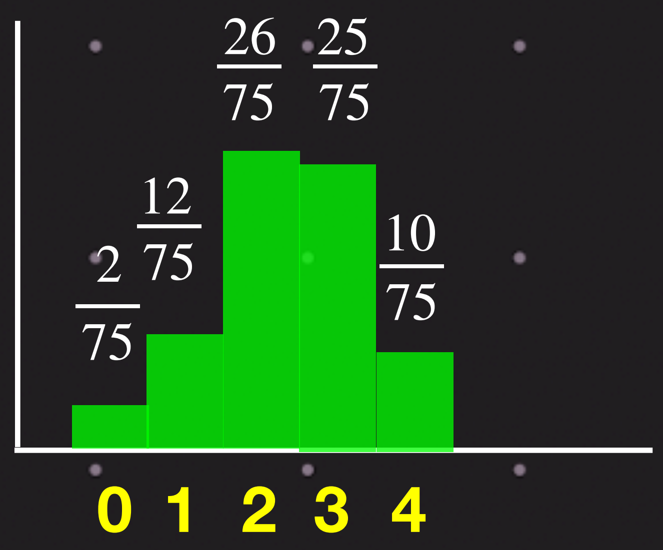
Let “X” denote the number of red balls when you draw 4 balls with replacement

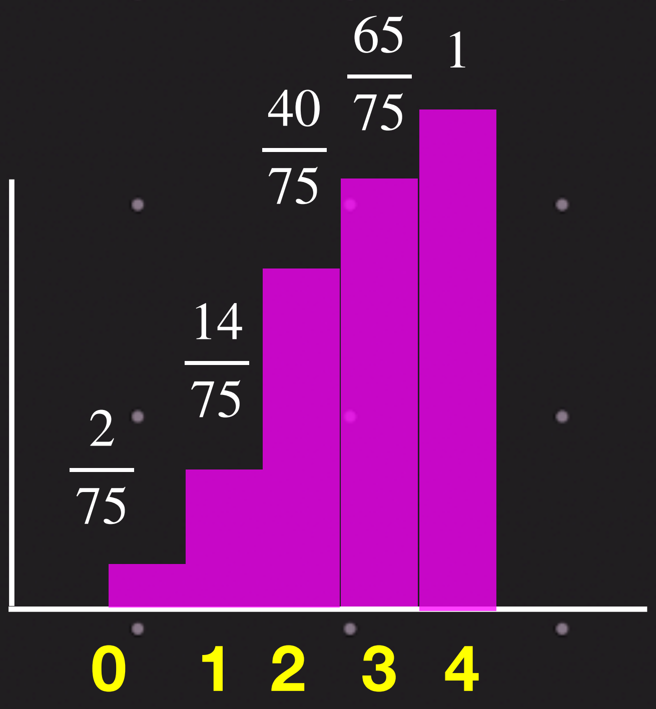
Empirical approach: Estimate probability using data

Data from 75 people

$X = 0$	2 people	$X \leq 0$	2 people
$X = 1$	12 people	$X \leq 1$	14 people
$X = 2$	26 people	$X \leq 2$	40 people
$X = 3$	25 people	$X \leq 3$	65 people
$X = 4$	10 people	$X \leq 4$	75 people

X	$P[X]$	$F[X]$
0	$\frac{2}{75}$	$\frac{2}{75}$
1	$\frac{12}{75}$	$\frac{14}{75}$
2	$\frac{26}{75}$	$\frac{40}{75}$
3	$\frac{25}{75}$	$\frac{65}{75}$
4	$\frac{10}{75}$	1





Cumulative distribution Function F[X]

Casino case study

A bag has 3 red and 2 blue balls.

You pick a ball, write its colour, and put it back in the bag. This is done 4 times in total.

Let "X" denote the number of red balls when you draw 4 balls with replacement

Theoretical approach: Compute probability using rules

X	Formula for $P[X]$	Code for $P[X]$	Value of $P[X]$	Value of $F[X]$	Code of $F[X]$
0	${}^4C_0\left(\frac{2}{5}\right)^4$	`binom.pmf(k=0, n=4, p=3/5)`	0.0256	0.0256	`binom.cdf(k=0, n=4, p=3/5)`
1	${}^4C_1\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1$	`binom.pmf(k=1, n=4, p=3/5)`	0.153	0.1792	`binom.cdf(k=1, n=4, p=3/5)`
2	${}^4C_2\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^2$	`binom.pmf(k=2, n=4, p=3/5)`	0.345	0.5248	`binom.cdf(k=2, n=4, p=3/5)`
3	${}^4C_3\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3$	`binom.pmf(k=3, n=4, p=3/5)`	0.345	0.8704	`binom.cdf(k=3, n=4, p=3/5)`
4	${}^4C_4\left(\frac{3}{5}\right)^4$	`binom.pmf(k=4, n=4, p=3/5)`	0.129	1	`binom.cdf(k=4, n=4, p=3/5)`

Suppose we float 10 quizzes with four options each.

Calculate the probability that a student, who randomly guesses, answers 3 or more questions correctly

$$P[X \geq 3] = 1 - P[X <= 2] = $$ `1 - binom.cdf(k=2, n=10, p=1/4)` $$= 0.474$$

Calculate the probability that a student, who randomly guesses, answers exactly 2 questions correctly

$$P[X = 2] = {}^{10}C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^8 = $$ `binom.pmf(k=2, n=10, p=1/4)` $$= 0.2815$$

Each light bulb manufactured is defective with probability 0.05

We buy 5 light bulbs

Find the probability that none are defective

$$P[X=0] = {}^5C_0\,(0.05)^0\,(0.95)^5 = \texttt{binom.pmf(k=0, n=5, p=0.05)} = 0.773$$

Find the probability that 2 or more are defective

$$P[X \geq 2] = 1 - P[X \leq 1] = \texttt{1 - binom.cdf(k=1, n=5, p=0.05)} = 0.0226$$